Discrete Variational Autoencoders with Relaxed Boltzmann Priors

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Variational Autoencoder

- Генеративная модель со скрытыми переменными.
- Отображает объекты в заданное скрытое пространство и, соответственно, генерирует из него новые объекты.

x - данные, ζ - скрытые переменные

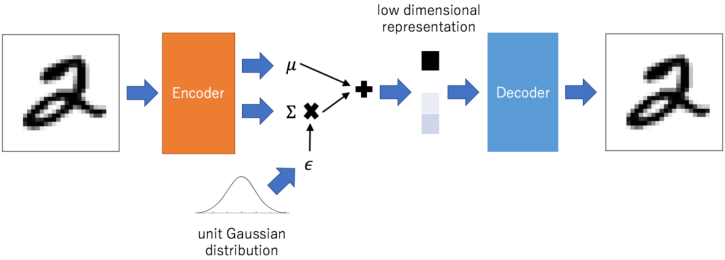
$$\log p(x) \geq \mathbb{E}_{q(\zeta|x)} \log p(x|\zeta) - \mathbb{KL}(q(\zeta|x)||p(\zeta))$$

$$p(\zeta) = N(0, I), q(\zeta|x; \theta_1) = N(\mu(x; \theta_1), \Sigma(x; \theta_1))$$

$$\mathbb{KL} = \frac{1}{2}(tr(\Sigma(x)) + \mu(x)^T \mu(x) - k - \log \det \Sigma(x))$$

$p(x|\zeta; \theta_2) = f(\zeta; \theta_2) + \epsilon, f(\zeta; \theta_2)$ - декодер

Variational Autoencoder



$z \in \{0, 1\}^D$ - бинарные латентные переменные
Больцман прайор: $p(z) = e^{-E_\theta(z)}/Z_\theta$
$E_\theta(z)$ - функция энергии, $\theta = \{W, a\}$

$$E_\theta(z) = -a^T z - \frac{1}{2} z^T W z$$

Z_θ- функция разделения

$$p(x, z, \zeta) = p(z)r(\zeta|z)p(x|\zeta)$$

$$r(\zeta|z) = \prod_i r(\zeta_i|z_i)$$

Сглаживающая трансформация в DVAE:

$$r(\zeta|z) = \begin{cases} \delta(\zeta) & \text{,z=0} \\ e^{\beta(\zeta-1)}/Z_\beta & , otherwise \end{cases}$$

$\delta(\zeta)$ - Дельта-функция Дирака
Z_β - нормировачная константа, $\zeta \in [0, 1]$

Сглаживающая трансформация в DVAE++:

$$r(\zeta|z) = \begin{cases} e^{-\beta\zeta}/Z_\beta & ,z{=}0 \\ e^{\beta(\zeta-1)}/Z_\beta & , otherwise \end{cases}$$

$$\log p(x) \geq \mathbb{E}_{q(\zeta|x)} \log p(x|\zeta) + H(q(z|x))$$
$$+\mathbb{E}_{q(z|x,\zeta)} \log p(z)$$

$$q(\zeta|x) = \prod_i q(\zeta_i|x) = \prod_i \sum_{z_i} q(z_i|x) r(\zeta_i|z_i)$$

$H(q(z|x))$ - энтропия

Importance weighting bound

$$\log p(x) \geq \mathcal{L}_K(x)$$

$$\mathcal{L}_K(x) = \mathbb{E}_{\zeta^{(k)} \sim q(\zeta|x)} \log\left(\frac{1}{K}\sum_{k=1}^{K}\right)\frac{p(\zeta^{(k)})p(x|\zeta^{(k)})}{q(\zeta^{(k)}|x)}$$

$$q(\zeta|x) = \prod_i q(\zeta_i|x) = \prod_i \sum_{z_i} q(z_i|x) r(\zeta_i|z_i)$$

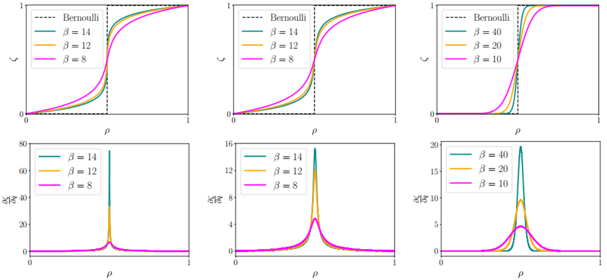
$H(q(z|x))$ - энтропия

Overlapping Relaxations

$$\log p(\zeta) = \log(\sum_x e^{-E_\theta(z)+b^\beta(\zeta)^T z + c^\beta(\zeta)}) - \log Z_\theta$$

$$b_i^\beta(\zeta) = \beta(2\zeta_i - 1), c_i^\beta(\zeta) = -\beta\zeta_i - \log Z_\beta$$

$$\hat{E}_{\theta,\zeta}^\beta = -E_\theta(z) + b^\beta(\zeta)^T z + c^\beta(\zeta)$$



(a) Exponential Transformation (b) Uniform+Exp Transformation (c) Power-Function Transformation

Table 1: The performance of DVAE# is compared against DVAE and DVAE++ on MNIST and OMNIGLOT. Mean±standard deviation of the negative log-likelihood for five runs are reported.

