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SGDR: STOCHASTIC GRADIENT DESCENT WITH RESTARTS

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ABSTRACT

Restart techniques are common in gradient-free optimization to deal with multi-modal functions. Partial restarts are also gaining popularity in gradient-based optimization to improve the rate of convergence in accelerated gradient schemes to deal with ill-conditioned functions. In this paper, we propose a simple restart technique for stochastic gradient descent to improve its anytime performance when training deep neural networks. We empirically study its performance on CIFAR-10 and CIFAR-100 datasets where we demonstrate new state-of-the-art results below 4% and 19%, respectively. Our source code is available at <https://github.com/loshchil/SGDR>.

1 INTRODUCTION

Deep neural networks (DNNs) are currently the best-performing method for many classification problems, such as object recognition from images (Krizhevsky et al., 2012; Donahue et al., 2014) or speech recognition from audio data (Deng et al., 2013). Their training on large datasets (where DNNs perform particularly well) is the main computational bottleneck: it often requires several days, even on high-performance GPUs, and any speedups would be of substantial value.

The training of a DNN with n free parameters can be formulated as the problem of minimizing a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$. The commonly used procedure to optimize f is to iteratively adjust $\mathbf{x}_t \in \mathbb{R}^n$ (the parameter vector at time step t) using gradient information $\nabla f_t(\mathbf{x}_t)$ obtained on a relatively small t -th batch of b datapoints. The Stochastic Gradient Descent (SGD) procedure then becomes an extension of the Gradient Descent (GD) to stochastic optimization of f as follows:

$$\mathbf{x}_{t+1} = \mathbf{x}_t - \eta_t \nabla f_t(\mathbf{x}_t), \quad (1)$$

where η_t is a learning rate. One would like to consider second-order information

$$\mathbf{x}_{t+1} = \mathbf{x}_t - \eta_t \mathbf{H}_t^{-1} \nabla f_t(\mathbf{x}_t), \quad (2)$$

but this is often infeasible since the computation and storage of the inverse Hessian \mathbf{H}_t^{-1} is intractable for large n . The usual way to deal with this problem by using limited-memory quasi-Newton methods such as L-BFGS (Liu & Nocedal, 1989) is not currently in favor in deep learning, not the least due to (i) the stochasticity of $\nabla f_t(\mathbf{x}_t)$, (ii) ill-conditioning of f and (iii) the presence of saddle points as a result of the hierarchical geometric structure of the parameter space (Fukumizu & Amari, 2000). Despite some recent progress in understanding and addressing the latter problems (Bordes et al., 2009; Dauphin et al., 2014; Choromanska et al., 2014; Dauphin et al., 2015), state-of-the-art optimization techniques tend to approximate the inverse Hessian in a reduced way, e.g., by considering only its diagonal to achieve adaptive learning rates. AdaDelta (Zeiler, 2012) and Adam (Kingma & Ba, 2014) are notable examples of such methods.

Intriguingly enough, the current state-of-the-art results on CIFAR-10, CIFAR-100, SVHN, ImageNet, PASCAL VOC and MS COCO datasets were obtained by Residual Neural Networks

