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Concave Down: Definition, Function & Graph

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Learn about when a function is concave down. Discover graphs and examples of functions which are concave down and generate tables of values to see their numbers.

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Concave Down

Functions can be analyzed graphically, numerically, and analytically (or algebraically). One way to graphically describe the shape of a curve is to refer to its shape, or **concavity**. The graphs of curves can be **concave up** or **concave down**. A simple way to describe the differences between a graph being concave up or down is to use the shape of a bowl. Curves that are concave up look like an upright bowl, curves that are concave down look like an upside-down bowl. In the simplest terms, it could also be like describing a smile and a frown.

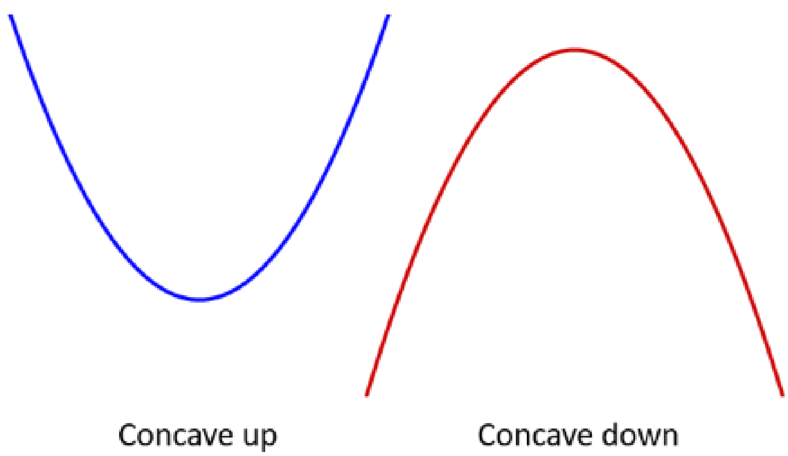


Image of a concave up curve and a concave down curve.

In calculus, concavity is defined as follows:

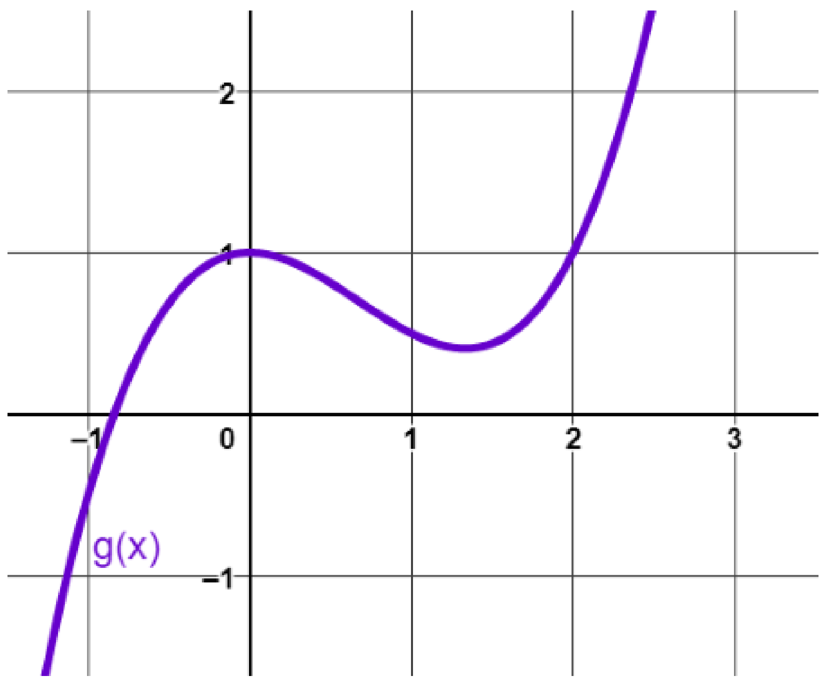
Let f be differentiable on an interval I . The graph of f is concave up on I if f' is increasing. The graph of f is concave down on I if f' is decreasing. If f' is constant then the graph of f is said to have no concavity.

Therefore, concave down means that the **slopes** of the function are decreasing, or rather the **derivative** of the function is decreasing. Furthermore, the second derivative can also determine concavity. If the second derivative of a function is negative on some interval, then the function is concave down on that interval. Similarly, if the second derivative is positive, then the function is concave up. The graphs of functions can differ in shape. Some curves will be concave up and concave down or only concave up or only concave down or not have any concavity at all. The curve of the cubic function $g(x) = \frac{1}{2}x^3 - x^2 + 1$ is both concave down and concave up.

An example of a function that is concave down is the quadratic function $f(x) = x^2 + 4$. Finally, a linear function such as $y = 3x - 5$ is neither concave up or down because it is the equation of a straight line.

