New critical and collective phenomena in the random networks.

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  Phys.Rev E96,062303(2017) + arXiv 1801.03912 + to appear

Outline of the talk

- Motivation and key questions

- Numerical findings for constrained Erdos-Renyi network(CERN) and regular random graph (RRG)

- Spectral analysis. Eigenvalue tunneling and the ground state of constrained networks.

- Propagation of the excitation and spectral properties

- Application. Critical phenomena in generalized Schelling model of segregation

- Application to connectome

Properties of networks

- Degree distribution
- Clusterization
- Typical path on the network
- Number of layers and interlayer interaction
- Synchronization
- Propagation of the signal on the network
- Phase transition in the exponential networks

Network Models: Summary

Erdös-Renyi model

- short path lengths
- Poisson distribution (no hubs)
- no clustering

Barabási-Albert scale-free model

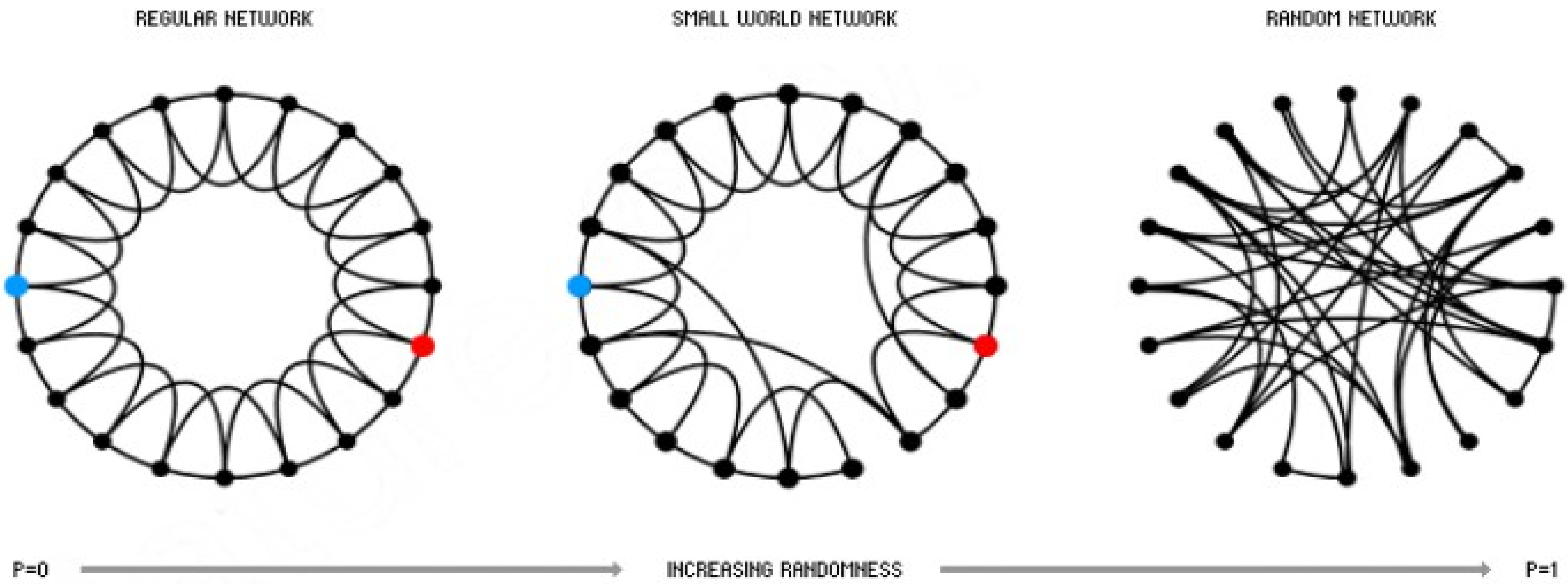
- short path lengths
- power-law distribution for degrees
- robustness
- no clustering (may be fixed)

Watts-Strogatz Small World model

- short path lengths
- high clustering (N independent)
- almost constant degrees

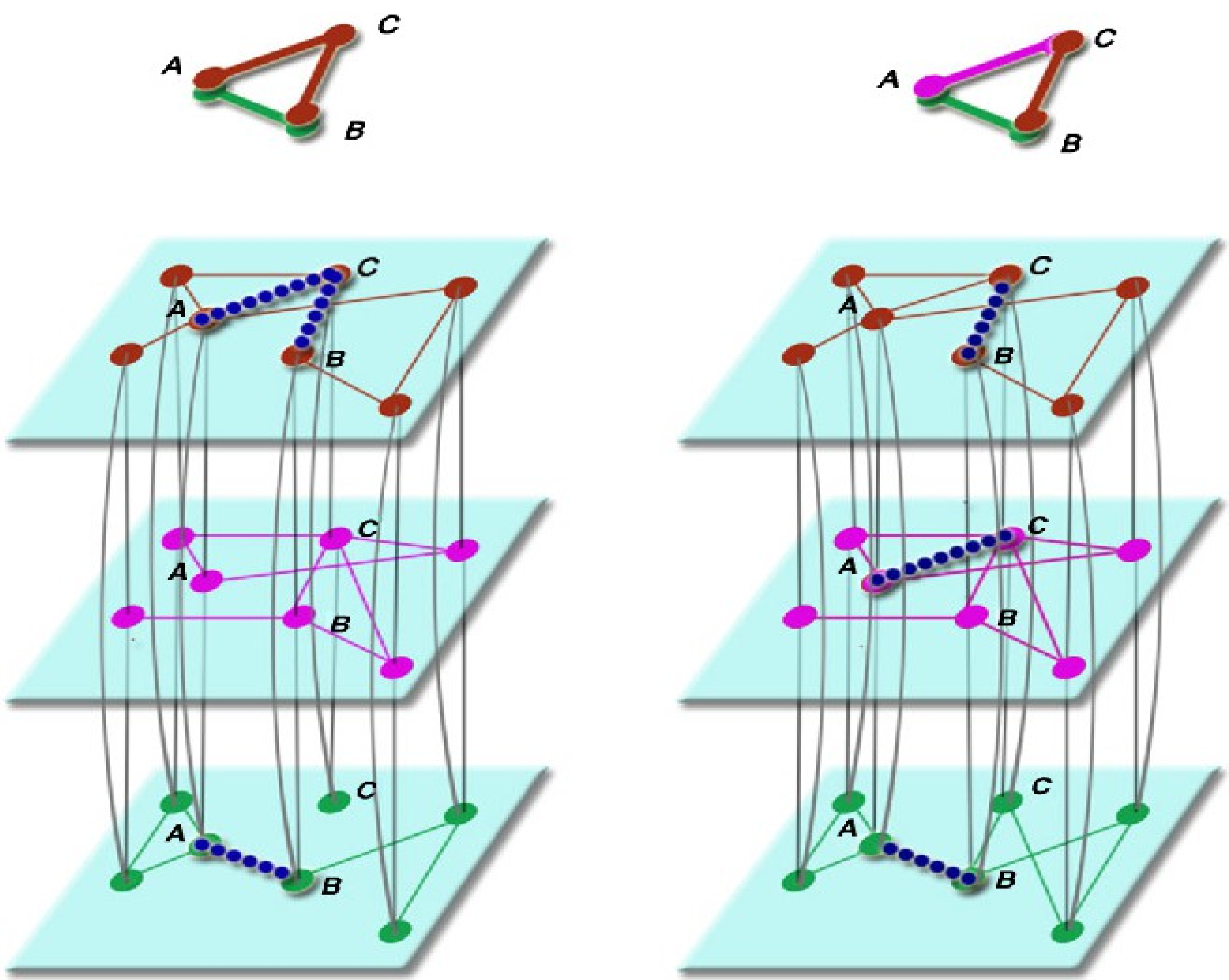
Real-world networks

- short path lengths
- high clustering
- broad degree distributions, often power laws

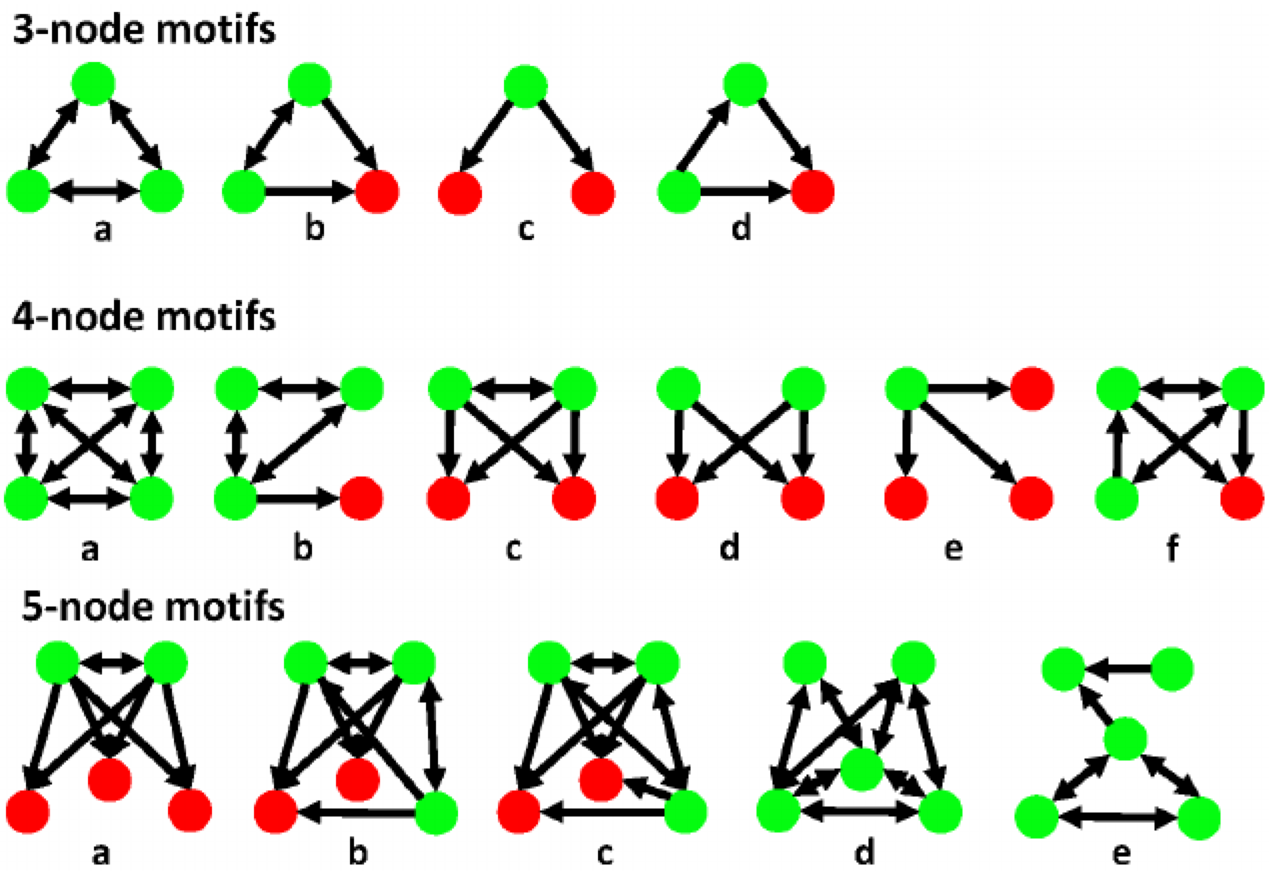


Typical examples of network organization

Multilayer networks



Exponential networks — network ensembles with weights for some motifs



Mixed network ensembles

- Investigation of the mixed network ensemble. Set of local constraints(degree conservation) + additional chemical potential for some motif(substructure)
- The degree of the vertex fixed. Standard condition in chemistry and biology.May be fixed by some topological arguments
- Properties of the mixed ensembles differ from the properties of canonical or microcaconical ensembles

Multicluster networks

- How multicluster networks emerge?
- How interaction between the clusters depends on the parameters of networks? When the clusters get synchronized?
- How the external probe and excitations behave on the clustered networks?

