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# Modern Deep Learning through Bayesian Eyes

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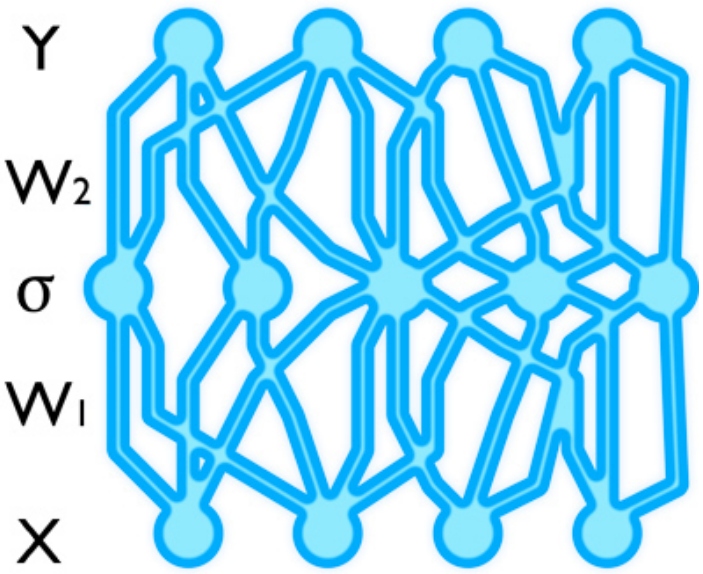
To keep things interesting, a photo or an equation in every slide! (unless specified otherwise, photos are either original work or taken from Wikimedia, under Creative Commons license)

# Modern deep learning




*Conceptually simple models...*

- Attracts **tremendous attention** from popular media,
- **Fundamentally affected** the way ML is used in industry,
- Driven by **pragmatic** developments...
- of **tractable** models...
- that **work** well...
- and **scale** well.

# But many unanswered questions...

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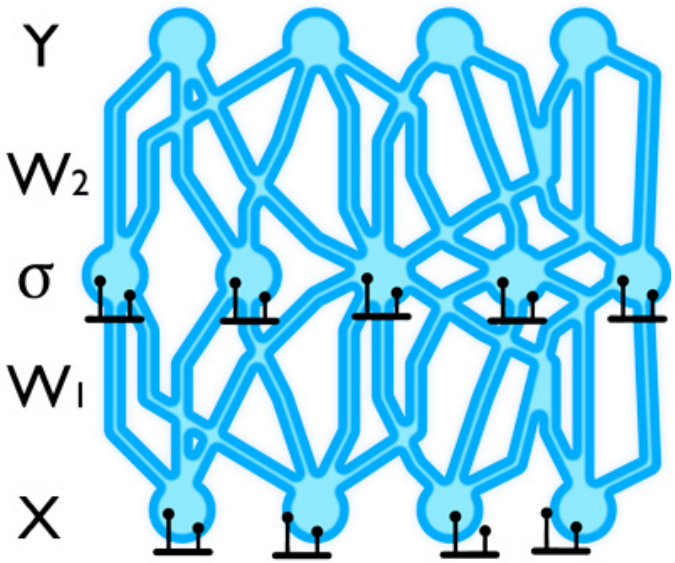
- **Why** does my model work

**We don't understand many of the tools that we use...**

E.g. stochastic reg. techniques (*dropout*, MGN[1]) are used in most deep learning models to avoid over-fitting. Why do they work?




- **What** does my model know?
- **Why** does my model predict this and not that?

[1] Wager et al. (2013) and Baldi and Sadowski (2013) attempt to explain dropout as sparse regularisation but cannot generalise to other techniques.

# But many unanswered questions...

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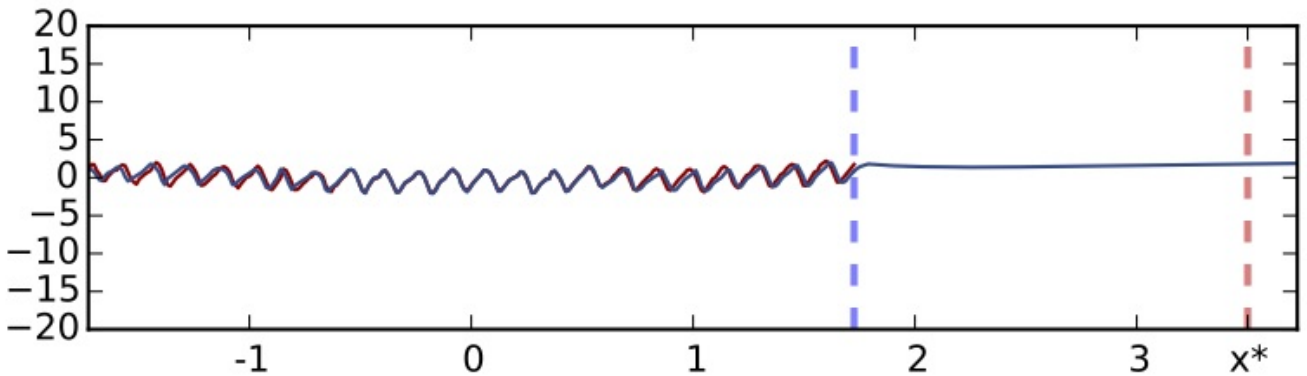
- **Why** does my model work
- **What** does my model know?

**We can't tell whether our models are certain or not...**

E.g. what would be the $CO_2$ concentration level in Mauna Loa, Hawaii, *in 20 years' time*?




- **Why** does my model predict this and not that?

# But many unanswered questions...

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- **Why** does my model work
- **What** does my model know?
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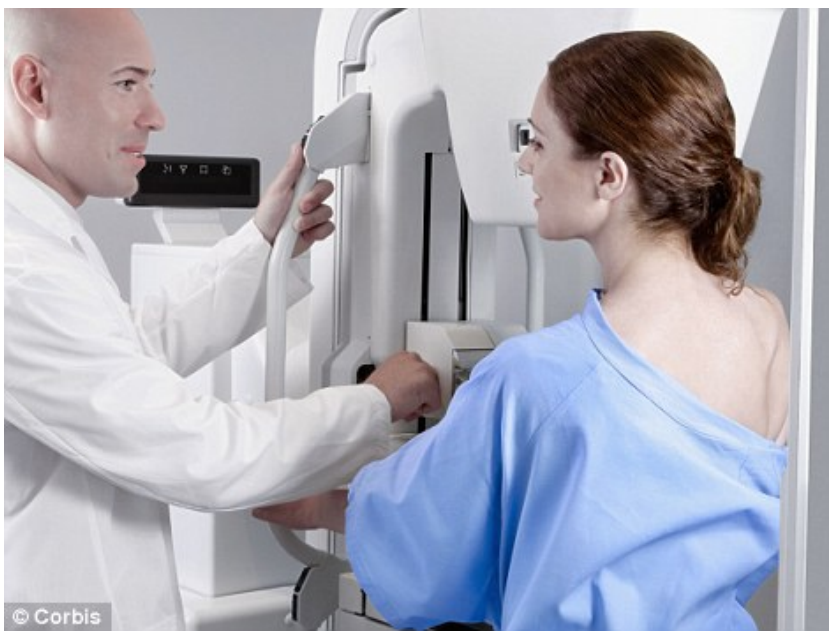
**Our models are black boxes and not interpretable...**

Physicians and others need to understand *why* a model predicts an output.

# But many unanswered questions...

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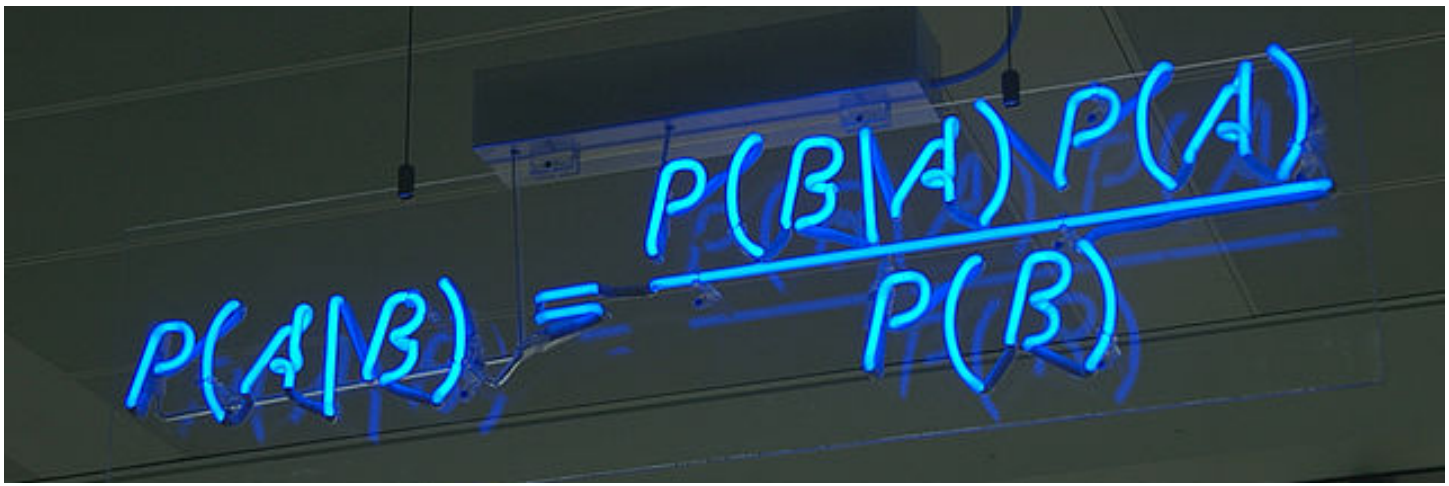


Surprisingly, we can use **Bayesian modelling** to answer the questions above

# Outline



- Many unanswered questions
- Why does my model work?
- What does my model know?
- Why does my model predict this and not that, and other open problems
- Conclusions

# Outline



- Many unanswered questions
- Why does my model work?
  - Bayesian modelling and neural networks
  - Modern deep learning as approximate inference
  - Real-world implications
- What does my model know?
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# Bayesian modelling and inference

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- Observed inputs $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^N$ and outputs $\mathbf{Y} = \{\mathbf{y}_i\}_{i=1}^N$
- Capture stochastic process believed to have generated outputs
- Def. $\omega$ model parameters as r.v.
- Prior dist. over $\omega$: $p(\omega)$
- Likelihood: $p(\mathbf{Y}|\omega, \mathbf{X})$
- Posterior: $p(\omega|\mathbf{X}, \mathbf{Y}) = \frac{p(\mathbf{Y}|\omega,\mathbf{X})p(\omega)}{p(\mathbf{Y}|\mathbf{X})}$ (Bayes' theorem)
- Predictive distribution given new input $\mathbf{x}^*$

$$p(\mathbf{y}^*|\mathbf{x}^*, \mathbf{X}, \mathbf{Y}) = \int p(\mathbf{y}^*|\mathbf{x}^*, \omega) \underbrace{p(\omega|\mathbf{X}, \mathbf{Y})}_{\text{posterior}} \mathrm{d}\omega$$

- But... $p(\omega|\mathbf{X}, \mathbf{Y})$ is often intractable

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# Approximate inference

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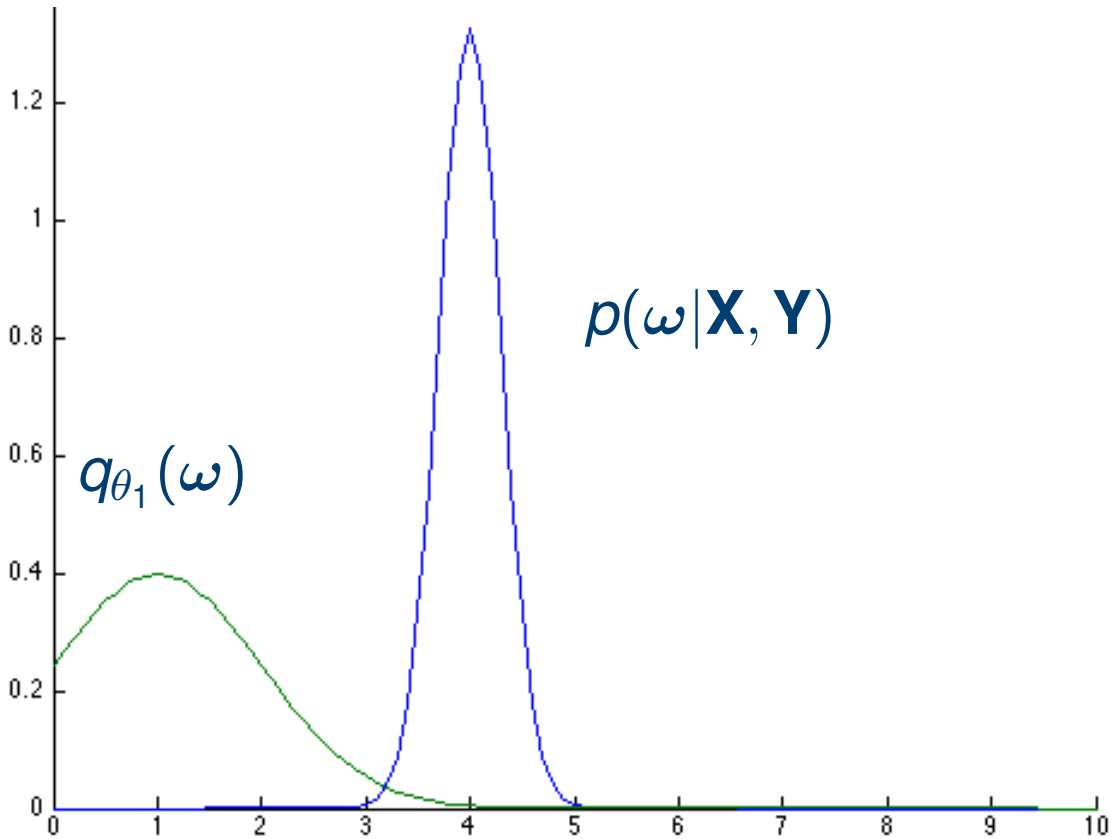
- Approximate $p(\omega|\mathbf{X}, \mathbf{Y})$ with simple dist. $q_\theta(\omega)$
- Minimise divergence from posterior w.r.t. $\theta$

$$\mathrm{KL}(q_\theta(\omega) \,||\, p(\omega|\mathbf{X}, \mathbf{Y}))$$



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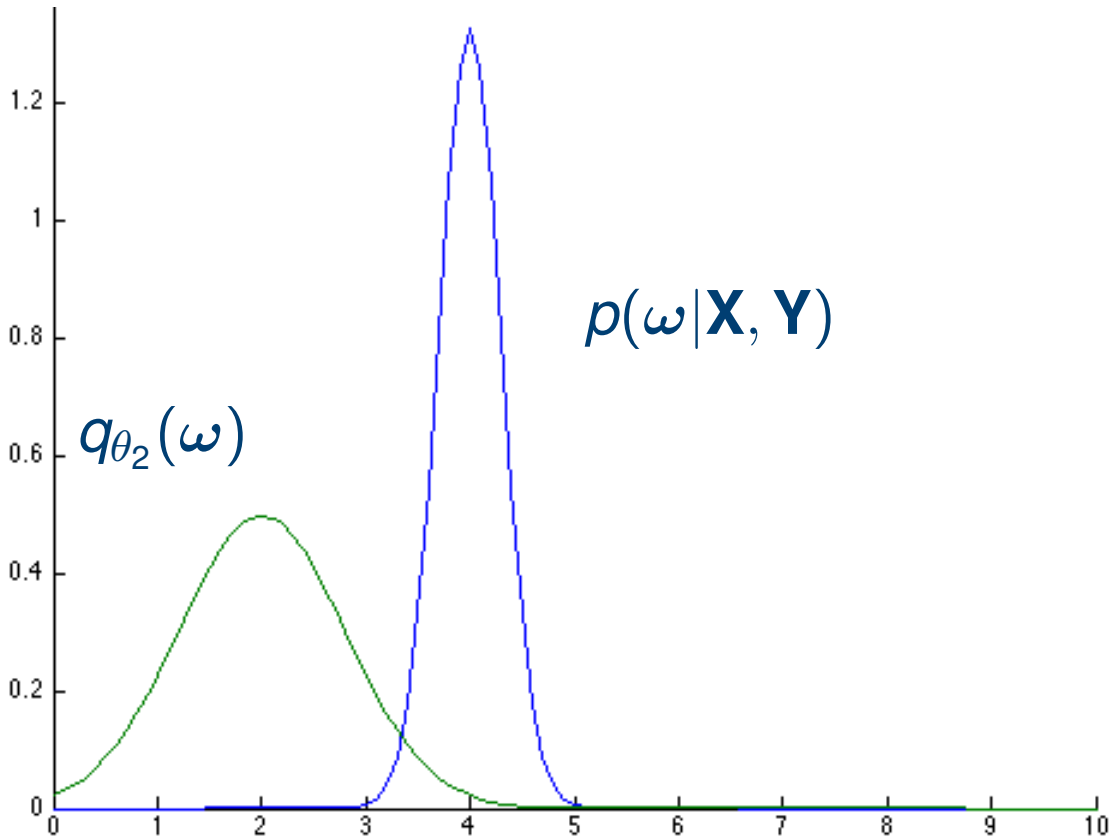

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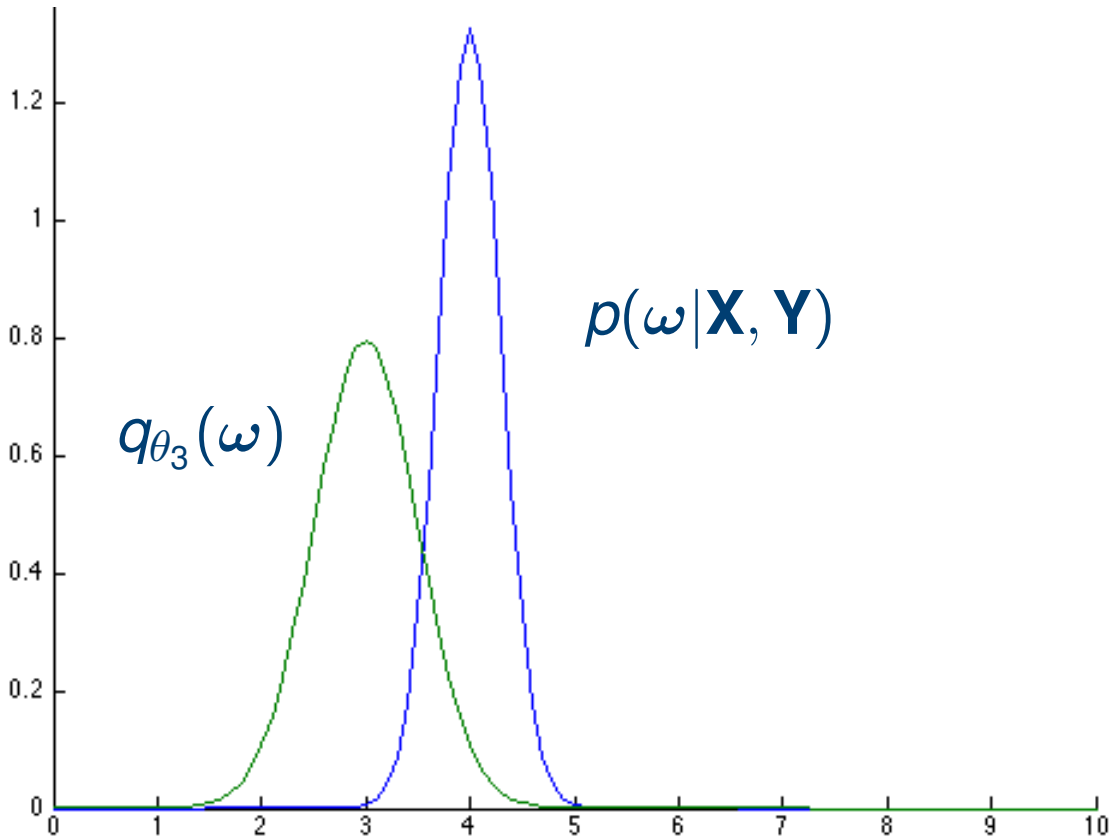

