1. What is the task tackled in this paper?

- entity alignment

2. What is the research question in this paper?

- Previous work has too hard constraints on the tranformation mapping.
- Previous work can only use the alignment information of two-graphs, cannot make full use of multi-graphs(>2).
- High cost when aligning between N graphs, in which order is O(N!).

3. How to solve it/what is the approach of this paper?

1) Model framework

The framework of model is still be a joint model of two parts :

- relational inference model : Regard as a controlled variable. We will use TransE.
- Alignment model : release hard constraints, such as linear transformation and translation constraint, to soft constraints - **nonlinear continuous mapping function**.

2) Nonlinear continuous mapping function

- **What is it?**

  It is the mapping between embedding spaces. It can be in two forms:

  - **Wasserstein transport mapping matrix** : $T \in \mathbb{R}^{m*n}$, which $\mathcal{X} = (x_1, \ldots, x_m) \in \Omega^m, \mathcal{Y} = (y_1, \ldots, y_n) \in \Omega^n$ and $T(\mathcal{X}) = \mathcal{Y}$.
  - **Global correspondence** of spaces $\mathcal{X}$ and $\mathcal{Y}$, which is a function $f : \mathcal{X} \to \mathcal{Y}$

  > Notes : We will use the first notation to explain the plan below.

- **What constraints conditions should it satisify?**

  Mapping must be **continuous and nonlinear** with respect to its space metric.

  (In other words, Continuous mapping between two spaces, maybe homeomorphism?)

- **What is it used for?**

  It is a part of Wasserstein metric, which is known as :

  $$\inf_T \int_\Omega c(x, T(x))\mu(dx)$$

- **How to optimize nonlinear continuous mapping function ?**

  Regards as a optimal transportation problem.

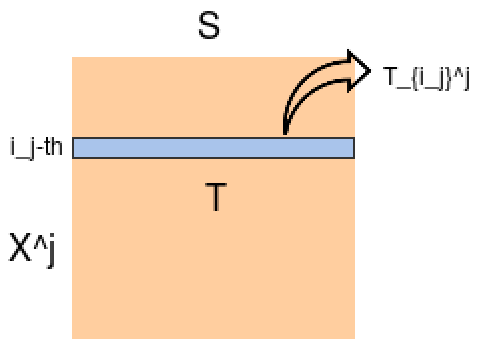
3) Use it in entity alignment task

- **Training process:**
  - Wasserstein metric when align **two graph embedding spaces** with alignment pair (x_i, y_j)
    - Simplily use Wasserstein distance as loss function.
    - The transport mapping should satisfy the **constraint** (the most difficult one) below:

      $T(x_i, y_j) = \sum_k^n T(x_i, y_k)\ (x_i \in \mathbb{R}^m, y_j \in \mathbb{R}^n)$
  - Wasserstein metric among **multi-graph embedding spaces** with alignment list $(x_{i_1}^1, \dots, x_{i_k}^k)$:
    - Firstly, Calculate the Wasserstein barycenter $S \in \Omega^x$ of n graph embedding spaces .
    - Secondly, The transport mapping should satisfy the **constraint** below:

      Each entity in alignment list $(x_{i_1}^1, \dots, x_{i_k}^k)$ has a coresponding distribution $P_{i_j}^j$ which is the i_j-th row $T_{i_j}^j$ of transport mapping matrix T^j between graph embedding space X^j and barycenter S.

      We note it as $(P_{i_1}^1, \dots, P_{i_k}^k)$. We need all the distributions in this list to be same.

      > Notes :
      >
      > 1. $P_{i_j}^j$ and $T_{i_j}^j$ is same here.
      > 2. Shown in graph:
      >
      > 
      >

- **Predicting process:**

  Predict in the way we trained it on different problems (Two graphs and N-graphs).

4. Summary & Questions

1) Questions

- How do you think about this plan, even it's only theoretical and not concrete now?
- **Just an opinion**: I think it may be unsatisfactory of using Wasserstein distance as alignment loss function. Because it only has very weak constraint on embedding space.

2) Summary

- It's a joint model, because alignment loss is related to relational inference loss.
- I think using the first and second items (in section2) as motivations is better than high computation cost deficiency.
- Anyway, The next step is to find the optimization method and do an experiment to verify the effect.