Experimental data for one color

- The CERN network or RRG
- The degree of all nodes in the network is conserved-constraint
- Chemical potential for the triangles. Mixed ensemble
- Phase transitions. Cliques formation above critical chemical potential
- For CERN network [1/p] cliques emerge (average number of links at vertex equals pN)

Model.Mixed ensemble

$$H = -\mu N_\Delta, \qquad (\mu > 0)$$ (= Tr A^3, A -adjacency matrix)

- The possible moves in the network(= kinetic term)

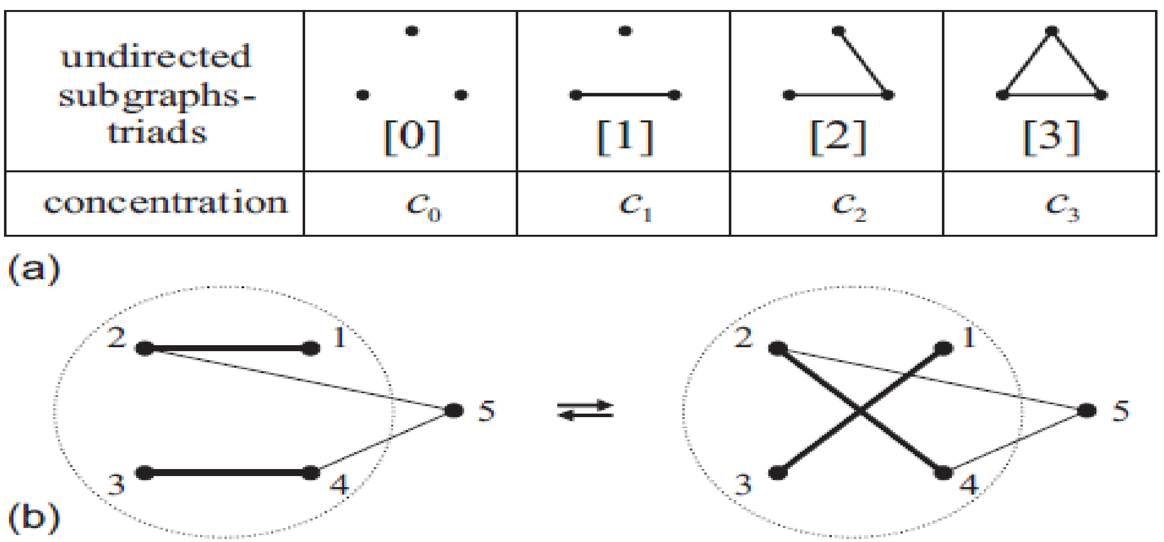


Figure 1: a) Possible triads in a non-directed network; b) Single link permutation: links (12) and (34) are removed, and links (13) and (24) are created. Triad {135} goes from type [0] to type [1], triads {125, 345} – from type [2] to type [1], and triad {245} – from type [2] to type [3]: three new triads of type [1] and one triad of type [3] are created instead of three triads of type [2] and one of type [0], compare to Eq.(1).

Maslov, Sneppen 2004

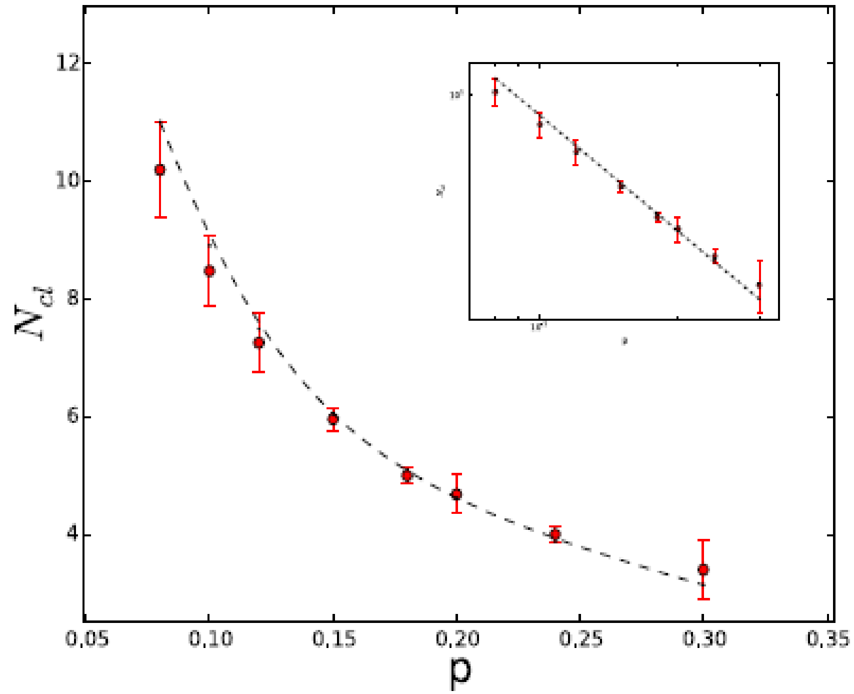


FIG. 1: The number of clusters N_{cl} as a function of the probability p in ER graph. The numerical data are obtained by averaging over 100 randomly generated graphs up to 512 vertices. Numerical values are fitted by the curve $p^{-0.95}$; the behavior in doubly logarithmic scale is shown in the insert.

The network is completely defragmented into the finite number of weakly coupled dense droplets above the critical point. Both for CERN and RRG

Model without degree conservation — Strauss model (1986), solved
In mean field approximation (Newman,Park -2004)

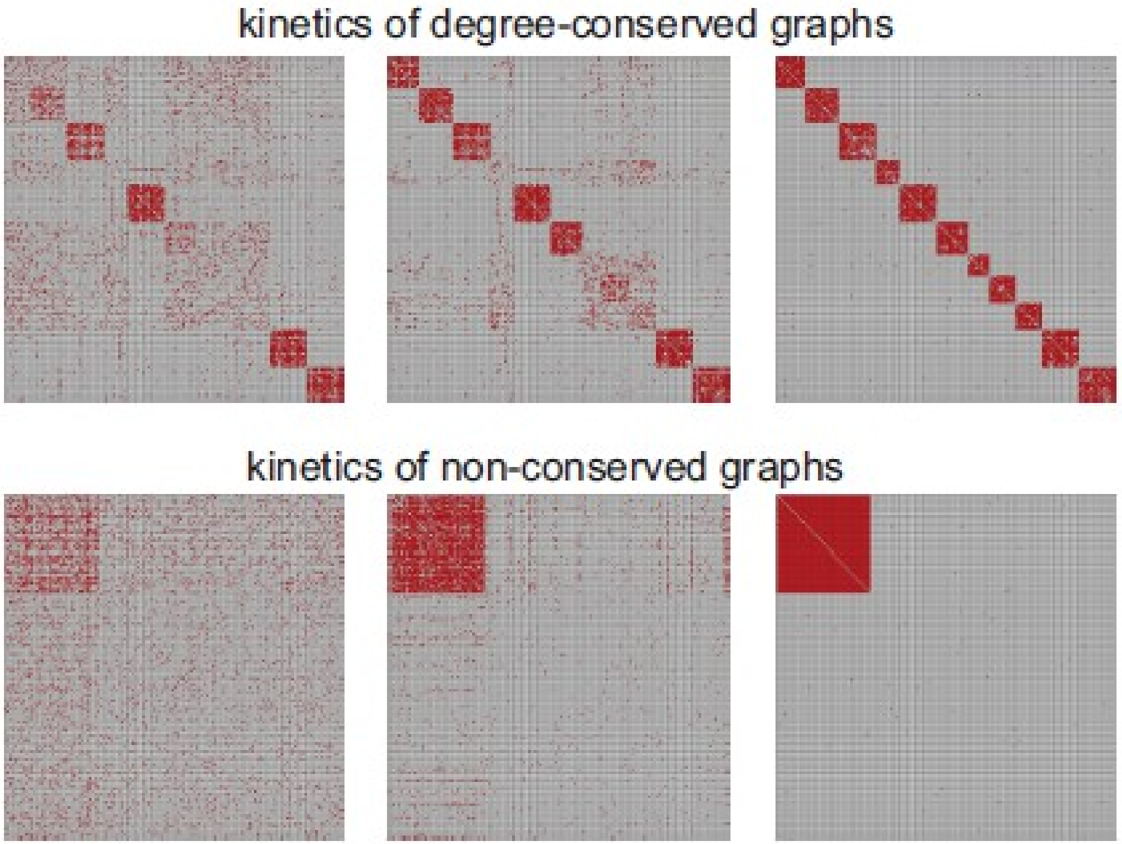
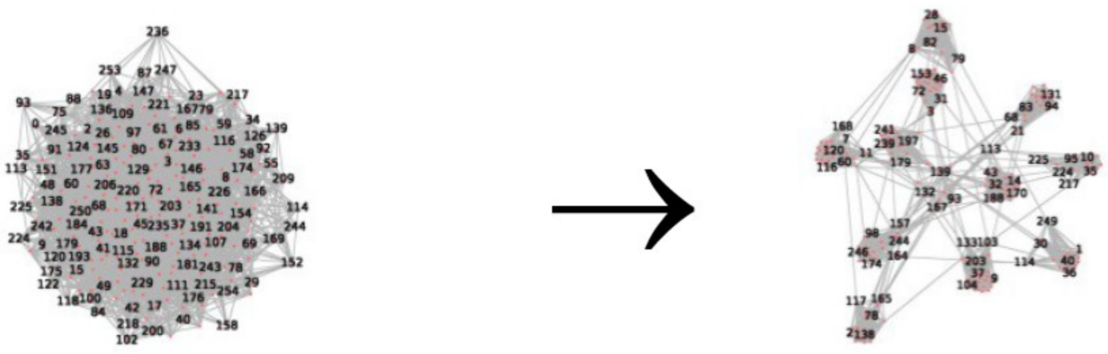


FIG. 5: Few typical samples of intermediate stages of the network evolution: upper panel – evolution with fixed vertex degree; lower panel – eolution with non-fixed vertex degree.

