# The Implicit Metropolis Hastings Algorithm

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#### **Abstract**

The implicit Metropolis-Hastings algorithm is introduced as a generalization of approaches of using the discriminator of a GAN to filter out unrealistic samples of the generator. Implicit Metropolis-Hastings operates by learning a discriminator to estimate the density-ratio and then generating a chain of samples. Since the approximation of density ratio introduces an error on every step of the chain, it is crucial to analyze the stationary distribution of such chain. The algorithm thus is an improvement to the current generative models (GANs).

Github repo: https://github.com/Nerkan78/tIMH

## 1. Introduction

Learning a generative model from an empirical target distribution is one of the key tasks in unsupervised machine learning. Currently, Generative Adversarial Networks (GANs) are among the most successful approaches in building such models. Unlike conventional sampling techniques, such as Markov Chain Monte-Carlo (MCMC), they operate by learning the implicit probabilistic model, which allows for sampling but not for a density evaluation.

In this project a recent article (Neklyudov K, 2019) is replicated. The authors confirm the theoretical findings of (Neklyudov K & D, 2018) and (Turner R & Yosinski, 2018). In this article, the researchers come up with a new GAN postprocessing approach that involves using the Implicit Metropolis-Hastings algorithm. The main idea of the article is to use the algorithm implicitly so that it is possible to sample from the posterior distribution to build a chain of samples. The researchers also derive theoretical implications that justify their findings.

## 1.1. Experiments.

The authors present an empirical evaluation of the proposed algorithm and theory for both independent and Markov proposals. In both cases sampling via the implicit MH algorithm is better than the straightforward sampling from a

# Algorithm 1 The Implicit Metropolis-Hastings algorithm.

```
Input: target dataset D
Input: implicit model q(x|y)
Input: learned discriminator d(.,.)
Initialize y from the dataset D
for i=0 to n do
sample proposal point x \sim q(x|y)
P = min\{1, \frac{d(x,y)}{d(y,x)}\}
x_i=x, \text{ with probability P}
x_i=y, \text{ with probability 1-P}
y \leftarrow x_i
end for
output \{x_0 \dots x_n\}
```

generator. For independent proposals, we validate our theoretical result by demonstrating monotonous improvements of the sampling procedure throughout the learning of the discriminator.

The authors conduct their experiments on CelebA and CIFAR-10 datasets, use a pre-trained generator from DC-GAN and WPGAN, VAE and demonstrate the results of the algorithm while sampling independently and following a Markov chain throughout learning of the discriminator.

# 2. Our experiments.

We follow the authors, reimplement their algorithm and validate the results of it on the CIFAR-10 dataset using the Inception score and the Frechet Inception distance - the metrics of choice of the authors of the article. We also used the same sampling approach as the authors. We conduct experiments with 2 different approaches just like the authors: one is independent sampling, the second one is using a Markov chain. We take a pre-trained DCGAN model on CIFAR-10 with 200 epochs and we train a discriminator from scratch. The experiments were conducted on a Colab GPU with 5-30 epochs of training, which was enough for a good quality discriminator.

Sampling via HMC from the Gaussian is equivalent to the interpolation between the previous accepted point  $z_y$  and the random vector  $\mathbf{v}$ :

$$z_x = cos(t)z_y + sin(t)v, v \sim N(0, I)$$

Further we proceed to estimate the metrics: Inception score and Frechet Inception distance by using a PyTorch implementation of the Inception v3 model. The metrics are estimated simultaneously for discriminator with the IMH applied and without, so we can see the difference in scores to see the improvement.

# 3. Results. (one page lower)

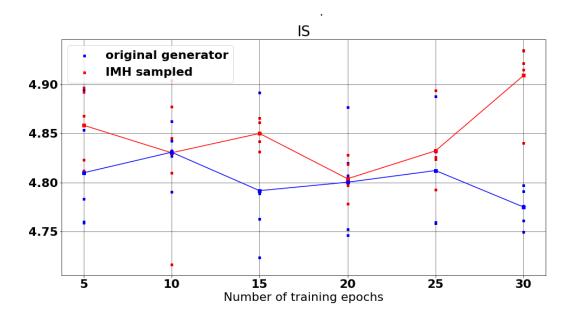


Figure 1. Inception scores for independent proposals.