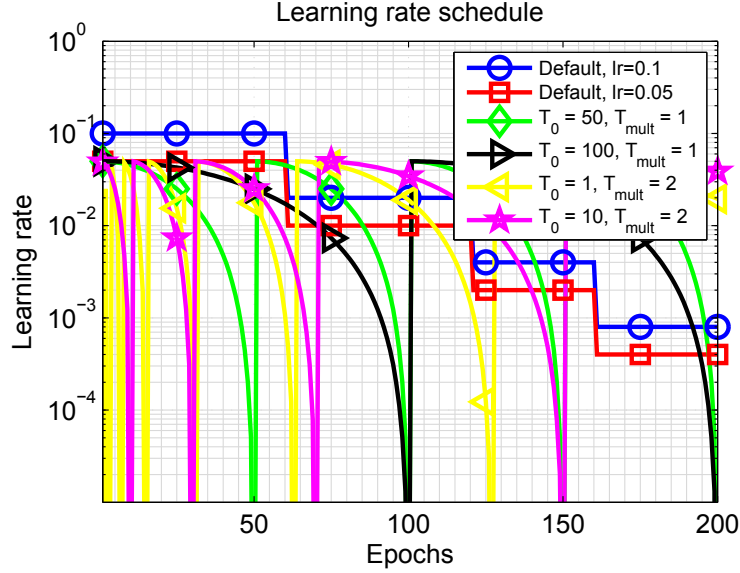


Figure 1: Alternative schedule schemes of learning rate η_t over batch index t : default schemes with $\eta_0 = 0.1$ (blue line) and $\eta_0 = 0.05$ (red line) as used in (Zagoruyko & Komodakis, 2016); restarts simulated every $T_0 = 50$ (green line) and $T_0 = 100$ (black line) epochs with η_t decaying during i -th run from $\eta_{max}^i = 0.05$ to $\eta_{min}^i = 0$ according to eq. (5); restarts starting from epoch $T_0 = 1$ (yellow line) and $T_0 = 10$ (magenta line) with doubling ($T_{mult} = 2$) periods T_i at every new restart.

(He et al., 2015; Huang et al., 2016; He et al., 2016; Zagoruyko & Komodakis, 2016) trained without the use of advanced methods such as AdaDelta and Adam. Instead, they simply use SGD with momentum¹:

$$\mathbf{v}_{t+1} = \mu_t \mathbf{v}_t - \eta_t \nabla f_t(\mathbf{x}_t), \quad (3)$$

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \mathbf{v}_{t+1}, \quad (4)$$

where \mathbf{v}_t is a velocity vector initially set to $\mathbf{0}$, η_t is a decreasing learning rate and μ_t is a momentum rate which defines the trade-off between the current and past observations of $\nabla f_t(\mathbf{x}_t)$. The main difficulty in training a DNN is then associated with the scheduling of the learning rate and the amount of L2 weight decay regularization employed. A common learning rate schedule is to use a constant learning rate and divide it by a fixed constant in (approximately) regular intervals. The blue line in Figure 1 shows an example of such a schedule, as used by Zagoruyko & Komodakis (2016) to obtain the current state-of-the-art results on CIFAR-10, CIFAR-100 and SVHN datasets.

In this paper, we propose to periodically simulate restarts of SGD, where in each restart the learning rate is initialized to some value and is scheduled to decrease. Four different instantiations of this new learning rate schedule are visualized in Figure 1. Our empirical results suggest that SGD with restarts requires $2\times$ to $4\times$ fewer epochs than the currently-used learning rate schedule schemes to achieve comparable or even better results. Using this technique, we obtain new state-of-the-art error rates below 4% for CIFAR-10 and below 19% for CIFAR-100.

2 RELATED WORK

2.1 RESTARTS IN GRADIENT-FREE OPTIMIZATION

When optimizing multimodal functions one may want to find all global and local optima. The tractability of this task depends on the landscape of the function at hand and the budget of func-

¹More specifically, they employ Nesterov’s momentum (Nesterov, 1983; 2013)

tion evaluations. Gradient-free optimization approaches based on niching methods (Preuss, 2015) usually can deal with this task by covering the search space with dynamically allocated niches of local optimizers. However, these methods usually work only for relatively small search spaces, e.g., $n < 10$, and do not scale up due to the curse of dimensionality (Preuss, 2010). Instead, the current state-of-the-art gradient-free optimizers employ various restart mechanisms (Hansen, 2009; Loshchilov et al., 2012). One way to deal with multimodal functions is to iteratively sample a large number λ of candidate solutions, make a step towards better solutions and slowly shape the sampling distribution to maximize the likelihood of successful steps to appear again (Hansen & Kern, 2004). The larger the λ , the more global search is performed requiring more function evaluations. In order to achieve good anytime performance, it is common to start with a small λ and increase it (e.g., by doubling) after each restart. This approach works best on multimodal functions with a global funnel structure and also improves the results on ill-conditioned problems where numerical issues might lead to premature convergence when λ is small (Hansen, 2009).