Concave Down Graphs

As previously mentioned, the graph of $g(x) = \frac{1}{2}x^3 - x^2 + 1$ can be described as being concave down and concave up. On the interval from $\left(-\infty, \frac{2}{3}\right)$, $g(x)$ is concave down, but then it is concave up from $\left(\frac{2}{3}, \infty\right)$.



Graph of a g(x), a cubic function that is both concave up and concave down.

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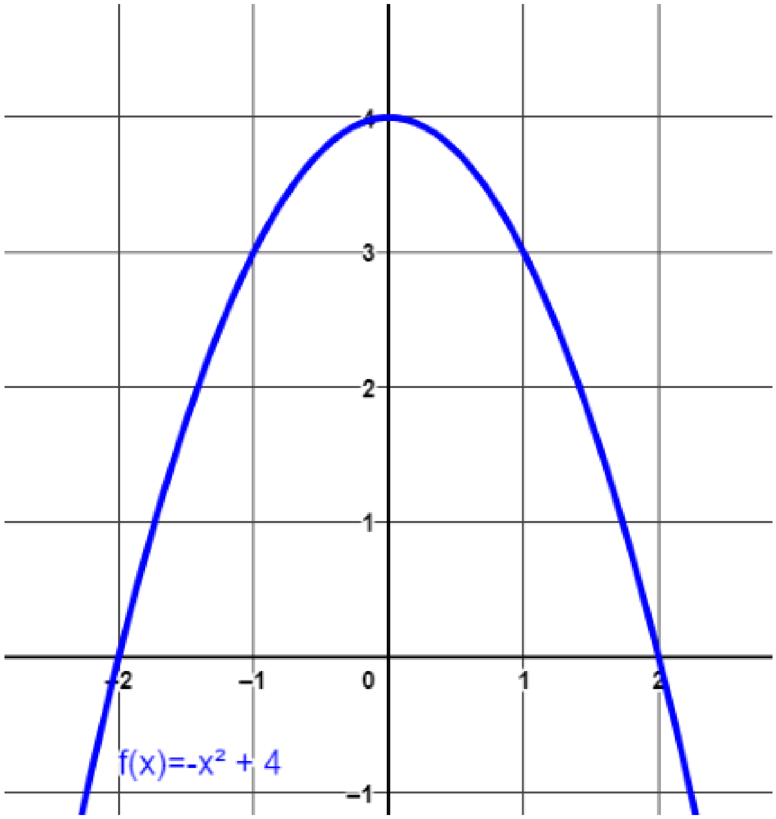
The graph of $f(x)$ is strictly concave down on $\left(-\infty, \infty\right)$. The shape of this graph is like an open umbrella.



Graph of a f(x)=x^2+4, a quadratic function that is concave down.

The Math Behind Concave Down

Looking at a table for $g(x)$, as the x values are getting bigger, the y values are also getting bigger, but then after $x = 0$, the y values shrink just to start going back up again after $x = 1.5$. The graph of $g(x)$ shown indicates the slopes at various points of the function . The green lines indicate a positive slope, the horizontal black line indicates a zero slope, and the red line indicates a negative slope. Remember that concave down means the derivative (or slopes) of a function is decreasing, and that concave up is when the slopes are increasing. As the slopes are read from left to right, it is obvious that the slopes are decreasing, then hit zero just to then increase again, reflecting the change in concavity of the graph of g(x) from concave down to concave up.

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x	f(x)
-2	-7
-1.5	-2.9375
-1	-0.5
-0.5	0.6875
0	1
0.5	0.8125
1	0.5
1.5	0.4375
2	1
2.5	2.5625
3	5.5

Table for g(x).