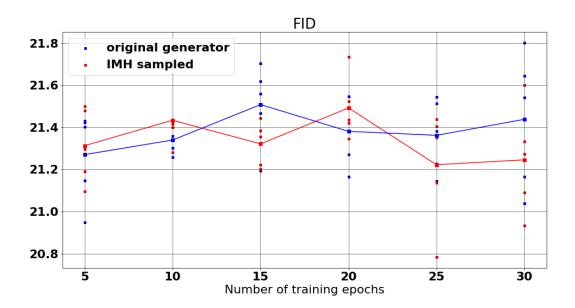


Figure 2. Frechet Inception distances for independent proposals.



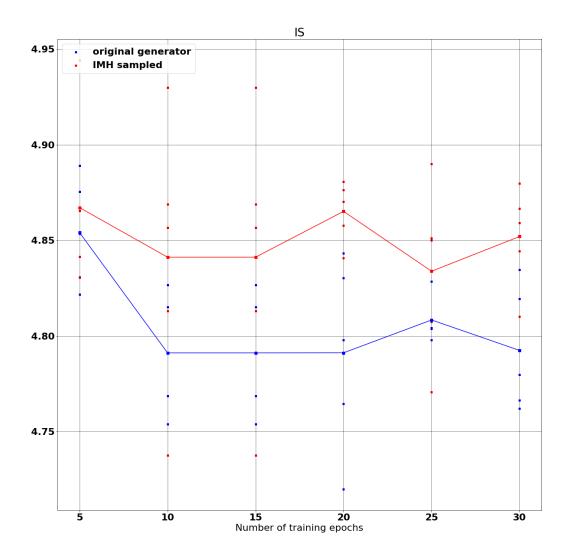


Figure 3. Inception scores for the Markov chain proposals.



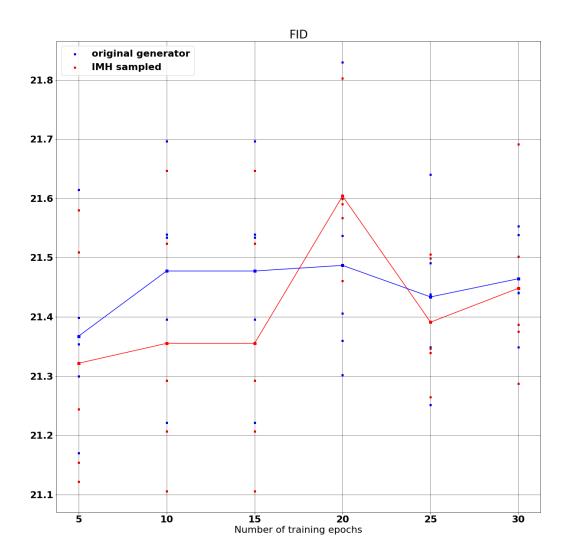


Figure 4. Frechet Inception distances for the Markov chain proposals.

#### 3.1. Discussion of the results.

To estimate the displayed metrics, we estimated 10k samples for each dot on the plot 5 times (scatterplot), and then averaged across the entries of the same method and number of training epochs (lines). Higher IS and lower FID are better. We obtained a better scoring result in both IS and FID - the only explanation can be that we took a really great pretrained GAN model (on 200 epochs, and it is unknown how good was the initial model of the authors). However, we still can see from the plots that the proposed sampling method provides a small improvement to the quality of the generator, given that discriminator trains for enough epochs (> 25). On average, generator with IMH sampling provides higher inceptions scores and lower FID. We obtain similar results for the Markov chain proposals. It seems the results of Markov chain training depend heavily on the loss function and we implemented a different from the researchers' loss function, which possibly explains the differences between our results and the article results.

## 4. Conclusion.

We conclude that the proposed method of postprocessing enhances the performance of GANs. However, the margin of improvement in terms of IS and FID scores is not that high in case of already well-trained models on CIFAR-10 with image size=32. IMH postprocessing account for improvement of several percent, while for a not-fully-trained generator the effect might be greater. The strong side of the proposed method is that it can be applied to literally any GAN as it does not depend on the proposing model itself and enhance its performance.

# References

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