1. if T is an affine transformation matrix it has the general form:

$$T = \begin{bmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

if T^{-1} exists:

$$T^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{b}{bc-ad} & \frac{dt_x - bt_y}{bc-ad} \\ \frac{c}{bc-ad} & \frac{a}{ad-bc} & \frac{ct_x - at_y}{ad-bc} \\ 0 & 0 & 1 \end{bmatrix}$$

T^{-1} also has the general form of affine transformation matrix
T^{-1} is affine.

2. (a) let homography H be

$$\begin{bmatrix} a & b & c \\ d & e & f \\ h & k & 1 \end{bmatrix}$$

$$H \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cong \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} a + c + (h+1) = 0 \\ d + f = 0 \end{cases} \quad ①$$

$$H \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cong \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} b + c - 2(k+1) = 0 \\ e + f = 0 \end{cases} \quad ②$$

$$H\begin{bmatrix}0\\0\\1\end{bmatrix} \cong \begin{bmatrix}1\\2\\1\end{bmatrix} \Rightarrow \begin{cases} c-1=0 \\ f-2=0 \end{cases} \quad ③$$

$$H\begin{bmatrix}3\\1\\1\end{bmatrix} \cong \begin{bmatrix}4\\2\\1\end{bmatrix} \Rightarrow \begin{cases} 3a+b+c-4(3h+k+1)=0 \\ 3d+e+f-2(3h+k+1)=0 \end{cases} \quad ④$$

based on ①, ②, ③, ④ we can solve
$(a, b, c, d, e, f, h, k) = (-2, -7, 1, -2, -2, 2, 0, -4)$

$$H = \begin{bmatrix} -2 & -7 & 1 \\ -2 & -2 & 2 \\ 0 & -4 & 1 \end{bmatrix}$$

(b) $$H \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}-8\\-2\\-3\end{bmatrix} = \begin{bmatrix}\frac{8}{3}\\\frac{2}{3}\\1\end{bmatrix}$$

(c) it is not an affline transformation. it does not has the general form.

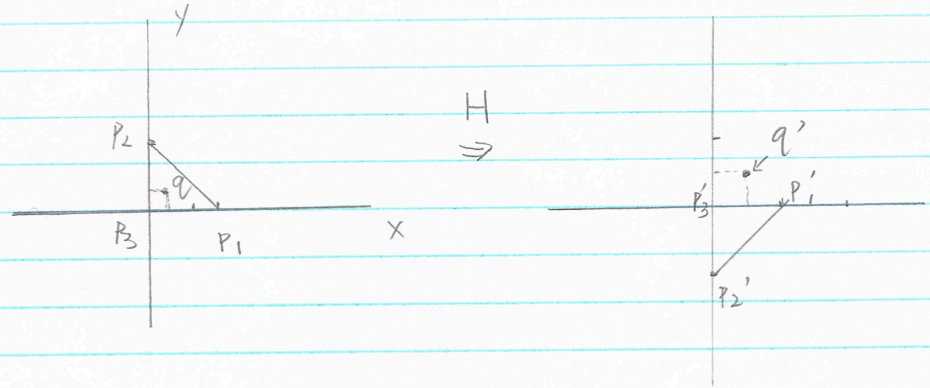
$$\left[\begin{array}{cc|c} \multicolumn{2}{c|}{A} & \bar{t} \\ \hline 0 & 0 & 1 \end{array}\right]$$ where A is 2×2 matrix, $\bar{t}$ is a vector

3 let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, let $q = \begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\\1\end{bmatrix}$, $P_1 = \begin{bmatrix}1\\0\\1\end{bmatrix}$ $P_2 = \begin{bmatrix}0\\1\\1\end{bmatrix}$ $P_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

q is the inner point of triangle P_1, P_2, P_3

$$H\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix}, \quad H\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}, \quad H\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\1\\-1\end{bmatrix} = \begin{bmatrix}0\\-1\\1\end{bmatrix}$$

$$H\begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\\\frac{1}{2}\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\\1\end{bmatrix}$$



after transformation q' is not the interior of triangle $\overline{P_1' P_2' P_3'}$

(b) it will preserve, let l_1 be the line go through p_1, p_2
let l_2 be the line go through p_2, p_3
let l_3 be the line go through p_3, p_1

must have. l_1' parallel to l_1 go through q, intersect triangle $\overline{p_1 p_2 p_3}$
l_2' parallel to l_2 go through q, intersect triangle $\overline{p_1 p_2 p_3}$
l_3' parallel to l_3 go through q intersect triangle $\overline{p_1 p_2 p_3}$

after affline transformation, $f(l_1')$, $f(l_2')$, $f(l_3')$ are still parallel to $f(l_1)$, $f(l_2)$, $f(l_3)$.

$f(l_1')$, $f(l_2')$, $f(l_3')$ cannot be outside of triangle $\overline{f(p_1) f(p_2) f(p_3)}$ otherwise they will have no intersection with it.

$\Rightarrow$ the intersection of $f(l_1')$, $f(l_2')$, $f(l_3')$ is inside of $\overline{f(p_1) f(p_2) f(p_3)}$

$\Rightarrow$ $f(q)$ is inside of $f(p_1) f(p_2) f(p_3)$

5 let $\begin{bmatrix} 1 & 0 \\ h & 1 \end{bmatrix}$ be the sheer in y axis.

$$\begin{bmatrix} 1 & 0 \\ h & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

it means sheer in y by h can be expressed as
① rotation by 90 degree
② sheer in x by $-h$
③ rotation by -90 degree.

6. tangent Vecter $= (X'(t), y'(t))$

$= (\cos(20\pi t) - 20\pi \sin(20\pi t), \ 20\pi \cos(20\pi t))$

normal = tangent vecter rotate by 90 degree

$$\begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -y'(t) \\ x'(t) \end{bmatrix}$$

$= (-20\pi \cos(20\pi t), \ \cos(20\pi t) - 20\pi \sin(20\pi t))$