$\theta = \cos^{-1} \xi$ with the negative ... dominant pole will give the location of the dominant pole. The value ... can be obtained from the magnitude condition.

6.5 EFFECT OF ADDING POLES AND ZEROS TO G(s)H(s)

The general problem of controller design in control systems may be treated as an investigation of the effects to the root loci when poles and zeros are added to the loop transfer function $G(s)H(s)$.

6.5.1 Addition of Poles to G(s)H(s)

Adding a pole to $G(s)H(s)$ has the effect of pushing the root loci towards the right half. The complex path of the root loci bends to the right, the angle of asymptotes reduces and the centroid is shifted to the left, and the system stability will be reduced. Even a system which was perfectly stable may become unstable as K increases.

In general, we can say that the addition of poles to $G(s)H(s)$ has the effect of moving the dominant portion of the root loci towards the right half of the s-plane. So the relative stability of the system is decreased by the addition of a pole.

6.5.2 Addition of Zeros to G(s)H(s)

Adding left-half plane zeros to the function $G(s)H(s)$ generally has the effect of moving and bending the root loci towards the left-half of the s-plane. So the relative stability of the system is improved by the addition of a zero.

Example 6.4 Sketch the root locus plot for the system given below with K as a variable

Example 6.14 The characteristic equation of a feedback control system is

$$s^4 + 3s^3 + 12s^2 + (K - 16)s + K = 0$$

Sketch the root locus plot for $0 < K < \infty$ and show that the system is conditionally stable (stable only for a range of gain K). Determine the range of gain for which the system is stable.

Solution: The characteristic equation is

$$s^4 + 3s^3 + 12s^2 + (K - 16)s + K = 0$$

To sketch the root locus, we require the open-loop transfer function $G(s)H(s)$

$$1 + G(s)H(s) = s^4 + 3s^3 + 12s^2 - 16s + Ks + K$$

$$= s(s^3 + 3s^2 + 12s - 16) + K(s + 1) = 0$$

i.e.

$$1+\frac{K(s+1)}{s(s^3+3s^2+12s-16)}=1+\frac{K(s+1)}{s(s-1)(s^2+4s+16)}=0$$

$$G(s)H(s) = \frac{K(s+1)}{s(s-1)(s^2+4s+16)} = \frac{K(s+1)}{s(s-1)(s+2+j3.42)(s+2-j3.42)}$$

∴

For the obtained open-loop transfer function $G(s)H(s)$:
The open-loop poles are at $s = 0$, $s = 1$, $s = -2 + j3.42$, $s = -2 - j3.42$. Therefore, $n = 4$.
The open-loop zero is at $s = -1$. Therefore, $m = 1$.
Hence the number of branches of root locus $= n = 4$ and the number of asymptotes $= n - m = 4 - 1 = 3$.
The complete root locus is drawn as shown in Figure 6.22, as per the rules given below.

1. Since the pole-zero configuration is symmetrical with respect to the real axis, the root locus will be symmetrical with respect to the real axis.
2. The four branches of the root locus originate at the open-loop poles $s = 0$, $s = 1$, $s = -2 + j3.42$, and $s = -2 - j3.42$, where $K = 0$ and terminate at the open-loop zeros at $s = 1$, $s = \infty$, $s = \infty$, and $s = \infty$, where $K = \infty$.
3. Three branches of the root locus go to the zeros at infinity along asymptotes making angles of $\theta_q = \dfrac{(2q+1)\pi}{n-m}$, $q = 0, 1, 2$ with the real axis, i.e.

$$\theta_0 = \frac{\pi}{3}, \quad \theta_1 = \pi, \quad \theta_2 = \frac{5\pi}{3}$$

4. The point of intersection of the asymptotes on the real axis (centroid) is given by

$$-\sigma = \frac{\text{sum of real parts of poles} - \text{sum of real parts of zeros}}{\text{number of poles} - \text{number of zeros}} = \frac{(0-2-2+1)-(-1)}{4-1} = -0.66$$

5. The root locus exists on the real axis from $s = 1$ to $s = 0$ and to the left of $s = -1$.
6. The breakaway points are given by the solution of $\dfrac{dK}{ds} = 0$.

$$|G(s)H(s)| = \left|\frac{K(s+1)}{s(s-1)(s^2+4s+16)}\right| = 1$$

∴ $$K = \frac{s(s-1)(s^2+4s+16)}{s+1}$$

i.e. $$\frac{dK}{ds} = (s+1)\frac{d}{ds}(s^4+3s^3+12s^2-16s) - (s^4+3s^3+12s^2-16s)\frac{d}{ds}(s+1) = 0$$

i.e. $$(s+1)(4s^3+9s^2+24s-16) - s^4 - 3s^3 - 12s^2 + 16s = 0$$

$$4s^4 + 13s^3 + 33s^2 + 8s - 16 - s^4 - 3s^3 - 12s^2 + 16s = 0$$

$$3s^4 + 10s^3 + 21s^2 + 24s - 16 = 0$$

Therefore, $s = 0.45$ and $s = -2.26$ are the actual break points. Out of these, $s = 0.45$ is the breakaway point and $s = -2.26$ is the break-in point.
The break angles at $s = 0.45$ and $s = -2.26$ are