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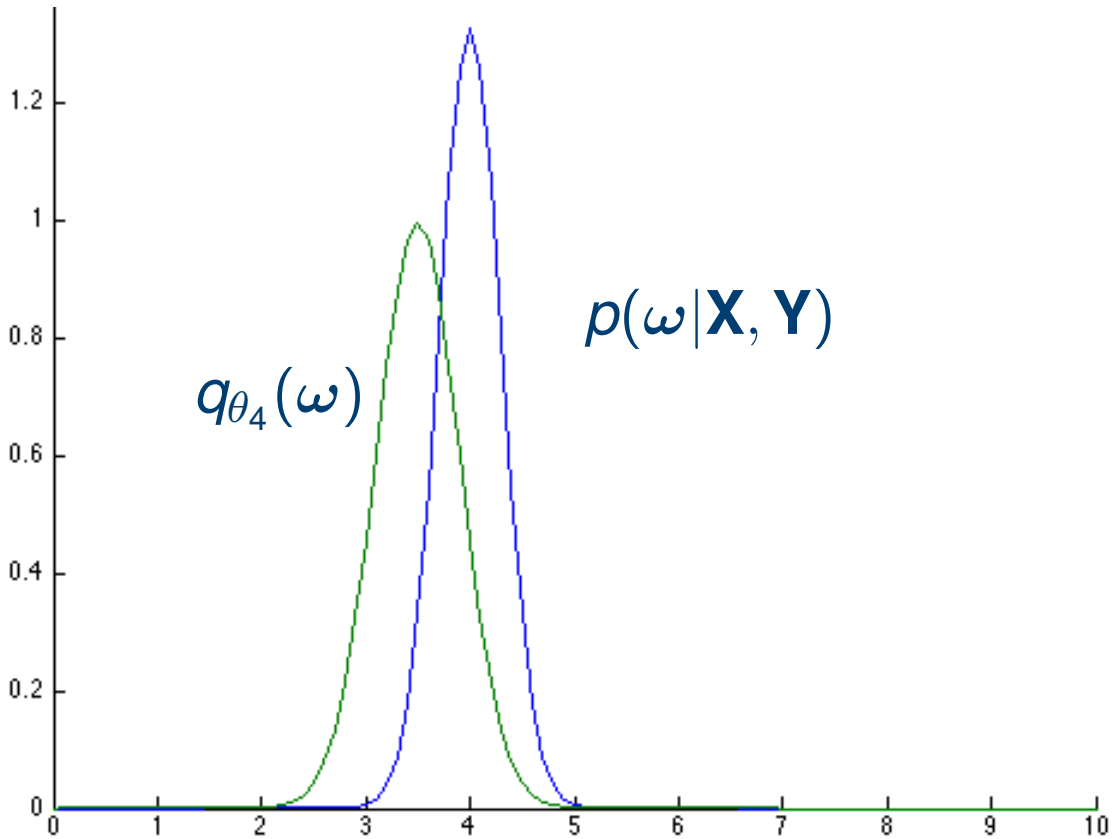

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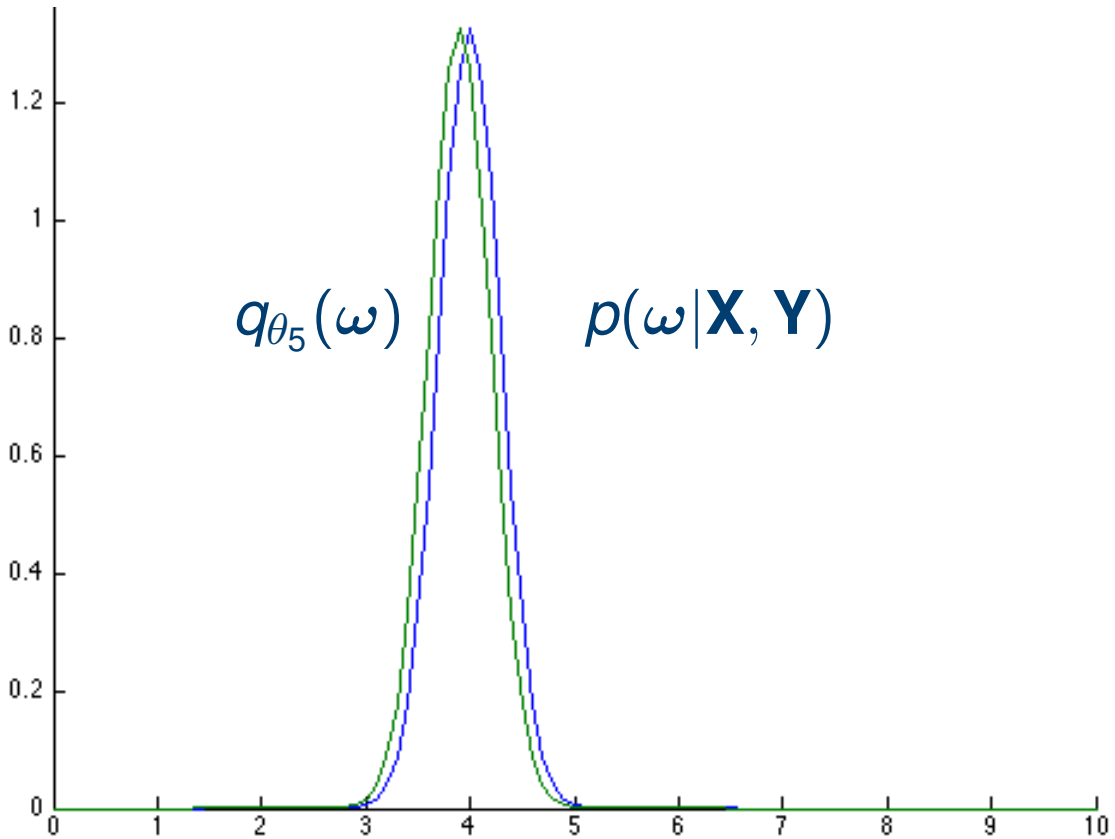

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- Approximate $p(\omega|\mathbf{X}, \mathbf{Y})$ with simple dist. $q_\theta(\omega)$
- Minimise divergence from posterior w.r.t. $\theta$

$$\mathrm{KL}(q_\theta(\omega) \,||\, p(\omega|\mathbf{X}, \mathbf{Y}))$$

- Identical to minimising

$$\mathcal{L}_{\mathrm{VI}}(\theta) := -\int q_\theta(\omega) \log \overbrace{p(\mathbf{Y}|\mathbf{X}, \omega)}^{\text{likelihood}} \mathrm{d}\omega + \mathrm{KL}(q_\theta(\omega)||\overbrace{p(\omega)}^{\text{prior}})$$

- We can approximate the **predictive distribution**

$$q_\theta(\mathbf{y}^*|\mathbf{x}^*) = \int p(\mathbf{y}^*|\mathbf{x}^*, \omega) q_\theta(\omega) \mathrm{d}\omega.$$

# What to this and deep learning?

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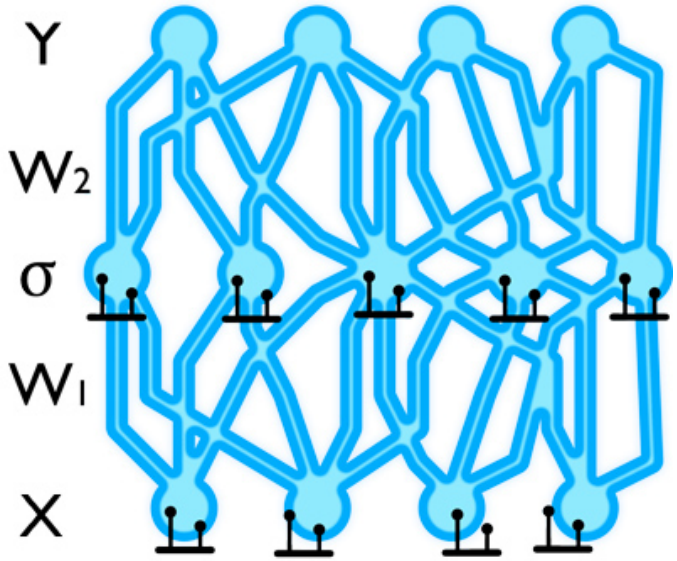
We'll look at dropout specifically:

- Used in **most modern deep learning models**




- It somehow circumvents **over-fitting**
- And improves **performance**

With Bayesian modelling we can explain **why**

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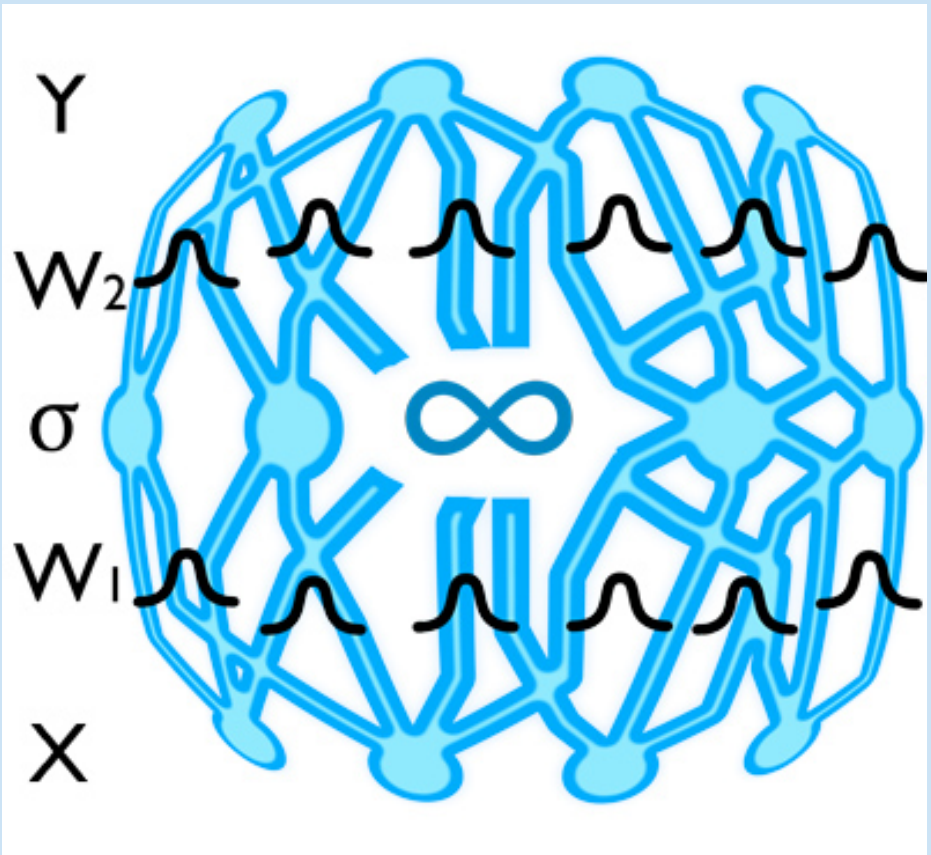
## Bayesian neural networks

- Place prior $p(\mathbf{W}_i)$:

$$\mathbf{W}_i \sim \mathcal{N}(0, \mathbf{I})$$

for $i \leq L$ (and write $\omega := \{\mathbf{W}_i\}_{i=1}^{L}$).



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- Output is a r.v. $\mathbf{f}(\mathbf{x}, \omega) = \mathbf{W}_L \sigma(...\mathbf{W}_2 \sigma(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1)...)$.

- Softmax likelihood for class.: $p(y|\mathbf{x}, \omega) = \text{softmax}(\mathbf{f}(\mathbf{x}, \omega))$
  or a Gaussian for regression: $p(\mathbf{y}|\mathbf{x}, \omega) = \mathcal{N}(\mathbf{y}; \mathbf{f}(\mathbf{x}, \omega), \tau^{-1}\mathbf{I})$.

- But difficult to evaluate posterior

$$p(\omega|\mathbf{X}, \mathbf{Y}).$$

**Many have tried...**

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**Many have tried...**

# Long history[1]

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- Denker, Schwartz, Wittner, Solla, Howard, Jackel, Hopfield (1987)
- Denker and LeCun (1991)
- MacKay (1992)
- Hinton and van Camp (1993)
- Neal (1995)
- Barber and Bishop (1998)

And more recently...

- **Graves (2011)**
- Blundell, Cornebise, Kavukcuoglu, and Wierstra (2015)
- **Hernandez-Lobato and Adam (2015)**

**But we don't use these... do we?**

[1] Complete references at end of slides

# Outline



- Many unanswered questions
- Why does my model work?
  - Bayesian modelling and neural networks
  - Modern deep learning as approximate inference
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## Approximate inference in Bayesian NNs

- Def $q_\theta(\omega)$ to approximate posterior $p(\omega|\mathbf{X},\mathbf{Y})$
- KL divergence to minimise:

$$\mathrm{KL}(q_\theta(\omega)\,||\,p(\omega|\mathbf{X},\mathbf{Y}))$$

$$\propto \boxed{-\int q_\theta(\omega)\log p(\mathbf{Y}|\mathbf{X},\omega)\mathrm{d}\omega} + \mathrm{KL}(q_\theta(\omega)\,||\,p(\omega))$$

$$=: \mathcal{L}(\theta)$$

- Approximate the integral with MC integration $\widehat{\omega} \sim q_\theta(\omega)$:

$$\widehat{\mathcal{L}}(\theta) := -\log p(\mathbf{Y}|\mathbf{X},\widehat{\omega}) + \mathrm{KL}(q_\theta(\omega)\,||\,p(\omega))$$

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## Stochastic approx. inference in Bayesian NNs

- Unbiased estimator:

$$E_{\widehat{\omega} \sim q_\theta(\omega)}\left(\widehat{\mathcal{L}}(\theta)\right) = \mathcal{L}(\theta)$$

- Converges to the same optima as $\mathcal{L}(\theta)$
- For inference, repeat:
  - Sample $\widehat{\omega} \sim q_\theta(\omega)$
  - And minimise (one step)

$$\widehat{\mathcal{L}}(\theta) = -\log p(\mathbf{Y}|\mathbf{X}, \widehat{\omega}) + \mathrm{KL}\left(q_\theta(\omega) \,||\, p(\omega)\right)$$

w.r.t. $\theta$.

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## Specifying $q_\theta(\cdot)$

- Given $\mathbf{z}_{i,j}$ Bernoulli r.v. and variational parameters $\theta = \{\mathbf{M}_i\}_{i=1}^{L}$ (set of matrices):

$$\mathbf{z}_{i,j} \sim \text{Bernoulli}(p_i) \text{ for } i = 1, ..., L,\ j = 1, ..., K_{i-1}$$

$$\mathbf{W}_i = \mathbf{M}_i \cdot \text{diag}([\mathbf{z}_{i,j}]_{j=1}^{K_i})$$

$$q_\theta(\omega) = \prod q_{\mathbf{M}_i}(\mathbf{W}_i)$$

In summary:

## Minimise divergence between $q_\theta(\omega)$ and $p(\omega|\mathbf{X}, \mathbf{Y})$:

- Repeat:
  - Sample $\widehat{\mathbf{z}}_{i,j} \sim \text{Bernoulli}(p_i)$ and set

$$\widehat{\mathbf{W}}_i = \mathbf{M}_i \cdot \text{diag}([\widehat{\mathbf{z}}_{i,j}]_{j=1}^{K_i})$$

$$\widehat{\omega} = \{\widehat{\mathbf{W}}_i\}_{i=1}^{L}$$

  - Minimise (one step)

$$\widehat{\mathcal{L}}(\theta) = -\log p(\mathbf{Y}|\mathbf{X}, \widehat{\omega}) + \text{KL}(q_\theta(\omega) \,||\, p(\omega))$$

w.r.t. $\theta = \{\mathbf{M}_i\}_{i=1}^{L}$ (set of matrices).

In summary:

**Minimise divergence between $q_\theta(\omega)$ and $p(\omega|\mathbf{X}, \mathbf{Y})$:**

- Repeat:
  - = Randomly set columns of $\mathbf{M}_i$ to zero
  - Minimise (one step)

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In summary:

**Minimise divergence between $q_\theta(\omega)$ and $p(\omega|\mathbf{X}, \mathbf{Y})$:**

- Repeat:
  - = Randomly set units of the network to zero
  - Minimise (one step)

$$\widehat{\mathcal{L}}(\theta) = -\log p(\mathbf{Y}|\mathbf{X}, \widehat{\omega}) + \mathrm{KL}(q_\theta(\omega) \,||\, p(\omega))$$

w.r.t. $\theta = \{\mathbf{M}_i\}_{i=1}^{L}$ (set of matrices).

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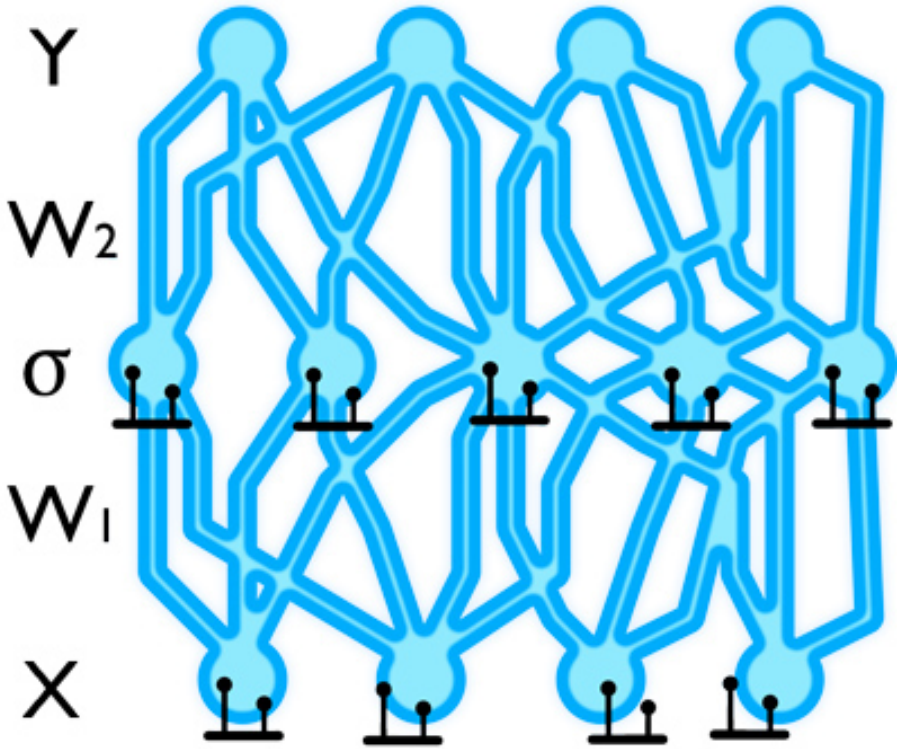
Sounds familiar?[2]




$$\widehat{\mathcal{L}}(\theta) = \overbrace{-\log p(\mathbf{Y}|\mathbf{X},\widehat{\omega})}^{=\text{loss}} + \overbrace{\mathrm{KL}\left(q_\theta(\omega) \,||\, p(\omega)\right)}^{=L_2\text{ reg}}$$

[2] For more details see appendix of Gal and Ghahramani (2015) – yarin.co/dropout

# Why does my model work?

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Now we can answer: “Why does dropout work?”