Intermediate «spin glass- like» pattern?

Typical evolution of the random initial network to the ground state

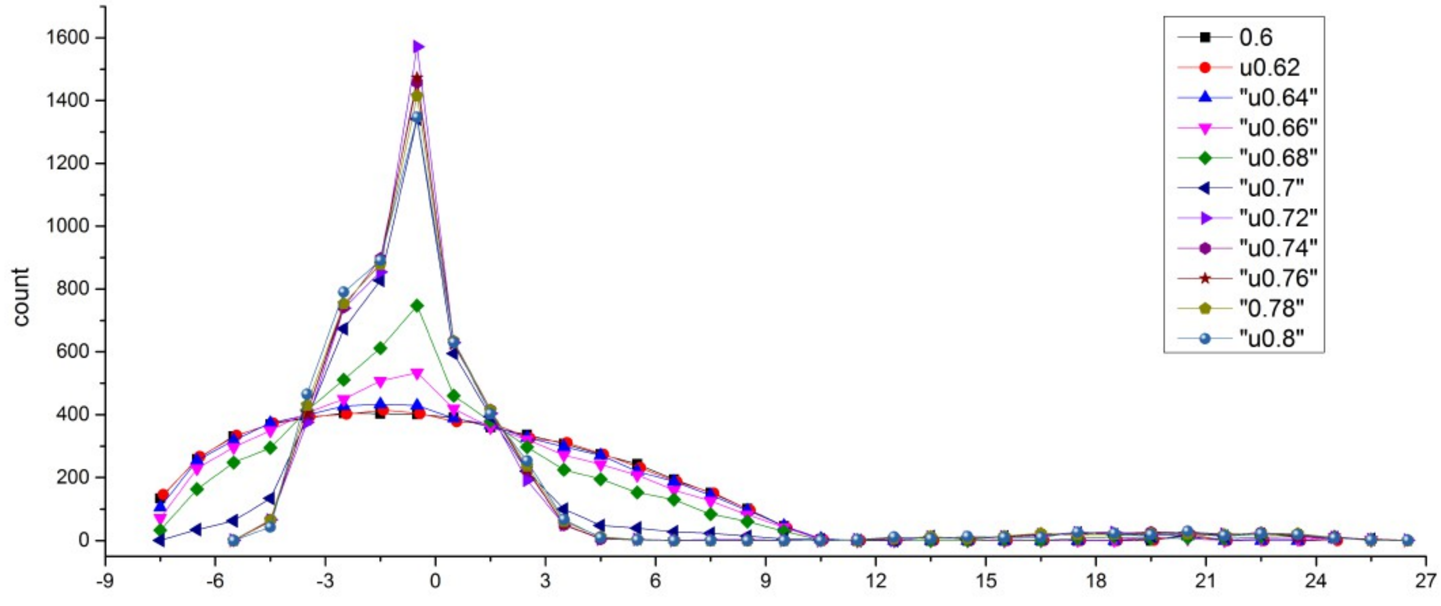


Number of the droplets in the ground state can be predicted!
All clusters are almost complete graphs.

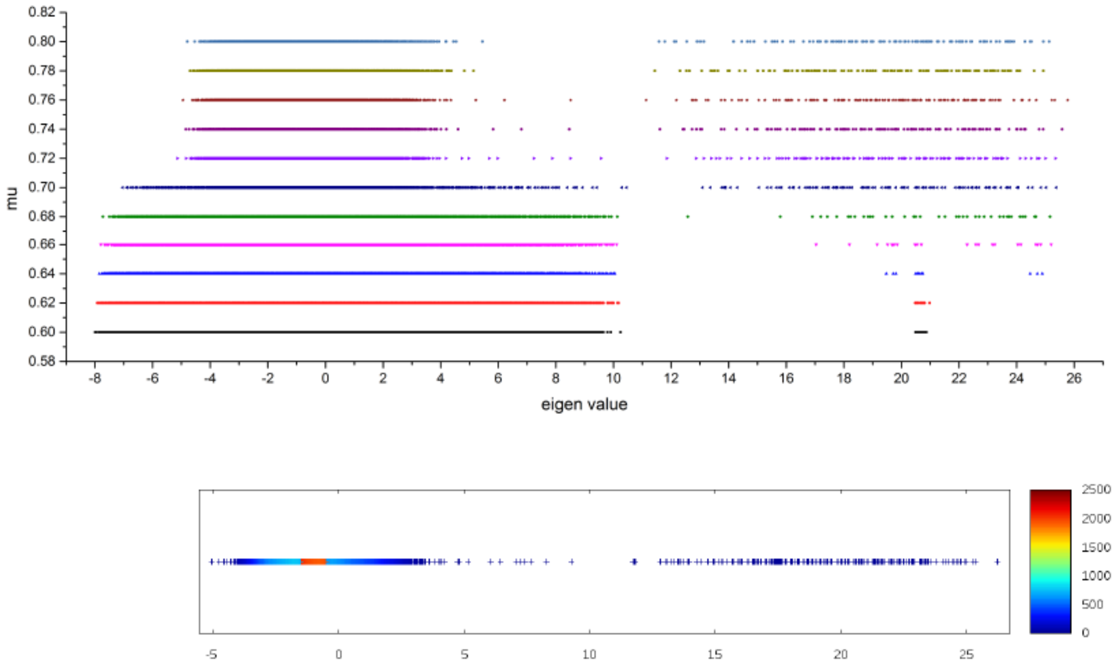
Spectral anatomy of transitions

- Second zone formation from the separated eigenvalues moving from the central zone.
- Number of isolated eigenvalues equals the number of clusters (Newman.et al, 13)
- Semicircle distribution before the phase transition. «Trianlge»-shape density in the central zone after the phase transition + second zone.
- Strong effect of intercluster interaction.

Spectral density of adjacency matrix before and after phase transition



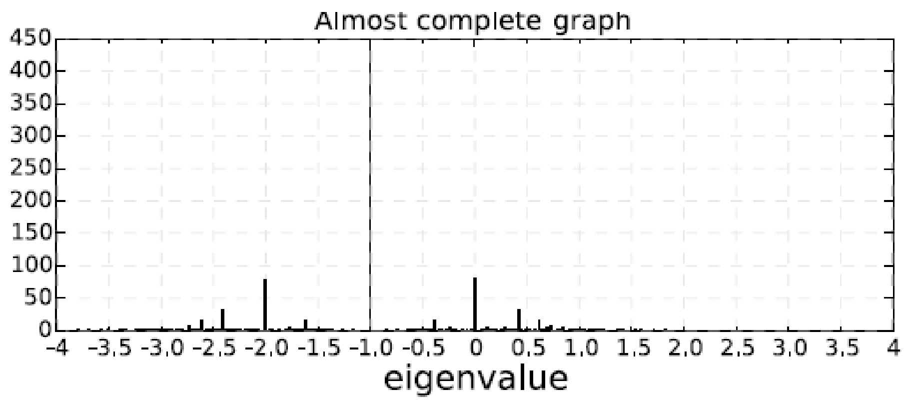
Second zone formation from clusters. Number of the Isolated eigenvalues of the adjacency matrix equals to the number of clusters



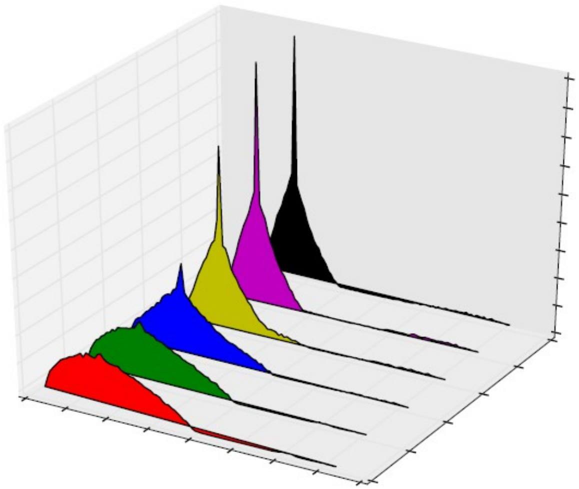
The second zone in the spectrum of adjecency matrix corresponds to the soft modes in spectrum of graph Laplacian

Spectral density for unicolor case

- Spectral density for each cluster



Account of intercluster entanglement
amounts to the very different total spectral density