$$\pm\frac{\pi}{r} = \pm\frac{180^\circ}{2} = \pm 90^\circ$$

7. The angle of departure of the root locus from the open-loop pole at $s = -2 + j3.42$ is

$$\theta_d = (2q + 1)\pi + \phi, \text{ where } \phi = \theta_4 - (\theta_1 + \theta_2 + \theta_3)$$

$$= 106.29^\circ - (131.25^\circ + 120.31^\circ + 90^\circ) = -235.27^\circ$$

Therefore, the angle of departure of the root locus from the open-loop pole at $s = -2 - j3.42$ is $\theta_d = 180^\circ - 235.27^\circ = -55.27^\circ$.

8. The point of intersection of the root locus with the $j\omega$-axis and the critical value of K can be obtained using the Routh criterion. The characteristic equation is

$$s^4 + 3s^3 + 12s^2 + (K - 16)s + K = 0$$

The Routh table is as follows:

s^4	1	12	K
s^3	3	$K - 16$	
s^2	$\dfrac{36 - K + 16}{3}$	K	
s^1	$\dfrac{\frac{(52-K)}{3}(K-16) - 3K}{\frac{52-K}{3}}$		
s^0	K		

For stability, all the elements in the first column of the Routh array must be positive. Therefore,

$$K > 0$$

$$52 - K > 0$$

i.e. $K < 52$

and $52K + 16K - K^2 - 832 - 9K > 0$

i.e. $K^2 - 59K + 832 < 0$

$\therefore$ $K > 23.3$ and $K < 35.7$

So the system is conditionally stable, and the range of values of K for stability 23.3 < K < 35.7. The corresponding oscillation frequencies are 1.55 rad/s and 2.6 rad/ respectively.

The complete root locus is shown in Figure 6.22.

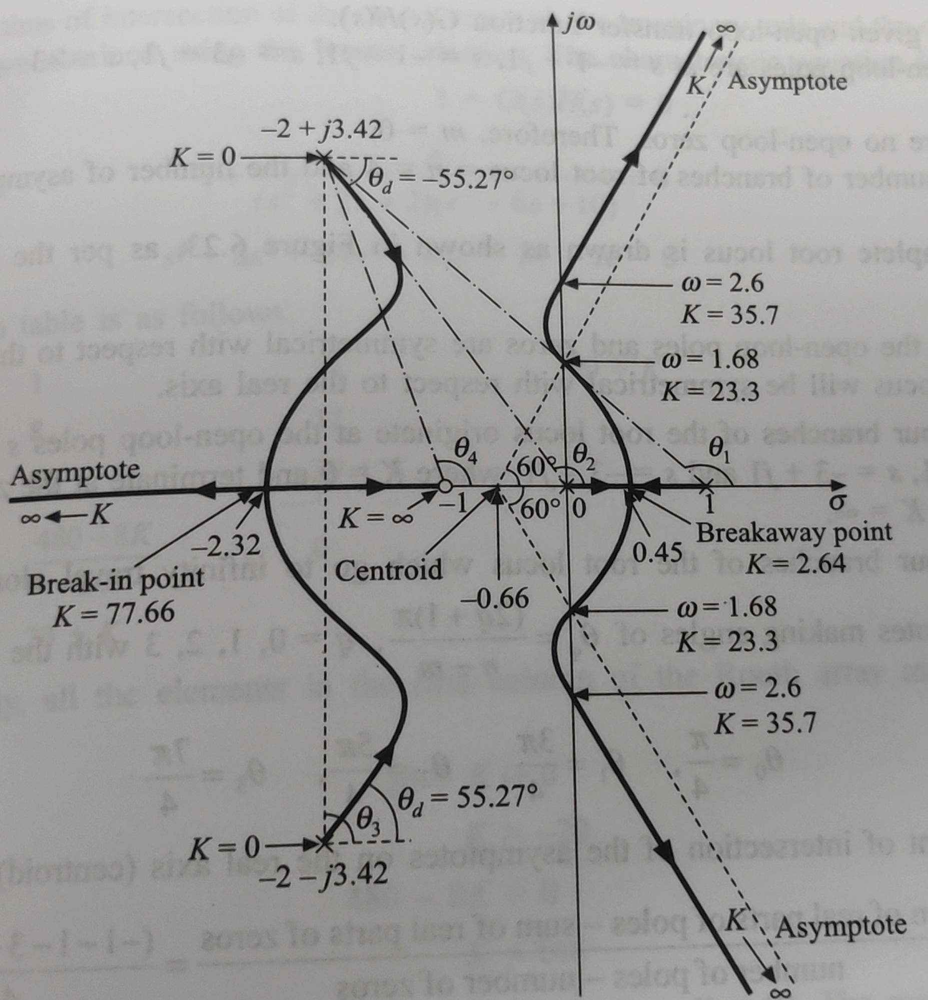

Figure 6.22 Example 6.14: Root locus.