- ~~It adds noise~~
- ~~Sexual reproduction~~[3]

> **2. Motivation**
>
> A motivation for dropout comes from a theory of the role of sex in evolution (Livnat et al., 2010). Sexual reproduction involves taking half the genes of one parent and half of the

- Because it approximately **integrates** over model parameters
- The **noise is a side-effect** of approx. integration
- Explains model **over specification**, “adaptive model capacity”
- We **fit the process** that generated our data

[3] Srivastava et al. (2014)

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- Explains model **over specification**, “adaptive model capacity”
- We **fit the process** that generated our data

[3] Srivastava et al. (2014)

# Why does my model work?

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Now we can answer: "Why does dropout work?"

- ~~It adds noise~~
- ~~Sexual reproduction[3]~~

> **2. Motivation**
>
> A motivation for dropout comes from a theory of the role of sex in evolution (Livnat et al., 2010). Sexual reproduction involves taking half the genes of one parent and half of the

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# Diving deeper into dropout

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- "Why this $q_\theta(\cdot)$?"
  - Bernoullis are cheap
  - Dropout at test time $\approx$ propagate the mean $\mathbb{E}(\mathbf{W}_i) = p_i \mathbf{M}_i$
  - Constrains the weights to near the origin:
    - Posterior uncertainty decreases with more data
    - $\text{Var}(\mathbf{W}_i) = \mathbf{M}_i \mathbf{M}_i^T (p_i - p_i^2)$
    - For fixed $p_i$, to decrease uncertainty must decrease $||\mathbf{M}_i||$
  - Smallest $||\mathbf{M}_i||$ = strongest reg. at $p_i = 0.5$.

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# Other stochastic reg. techniques

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- Multiplicative Gaussian noise (Srivastava et al. 2014) –
- Multiply network units by $\mathcal{N}(1,1)$
- Same performance as dropout

$$\Updownarrow$$

## Multiplicative Gaussian noise as approximate inference[4]

$$\mathbf{z}_{i,j} \sim \mathcal{N}(1,1) \text{ for } i = 1,...,L,\ j = 1,...,K_{i-1}$$
$$\mathbf{W}_i = \mathbf{M}_i \cdot \text{diag}([\mathbf{z}_{i,j}]_{j=1}^{K_i})$$
$$q_\theta(\omega) = \prod q_{\mathbf{M}_i}(\mathbf{W}_i)$$

Similarly for **drop-connect** (Wan et al., 2013), **hashed neural networks** (Chen et al., 2015)

[4] See Gal and Ghahramani (2015) and Kingma et al. (2015)

# Outline

- Many unanswered questions
- Why does my model work?
  - Bayesian modelling and neural networks
  - Modern deep learning as approximate inference
  - Real-world implications
- What does my model know?
- Why does my model predict this and not that, and other open problems
- Conclusions

# Real world implications

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*“A theory is worth nothing if you can’t use it to make better code.”*

– DeadMG Jun 10 ’12, stackexchange

- Better use of dropout
- Model structure selection
  - (No time: use Bayesian statistics to understand model architecture)

# Better use of dropout

How do we use dropout with convolutional neural networks (convnets)?

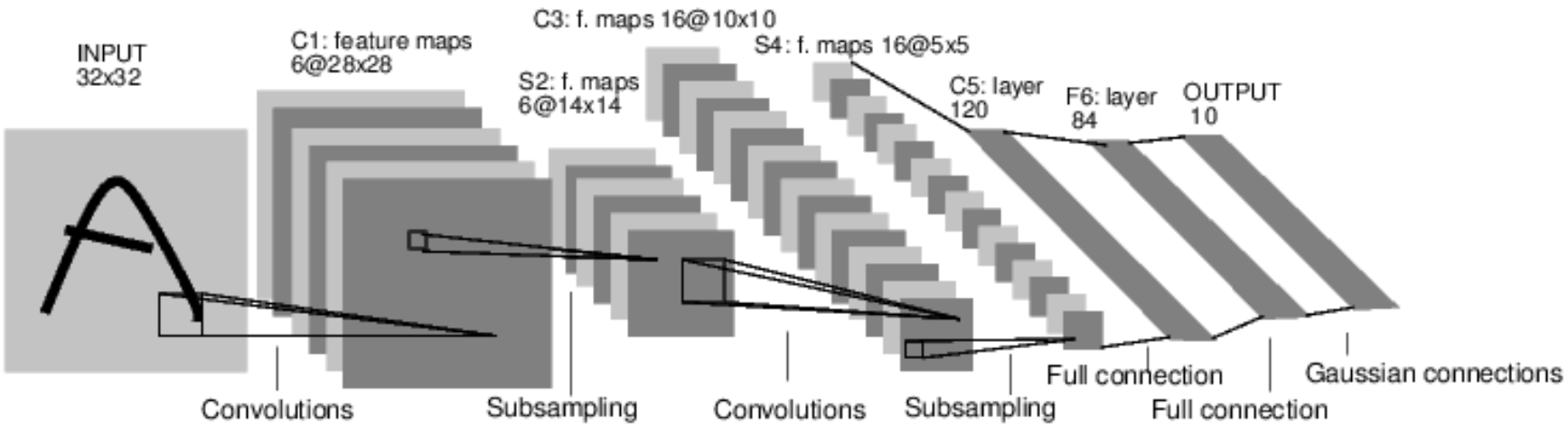


Figure : LeNet convnet structure

Image Source: LeCun et al. (1998)

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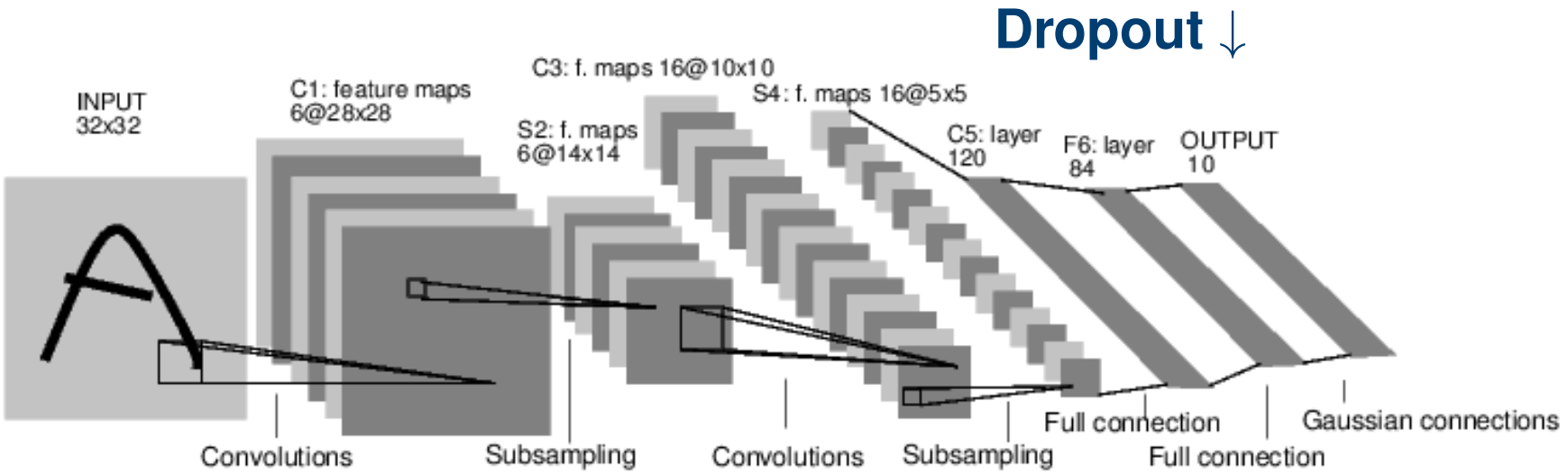


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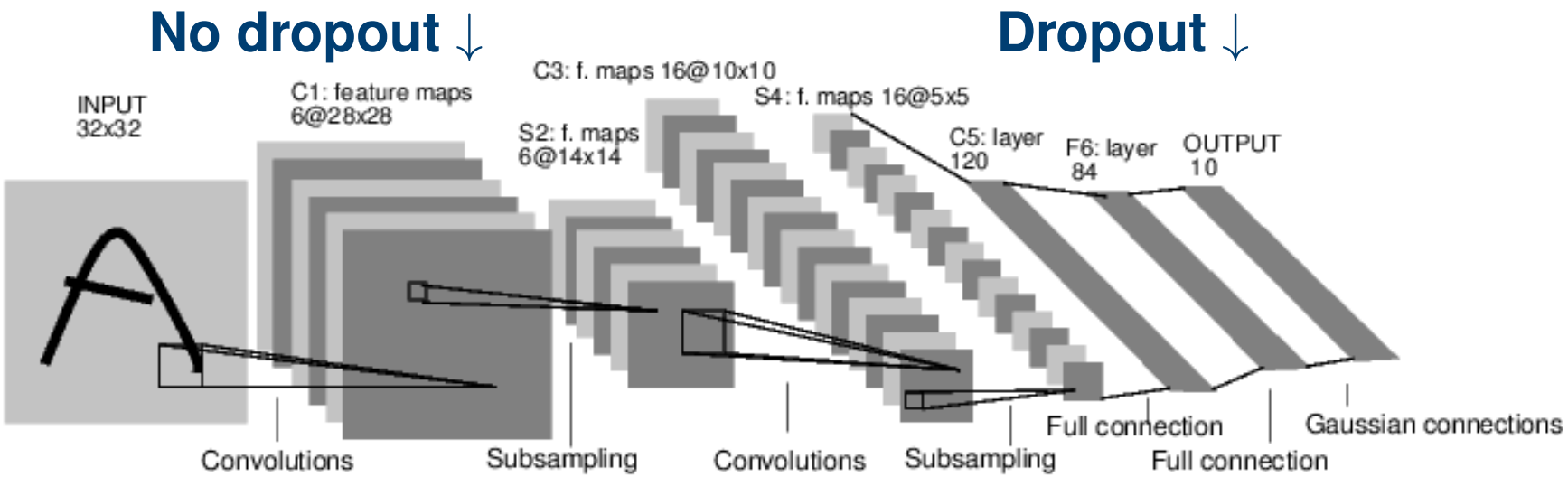


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# Better use of dropout

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Why not use dropout et al. with convolutions?

- ~~It doesn't work~~
- ~~Low co-adaptation in convolutions~~
- Because it's not used correctly
  - Standard dropout averages **weights** at test time

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# Better use of dropout

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Instead, **predictive mean**, approx. with MC integration:

$$\mathbb{E}_{q_\theta(\mathbf{y}^*|\mathbf{x}^*)}(\mathbf{y}^*) = \frac{1}{T}\sum_{t=1}^{T}\widehat{\mathbf{y}}(\mathbf{x}^*, \widehat{\omega}_t).$$

with $\widehat{\omega}_t \sim q_\theta(\omega)$.

- In practice, **average stochastic forward passes through the network** (referred to as "MC dropout").[5]
- Dropout after convolutions and averaging forward passes = **approximate inference in Bayesian convnets**.[6]

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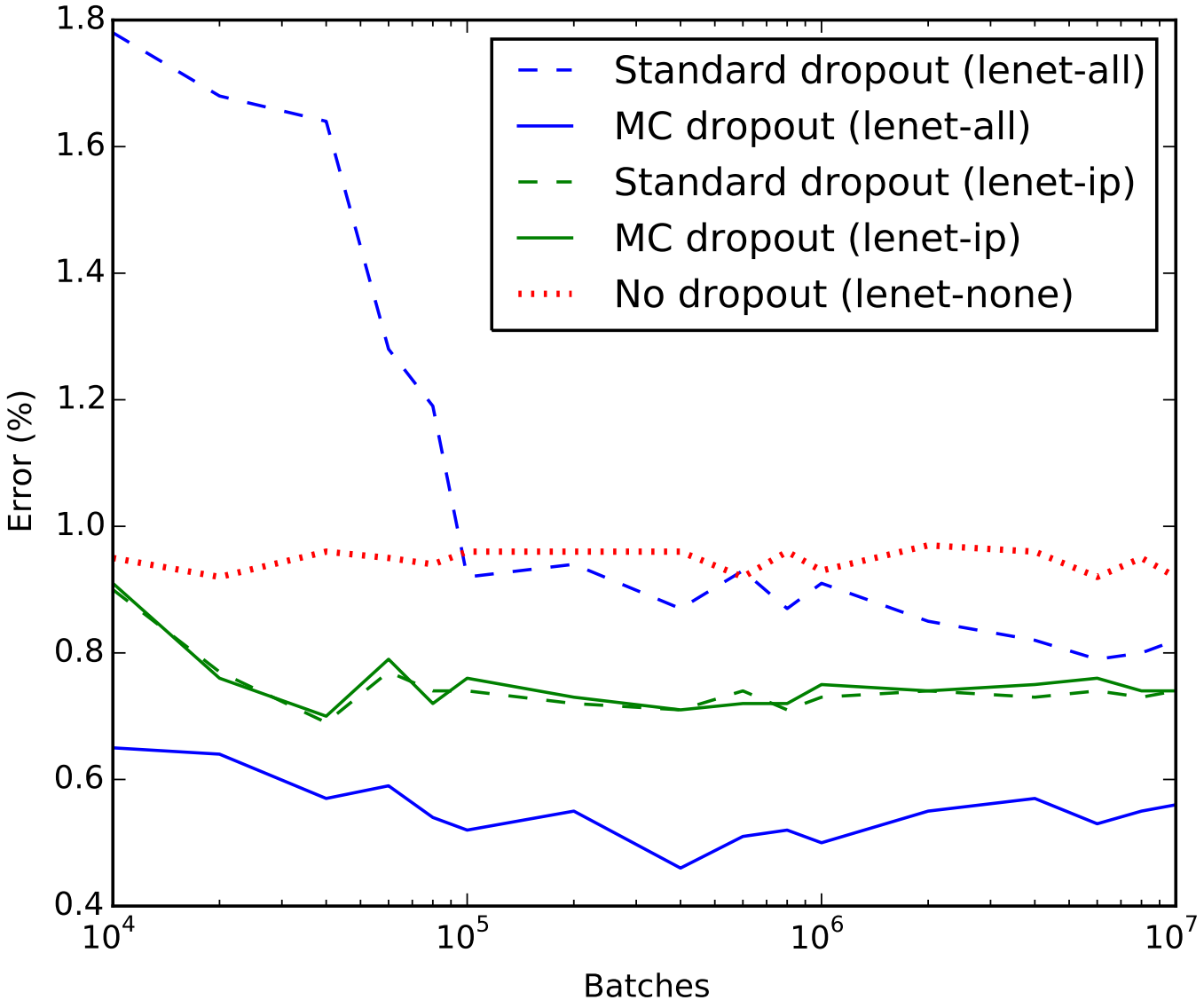


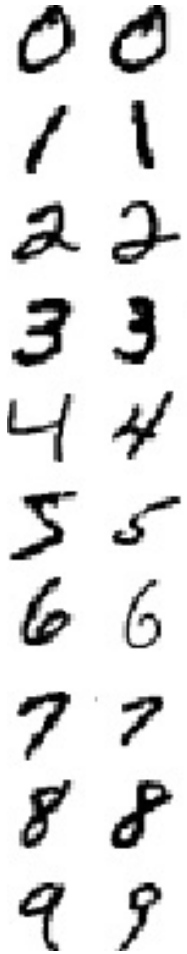

**Red**: standard LeNet (no dropout)

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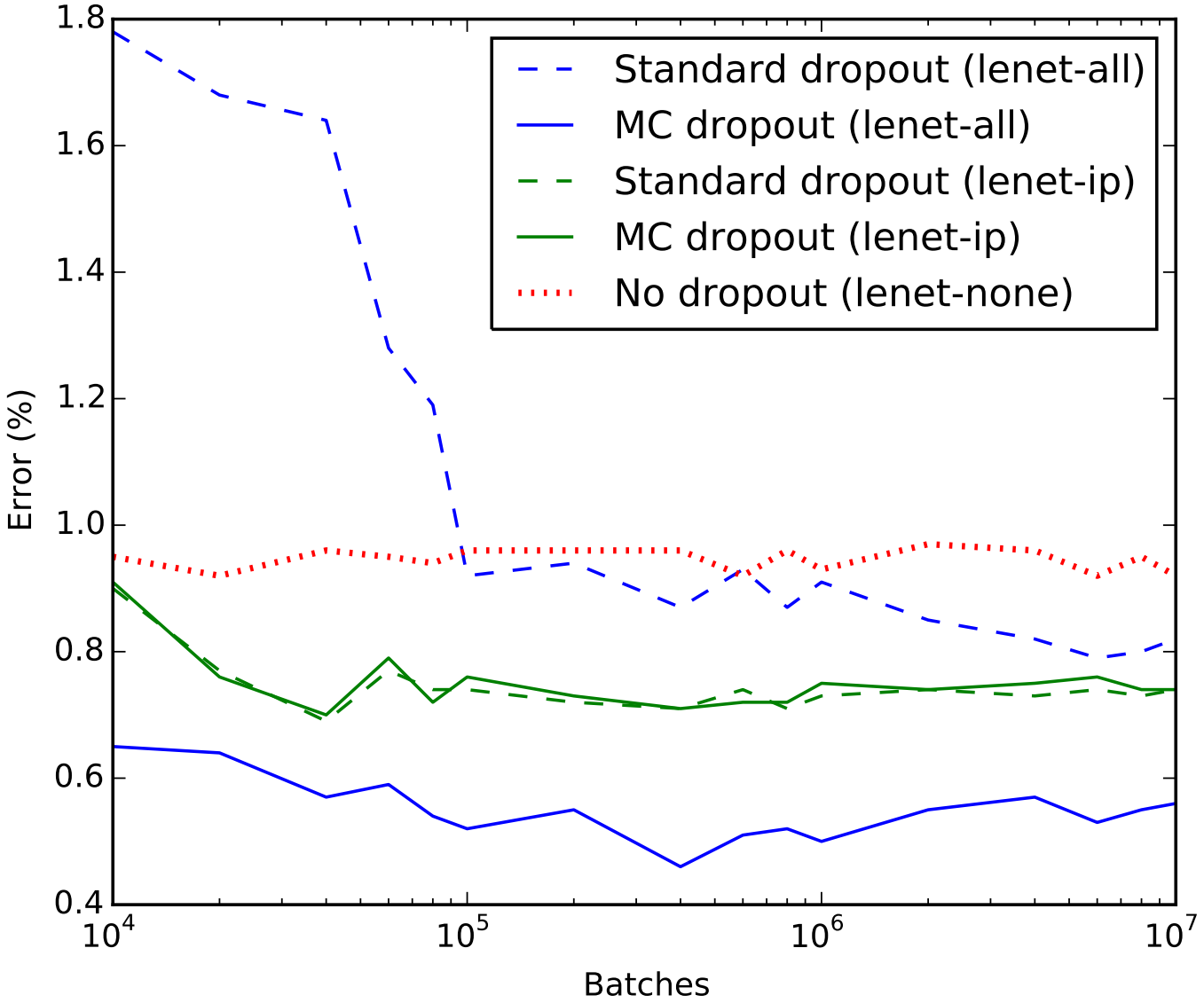