2.2 RESTARTS IN GRADIENT-BASED OPTIMIZATION

Gradient-based optimization algorithms such as BFGS also perform restarts to deal with multimodal functions (Ros, 2009). In large-scale settings when the usual number of variables n is on the order of $10^3 - 10^9$, the availability of gradient information provides a speedup of a factor of n w.r.t. gradient-free approaches. Restarts are usually employed to improve the convergence rate rather than to deal with multimodality: often it is sufficient to approach any local optimum to a given precision and in many cases the problem at hand is unimodal. Fletcher & Reeves (1964) proposed to flesh the history of conjugate gradient, the conjugate gradient method every n or $(n + 1)$ iterations. Powell (1977) proposed to check whether enough orthogonality between $\nabla f(\mathbf{x}_{t-1})$ and $\nabla f(\mathbf{x}_t)$ has been lost to warrant another restart. Recently, O’Donoghue & Candes (2012) noted that the iterates of accelerated gradient schemes proposed by Nesterov (1983; 2013) exhibit a periodic behavior if momentum is overused. The period of the oscillations is proportional to the square root of the local condition number of the (smooth convex) objective function. The authors showed that fixed restarts of the algorithm with a period proportional to the conditional number achieves the optimal linear convergence rate of the original accelerated gradient scheme. Since the condition number is an unknown parameter and its value may vary during the search, they proposed two adaptive restart techniques (O’Donoghue & Candes, 2012):

- **The function scheme** restarts whenever the objective function increases.
- **The gradient scheme** restarts whenever the angle between the momentum term and the negative gradient is obtuse, i.e., when the momentum seems to be taking us in a bad direction, as measured by the negative gradient at that point. This scheme resembles the one of Powell (1977) for the conjugate gradient method.

O’Donoghue & Candes (2012) showed (and it was confirmed in a set of follow-up works) that these simple schemes provide an acceleration on smooth functions and can be adjusted to accelerate state-of-the-art methods such as FISTA on nonsmooth functions.

Smith (2015) recently introduced cyclical learning rates for deep learning, his approach is closely-related to our approach in its spirit and formulation but does not focus on restarts.

3 STOCHASTIC GRADIENT DESCENT WITH RESTARTS (SGDR)

The existing restart techniques can also be used for stochastic gradient descent if the stochasticity is taken into account. Since gradients and loss values can vary widely from one batch of the data to another, one should denoise the incoming information: by considering averaged gradients and losses, e.g., once per epoch, the above-mentioned restart techniques can be used again.

In this work, we consider one of the simplest restart approaches. We simulate a new warm-started run / restart of SGD once T_i epochs are performed, where i is the index of the run. Importantly, the restarts are not performed from scratch but emulated by increasing the learning rate η_t while the old value of \mathbf{x}_t is used as an initial solution. The amount of this increase controls to which extent the previously acquired information (e.g., momentum) is used.

Within the i -th run, we decay the learning rate for each batch as follows:

$$\eta_t = \eta_{min}^i + \frac{1}{2}(\eta_{max}^i - \eta_{min}^i)(1 + \cos(\frac{T_{cur}}{T_i}\pi)), \quad (5)$$

where η_{min}^i and η_{max}^i are ranges for the learning rate, and T_{cur} accounts for how many epochs have been performed since the last restart. Thus, $\eta_t = \eta_{max}^i$ when $t = 0$ and $T_{cur} = 0$. Once $T_{cur} = T_i$, the \cos function will output -1 and thus set $\eta_t = \eta_{min}^i$. The decrease of the learning rate is shown in Figure 1 for fixed $T_i = 50$ and $T_i = 100$; note that the logarithmic axis obfuscates the typical shape of the cosine function.

In order to improve any-time performance, we suggest an option to start with an initially small T_i and increase it by a factor of T_{mult} at every restart (see, e.g., Figure 1 for $T_0 = 1, T_{mult} = 2$ and $T_0 = 10, T_{mult} = 2$). It might be of great interest to decrease η_{max}^i and η_{min}^i at every new restart. However, for the sake of simplicity, here, we keep η_{max}^i and η_{min}^i the same for every i to reduce the number of hyperparameters involved.

Since our simulated restarts (the increase of the learning rate) often temporarily worsen performance, we do not always use the last \mathbf{x}_t as our recommendation for the best solution (also called the *incumbent solution*). While our recommendation during the first run (before the first restart) is indeed the last \mathbf{x}_t , our recommendation after this is *a solution obtained at the end of the last performed run at* $\eta_t = \eta_{min}^i$. We emphasize that with the help of this strategy, our method does not require a separate validation data set to determine a recommendation.

4 EXPERIMENTAL RESULTS

4.1 EXPERIMENTAL SETTINGS

We consider the problem of training Wide Residual Neural Networks (WRNs; see Zagoruyko & Komodakis (2016) for details) on the CIFAR-10 and CIFAR-100 datasets (Krizhevsky, 2009). We will use the abbreviation WRN- d - k to denote a WRN with depth d and width k . Zagoruyko & Komodakis (2016) obtained the best results with a WRN-28-10 architecture, i.e., a Residual Neural Network with $d = 28$ layers and $k = 10$ times more filters per layer than used in the original Residual Neural Networks (He et al., 2015; 2016).