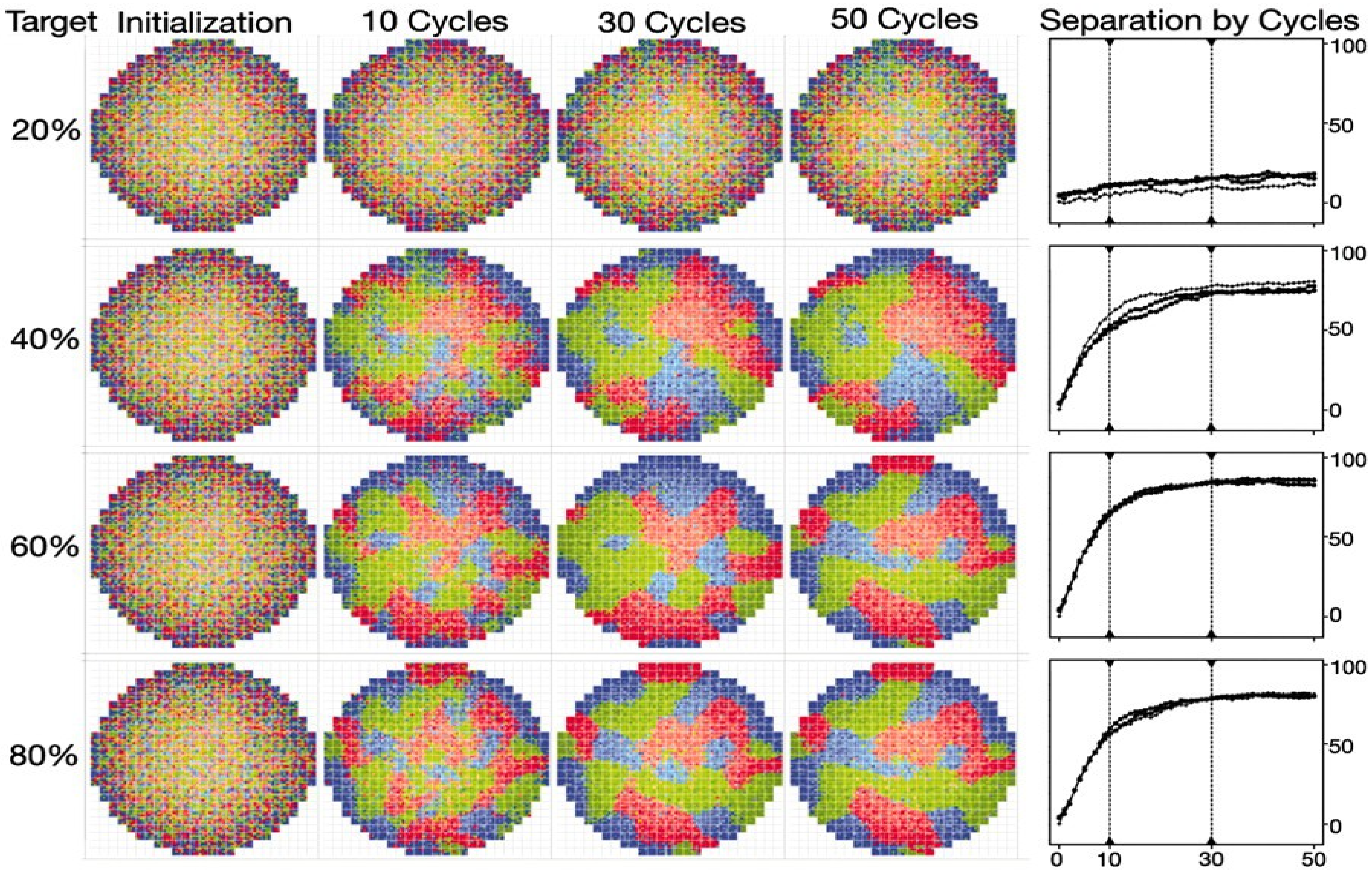


Application of the random network theory

- Transport networks
- Linguistic networks
- Evolutionary network
- Social networks and segregation models.
- Connectome- brain network

- Network of space-time and «Holographic Universe».

Schelling model of segregation. Single parameter-level of tolerance

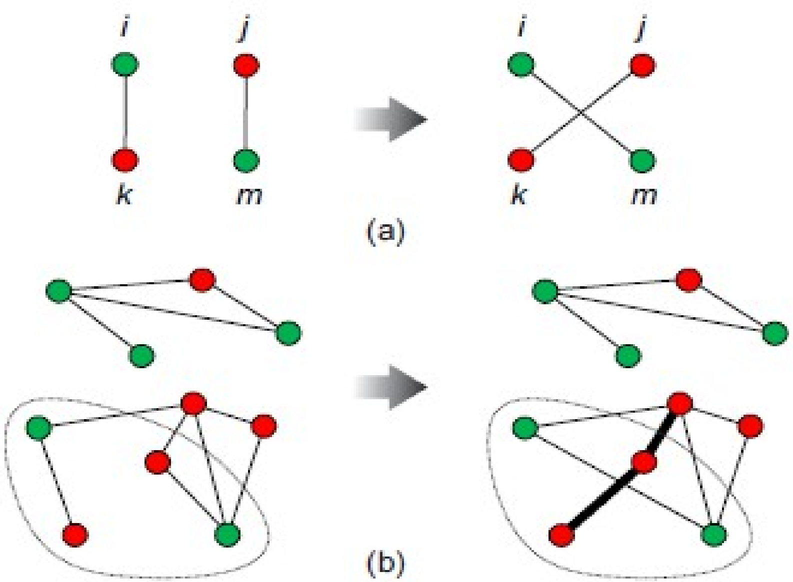


Model for two colors-communities

- Chemical potentials for trimers are the same for all colors. Degrees of nodes conserved
- Two possibilities. Colored CERN network and colored RRG. The results for two cases are the same.
- Communities — countries,genders etc

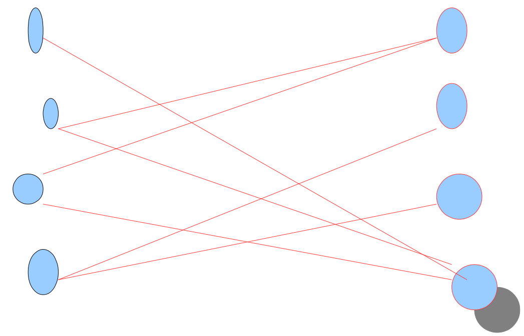
Generalization of Schelling model with two parameters.
One parameter- the weight for the intercommunity links
Second parameter — the weight for the formation
of triad inside community.

$$Z = \sum e^{-(\mu_G N_G + \mu_R N_R + \nu D N_{RG})}$$

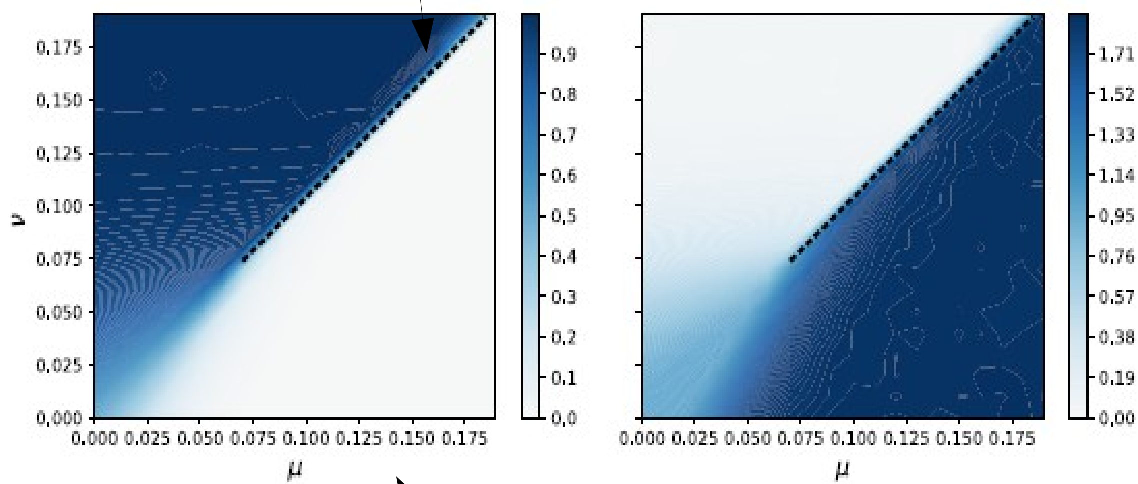


Phase1. Bipartite phase

(intercommunity links win)



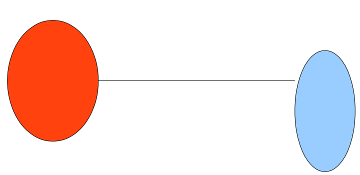
The line of phase transition



Phase 2.

Network = 2 unicolor clusters connected with
A few links

(intra-connections win)



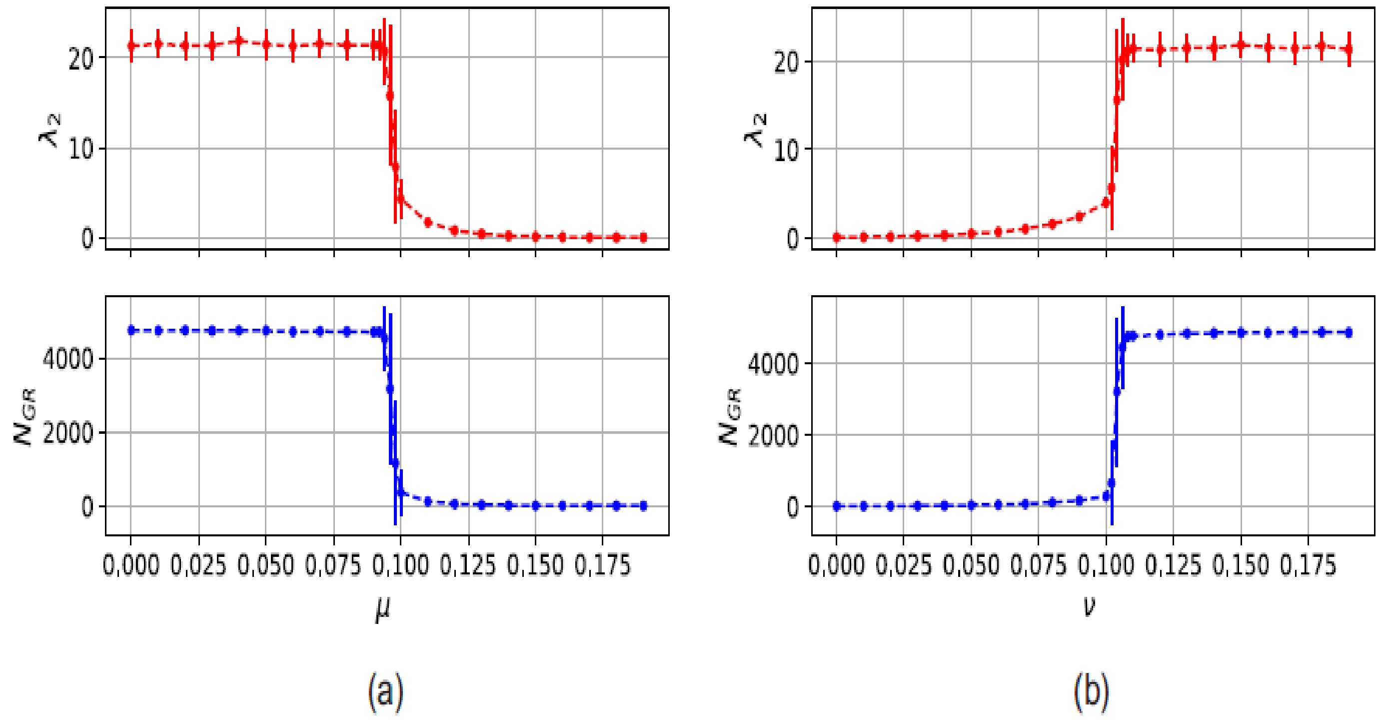
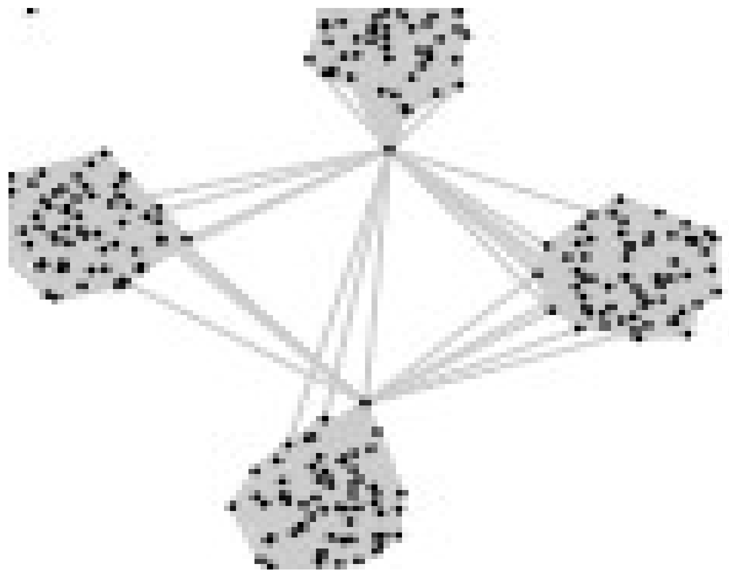


FIG. 3: (a) N_{RG} at $\nu = 0.1$ and the behavior of the second eigenvalue of Laplacian matrix, λ_2 as functions of μ; (b) N_{RG} as a function of ν at $\mu = 0.1$ and the behavior of the second eigenvalue of Laplacian matrix.