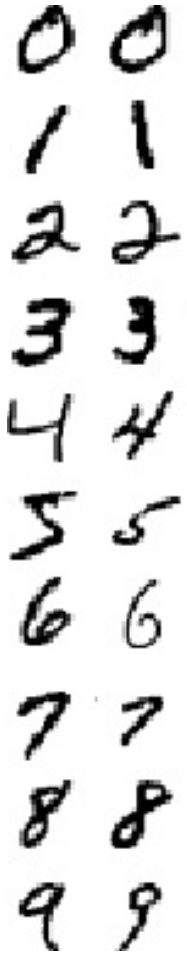

**Green**: standard dropout LeNet (dropout at the end)

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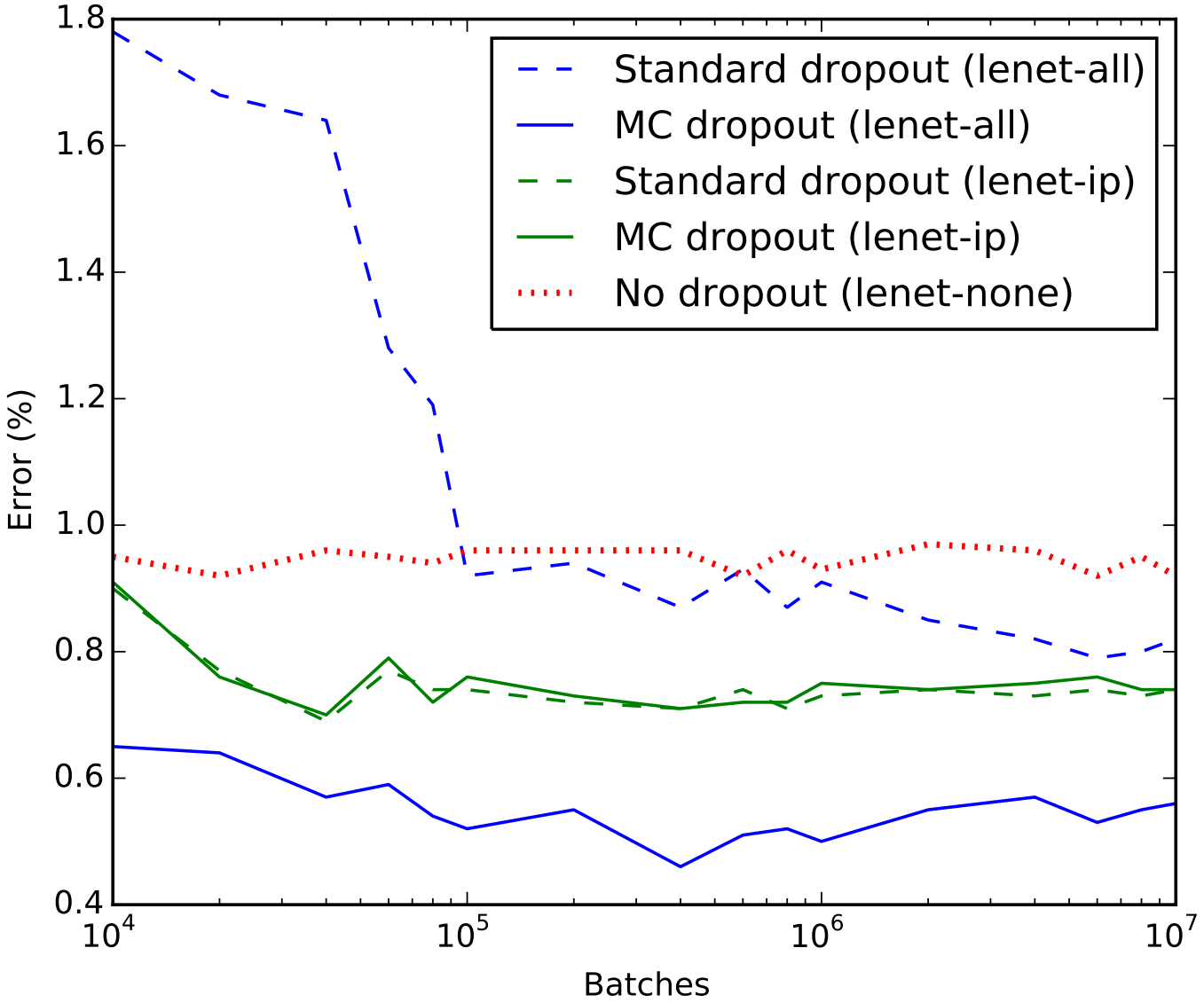


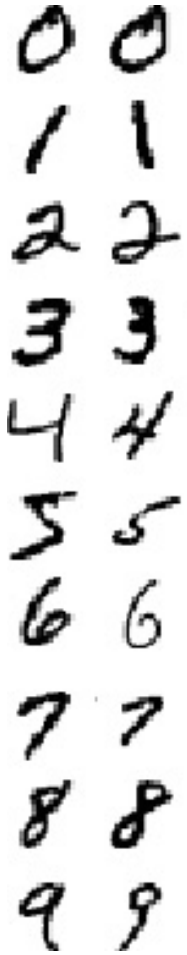

**Dashed blue**: Bayesian LeNet (weight averaging – FAIL)

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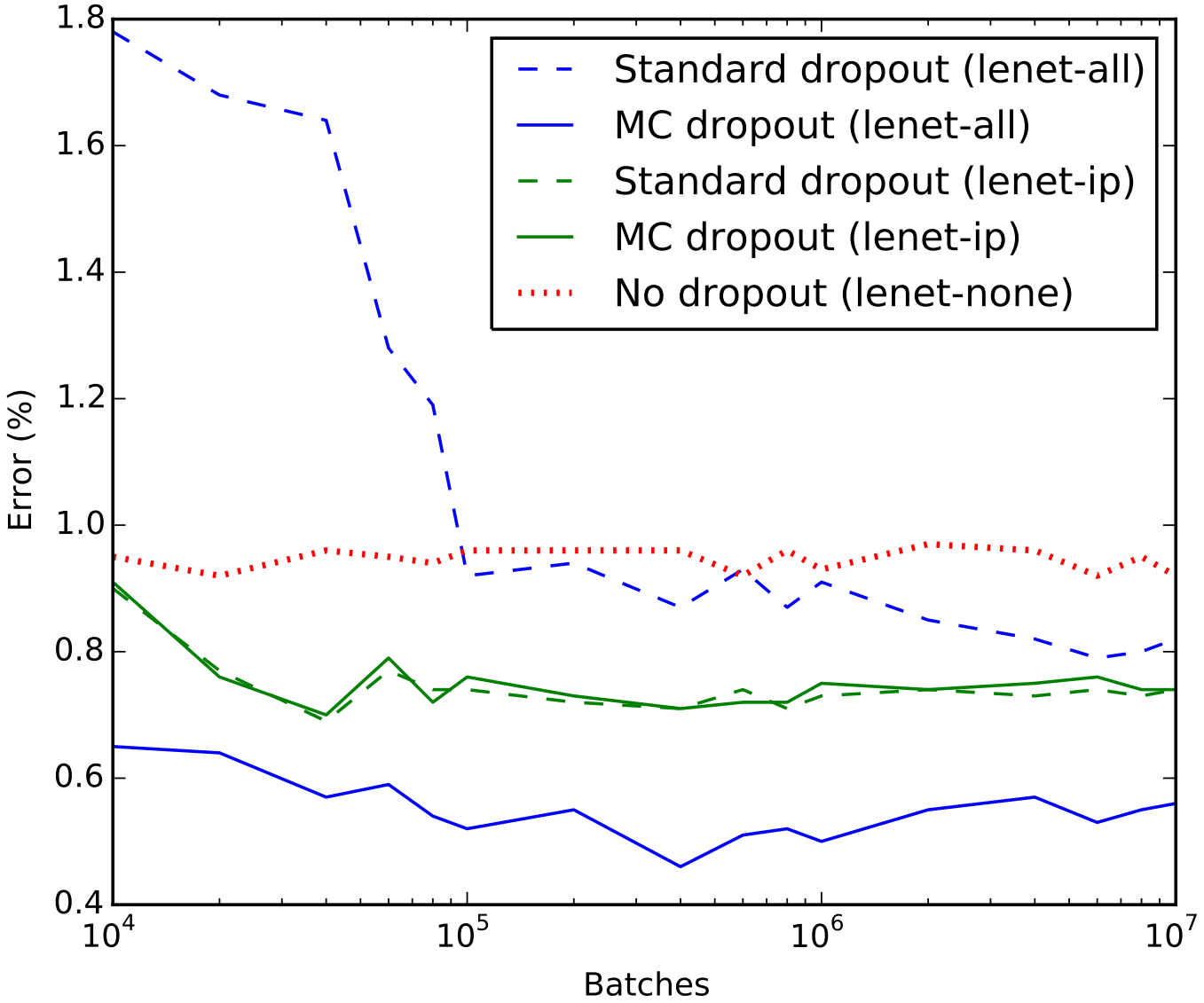


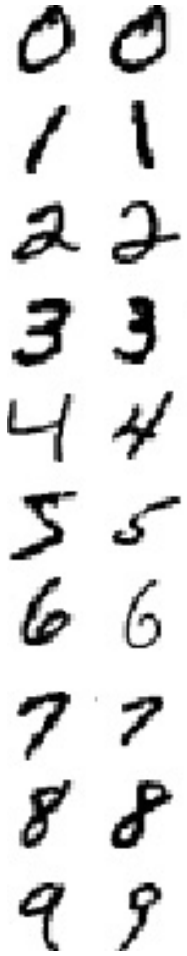

**Solid blue**: Bayesian LeNet (MC dropout)

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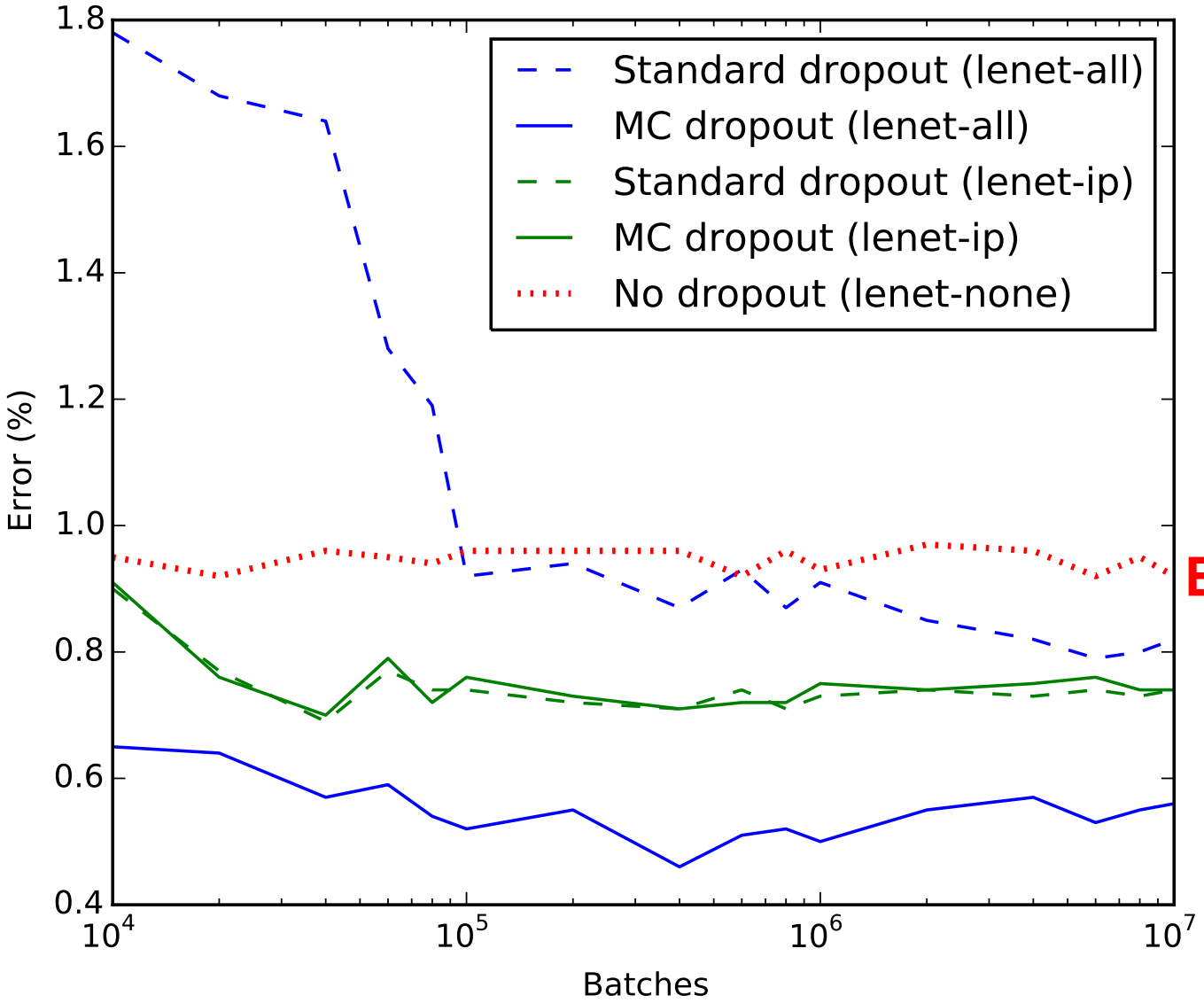


**Before dropout**

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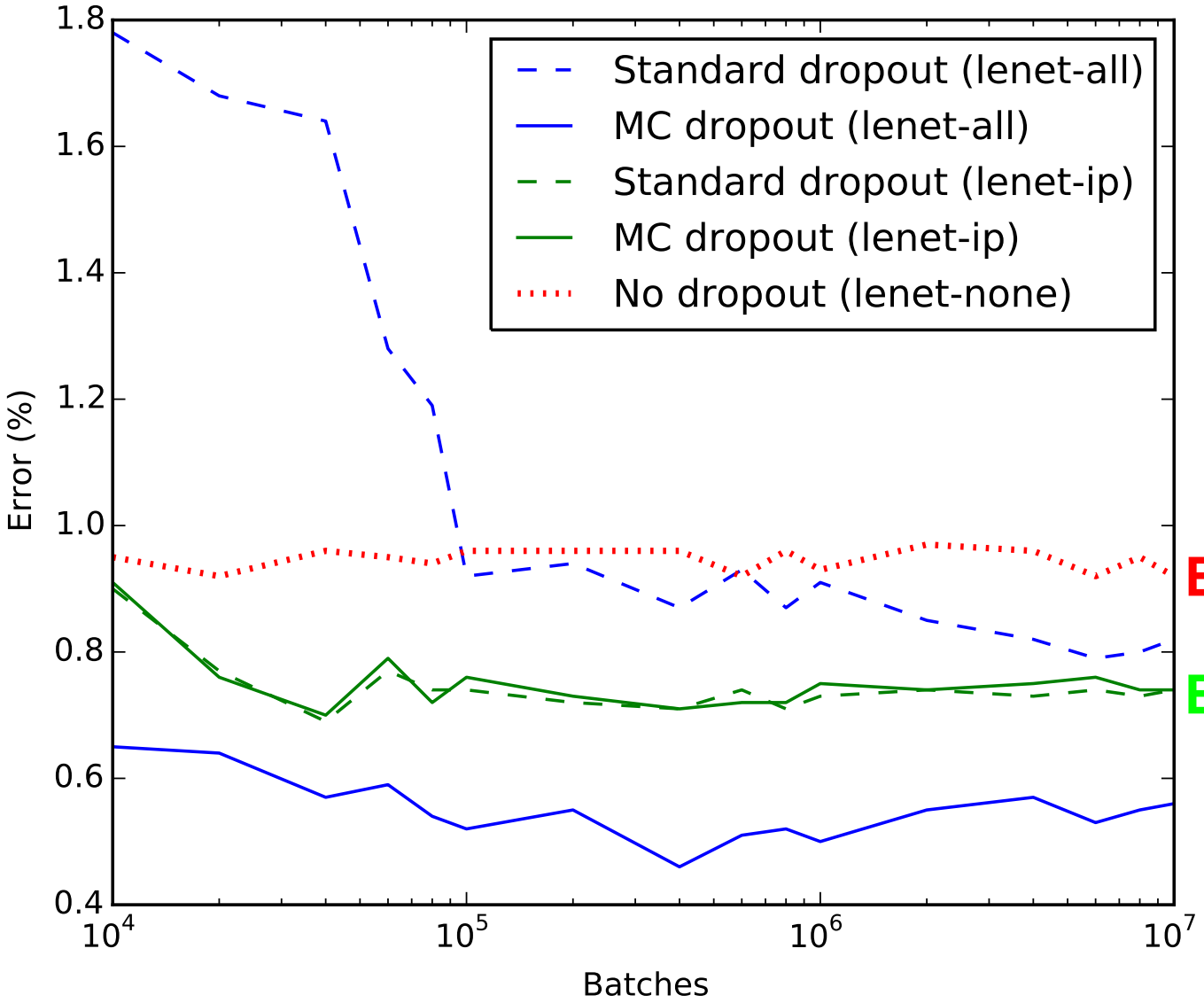
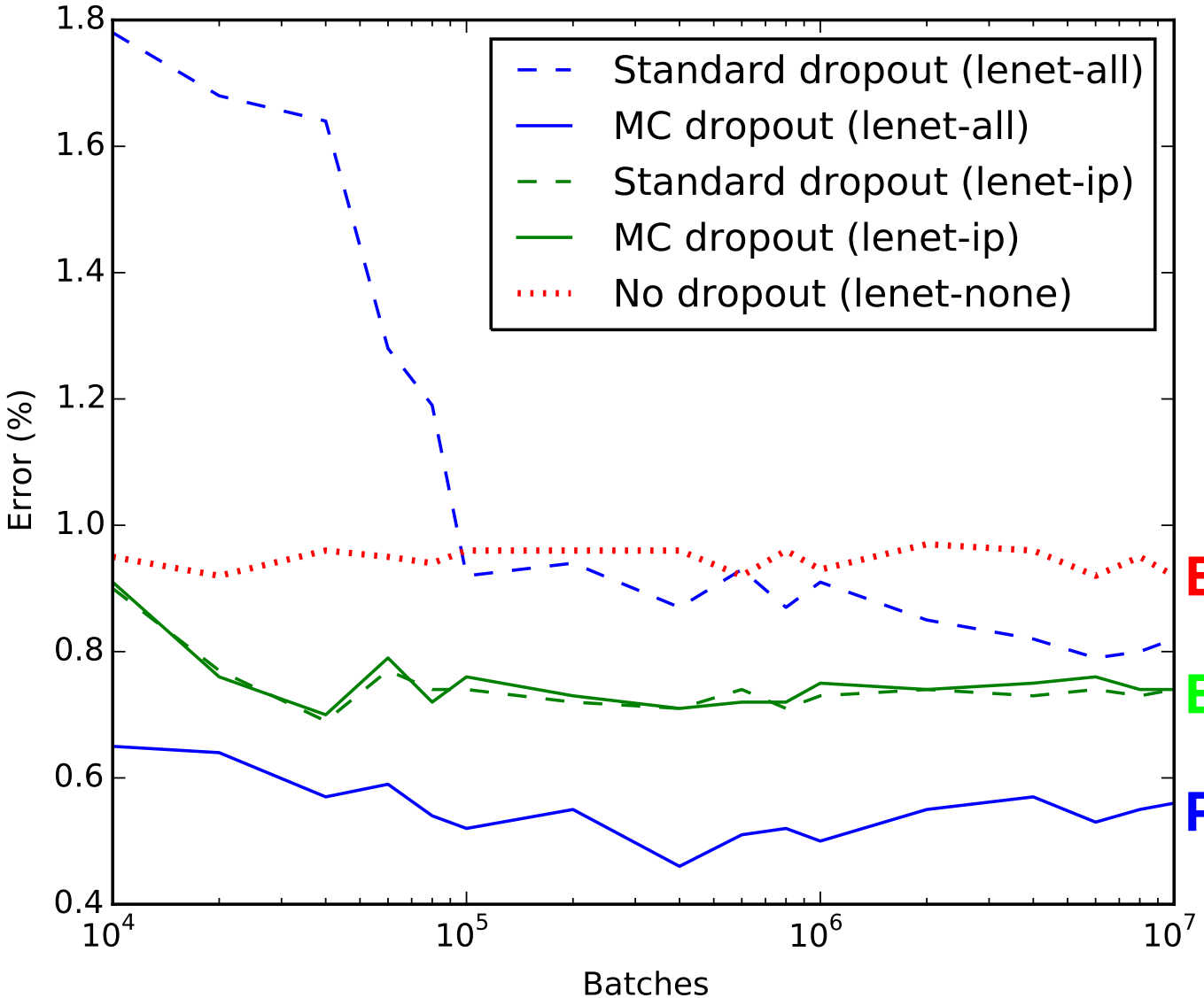



