Exercise

Apply sampling principles to a "wave packet"

$$s(t) = e^{-\frac{t}{\Delta t}} Sin(2\,\pi\nu_o t)\Theta(t)$$

Take $\Delta t = 10\ s$ and $\nu_o = 10\ Hz$

Calculate continuous Fourier Transform

Sample and estimate alias for $\nu_s = 20, 21, 50, 100\ Hz$

Truncate at $t = [-1, +20]s$ and $[-1, +50]s$ and estimate error within the data range

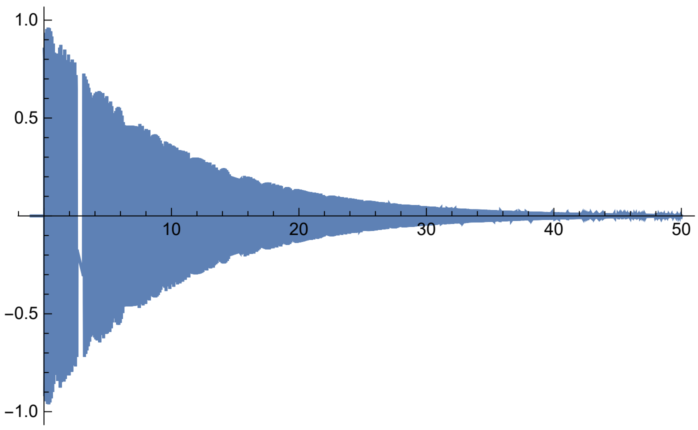
```
In[318]:= ΔT = 10; (* seconds *)
          ν0 = 10; (* hertz *)
          νs = {20, 21, 50, 100} ; (* sample frequencies *)
```

$$\text{In[321]:= } s[t\_] = E^{-\frac{t}{\Delta T}}\ Sin[2\ Pi\ \nu_0\ t]\ HeavisideTheta[t]$$

$$\text{Out[321]= } e^{-t/10}\ HeavisideTheta[t]\ Sin[20\ \pi\ t]$$

```
In[322]:= Plot[s[t], {t, -1, 50}, PlotRange → Full]
```

Out[322]=



In[323]:=

```
In[324]:= (* Fourier transform *)
          sft[ω_] = FourierTransform[s[t], t, ω]
```

$$\text{Out[324]= } \frac{1000\sqrt{2\,\pi}}{1 + 40\,000\,\pi^2 - 20\,i\,\omega - 100\,\omega^2}$$

In[325]:=

In[326]:=

```
In[327]:= (* Sample and estimation of aliases *)
```

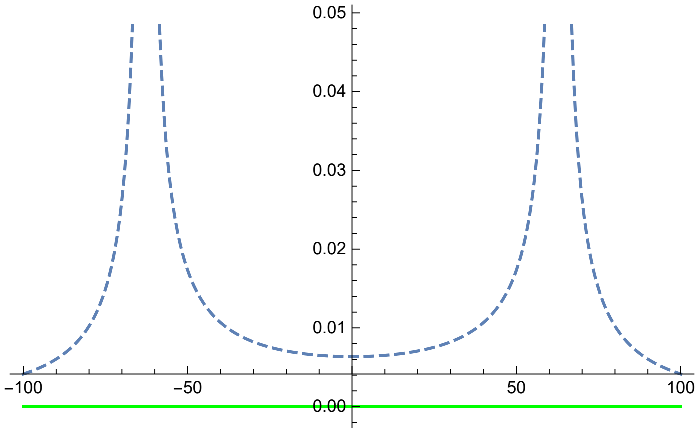
In[328]:=
```
sft-rec[ω_, T_] := (HeavisideTheta[ω + Pi/T] - HeavisideTheta[ω - Pi/T])
    Sum[ sft[ω + n (2 Pi)/T], {n, -200, +200} ];
aliasing = {0, 0, 0, 0}
```

Out[329]= {0, 0, 0, 0}

In[330]:=
```
(* 20 Hz *)
Show[ {Plot[ Abs[sft[ω]],                        {ω, -100, +100}, PlotStyle → Dashed ],
       Plot[ Abs[sft-rec[ω, 1/νs[[1]]] ], {ω, -100, +100} , PlotStyle → Green]},
  PlotRange → All]

N[Sum[ (Abs[sft[ω]] - Abs[sft-rec[ω, 1/νs[[1]]] ])^2 , {ω, -100, +100} ] , 6]
```

