CorrecionexamenEnero2022.pdf

Anónimo

Matemáticas I

1º Grado en Bioquímica y Ciencias Biomédicas

Facultad de Ciencias Biológicas
Universitat de València

Corrección de examen
Enero 2022

$\boxed{1}$

$$\lim_{x\to 0} \frac{ae^{ax}x + x}{\sin(ax)\cos x} = 2$$

Desarrollo de McLaurin de la función $y = f(x)$ hasta orden 3:

$$f(x) \approx f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

a) Para $f(x) = e^{ax}$

$$e^{ax} \approx 1 + ax + \frac{a^2x^2}{2} + \frac{a^3x^3}{3!}$$

b) Para $f(x) = \sin(ax)$

$$\sin(ax) \approx ax - \frac{a^3x^3}{6}$$

b) Para $f(x) = \cos x$

$$\cos x \approx 1 - \frac{x^2}{2}$$

Así:

$$\lim_{x\to 0} \frac{ae^{ax}x + x}{\sin(ax)\cos x} = \lim_{x\to 0} \frac{\cancel{x}(a + a^2x + a^3x^2/2 + a^4x^3/6 + 1)}{\cancel{x}(a - a^3x^3/6)(1 - x^2/2)} =$$

$$= \frac{a+1}{a}$$

$$\frac{a+1}{a} = 2 \rightarrow \boxed{a = 1}$$

$\boxed{2}$

$$y = x^{3/2} \longrightarrow y' = \frac{3}{2} x^{1/2}$$

$$\ell = \int_0^1 \sqrt{1 + f'(x)^2}\, dx = \int_0^1 \sqrt{1 + \left(\frac{3}{2}\sqrt{x}\right)^2}\, dx =$$

$$= \int_0^1 \sqrt{1 + \frac{9}{4}x}\, dx = \frac{4}{9} \cdot \frac{2}{3} \left(1 + \frac{9}{4}x\right)^{3/2} \Big|_0^1 = \frac{13\sqrt{3} - 8}{27}$$

$\boxed{3}$

a) $y(0) = -2 \xrightarrow[y=-2]{x=0} -4a + 4b = 0 \longrightarrow a = b$

$y(-1) = 1 \xrightarrow[y=1]{x=-1} 2a + b - 3 = 0 \longrightarrow 3a = 3 \longrightarrow a = 1 = b$

b)

$$y' = -\frac{3(1+x)}{1+y}$$

$y' = 0 \longrightarrow x = -1 \longrightarrow 2y + y^2 - 3 = 0$ $\begin{cases} \longrightarrow y = -3 \longrightarrow (-1, -3) \\ \longrightarrow y = 1 \longrightarrow (-1, 1) \end{cases}$

c)

$1 + y = 0 \longrightarrow y = -1 \longrightarrow 3x^2 + 6x - 1 = 0 \longrightarrow x = \frac{1}{3}(-3 \pm \sqrt{12})$

$\longrightarrow \left(+\frac{1}{3}(-3 \pm \sqrt{12}), -1\right)$

$\boxed{4}$

$$y\cos^2 y\,dy = e^x(\sin(x+y) - \cos x \sin y)\,dx$$
$$= e^x(\sin x\cos y + \cancel{\cos x \sin y} - \cancel{\cos x \sin y})\,dx$$
$$= e^x \sin x \cos y\,dx$$

$\downarrow$

$$\frac{y\cos^{\cancel{2}} y}{\cancel{\cos y}}\,dy = e^x \sin x\,dx \rightarrow y\cos y\,dy = e^x \sin x\,dx$$

$$\int y\cos y\,dy = y\sin y + \cos y$$

$$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + c$$

$\rightarrow$

$$\rightarrow y\sin y + \cos y = \frac{1}{2}e^x(\sin x - \cos x) + c$$

$\downarrow$ $y(0)=0$

$$1 = \frac{1}{2}(-1) + c \rightarrow c = \frac{3}{2}$$

$\downarrow$

$$y\sin y + \cos y = \frac{1}{2}e^x(\sin x - \cos x) + \frac{3}{2}$$

5

$P(t) = \{$Población en el año "t"$\}$

$$\frac{dP}{dt} = -0'1P + 100$$

a) $-0'1P + 100 = 0 \rightarrow P = \frac{100}{0'1} = 1000$ pto. equilibrio

$g(P) = -0'1P + 100 \rightarrow g'(P) = -0'1 < 0 \rightarrow 1000$ estable

b) $\frac{dP}{-0'1P+100} = dt \rightarrow -\frac{1}{0'1} Ln|-0'1P+100| = t + c \rightarrow$

$\rightarrow -0'1P + 100 = ce^{-0'1t}$

$\rightarrow P(t) = 1000 + ce^{-0'1t}$

$\left.\begin{array}{l} P(0) = 300 \\ P(0) = 1000 + c \end{array}\right| \rightarrow c = -700 \rightarrow P(t) = 1000 - 700e^{-0'1t}$

c) $\lim_{t \to +\infty} P(t) = 1000 \rightarrow$ No se extinguirá

d) $P(30) = 1000 - 700e^{-0'1 \cdot 30}$

$\boxed{6}$

$Q(t)$: { Cantidad de sal en el instante "t" }

$$\left.\begin{array}{l} Q(0)=1 \\ C_e = 2 \\ V_e = 3 \\ V_s = 2 \end{array}\right| \rightarrow \frac{dQ}{dt} = 2\cdot 3 - \frac{2Q}{V+t} = 6 - \frac{2}{V+t}Q \rightarrow$$

$Q(1) = 14/3$

$$\rightarrow \frac{dQ}{dt} + \frac{2}{V+t}Q = 6$$

Por tanto:

$$\rho(t) = e^{\int \frac{2}{V+t}dt} = e^{2Ln|V+t|} = e^{Ln|(V+t)^2|} = (V+t)^2$$

$$P(t) = \frac{1}{\rho(t)}\int \rho(t)\cdot 6dt = \frac{6}{(V+t)^2}\int (V+t)^2 dt =$$

$$= \frac{6}{(V+t)^2}\left(\frac{(V+t)^3}{3} + C\right)$$

$$= 2(V+t) + \frac{C}{(V+t)^2}$$

$$\left.\begin{array}{l} Q(0)=1 \\ Q(0) = \rightarrow 2V + \frac{C}{V^2} \end{array}\right| \rightarrow 1 = 2V + \frac{C}{V^2} \rightarrow C = V^2(1-2V)$$

$$\left.\begin{array}{l} Q(1) = 14/3 \\ Q(1) = 2(V+1) + \frac{C}{(V+1)^2} \end{array}\right| \rightarrow 2(V+1) + \frac{V^2(1-2V)}{(1+V)^2} = \frac{14}{3} \rightarrow$$

$$\rightarrow \frac{7V^2+6V+2}{(1+V)^2} = \frac{14}{3} \rightarrow 7V^2 - 10V - 8 = 0 \rightarrow$$

$$\rightarrow V = \frac{10 \pm \sqrt{100+224}}{14} = \frac{10\pm 18}{14} \begin{array}{l} \nearrow V = 2 \rightarrow C = -12 \\ \searrow V < 0 \end{array}$$

b) $Q(10) = 2(2+10) + \frac{4(1-4)}{(2+10)^2}$

c)

$v + t = 10 \longrightarrow 2 + t = 10 \longrightarrow \boxed{t = 8}$

d)

$C = \frac{\text{cantidad}}{\text{Volumen}} = \frac{Q(8)}{10}$