**Before dropout**

**Empirical dropout**

# Huge improvement (MNIST)




**Before dropout**

**Empirical dropout**

**Principled dropout**

# Over-fitting on small data

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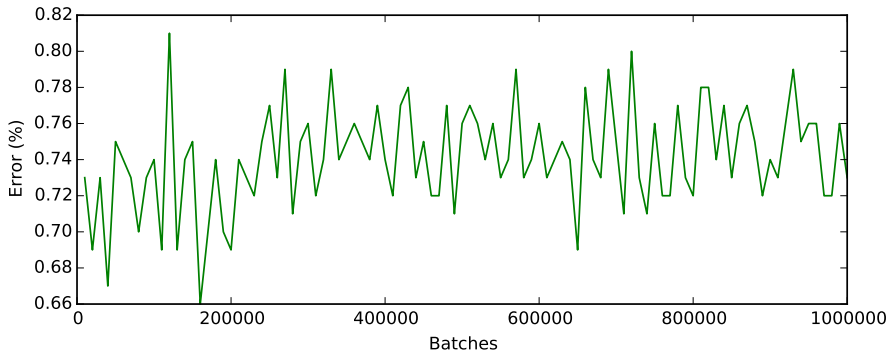
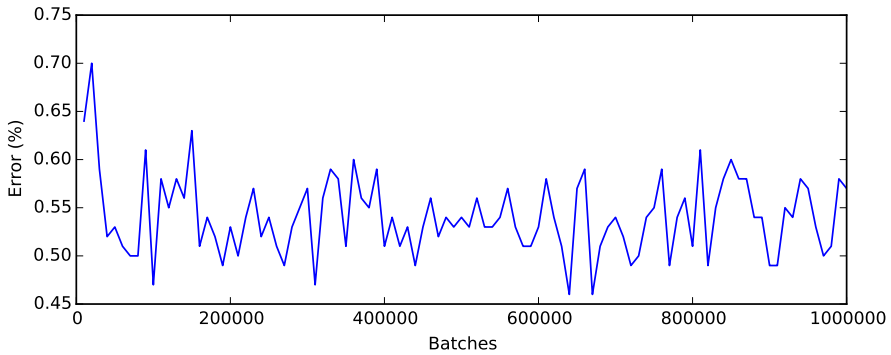
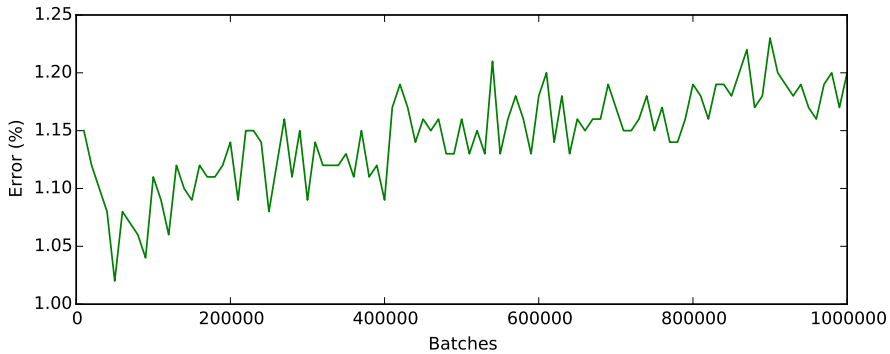
- Robustness to over-fitting on smaller datasets:



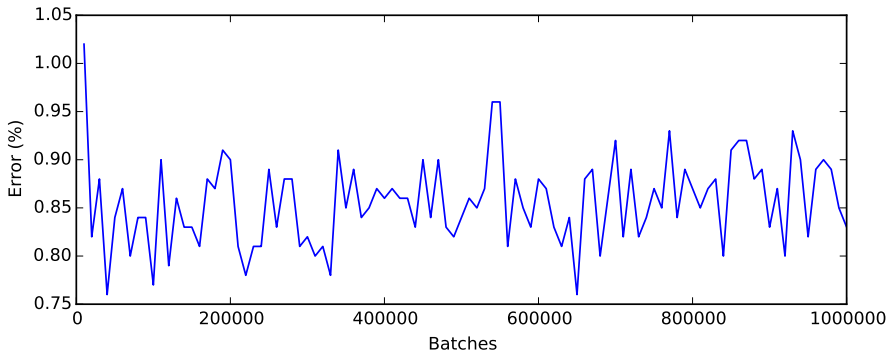

(a) Entire MNIST
Standard dropout convnet




(b) Entire MNIST
Bayesian convnet




(c) 1/4 of MNIST
Standard dropout convnet




(d) 1/4 of MNIST
Bayesian convnet

# State-of-the-art on CIFAR-10

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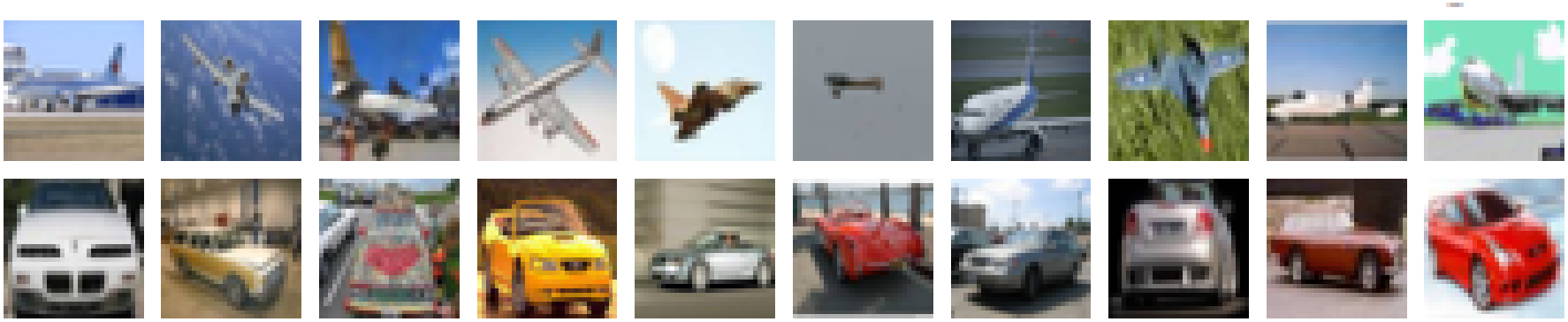
| | CIFAR Test Error (and Std.) | |
|---|---|---|
| **Model** | **Standard Dropout** | **MC Dropout** |
| NIN | 10.43 (Lin et al., 2013) | **10.27 ± 0.05** |
| DSN | 9.37 (Lee et al., 2014) | **9.32 ± 0.02** |
| Augmented-DSN | 7.95 (Lee et al., 2014) | **7.71 ± 0.09** |

Table : Bayesian techniques (MC dropout) with existing state-of-the-art



# RW: Camera pose localisation

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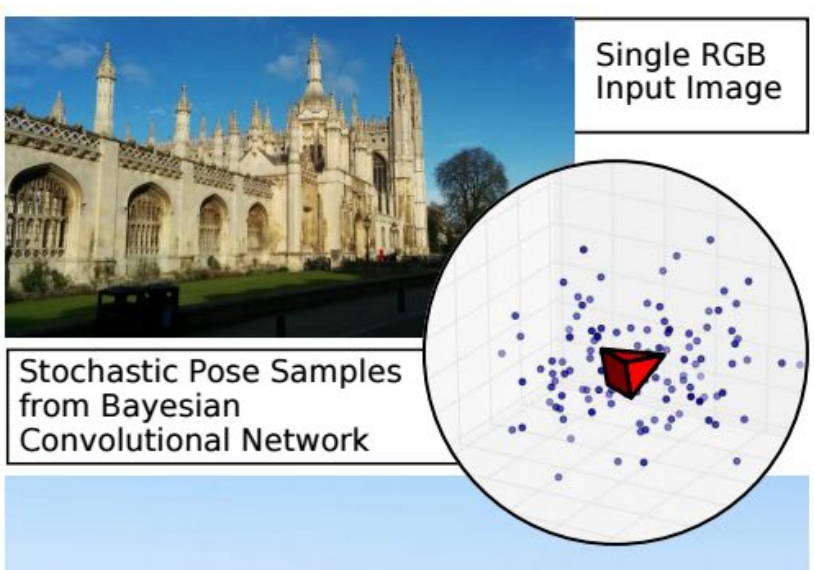
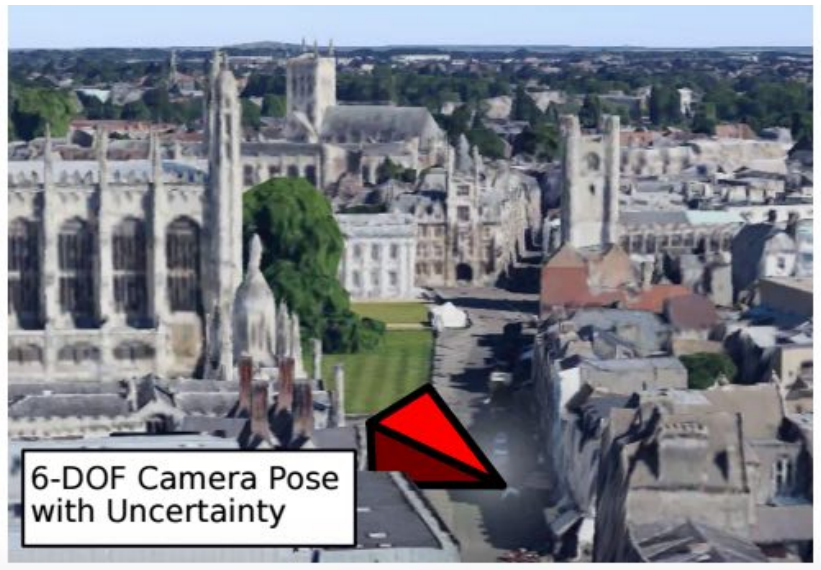
- Find the location from which a picture was taken[7]




- Kendall and Cipolla (2015) show **10–15%** improvement on **state-of-the-art** with Bayesian convnets

[7] Figures used with author permission

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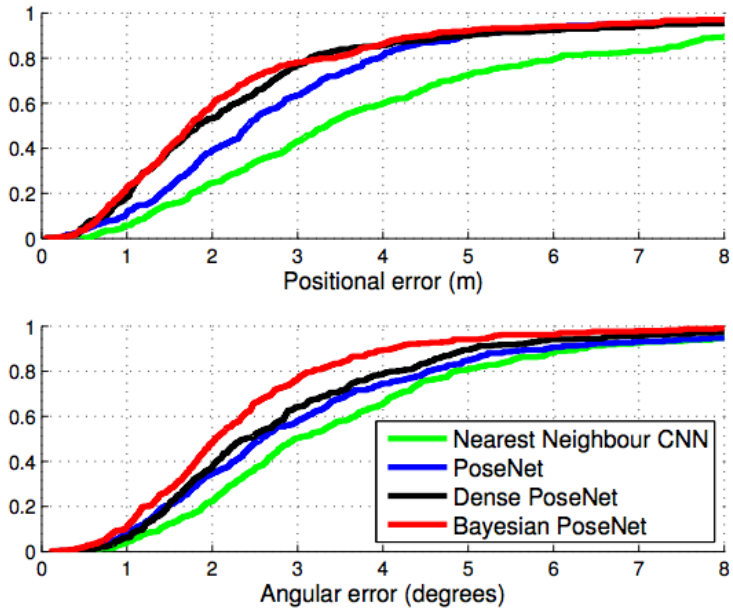
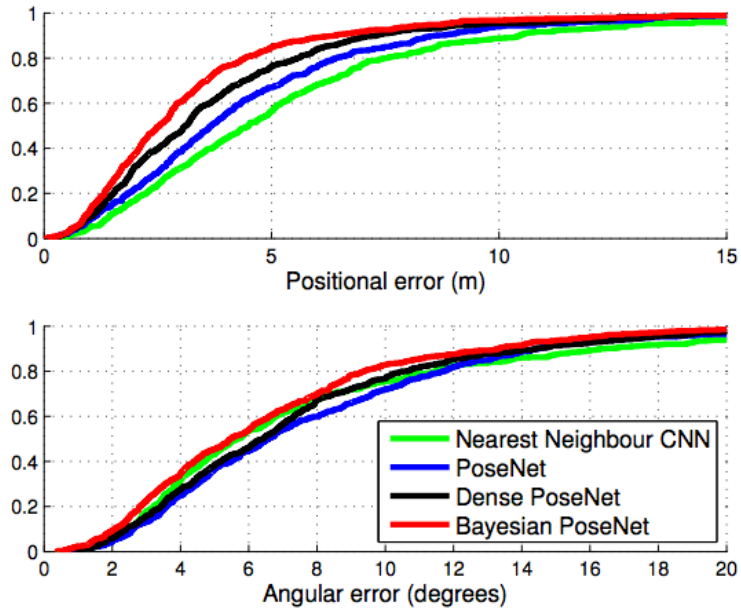



(a) King's College




(b) St Mary's Church

Localisation accuracy for different error thresholds

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# Outline

- Many unanswered questions
- Why does my model work?
- What does my model know?
  - Why should I care about uncertainty?
  - How can I get uncertainty in deep learning?
  - What does this uncertainty look like?
  - Real-world implications
- Why does my model predict this and not that, and other open problems
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# Why should I care about uncertainty?

UNIVERSITY OF CAMBRIDGE

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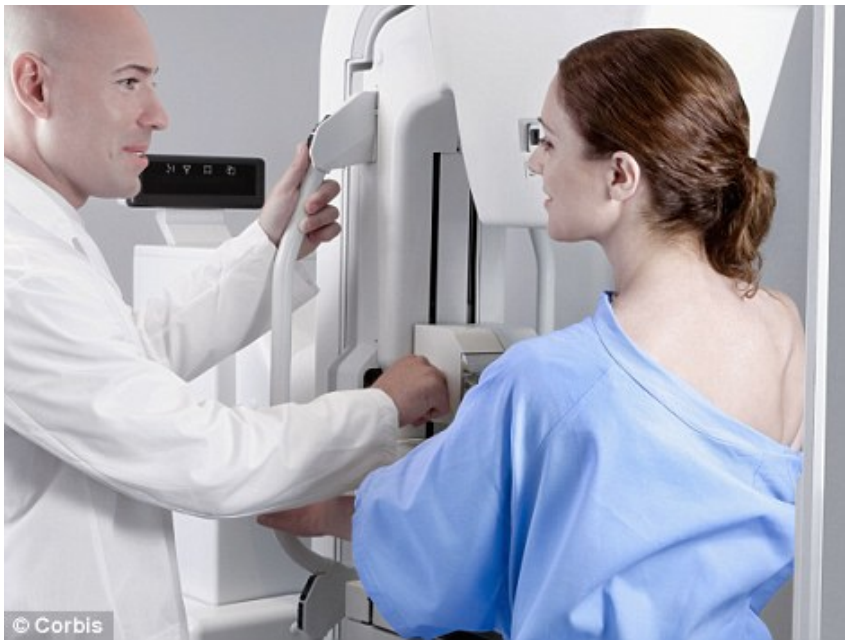


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- For the practitioner: pass inputs with low confidence to specialised models
- But I already have uncertainty in classification! well... no
- We need to be able to tell **what our model knows** and what it doesn't.[9]

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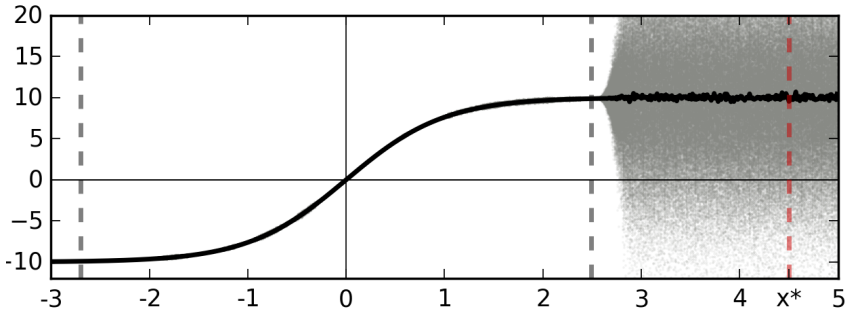
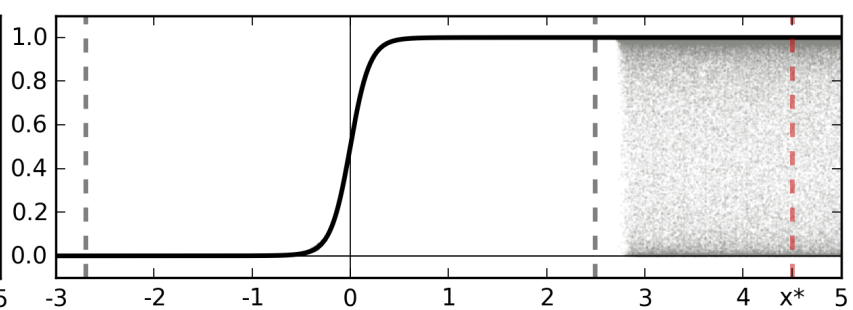



(a) Softmax *input* as a function of data $\mathbf{x}$: $f(\mathbf{x})$




(b) Softmax *output* as a function of data $\mathbf{x}$: $\sigma(f(\mathbf{x}))$

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- We fit a **distribution**; Already used its first moment:

$$\mathbb{E}(\mathbf{y}^*) = \frac{1}{T}\sum_{t=1}^{T}\widehat{\mathbf{y}}(\mathbf{x}^*, \widehat{\omega}_t)$$

with $\widehat{\omega}_t \sim q_\theta(\omega)$.

- For uncertainty (in regression) look at the **second moment**:

$$\boxed{\mathrm{Var}(\mathbf{y}^*)} = \tau^{-1}\mathbf{I} + \frac{1}{T}\sum_{t=1}^{T}\widehat{\mathbf{y}}(\mathbf{x}^*, \widehat{\omega}_t)^T\widehat{\mathbf{y}}(\mathbf{x}^*, \widehat{\omega}_t) - \mathbb{E}(\mathbf{y}^*)^T\mathbb{E}(\mathbf{y}^*)$$

- As simple as looking at the **sample variance** of stochastic forward passes through the network (plus obs. noise).[10]

[10] See `yarin.co/dropout` for more details

# Outline



- Many unanswered questions
- Why does my model work?
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# What does this uncertainty look like?