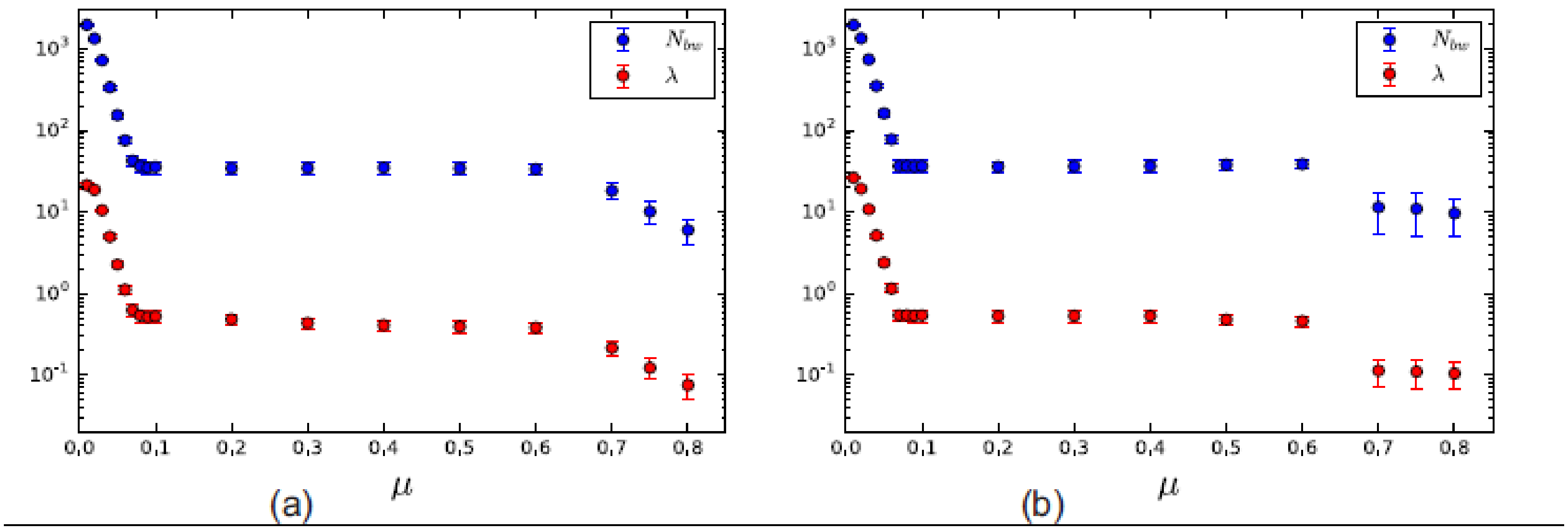
Near the transition between the in-dominated and Inter-dominated phases the interaction occurs via hubs



What it means for social dynamics?

- If the weights for the preferences of intra- or inter- connections are high the transitions between phases only via sharp phase transition. The playground for manipulations
- Any evolutionary transition occurs in the crossover region of low weights.
- When the number of links between the communities is small the interaction occurs via the hubs- ambassadors.

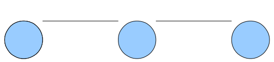
A bit of spectral analysis



CERN

RRG

The dependence of the first nonvanishing eigenvalue of Laplacian matrix (red) and dependence of the number of interlayer links (blue) on the chemical potential for trimers



Trimer

$$\lambda_2(\gamma) = cN_{bw}(\gamma)$$

c- some constant

The problem of evolution of λ_2.

The key phenomena — intersection and rearrangements of the spectrum of first modes In the block matrices

$$det \begin{pmatrix} (A-\lambda) & C \\ C^T & (D-\lambda) \end{pmatrix}, = det(A-\lambda)det(D-C^T(A-\lambda)^{-1}C) = 0$$

Chemical potential for trimers

Condition for the plateau entrance

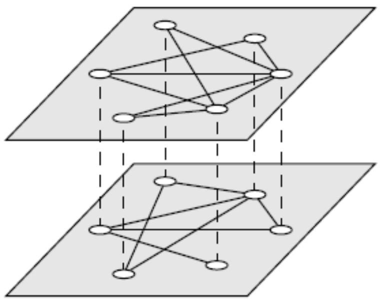
$$\lambda_2^A(\gamma) = \lambda_3(L)$$

Eigenvalue of the one layer Laplacian

Eigenvalue of the Laplacian of the whole network

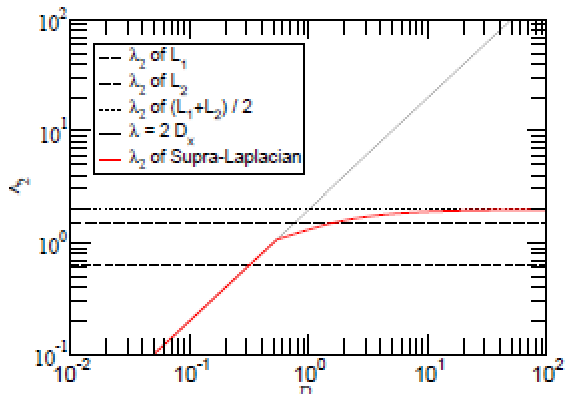
Two layers are absolutely correlated at the plateau!

The semiinfinite plateau in more
simple two-layer network only with interlayer interaction- phenomena of superdiffusion (Arenas et al 2013)



$$\mathcal{L} = \left(\begin{array}{c|c} D_1 L_1 + D_x I & -D_x I \\ \hline -D_x I & D_2 L_2 + D_x I \end{array} \right)$$

Laplacian of the whole network



Analogue of our plateau in the model with
One-one interaction between layers

The layers are absolutely synchronized at plateau. Phenomena of Superdiffusion due to synchronization

Matrix model description

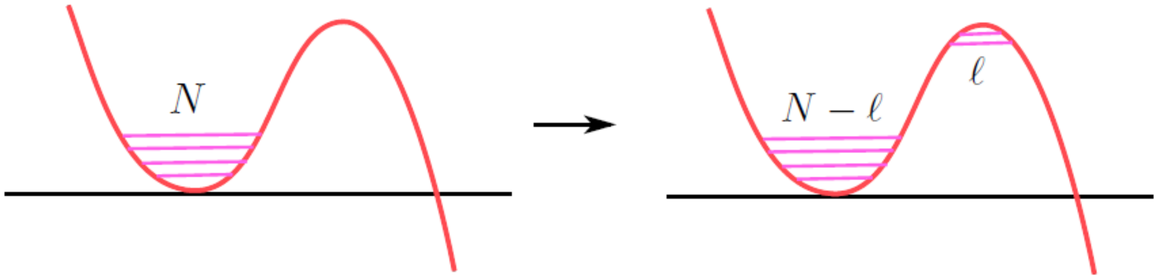
Adjacency matrix is symmetric random matrix involving 1 and 0 only

$$Z = \int dM exp(aTrM^2 - \mu TrM^3)$$

Additional constraint: sum of elements in each row and each column is fixed

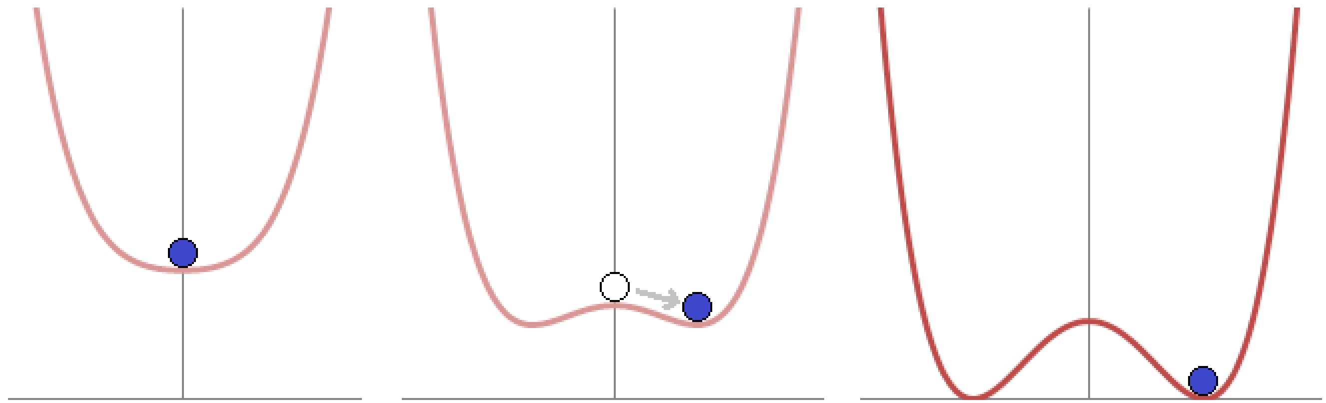
a- parameter of the network . Chemical potential for the number of triangles yields «interaction cubic term».