The CIFAR-10 and CIFAR-100 datasets (Krizhevsky, 2009) consist of 32×32 color images drawn from 10 and 100 classes, respectively, split into 50,000 train and 10,000 test images. For image preprocessing Zagoruyko & Komodakis (2016) performed global contrast normalization and ZCA whitening. For data augmentation they performed horizontal flips and random crops from the image padded by 4 pixels on each side, filling missing pixels with reflections of the original image.

For training, Zagoruyko & Komodakis (2016) used SGD with Nesterov’s momentum with initial learning rate set to $\eta_0 = 0.1$, weight decay to 0.0005, dampening to 0, momentum to 0.9 and minibatch size to 128. The learning rate is dropped by a factor of 0.2 at 60, 120 and 160 epochs, with a total budget of 200 epochs. We reproduce the results of Zagoruyko & Komodakis (2016) with the same settings except that i) we subtract per-pixel mean only and do not use ZCA whitening; ii) we use SGD with momentum as described by eq. (3-4) and not Nesterov’s momentum.

The schedule of η_t used by Zagoruyko & Komodakis (2016) is depicted by the blue line in Figure 1. The same schedule but with $\eta_0 = 0.05$ is depicted by the red line. The schedule of η_t used in SGDR is also shown in Figure 1, with two initial learning rates T_0 and two restart doubling periods.

4.2 RESULTS

Table 1 shows that our experiments reproduce the results given by Zagoruyko & Komodakis (2016) for WRN-28-10 both on CIFAR-10 and CIFAR-100. These “default” experiments with $\eta_0 = 0.1$ and $\eta_0 = 0.05$ correspond to the blue and red lines in Figure 2. The results for $\eta_0 = 0.05$ show better performance, and therefore we use $\eta_0 = 0.05$ in our later experiments.

SGDR with $T_0 = 50, T_{mult} = 1$ and $T_0 = 100, T_{mult} = 1$ performs restarts every 50 and 100 epochs, respectively. A single run of SGD with the schedule given by eq. (5) for $T_0 = 100$ show the

	depth- k	# params	# runs	CIFAR-10	CIFAR-100
original-ResNet (He et al., 2015)	110	1.7M	mean of 5	6.43	25.16
	1202	10.2M	mean of 5	7.93	27.82
stoc-depth (Huang et al., 2016)	110	1.7M	1 run	5.23	24.58
	1202	10.2M	1 run	4.91	n/a
pre-act-ResNet (He et al., 2016)	110	1.7M	med. of 5	6.37	n/a
	164	1.7M	med. of 5	5.46	24.33
	1001	10.2M	med. of 5	4.62	22.71
WRN (Zagoruyko & Komodakis, 2016)	16-8	11.0M	1 run	4.81	22.07
	28-10	36.5M	1 run	4.17	20.50
	with dropout	28-10	36.5M	n/a	20.04
WRN (ours)					
default with $\eta_0 = 0.1$	28-10	36.5M	med. of 5	4.24	20.33
default with $\eta_0 = 0.05$	28-10	36.5M	med. of 5	4.13	20.21
$T_0 = 50, T_{mult} = 1$	28-10	36.5M	med. of 5	4.17	19.99
$T_0 = 100, T_{mult} = 1$	28-10	36.5M	med. of 5	4.07	19.87
$T_0 = 1, T_{mult} = 2$	28-10	36.5M	med. of 5	4.09	19.74
$T_0 = 10, T_{mult} = 2$	28-10	36.5M	med. of 5	4.03	19.58
default with $\eta_0 = 0.1$	28-20	145.8M	med. of 2	4.08	19.53
	28-20	145.8M	med. of 2	3.96	19.67
	28-20	145.8M	med. of 2	4.01	19.28
	28-20	145.8M	med. of 2	3.77	19.24
	28-20	145.8M	med. of 2	3.91	18.90
	28-20	145.8M	med. of 2	3.74	18.70
	28-20	145.8M	med. of 2		

Table 1: Test errors of different methods on CIFAR-10 and CIFAR-100 with moderate data augmentation (flip/translation). In the second column k is a widening factor for WRNs. Note that the computational and memory resources used to train all WRN-28-10 are the same. In all other cases they are different, but WRNs are usually faster than original ResNets to achieve the same accuracy (e.g., up to a factor of 8 according to Zagoruyko & Komodakis (2016)). Bold text is used only to highlight better results and is not based on statistical tests (too few runs).

best results suggesting that the original schedule of WRNs might be suboptimal w.r.t. the test error in these settings.