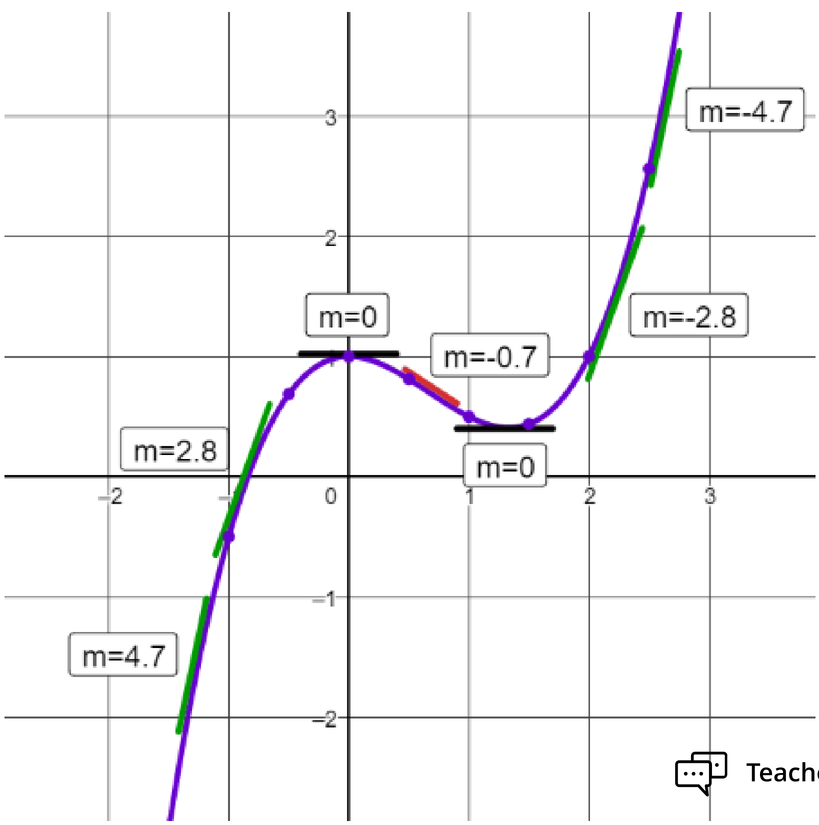

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Table for g(x).

In comparison, the graph of the function of f(x) is only concave down. As the x values for the graph of $f(x)$ are getting bigger, the y values also increase then at $x=0$ start to decrease but never go back up again. Looking at the slopes at various points of the graph of $f(x)$, it can be noted that the slopes are decreasing from left to right.

x	f(x)
-3	-5
-2	0
-1	3
0	4
1	3
2	0
3	-5

Table of values for f(x)

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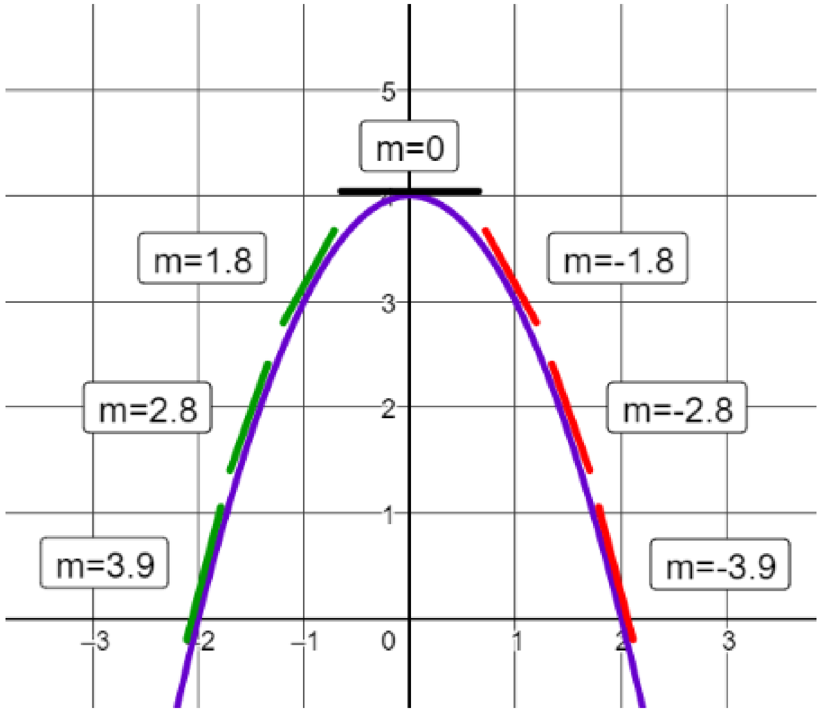


Graph of f(x) with slopes.

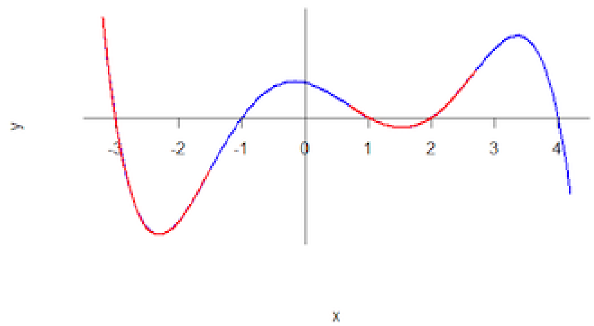
Lesson Summary

The shape of a curve can be described by its **concavity**. The graphs of curves can be **concave up** or **concave down**. Think of concave up like an upright bowl. Concave down looks like an upside-down bowl or an open umbrella. Concave down means that the **slopes** of the function are decreasing, or rather the **derivative** of the function is decreasing. The second derivative can also determine concavity. If the second derivative of a function is negative on some interval, then the function is concave down on that interval. Similarly, if the second derivative is positive, then the function is concave up.