The matrix model counterpart of the cluster formation.
Eigenvalue tunneling — instanton, nonperturbative phenomena in many physical situations. Formation of stable D-branes in the string theory from unstable branes.
Baby-universes in 2d gravity

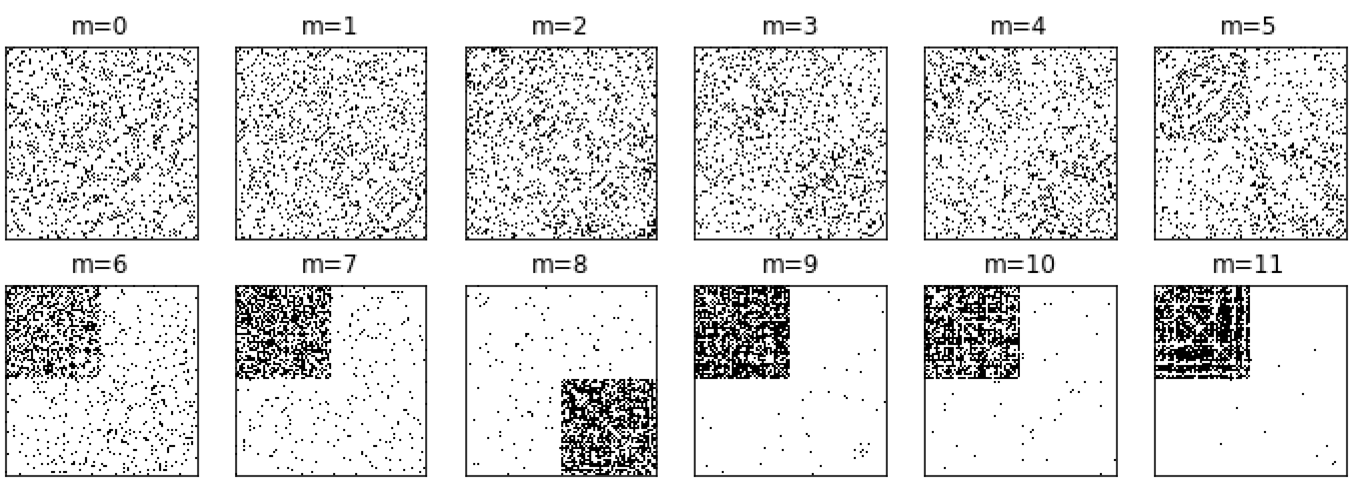


Spontaneous breaking of the symmetry in the two-color network

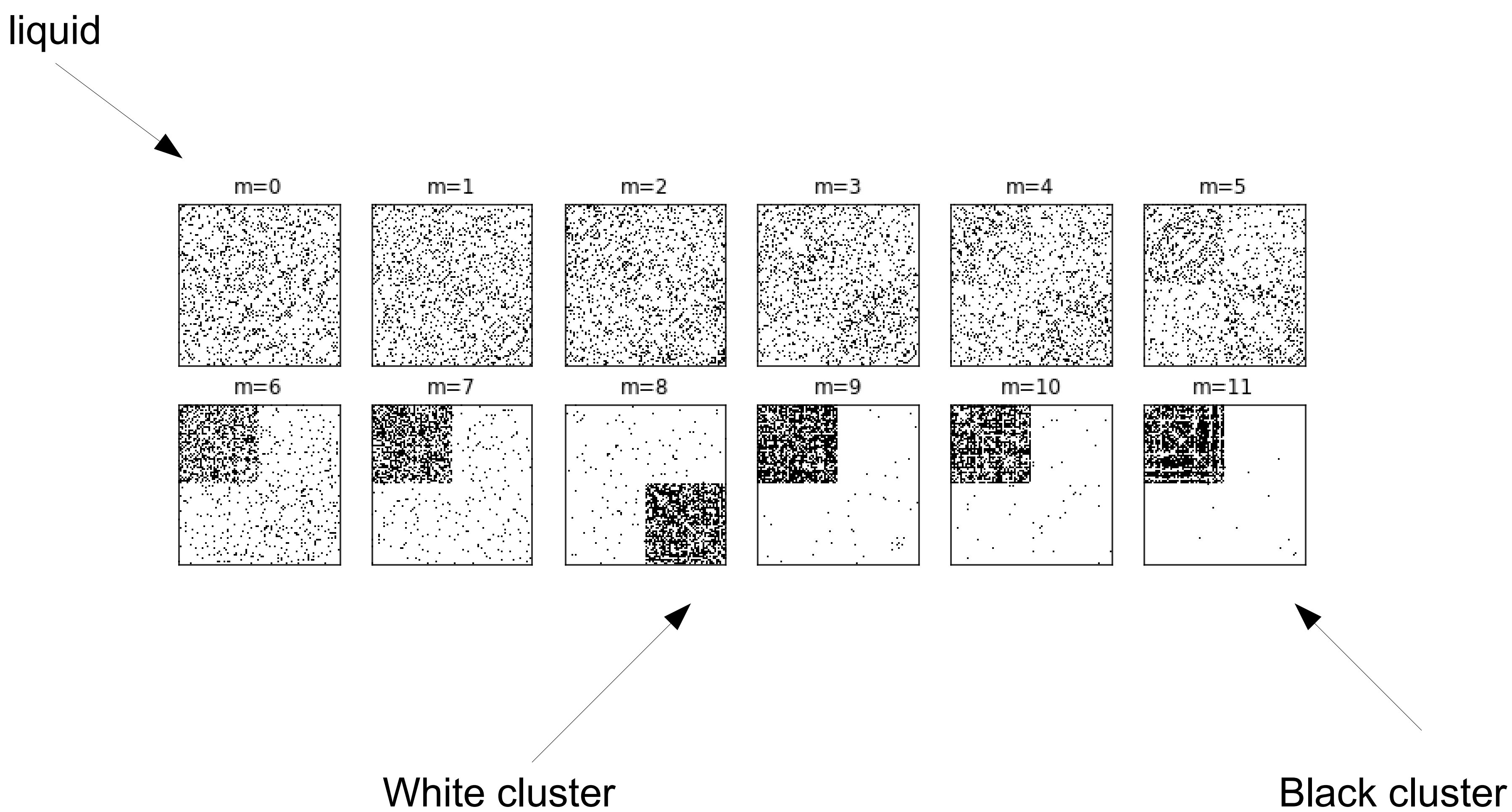
(no degree conservation)



The parameters of the black and white subsystems are the same!



Phase transition in two-color network above the critical weight for triads (no degree conservation)



Transport on the network

- How excitations behave on the network?
- Diffusion on the network is defined by the largest eigenvalue of the adjacency matrix
- Is there localization of the excitation on the network?

Standard criteria for localization

- Level spacing distribution (between the neigbohr levels)

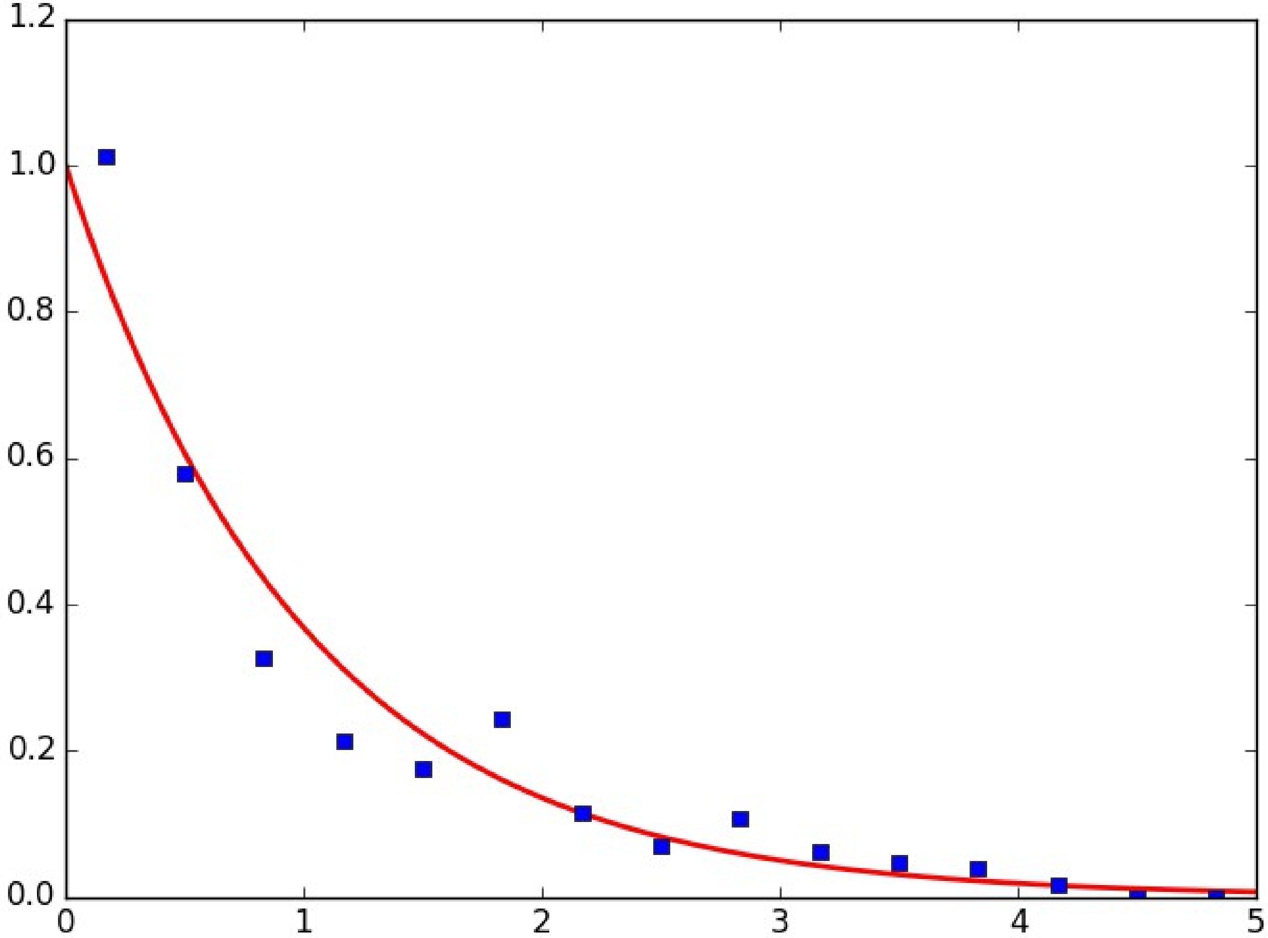
$$\begin{cases} P_{deloc}(s) = s\, e^{-as^2} \\ P_{loc}(s) = e^{-s} \end{cases}$$