Out[330]=

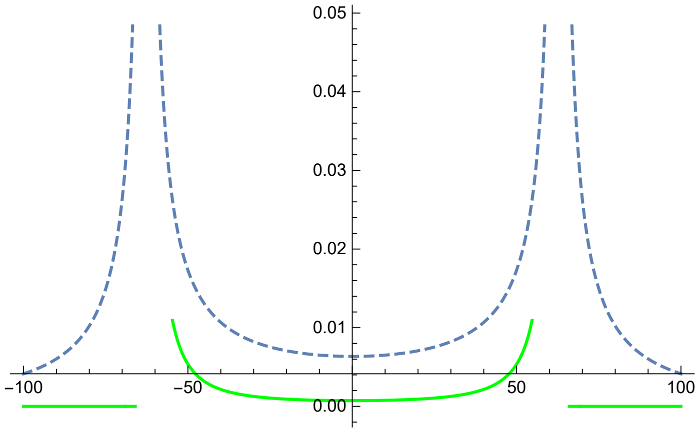


Out[331]= 2.34838

In[332]:=
```
(* 21 Hz *)
Show[ {Plot[ Abs[sft[ω]],                        {ω, -100, +100}, PlotStyle → Dashed ],
       Plot[ Abs[sft-rec[ω, 1/νs[[2]]] ], {ω, -100, +100} , PlotStyle → Green]},
  PlotRange → All]

N[Sum[ (Abs[sft[ω]] - Abs[sft-rec[ω, 1/νs[[2]]] ])^2 , {ω, -100, +100} ] , 6]
```

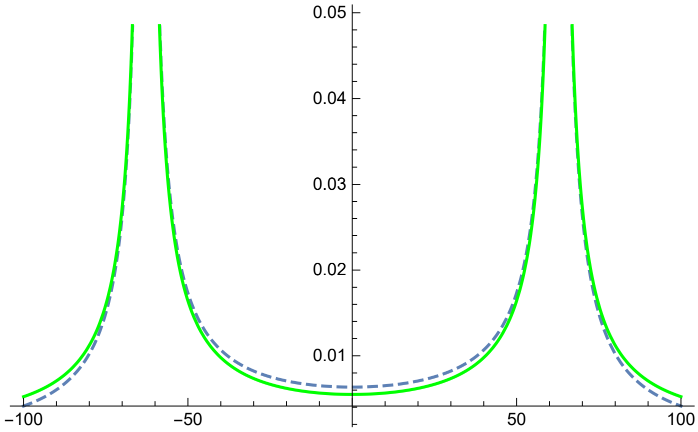
Out[332]=



Out[333]= 0.0487401

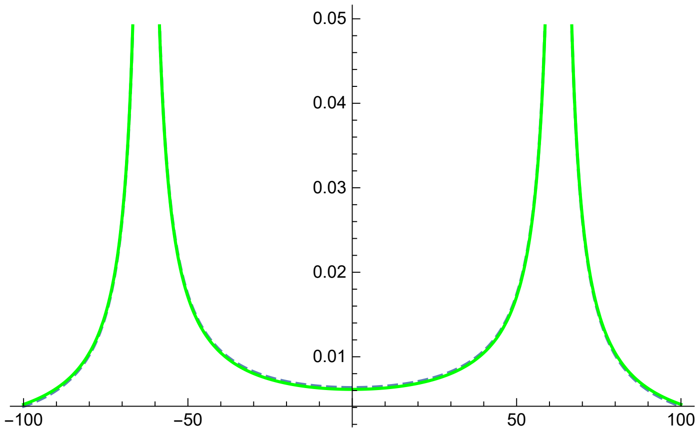
In[334]:=
```
(* 50 Hz *)
Show[ {Plot[ Abs[sft[ω]],                          {ω, -100, +100}, PlotStyle → Dashed ],
       Plot[ Abs[sft-rec[ω, 1/νs[[3]]] ], {ω, -100, +100} , PlotStyle → Green]},
 PlotRange → All]

N[Sum[ (Abs[sft[ω]] - Abs[sft-rec[ω, 1/νs[[3]]] ])^2 , {ω, -100, +100} ] , 6]
```

Out[334]=


Out[335]= 0.000173487

In[336]:=
```
(* 100 Hz *)
Show[ {Plot[ Abs[sft[ω]],                          {ω, -100, +100}, PlotStyle → Dashed ],
       Plot[ Abs[sft-rec[ω, 1/νs[[4]]] ], {ω, -100, +100} , PlotStyle → Green]},
 PlotRange → All]
N[Sum[ (Abs[sft[ω]] - Abs[sft-rec[ω, 1/νs[[4]]] ])^2 , {ω, -100, +100} ] , 6]
```