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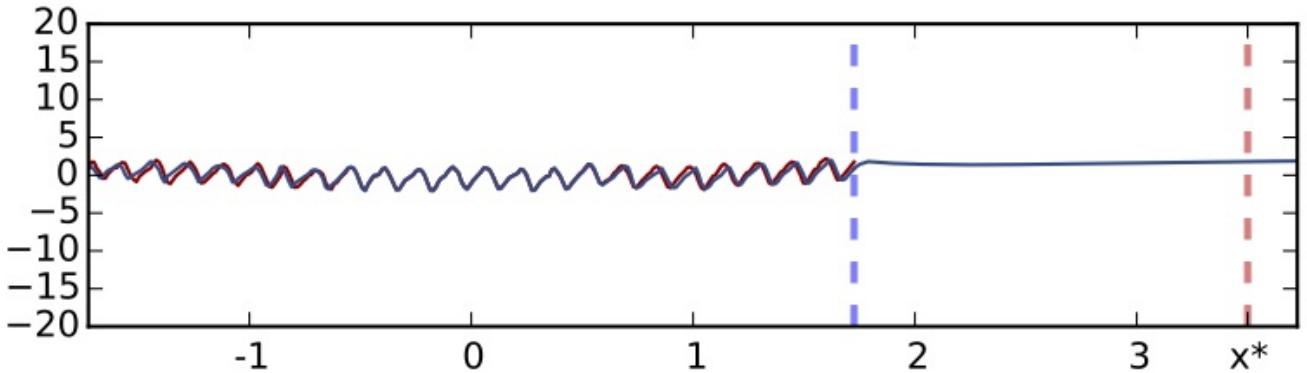
**What would be the $CO_2$ concentration level in Mauna Loa, Hawaii, *in 20 years' time*?**

- Normal dropout (weight averaging, 5 layers, ReLU units):

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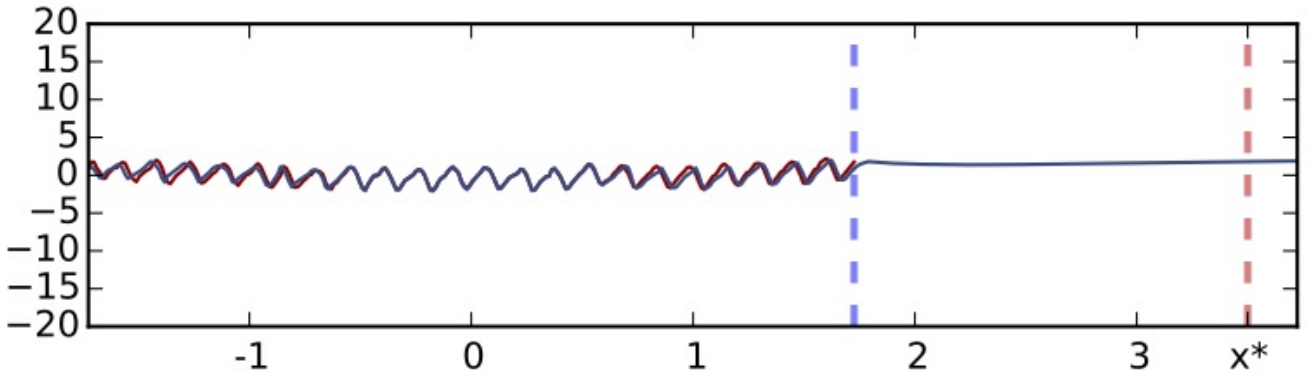


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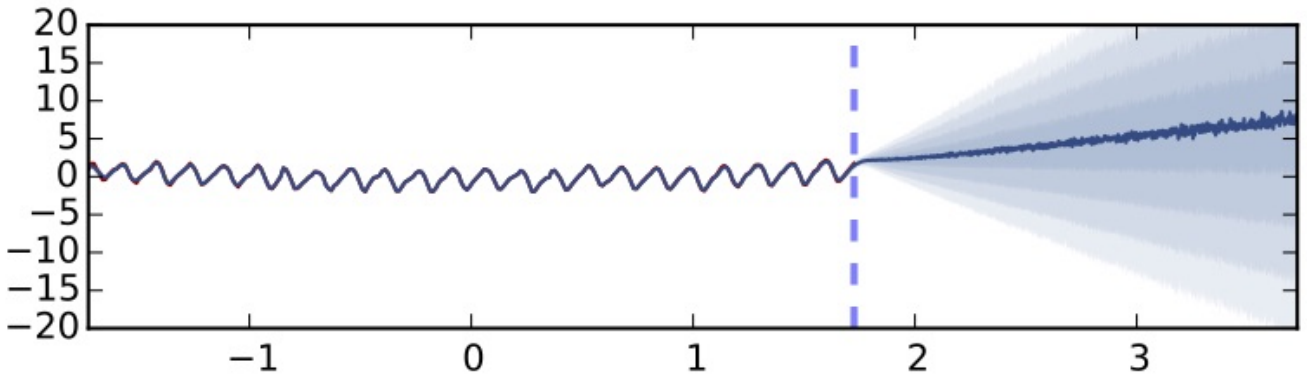

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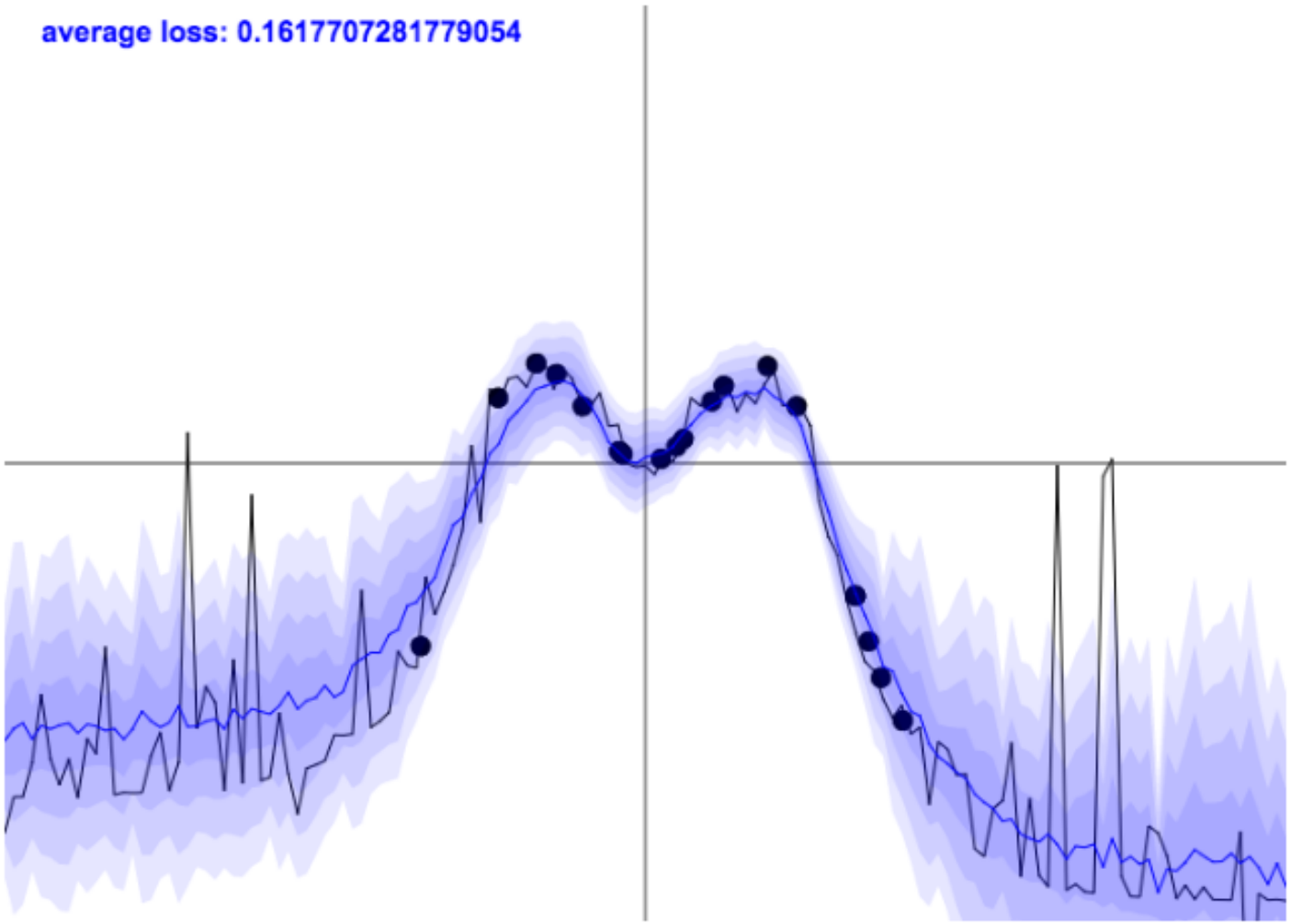


**[Online demo]** [11]

[11] yarin.co/blog

# What does this uncertainty look like?

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- How good is our uncertainty estimate?

| | Avg. Test RMSE and Std. Errors | | | Avg. Test LL and Std. Errors | | |
|---|---|---|---|---|---|---|
| **Dataset** | **VI** | **PBP** | **Dropout** | **VI** | **PBP** | **Dropout** |
| Boston Housing | 4.32 ±0.29 | 3.01 ±0.18 | **2.97 ±0.85** | -2.90 ±0.07 | -2.57 ±0.09 | **-2.46 ±0.25** |
| Concrete Strength | 7.19 ±0.12 | 5.67 ±0.09 | **5.23 ±0.53** | -3.39 ±0.02 | -3.16 ±0.02 | **-3.04 ±0.09** |
| Energy Efficiency | 2.65 ±0.08 | 1.80 ±0.05 | **1.66 ±0.19** | -2.39 ±0.03 | -2.04 ±0.02 | **-1.99 ±0.09** |
| Kin8nm | **0.10 ±0.00** | **0.10 ±0.00** | **0.10 ±0.00** | 0.90 ±0.01 | 0.90 ±0.01 | **0.95 ±0.03** |
| Naval Propulsion | **0.01 ±0.00** | **0.01 ±0.00** | **0.01 ±0.00** | 3.73 ±0.12 | 3.73 ±0.01 | **3.80 ±0.05** |
| Power Plant | 4.33 ±0.04 | 4.12 ±0.03 | **4.02 ±0.18** | -2.89 ±0.01 | -2.84 ±0.01 | **-2.80 ±0.05** |
| Protein Structure | 4.84 ±0.03 | 4.73 ±0.01 | **4.36 ±0.04** | -2.99 ±0.01 | -2.97 ±0.00 | **-2.89 ±0.01** |
| Wine Quality Red | 0.65 ±0.01 | 0.64 ±0.01 | **0.62 ±0.04** | -0.98 ±0.01 | -0.97 ±0.01 | **-0.93 ±0.06** |
| Yacht Hydrodynamics | 6.89 ±0.67 | **1.02 ±0.05** | 1.11 ±0.38 | -3.43 ±0.16 | -1.63 ±0.02 | **-1.55 ±0.12** |
| Year Prediction MSD | 9.034 ±NA | 8.879 ±NA | **8.849 ±NA** | -3.622 ±NA | -3.603 ±NA | **-3.588 ±NA** |

Table 1: **Average test performance in RMSE and predictive log likelihood** for a popular variational inference method (VI, Graves [20]), Probabilistic back-propagation (PBP, Hernández-Lobato and Adams [27]), and dropout uncertainty (Dropout).

# Outline

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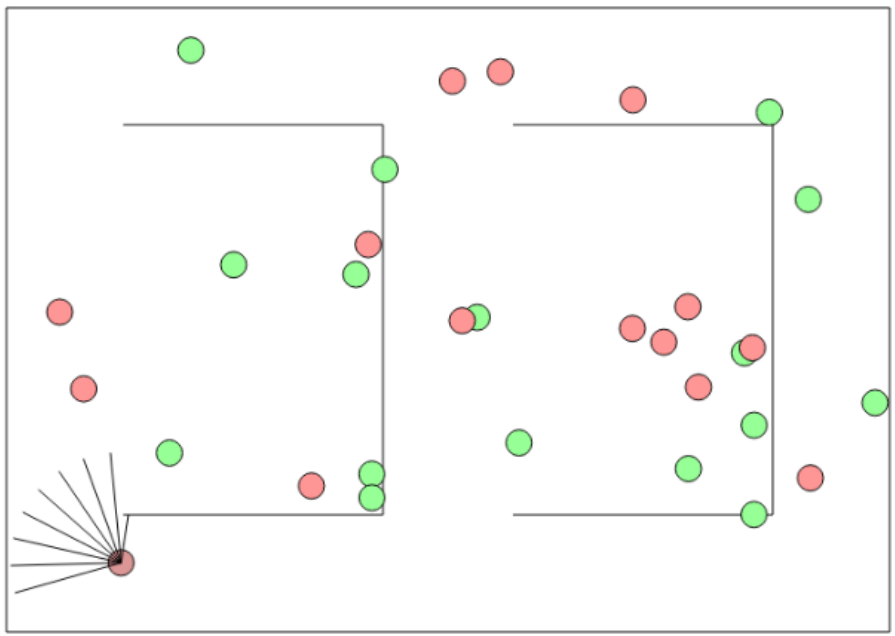
- We have a "Roomba"[12]
- Penalised $-5$ for walking into a wall, $+10$ reward for collecting dirt
- Our environment is stochastic and ever changing
- We want a net to learn what actions to do in different situations



[12] Code based on Karpathy and authors. `github.com/karpathy/convnetjs`

Behavioural policies:

- **Epsilon-greedy** – take random actions with probability $\epsilon$ and optimal actions otherwise
- Using uncertainty we can learn faster
- **Thompson sampling** – draw realisation from current belief over world, choose action with highest value
- In practice: simulate a stochastic forward pass through the dropout network and choose action with highest value

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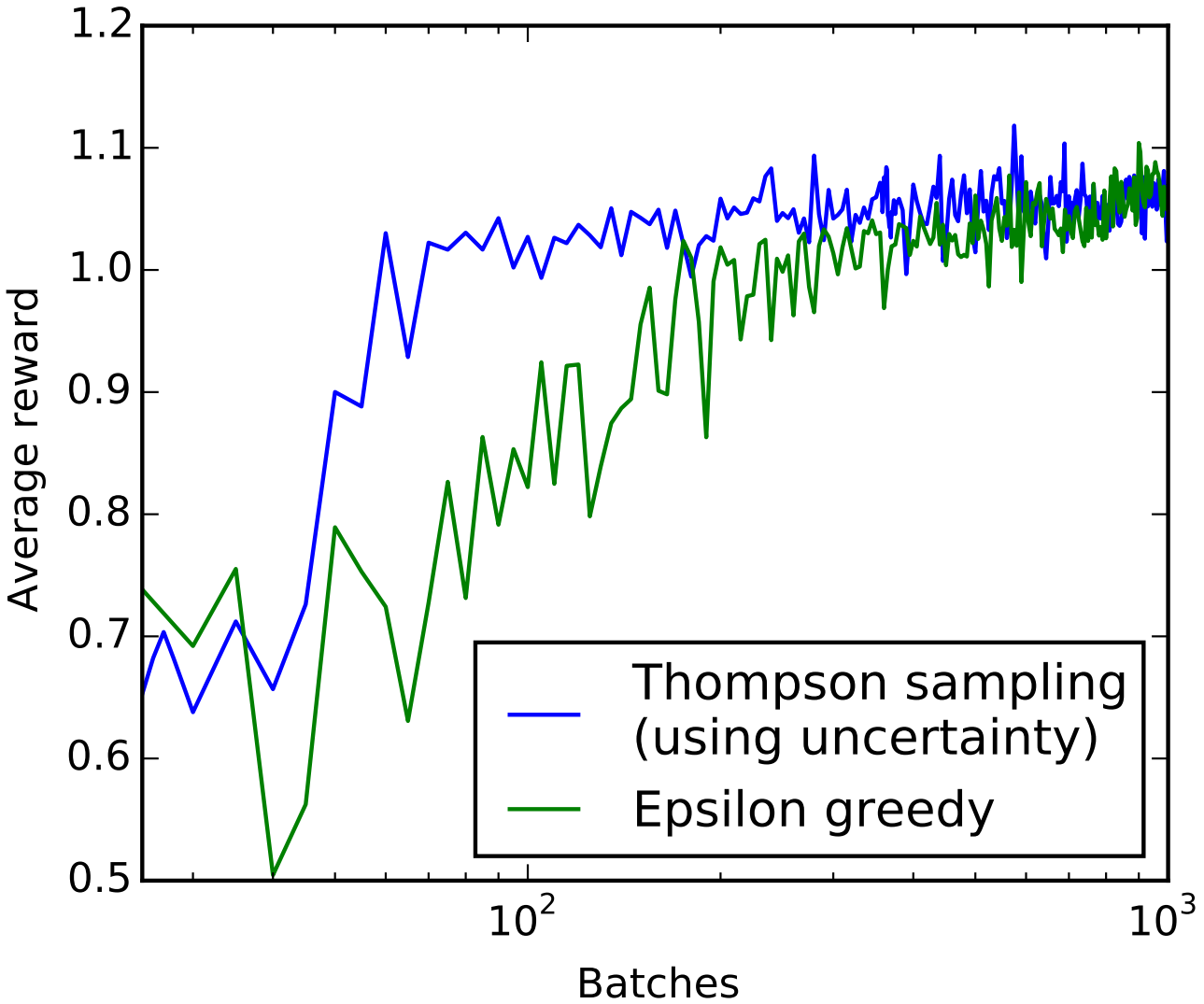


Average reward over time (log scale)

# RW: Camera local. (with uncertainty!)

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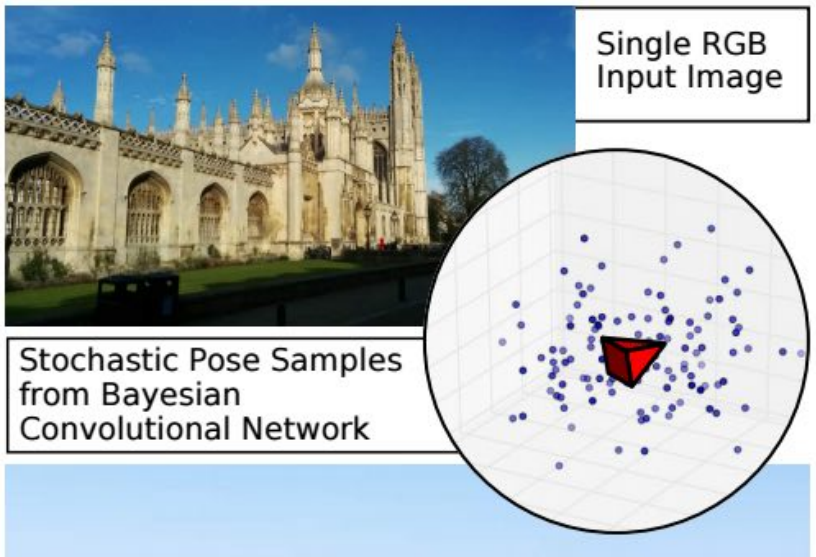
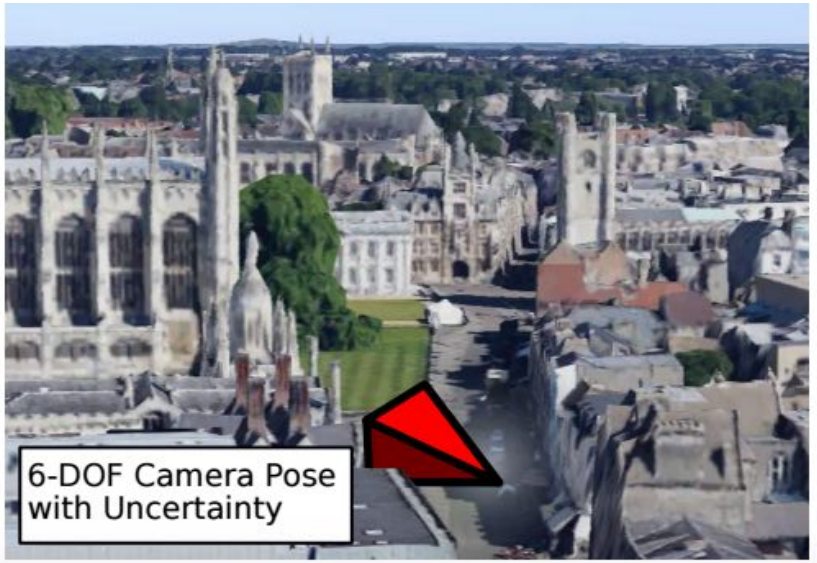
- Where was a picture taken? (Kendall and Cipolla, 2015)[14]