- Participation ratio or inverse participation ratio

- Area or volume law for the entanglement

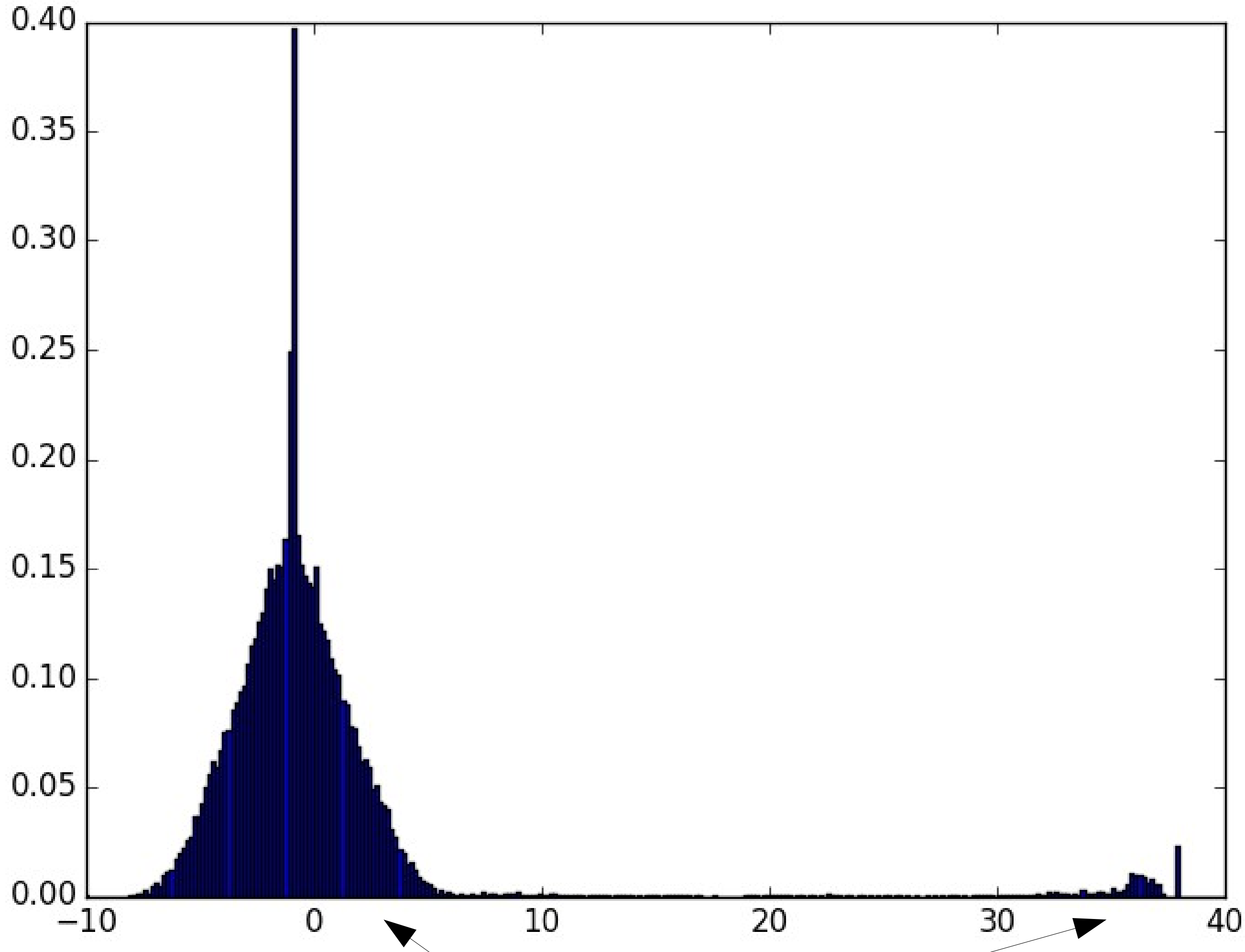
Let us treat our RRG and CERN as the «Fock space» for some interacting many-body system. Could we say smth about this many-body systems using our new critical phenomena?

First check- level spacing distribution



Level spacing distribution in the «nonperturbative» zone.
Poisson distribution — insulator zone.

The spectral density of adjacency matrix above the phase transition



The delocalization in «perturbative» and localiation in «nonperturbative» bands

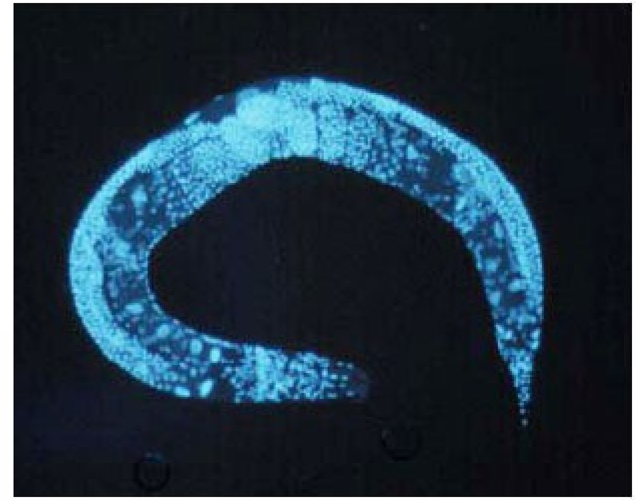
Non-ergodicity of delocalized modes

– We have considered the artificial network of clusters with the same parameters as our initial random network. Its spectral Density is different from the network prepared from the initial random network. Hence in the delocalized regime there is dependence on the initial condition! Mark of non-ergodicity.

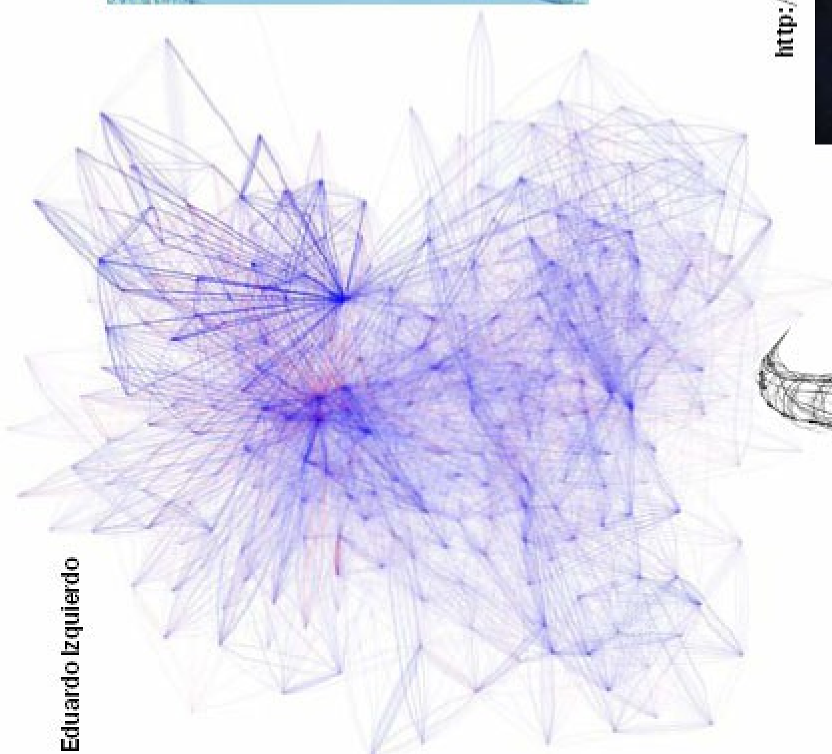
Connectome

- Only the connectome of C.Elegance is completely known -312 neurons(nodes)
- Connectome Program(USA-3 млд $, 2015-2025 гг)
- The number of neurons increases during first 2 years, is approximately stable till 60 and then starts to decrease
- There are structural and functional networks in connectome