SGDR with $T_0 = 1, T_{mult} = 2$ and $T_0 = 10, T_{mult} = 2$ performs its first restart after 1 and 10 epochs, respectively. Then, it doubles the maximum number of epochs for every new restart. The main purpose of this doubling is to reach good test error as soon as possible, i.e., achieve good anytime performance. Figure 2 shows that this is achieved and test errors around 4% on CIFAR-10 and around 20% on CIFAR-100 can be obtained about 2-4 times faster than with the default schedule used by Zagoruyko & Komodakis (2016).

Since SGDR achieves good performance faster, it may allow us to train larger networks. We therefore investigated whether results on CIFAR-10 and CIFAR-100 can be further improved by making WRNs two times wider, i.e., by training WRN-28-20 instead of WRN-28-10. Table 1 shows that the results indeed improved, by about 0.25% on CIFAR-10 and by about 0.5-1.0% on CIFAR-100. While network architecture WRN-28-20 requires roughly four times more computation than WRN-28-10, the aggressive learning rate reduction of SGDR nevertheless allowed us to achieve a better error rate in the same time on WRN-28-20 as we spent on 200 epochs of training on WRN-28-10. Specifically, Figure 2 (right middle and right bottom) show that after only 50 epochs, SGDR (even without restarts, using $T_0 = 50, T_{mult} = 1$) achieved an error rate below 19% (whereas none of the learning methods performed better than 19.5% on WRN-28-10). We therefore have hope that – by