			DVAE	DVAE++		DVAE#				
	Struct.	K	Spike-Exp	Exp	Power	Gauss. Int	Gaussian	Exp	Un+Exp	Power
MNIST	1 —	1	**89.00±0.09**	90.43±0.06	**89.12±0.05**	92.14±0.12	91.33±0.13	90.55±0.11	89.57±0.08	89.35±0.06
		5	89.15±0.12	90.13±0.03	89.09±0.05	91.32±0.09	90.15±0.04	89.62±0.08	88.56±0.04	**88.25±0.03**
		25	89.20±0.13	89.92±0.07	89.04±0.07	91.18±0.21	89.55±0.10	89.27±0.09	88.02±0.04	**87.67±0.07**
	1 ∼	1	85.48±0.06	85.13±0.06	85.05±0.02	86.23±0.05	86.24±0.05	85.37±0.05	85.19±0.05	**84.93±0.02**
		5	85.29±0.03	85.13±0.09	85.29±0.10	84.99±0.03	84.91±0.07	84.83±0.03	84.47±0.02	**84.21±0.02**
		25	85.92±0.10	86.14±0.18	85.59±0.10	84.36±0.04	84.30±0.04	84.69±0.08	84.22±0.01	**83.93±0.06**
	2 ∼	1	83.97±0.04	84.15±0.07	83.62±0.04	84.30±0.05	84.35±0.04	83.96±0.06	83.54±0.06	**83.37±0.02**
		5	83.74±0.03	84.85±0.13	83.57±0.07	83.68±0.02	83.61±0.04	83.70±0.04	83.33±0.04	**82.99±0.04**
		25	84.19±0.21	85.49±0.12	83.58±0.15	83.39±0.04	83.26±0.04	83.76±0.04	83.30±0.04	**82.85±0.03**
	4 ∼	1	84.38±0.03	84.63±0.11	83.44±0.05	84.59±0.06	84.81±0.19	84.06±0.06	83.52±0.06	**83.18±0.05**
		5	83.93±0.07	85.41±0.09	83.17±0.09	83.89±0.09	84.20±0.15	84.15±0.05	83.41±0.04	**82.95±0.07**
		25	84.12±0.07	85.42±0.07	83.20±0.08	83.52±0.06	83.80±0.04	84.22±0.13	83.39±0.04	**82.82±0.02**
OMNIGLOT	1 —	1	**105.11±0.11**	106.71±0.08	105.45±0.08	110.81±0.32	106.81±0.07	107.21±0.14	105.89±0.06	105.47±0.09
		5	**104.68±0.21**	106.83±0.09	105.34±0.05	112.26±0.70	106.16±0.11	106.86±0.10	104.94±0.05	**104.42±0.09**
		25	104.38±0.15	106.85±0.07	105.38±0.14	111.92±0.30	105.75±0.10	106.88±0.09	104.49±0.07	**103.98±0.05**
	1 ∼	1	102.95±0.07	101.84±0.08	101.88±0.06	103.50±0.06	102.74±0.08	102.23±0.08	101.86±0.06	**101.70±0.01**
		5	102.45±0.08	102.13±0.11	101.67±0.07	102.15±0.04	102.00±0.09	101.59±0.06	101.22±0.05	**101.00±0.02**
		25	102.74±0.05	102.66±0.09	101.80±0.15	101.42±0.04	101.60±0.09	101.48±0.04	100.93±0.07	**100.60±0.05**
	2 ∼	1	103.10±0.31	101.34±0.04	100.42±0.03	102.07±0.16	102.84±0.23	100.38±0.09	**99.84±0.06**	**99.75±0.05**
		5	100.88±0.13	100.55±0.09	99.51±0.05	100.85±0.02	101.43±0.11	99.93±0.07	99.57±0.06	**99.24±0.05**
		25	100.55±0.08	100.31±0.15	99.49±0.07	100.20±0.02	100.45±0.08	100.10±0.28	99.59±0.16	**98.93±0.05**
	4 ∼	1	104.63±0.47	101.58±0.22	100.42±0.08	102.91±0.25	103.43±0.10	100.85±0.12	99.92±0.11	**99.65±0.09**
		5	101.77±0.20	101.01±0.09	99.52±0.09	101.79±0.25	101.82±0.13	100.32±0.19	99.61±0.07	**99.13±0.10**
		25	100.89±0.13	100.37±0.09	99.43±0.14	100.73±0.08	100.97±0.21	99.92±0.30	99.36±0.09	**98.88±0.09**

Литература

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