- Uncertainty increases as a test photo diverges from training distribution
- Test photos with high uncertainty (strong occlusion from vehicles, pedestrians or other objects)
- Localisation error correlates with uncertainty

[14] Figures used with author permission

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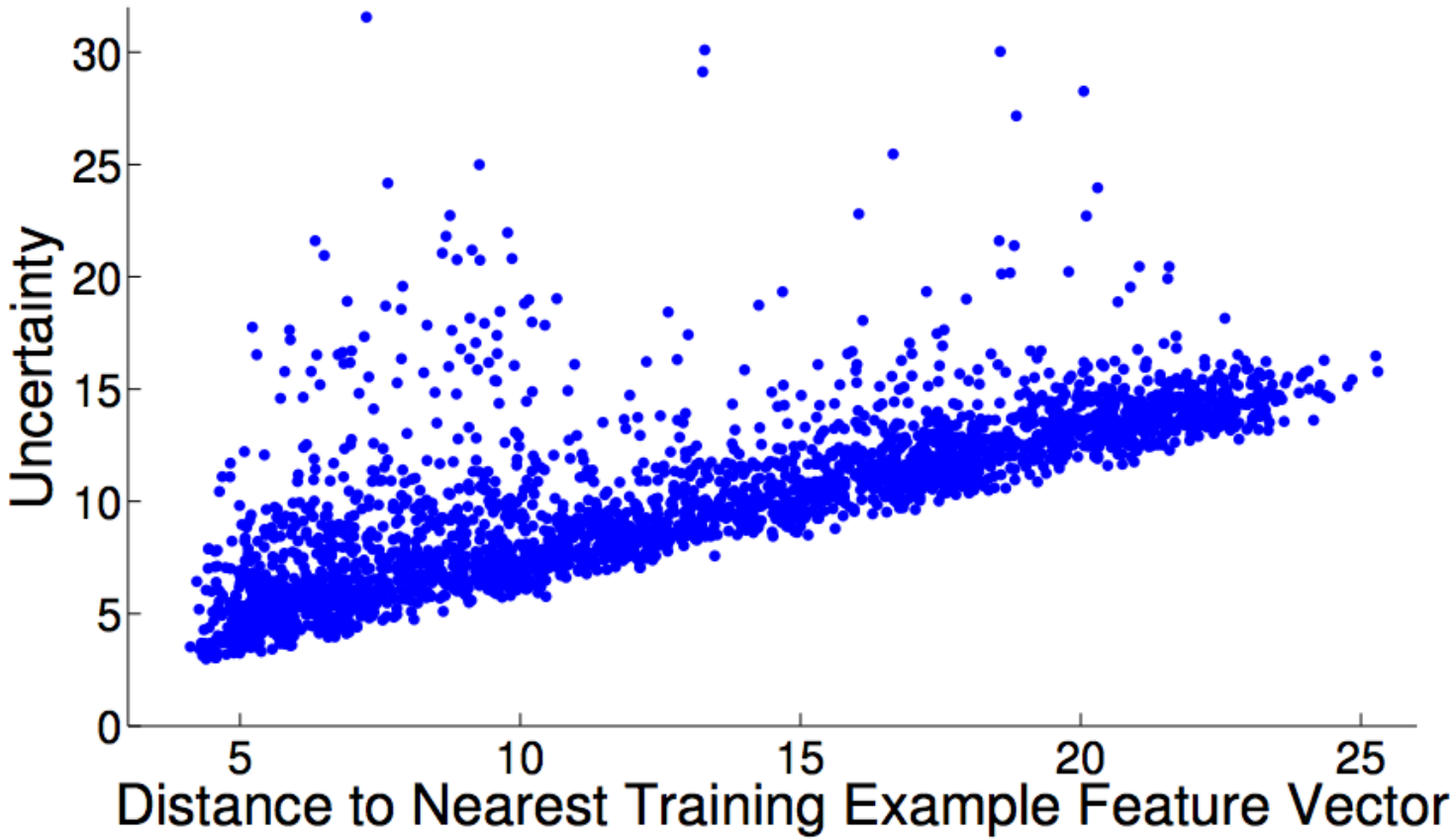


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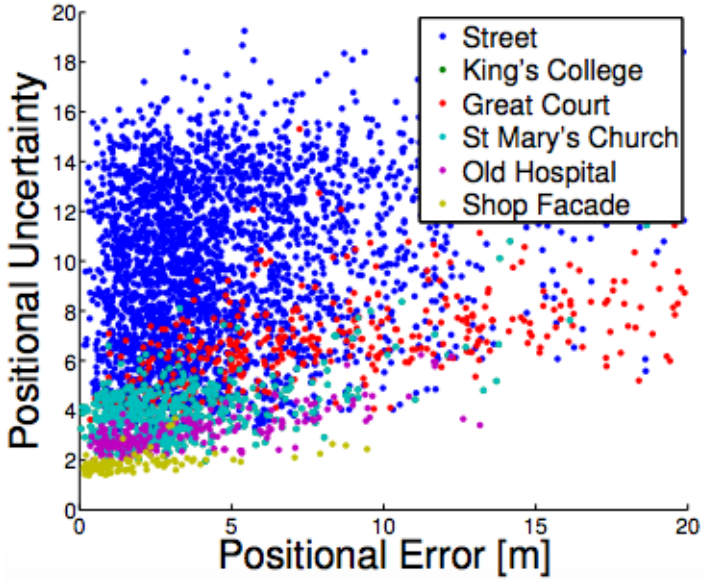


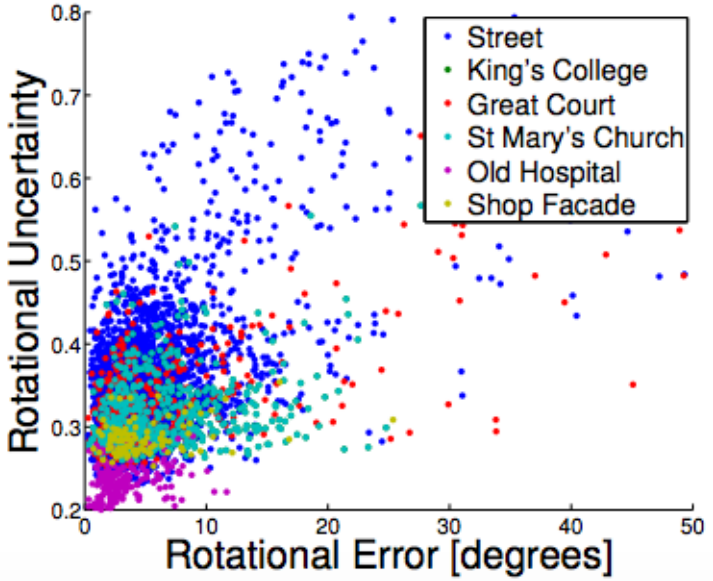


[14] Figures used with author permission

# RW: Image segmentation

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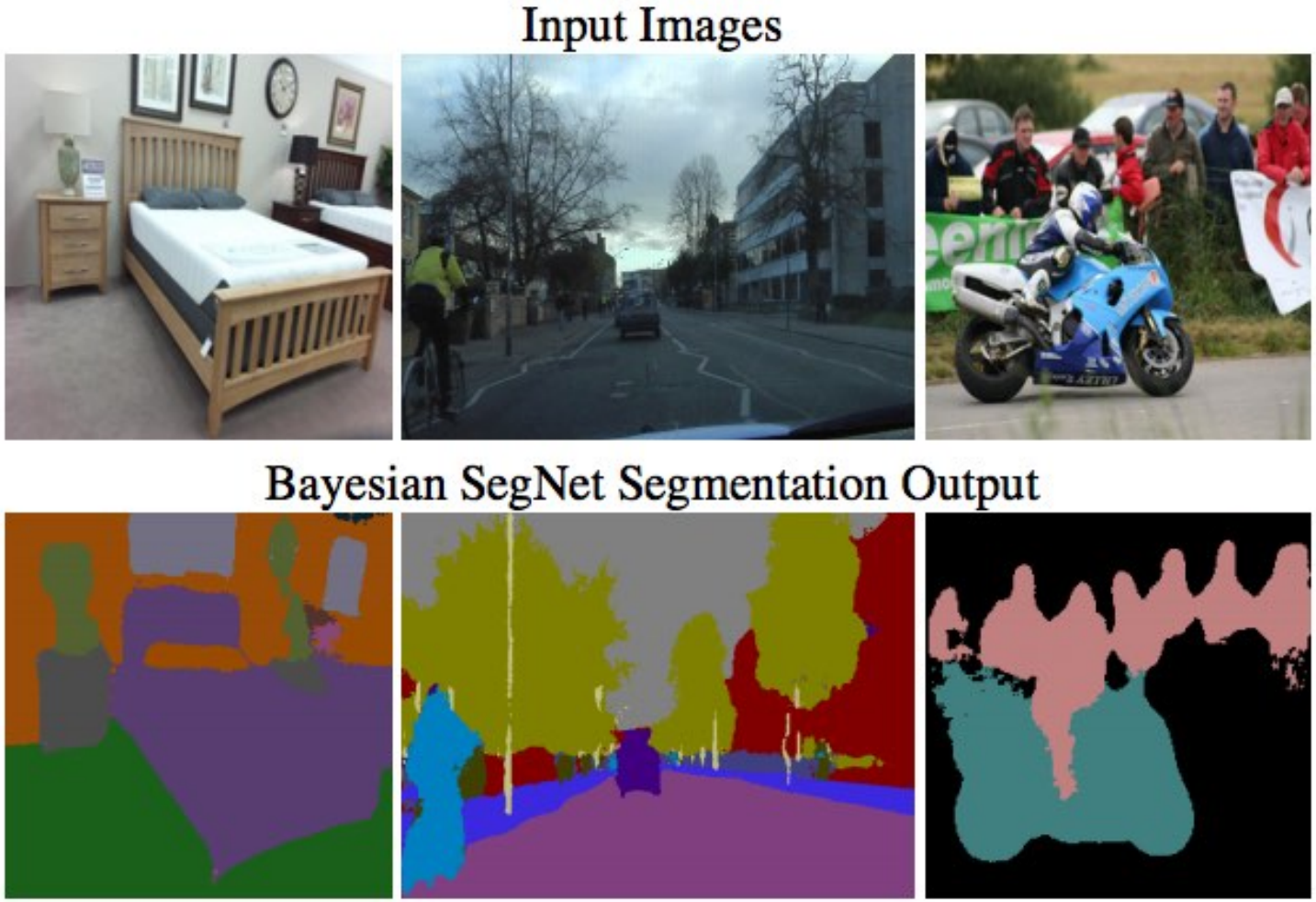
- Scene understanding: what's in a photo and where? (Kendall, Badrinarayanan, and Cipolla, 2015)[15]

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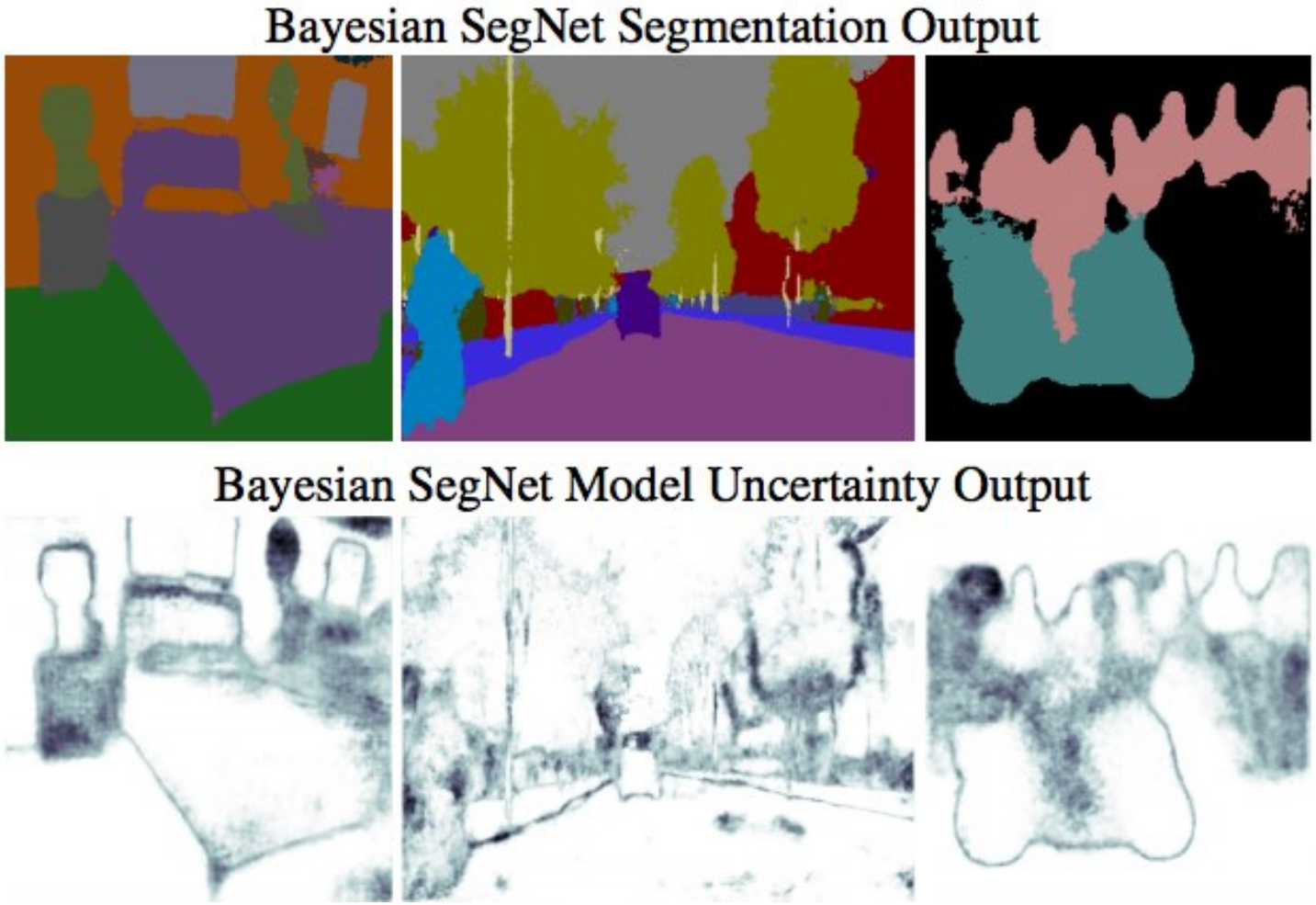


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# RW: DNA methylation

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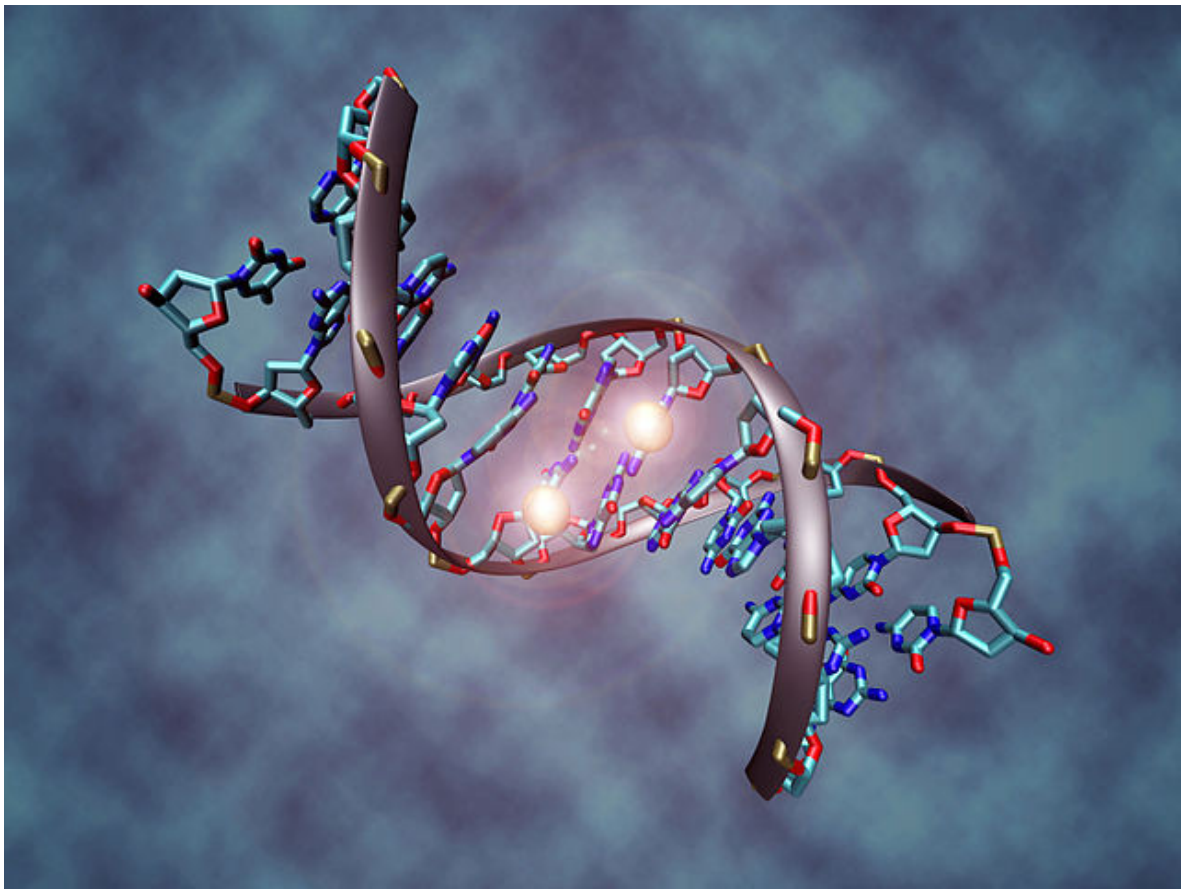
- Angermueller and Stegle (2015) fit a network to predict **DNA methylation** – used for gene regulation



- Look at methylation rate of different embryonic stem cells. **Uncertainty increases** in genomic contexts that are hard to predict (e.g. LMR or H3K27me3)