Video Transcript

Definition of Concave Down

A **function**, which is a fancy word for equation, is **concave down** in some region if it looks like an upside down bowl, or the inside of an umbrella, in that region. The *y*-values in that region become bigger at a slower and slower rate as you move from left to right on the curve. Using calculus, you would find that the first derivative was decreasing in that interval and the second derivative was less than zero. Check out this image to help you visualize concave down.



Picturing Concave Down

The blue regions in the graph are concave down, while the red regions are concave up. The concave down sections look like the inside of an umbrella. Rainwater would roll off the top of them.

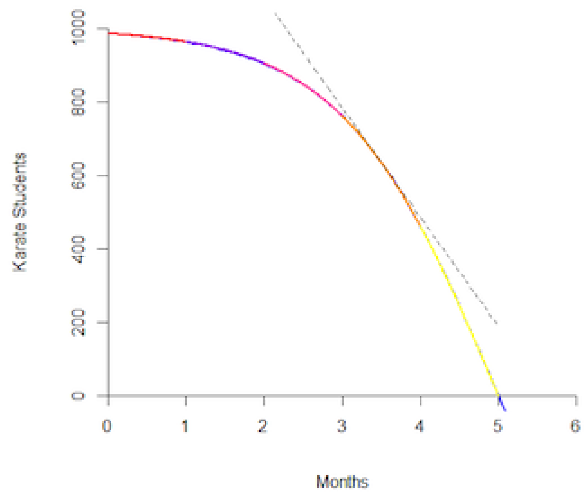
What Concave Down Means

Imagine that you start a brand new online karate school. After a huge promotion, you have 1,000 virtual students. You're going to get rich!

You quickly find out, however, that students start dropping like flies. Maybe the fact that you don't actually know karate becomes obvious to them. Or maybe the Internet just isn't a great way to teach martial arts. In any case, your karate enrollment drops quickly for a few months until you have no students left.

If you look at the karate student graph, you will see that it is concave down. During the first month (red on the curve), you lose students very slowly - your enrollment is now a little under 1,000. During the second month (purple), slightly more students drop out. You are down to about 900 students. The purple section of the curve looks a little steeper than the red section.

During the third month (pink) your enrollment drops even more. Maybe the word is starting to spread that your course is a scam, or maybe you just got them so excited about karate that they quit your course and sign up at the live class at the local strip mall. The fourth month (orange) and the fifth month (yellow) get even steeper. You can see by the length of the yellow piece of the curve what a big drop you had.



Over time, your decrease in students is getting bigger. You're not only continuing to lose students - you're losing them faster and faster.

If you were to draw a **tangent** line, which is a line adjacent to the curve (like the dotted line shown in our karate school example), at various points, you would find the slope of the line was decreasing as you moved from left to right. In this example, a tangent line drawn in the red section would be flatter than one in the orange or yellow section. Because the function is decreasing (it curves down as you move to the right), the slope is always negative. A steeper negative slope suggests a 'more negative' number. A slope of -6 is smaller (less quantity) than a slope of -2. So, the slope is decreasing.

The Mathematics Behind Concave Down

In calculus, the slope of the tangent line at any point is found from the first derivative. If a function is concave down in some region, then the first derivative should decrease as you move from left to right. If $f(x)$ is your function, then $f'(x)$ should get smaller as x increases anywhere that the function is concave down.

Another way to use calculus to find out if a function is concave down is to find the second derivative. If the first derivative represents change (such as losing students), then the second derivative represents change in change (such as losing students faster and faster). The second derivative will be negative in any concave down region. In mathematical notation, $f(x) < 0$.

Lesson Summary

Let's take a few moments to review what we've learned about concave down in this lesson. Concavity in a **function**, which is a fancy word for equation, tells you how the steepness of the curve is changing as x changes. If a curve is **concave down**, then the slope of a tangent line to the curve is decreasing as x increases. The curve will look like a bowl facing downward, or an umbrella, in that region.

So, following this lesson, you should be able to do the following:

- Explain what concave down on a graph looks like

- Describe the meaning of concave down using a tangent line
- Summarize how to explain concave down in calculus terms

Frequently Asked Questions

Does concave down mean negative?

Concave down means that the second derivative is negative. It also means that the slopes of the function are decreasing which may be thought of as trending negatively.

What is concave down?

Concave down means that the **slopes** of a function are decreasing, or rather the **derivative** of the function is decreasing.

How do you know when a function is concave down?

Using the slopes, a function can be determined to be concave down, if the slopes are decreasing. Also, if the second derivative is negative then the the function will be concave down on the same interval. Lastly, if looking at a graph, then the function is concave down wherever the graph appears to have the shape of an upside down bowl.