In[]:=
$$u_1[\rho\_, b\_] := \rho\, e^{-b\rho}$$
$$u_2[\rho\_, b\_] := \frac{\rho}{b^2 + \rho^2}$$
$$u_3[\rho\_, b\_] := \rho^2\, e^{-b\rho}$$

In[]:=
$$\text{Energy}_1[b\_] = \text{Integrate}\left[u_1[\rho, b]\left(-\partial_{\{\rho,2\}} u_1[\rho, b] - \frac{2\,u_1[\rho, b]}{\rho}\right), \{\rho, 0, \infty\}, \text{Assumptions} \to b > 0 \&\& b \in \mathbb{R}\right] \Big/ \text{Integrate}[u_1[\rho, b]\, u_1[\rho, b], \{\rho, 0, \infty\}, \text{Assumptions} \to b > 0 \&\& b \in \mathbb{R}]$$

Out[]=
$$(-2 + b)\, b$$

In[]:=
$$\text{Energy}_2[b\_] = \text{Integrate}\left[u_2[\rho, b]\left(-\partial_{\{\rho,2\}} u_2[\rho, b] - \frac{2\,u_2[\rho, b]}{\rho}\right), \{\rho, 0, \infty\}, \text{Assumptions} \to b > 0 \&\& b \in \mathbb{R}\right] \Big/ \text{Integrate}[u_2[\rho, b]\, u_2[\rho, b], \{\rho, 0, \infty\}, \text{Assumptions} \to b > 0 \&\& b \in \mathbb{R}]$$

Out[]=
$$\frac{-8\,b + \pi}{2\,b^2\,\pi}$$

In[]:=
$$\text{Energy}_3[b\_] = \text{Integrate}\left[u_3[\rho, b]\left(-\partial_{\{\rho,2\}} u_3[\rho, b] - \frac{2\,u_3[\rho, b]}{\rho}\right), \{\rho, 0, \infty\}, \text{Assumptions} \to b > 0 \&\& b \in \mathbb{R}\right] \Big/ \text{Integrate}[u_3[\rho, b]\, u_3[\rho, b], \{\rho, 0, \infty\}, \text{Assumptions} \to b > 0 \&\& b \in \mathbb{R}]$$

Out[]=
$$\frac{1}{3}\,(-3 + b)\, b$$

In[]:=
$$\text{Minimize}[\text{Energy}_1[b], b]$$

Out[]=
$$\{-1, \{b \to 1\}\}$$

In[]:=
$$\text{Minimize}[\text{Energy}_2[b], b]$$

Out[]=
$$\left\{-\frac{8}{\pi^2}, \left\{b \to \frac{\pi}{4}\right\}\right\}$$

In[]:=
$$\text{Minimize}[\text{Energy}_3[b], b]$$

Out[]=
$$\left\{-\frac{3}{4}, \left\{b \to \frac{3}{2}\right\}\right\}$$

In[]:=
$$\int_0^\infty u_1[\rho, 1]\, \rho^2\, u_1[\rho, 1]\, d\rho$$

Out[]=
$$\frac{3}{4}$$

In[]:=
$$\int_0^\infty u_2\left[\rho, \frac{\pi}{4}\right] \rho^2\, u_2\left[\rho, \frac{\pi}{4}\right] d\rho$$

Integrate: Integral of $\dfrac{\rho^4}{\left(\frac{\pi^2}{16} + \rho^2\right)^2}$ does not converge on $\{0, \infty\}$.

In[]:=
$$\int_0^\infty u_3\left[\rho, \frac{3}{2}\right] \rho^2\, u_3\left[\rho, \frac{3}{2}\right] d\rho$$

Out[]=
$$\frac{80}{243}$$