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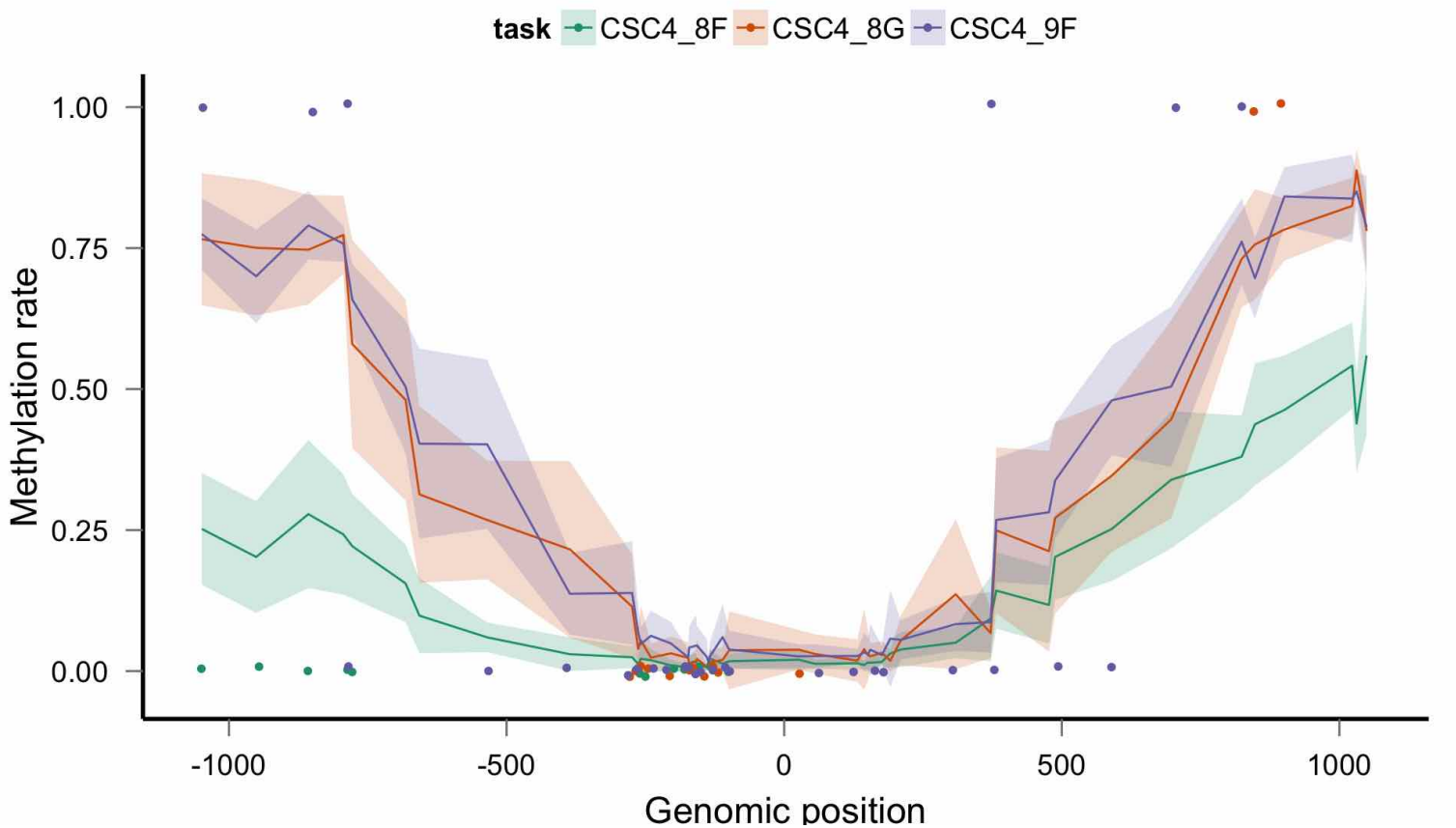

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# What lays ahead



Use the theory to answer many questions: **How can we...**

- ... build interpretable models?
- ... combine Bayesian techniques & deep models?
- ... practically use deep learning uncertainty in existing models?
- ... extend deep learning in a principled way?

# Many unanswered questions left

UNIVERSITY OF CAMBRIDGE

- Interpretable models?
  - Will *you* trust a decision made by a black-box?


  - Rich literature in interpretable Bayesian models (e.g. Sun (2006), Letham (2014))
  - Combine Bayesian and deep models in a principled way?
- Combine Bayesian techniques & deep models?
  - Unsupervised learning – Bayesian data analysis?
  - Bayesian models with complex data? (sequence data, image data)

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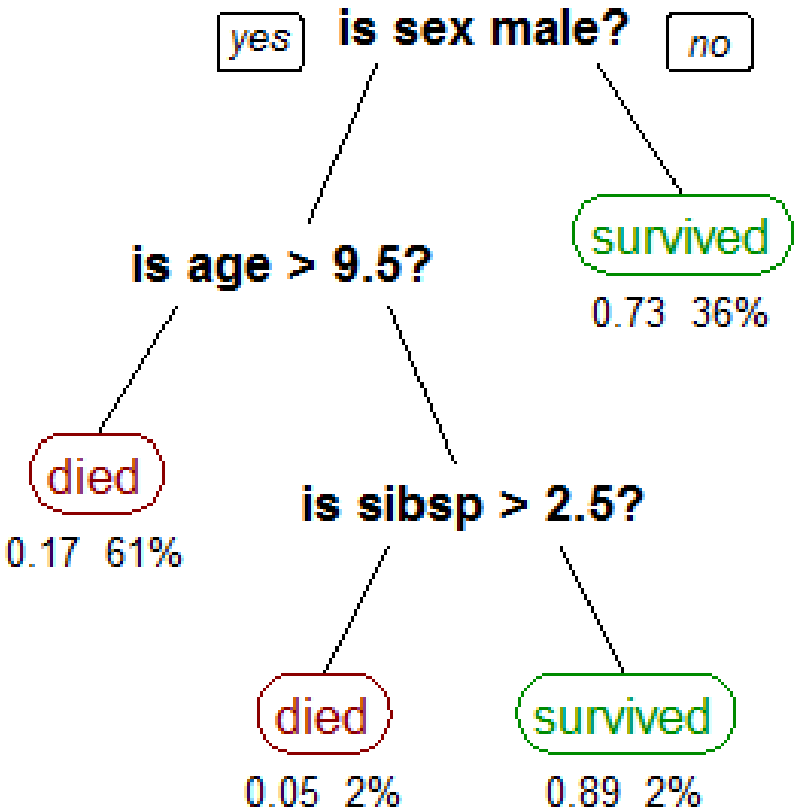

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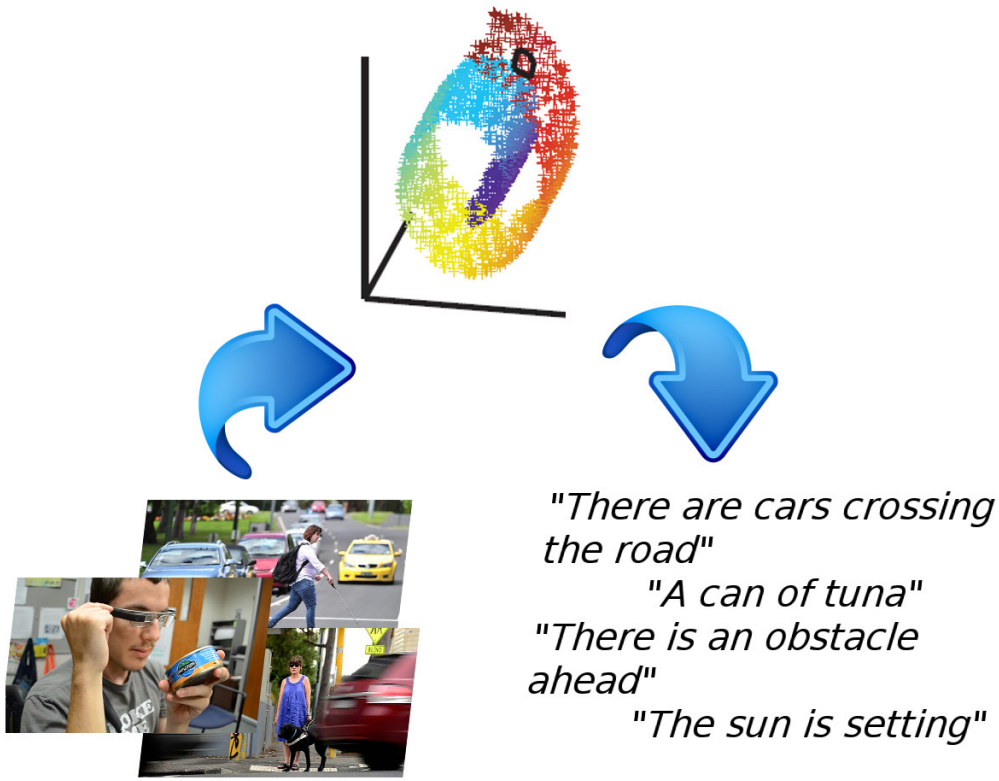

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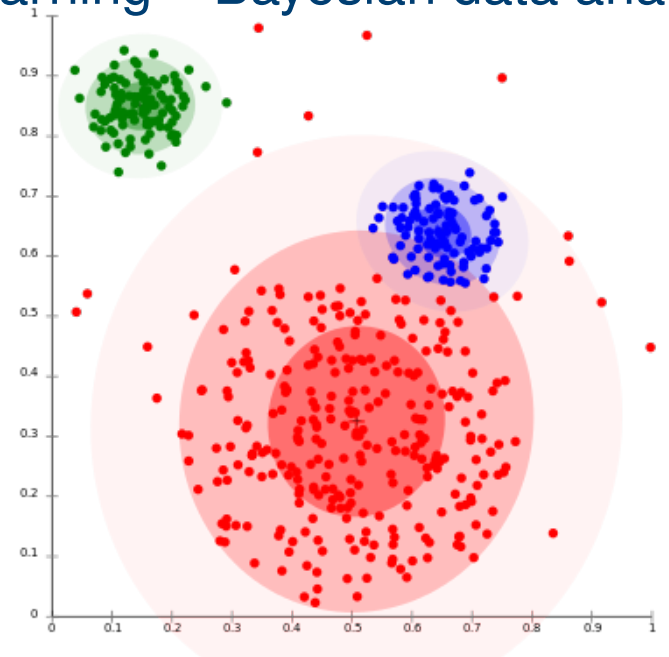

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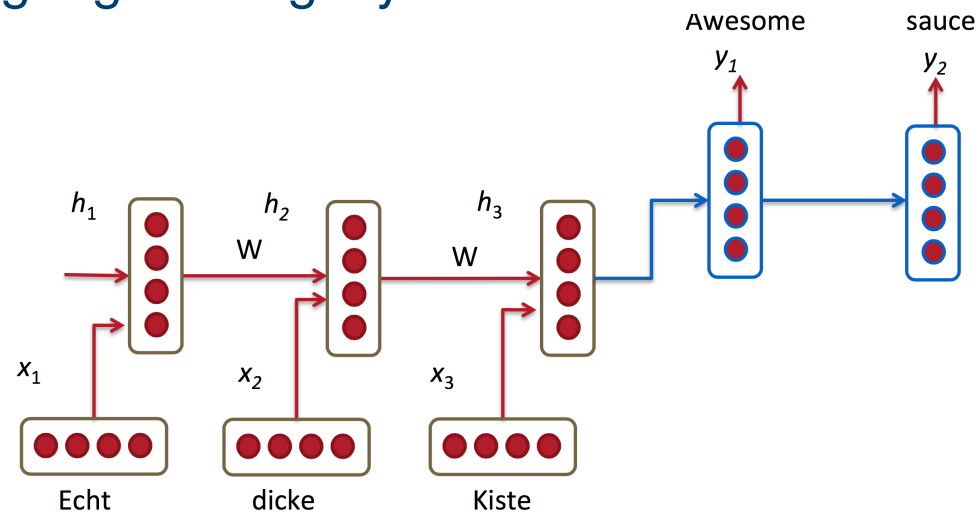


Image Source: `cs224d.stanford.edu/lectures/CS224d-Lecture8.pdf`

  - Weight uncertainty for model debugging?

- Principled extensions of deep learning?
  - Dropout in recurrent networks?
  - New appr. distributions = new stochastic reg. techniques?
  - Model compression: $W_i \sim$ discrete distribution w. continuous base measure?

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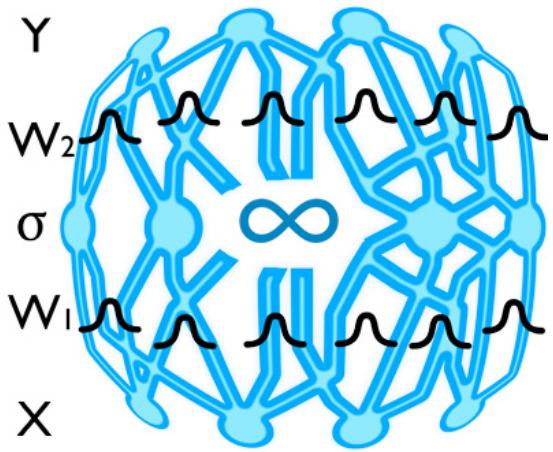


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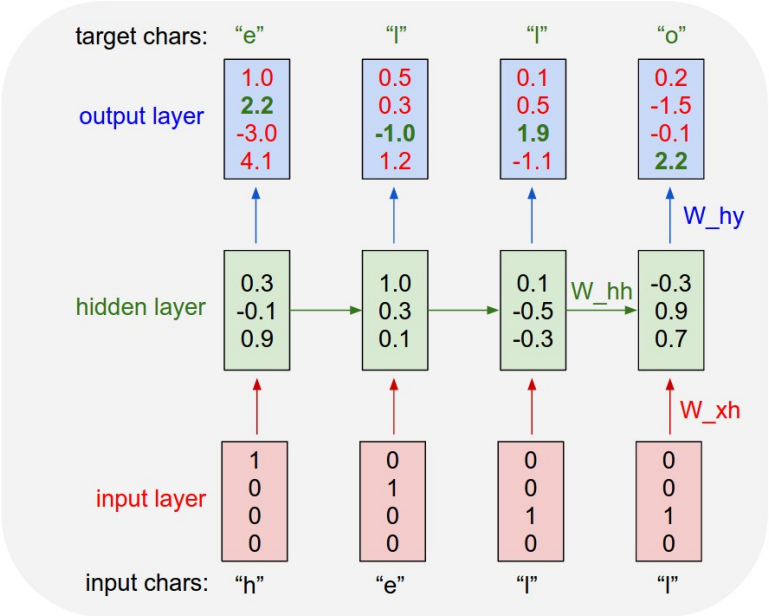


Image Source: `karpathy.github.io/2015/05/21/rnn-effectiveness`

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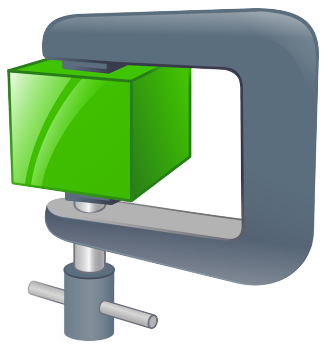

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Work in progress!

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- can use vast Bayesian statistics literature
- can be explained by mathematically rigorous theory
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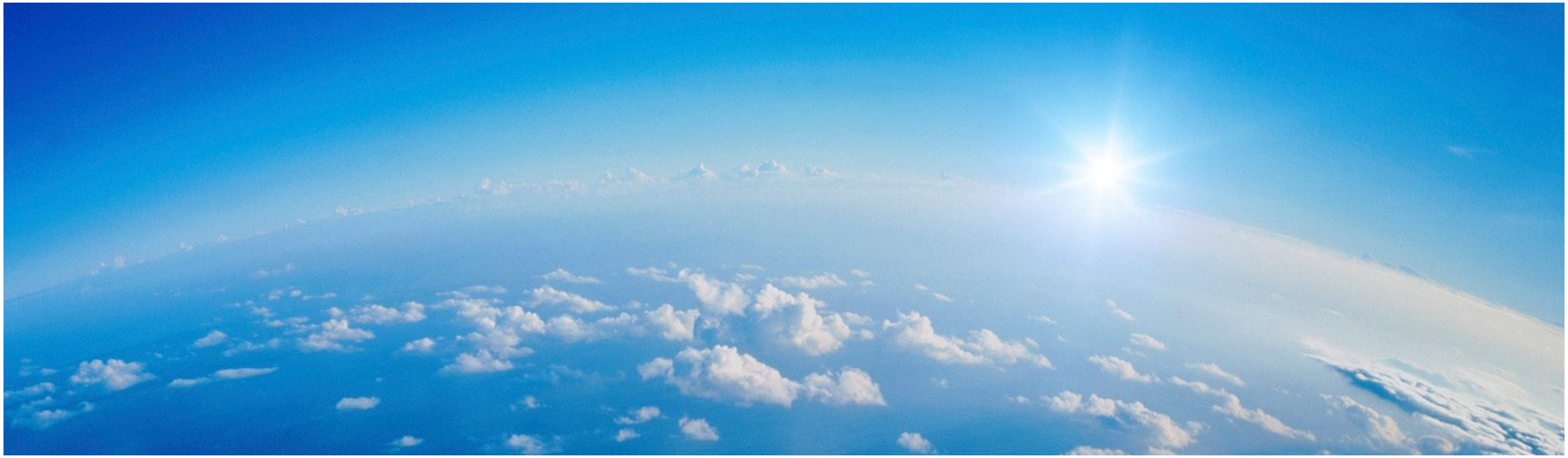
**But...**

# New horizons



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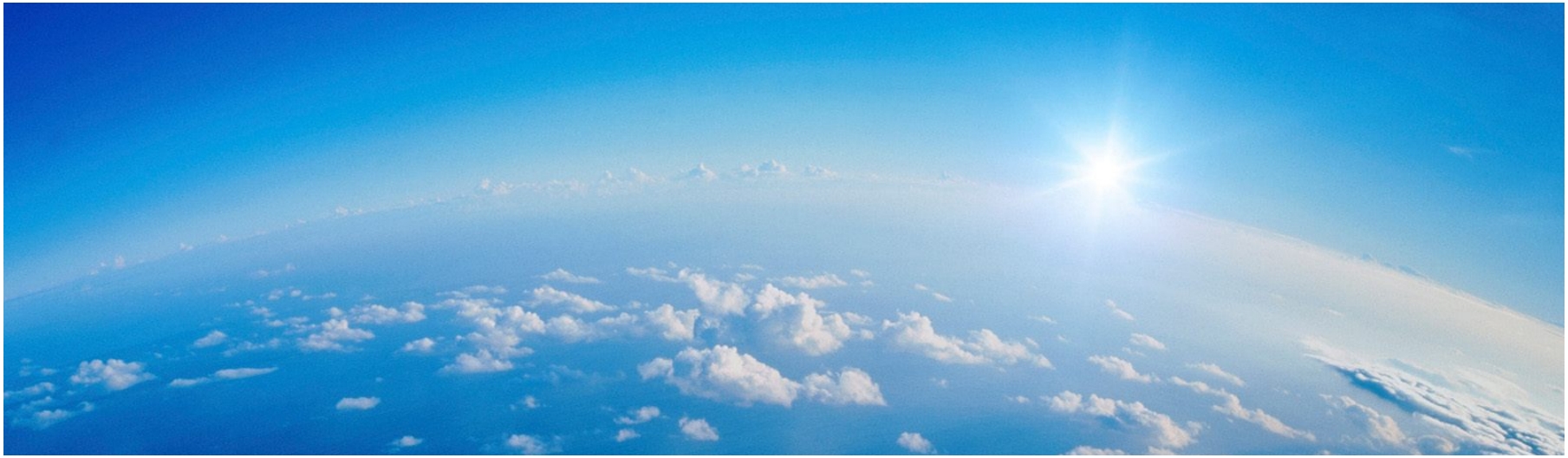

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**Thank you for listening.**

# References


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