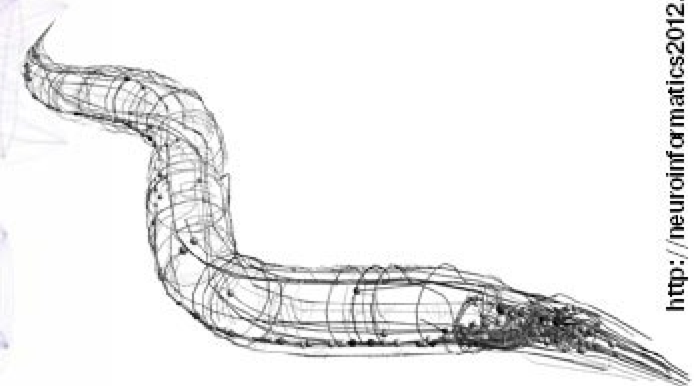
http://www.basis.ncl.ac.uk



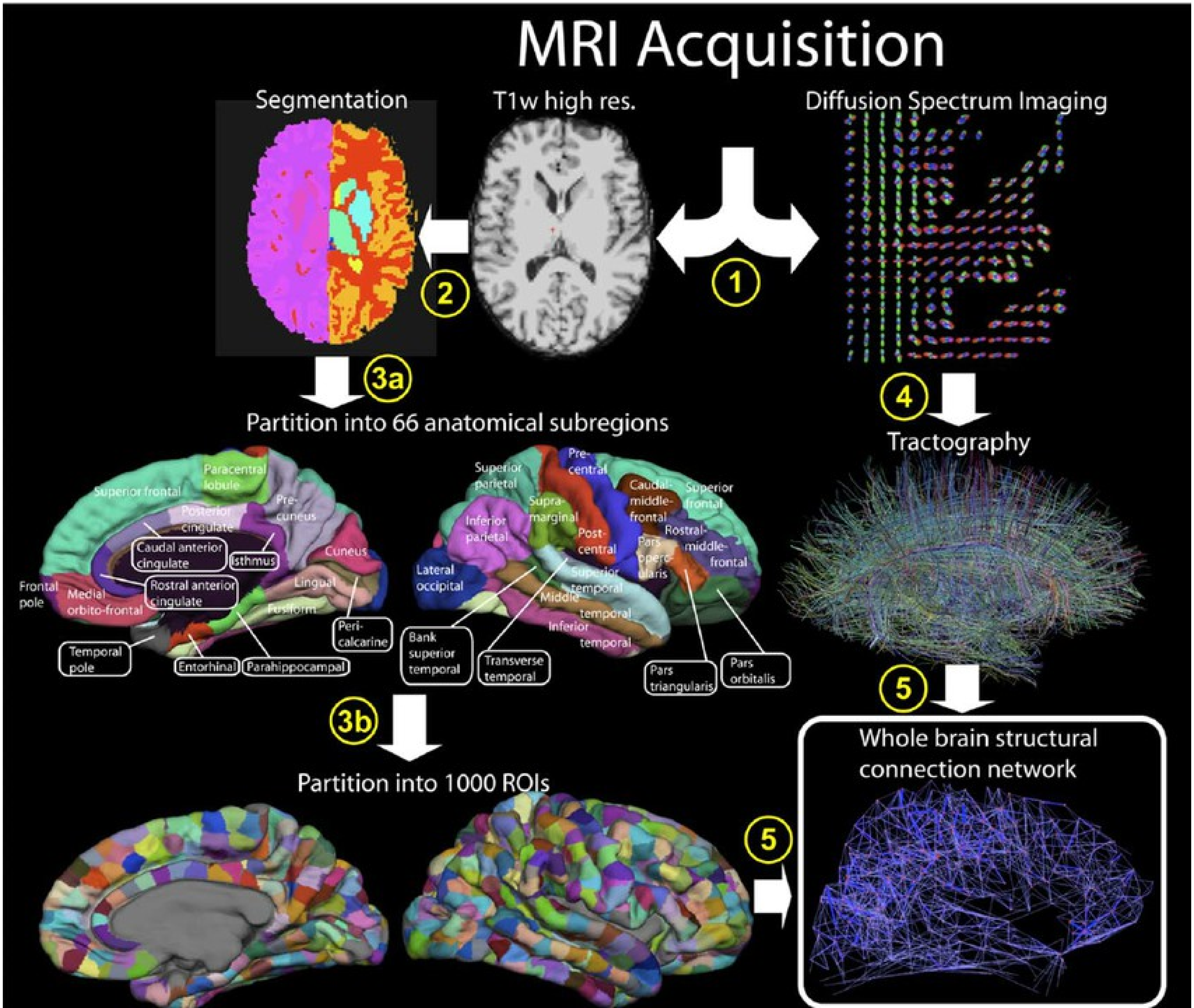
http://www.genome.gov/10000570



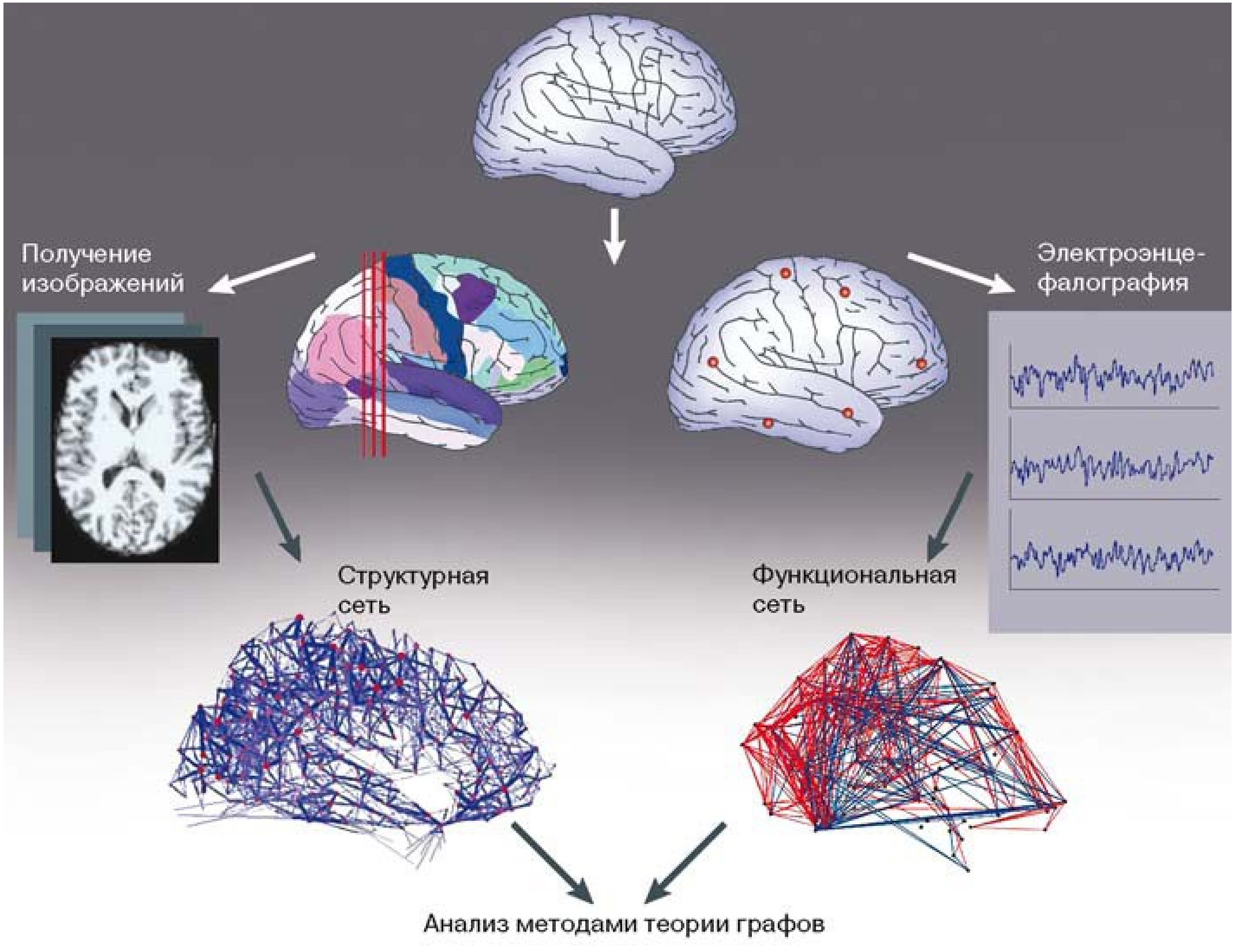
Eduardo Izquierdo



http://neuroinformatics2012.org/



MRI Acquisition
Segmentation
T1w high res.
Diffusion Spectrum Imaging
1
2
3a
Partition into 66 anatomical subregions
4
Tractography
Paracentral lobule
Superior frontal
Posterior cingulate
Pre-cuneus
Caudal anterior cingulate
Isthmus
Cuneus
Frontal pole
Medial orbito-frontal
Rostral anterior cingulate
Lingual
Fusiform
Peri-calcarine
Temporal pole
Entorhinal
Parahippocampal
Superior parietal
Pre-central
Caudal-middle-frontal
Superior frontal
Supra-marginal
Inferior parietal
Post-central
Rostral-middle-frontal
Pars operc-ularis
Lateral occipital
Superior temporal
Middle temporal
Inferior temporal
Bank superior temporal
Transverse temporal
Pars triangularis
Pars orbitalis
3b
5
Partition into 1000 ROIs
Whole brain structural connection network
5


Получение изображений
Электроэнцефалография
Структурная сеть
Функциональная сеть
Анализ методами теории графов

Connectom. What is known?

- Small-world network. Hyerarchical network
- Average clasterization
- Many hubs
- Anomalously large number of «open triads»
- Relation between the deseases and structural changes in connectome
- Complicated synchronization of functional networks

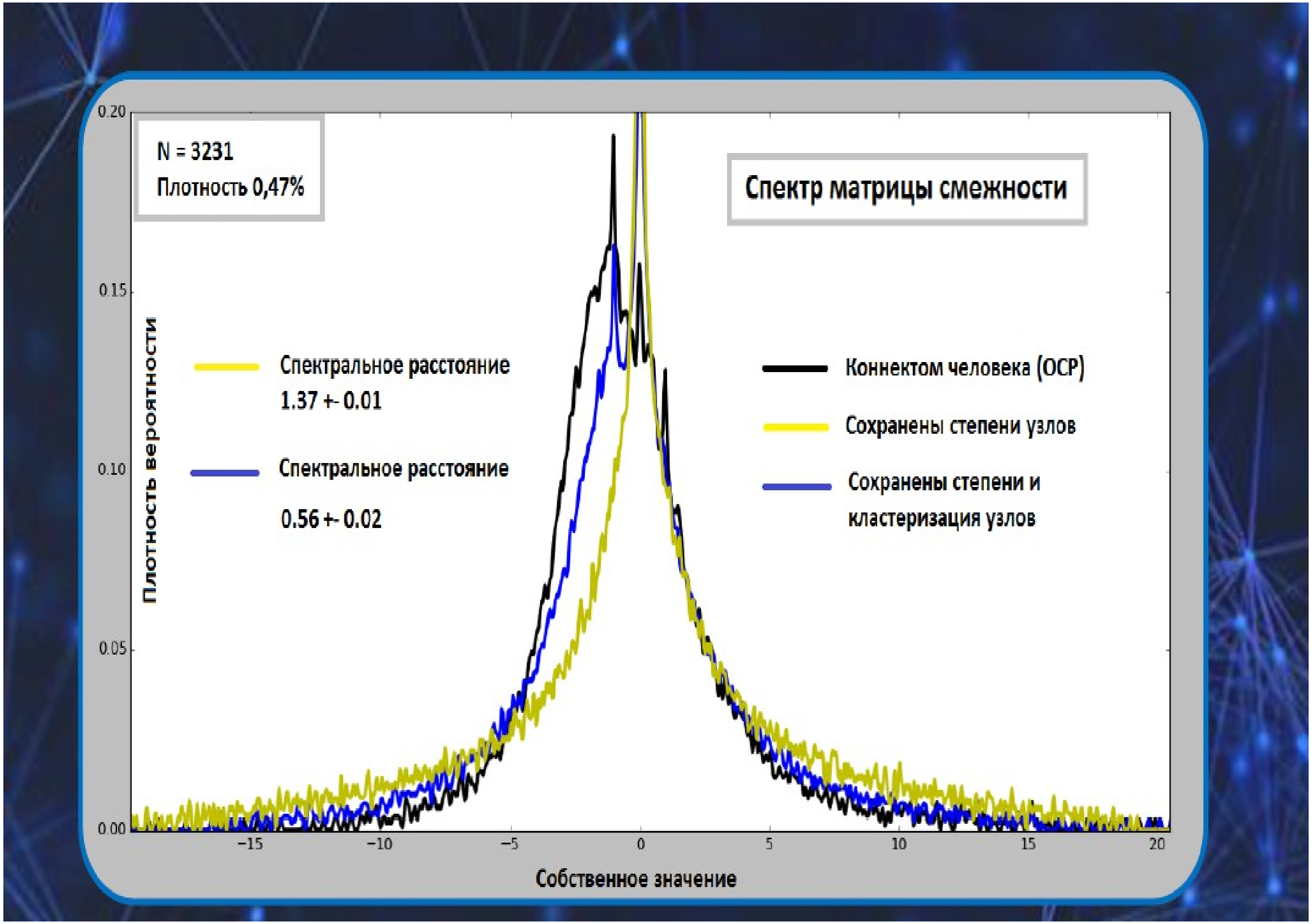

N = 3231
Плотность 0,47%
Спектр матрицы смежности
Плотность вероятности
Спектральное расстояние
1.37 +- 0.01
Спектральное расстояние
0.56 +- 0.02
Коннектом человека (OCP)
Сохранены степени узлов
Сохранены степени и
кластеризация узлов
0.20
0.15
0.10
0.05
0.00
−15
−10
−5
0
5
10
15
20
Собственное значение

Рис.3 Спектр коннектома человека и его рандомизированного аналога

New findings

- The human connectom is much more non-ergodic compared with other animals
- The local clusterization seems to be very important in connectom evolution(at least for human)
- The localization properties of the excitation in human connectome(level spacing distribution) are unusual(seems that mobility edge exists)
- K.Anokhin, V.Avetisov, S.Nechaev,N.Pospelov

  O.Valba, A.G.   To appear

И раньше догадывались,что случайные сети приносят неожиданные результаты

«Тятя,тятя,наши сети притащили мертвеца» (А.С. Пушкин)

И в советское время.

444. Браз А.
Случайная связь промелькнет как зарница... 1986