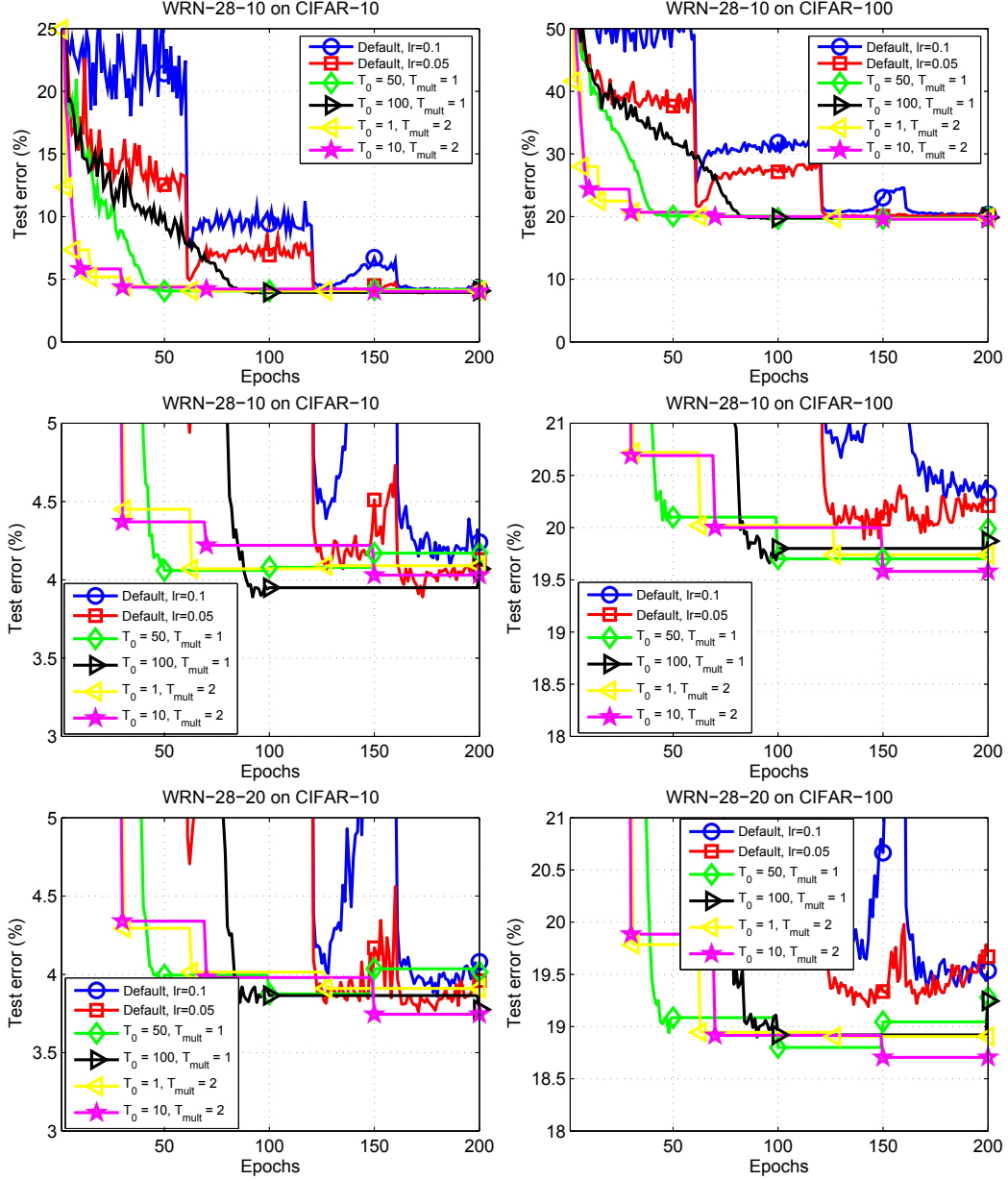


Figure 2: Test errors on CIFAR-10 (left column) and CIFAR-100 (right column) datasets. Note that for SGDR we only plot the recommended solutions. The top and middle rows show the same results on WRN-28-10, with the middle row zooming into the good performance region of low test error. The bottom row shows performance with a wider network, WRN-28-20.

The results of the default learning rate schedules of Zagoruyko & Komodakis (2016) with $\eta_0 = 0.1$ and $\eta_0 = 0.05$ are depicted by the blue and red lines, respectively. The schedules of η_t used in SGDR are shown with i) restarts every $T_0 = 50$ epochs (green line); ii) restarts every $T_0 = 100$ epochs (black line); iii) restarts with doubling ($T_{mult} = 2$) periods of restarts starting from the first epoch ($T_0 = 1$, yellow line); and iv) restarts with doubling ($T_{mult} = 2$) periods of restarts starting from the tenth epoch ($T_0 = 10$, magenta line).

enabling researchers to test new architectures faster – SGDR’s good anytime performance may also lead to improvements of the state of the art.

In a final experiment, Figure 3 compares SGDR and the default schedule with respect to training and test performance. As the figure shows, SGDR optimizes training loss faster than the standard default schedule until about epoch 120. After this, the default schedule overfits, as can be seen by an increase of the test error both on CIFAR-10 and CIFAR-100 (see, e.g., the right middle plot of Figure 3). In contrast, we only witnessed very mild overfitting for SGDR.

5 DISCUSSION

Our results suggest that *even without any restarts* the proposed aggressive learning rate schedule given by eq. (5) is competitive w.r.t. the default schedule when training WRNs on the CIFAR-10 and CIFAR-100 datasets. In practice, the proposed schedule requires only two hyper-parameters to be defined: the initial learning rate and the total number of epochs.

One *should not* suppose that the parameter values used in this study and many other works with (Residual) Neural Networks are selected to demonstrate the fastest decrease of the training error. Instead, the best validation or/and test errors are in focus. Notably, the validation error is rarely used when training Residual Neural Networks because the recommendation is defined by the final solution (in our approach, the final solution of each run). One could use the validation error to determine the optimal initial learning rate and then run on the whole dataset; this could further improve results.

The main purpose of the proposed restart scheme for SGD is to improve its any-time performance. While we mentioned that restarts can be useful to deal with multi-modal functions, *we do not claim* that we observe any effect related to multi-modality.

As we noted earlier, one could decrease η_{max}^i and η_{min}^i at every new simulated restart to control the amount of divergence. If new restarts are worse than the old ones w.r.t. validation error, then one might also consider to go back to the last best solution and perform a new restart with adjusted hyperparameters.

6 CONCLUSION

In this paper, we investigated a simple restart mechanism for SGD to accelerate the training of DNNs. Our SGDR simulates restarts by scheduling the learning rate to achieve competitive results on CIFAR-10 and CIFAR-100 roughly two to four times faster. We also achieved new state-of-the-art results with SGDR, mainly by using even wider WRNs. Future empirical studies should also consider the ImageNet and MS COCO datasets, for which Residual Neural Networks showed the best results so far. Alternative network structures should be also considered; e.g., Zhang et al. (2016) just reported results that are basically the same as our best ones (with $T_0 = 10, T_{mult} = 2$) on CIFAR-10 and (only) one percent worse than ours on CIFAR-100. Their new variant of Residual Neural Networks is very promising since it requires less parameters to achieve the same performance as WRNs and thus it should be tested whether our results can be further improved by using it instead of WRNs.

