1 Edge current calculation

Calculation of edge current, using trajectories from a point $(-L/2, y_i)$ at the left boundary through a fixed point $(x, y) = (0, \pm W/2)$ to a point at the right boundary $(+L/2, y_f)$. Assuming, trajectories are isotropically scattered from there to the right boundary with a transmission coefficient $\mathcal{T}_{\text{edge}}$.

1.1 Upper edge

Starting with the upper edge $(x, y) = (0, +W/2)$. Parametrization of trajectory by angles $\theta_{i,f}$:

$$\tan \theta_i = \frac{W - 2y_i}{L}, \quad \frac{d\theta_i}{dy_i} = -\frac{2L}{L^2 + (W - 2y_i)^2} \tag{1}$$

$$\tan \theta_f = \frac{W - 2y_f}{L}, \quad \frac{d\theta_f}{dy_f} = -\frac{2L}{L^2 + (W - 2y_f)^2}. \tag{2}$$

Effective magnetic phase

$$\frac{2\pi}{\Phi_0} \int d\mathbf{l} \cdot \mathbf{A} = -\frac{2\pi B}{\Phi_0} \left(\int_{-L/2}^{0} x \tan \theta_i \, dx + \int_{0}^{+L/2} x \tan \theta_f \, dx \right) \tag{3}$$

$$= -\frac{\pi B}{\Phi_0} \left(\frac{L}{2} \right)^2 (\tan \theta_i + \tan \theta_f) \tag{4}$$

$$= \frac{\pi \phi}{2W} (y_i + y_f) + \frac{\pi \phi}{2} \tag{5}$$

First the critical current for zero flux, where we have no effective phase. Using the parametrization above and the current-phase relation

$$\mathcal{J}(\chi) \simeq \mathcal{T} \sin \chi, \quad \mathcal{T} \ll 1, \tag{6}$$

we find for zero magnetic field:

$$I_{c0}^{\text{edge}} = \max_\chi \int d\theta_i \cos^2 \theta_i \int d\theta_f \cos \theta_f \mathcal{J}(\chi) \tag{7}$$

$$= \int_{-W/2}^{W/2} dy_i \frac{-2L^3}{(L^2 + (W - 2y_i)^2)^2} \int_{-W/2}^{W/2} dy_f \frac{-2L^2}{(L^2 + (W - 2y_f)^2)^{3/2}} \tag{8}$$

$$= \frac{W}{\sqrt{L^2 + 4W^2}} \arctan \left(\frac{2W}{L} \right) + \frac{2LW^2}{(L^2 + 4W^2)^{3/2}}. \tag{9}$$

For comparison the zero field critical current in the QPC setup is

$$I_{c0}^{\text{qpc}} = \frac{L}{\sqrt{L^2 + W^2}} \arctan \frac{W}{L} + \frac{L^2 W}{(L^2 + W^2)^{3/2}} \tag{10}$$

Now with finite, but small magnetic flux ϕ. Using the effective phase from eq. (5) for the Josephson relation in eq.(6) (leads to factor $\cos(\pi\phi/2)$):

$$\frac{I_c(\phi)}{I_{c0}^{\text{edge}}} = \frac{1}{I_{c0}} \max_\chi \int d\theta_i \cos^2 \theta_i \int d\theta_f \cos \theta_f \mathcal{J}(\tilde{\chi}(y_i, y_f)) \tag{11}$$

$$= 1 - \frac{\pi^2 \phi^2}{32} f_{\text{egde}}(W/L) \tag{12}$$

$$f_{\text{egde}}(x) = 2 - \frac{\sqrt{4x^2 + 1} - 1}{x^2} - \frac{4(2x^2 + 1)}{2x^2 + x(4x^2 + 1)\arctan(2x)} \tag{13}$$

$$- \frac{\sqrt{4x^2 + 1} \log \left(\sqrt{4x^2 + 1} - 2x \right)}{x^3} \tag{14}$$

QPC function:

$$\frac{I_c(\phi)}{I_{c0}^{\text{qpc}}} \simeq 1 - \frac{\pi^2 \phi^2}{32} f_0(W/L) \tag{15}$$

$$f_0(x) = \frac{\sqrt{x^2 + 1} \log \left(\sqrt{x^2 + 1} + x \right)}{x} - \frac{x}{x + (x^2 + 1)\arctan(x)} \tag{16}$$

In the limit of high fields, extending the integration over $y_{i,f}$ to infinity, using $\tilde{\phi} = \frac{\pi \phi}{2W}$ we get ...
to be completed ...