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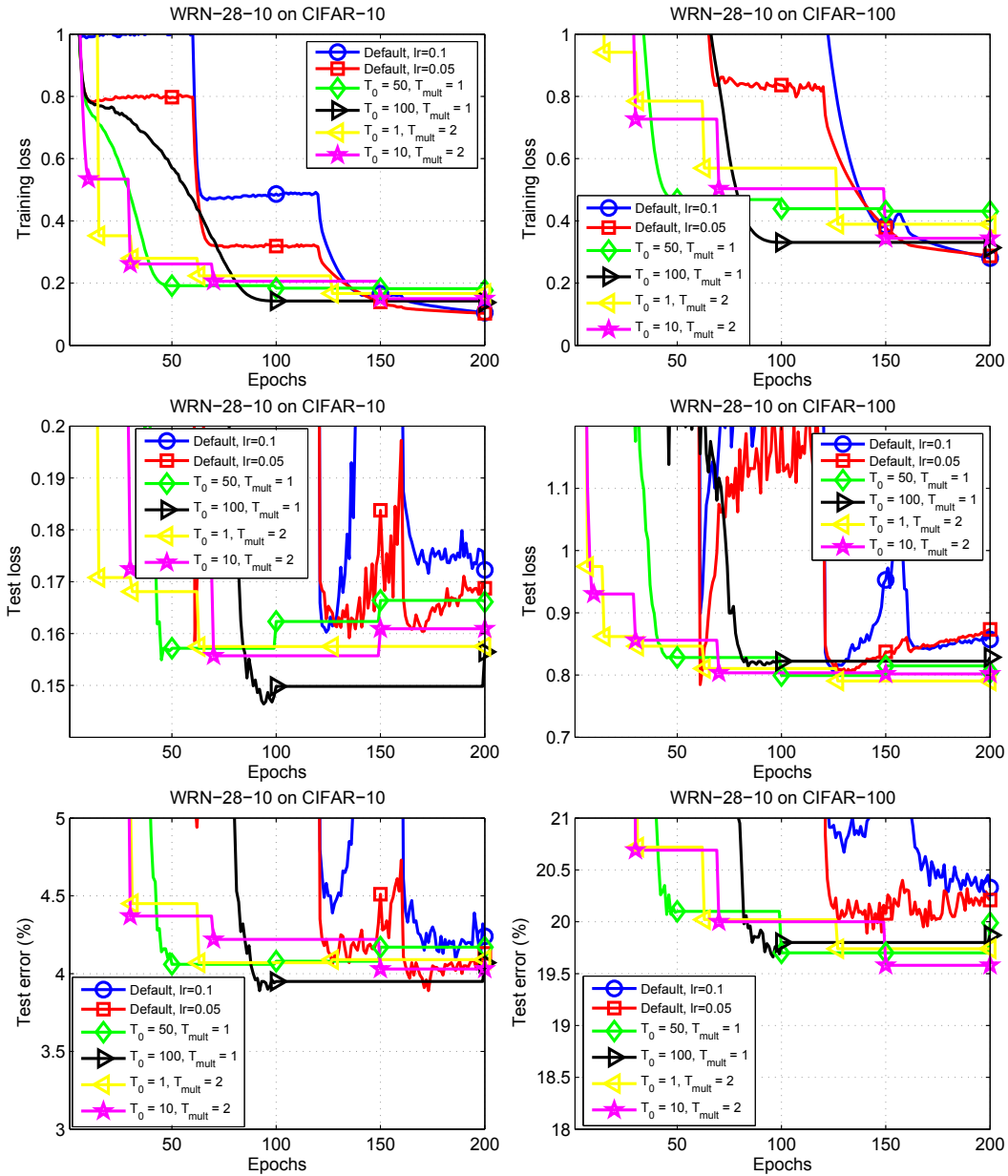


Figure 3: Training cross-entropy loss (top row), test loss (middle row) and test error (bottom row) on CIFAR-10 (left column) and CIFAR-100 (right column).

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