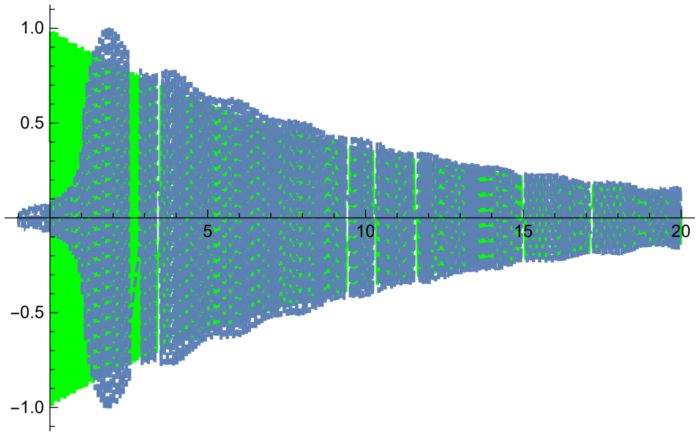
Out[336]=


Out[337]= 9.12534×10^{-6}

In[338]:=
```
(* Error estimation within ranges *)
srec[t_, samples_, T_] :=
  Sum[ samples[[k]] Sinc[Pi/T (t - k T)], {k, 1, Length@samples} ];
```

In[339]:=
```
(* νs=21 *)
points = Range[-1, 20, 1/νs[[2]] ];
samples = Map[s, points];
Show[ {Plot[ s[t],                              {t, -1, 20}, PlotStyle → Green ],
       Plot[ srec[t, samples, 1/νs[[2]]], {t, -1, 20}, PlotStyle → Dashed]},
 PlotRange → All]
N[Sum[ (s[t] - srec[t, samples, 1/νs[[2]]])^2, {t, -1, 20} ] , 6]
```

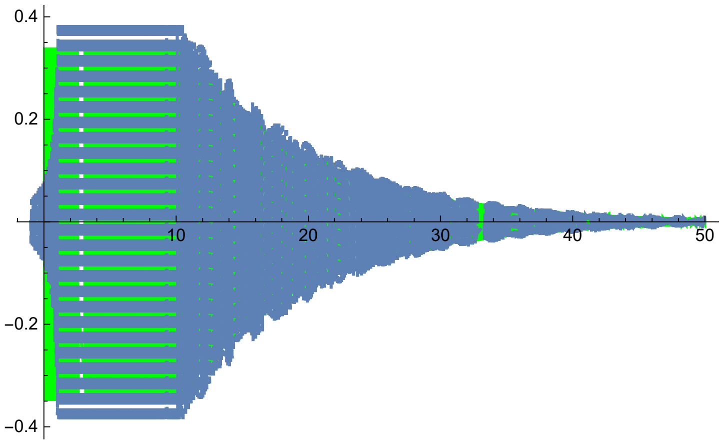
Out[341]=



Out[342]= 0.0990253

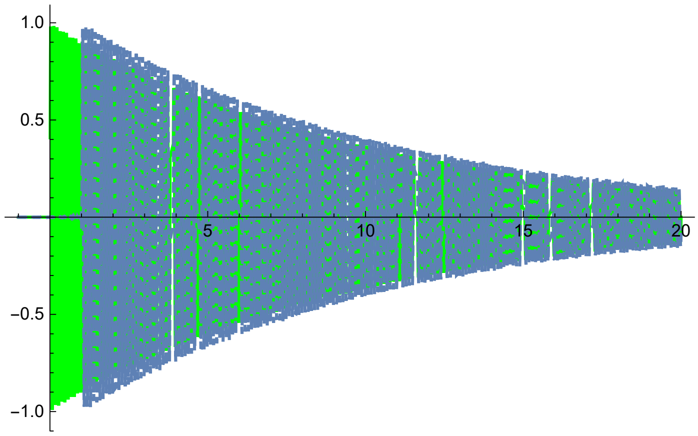
In[343]:=
```
points = Range[-1, 50, 1/νs[[2]] ];
samples = Map[s, points];
Show[ {Plot[ s[t],                              {t, -1, 50}, PlotStyle → Green ],
       Plot[ srec[t, samples, 1/νs[[2]]], {t, -1, 50}, PlotStyle → Dashed]},
 PlotRange → All]
N[Sum[ (s[t] - srec[t, samples, 1/νs[[2]]])^2, {t, -1, 50} ] , 6]
```

Out[345]=



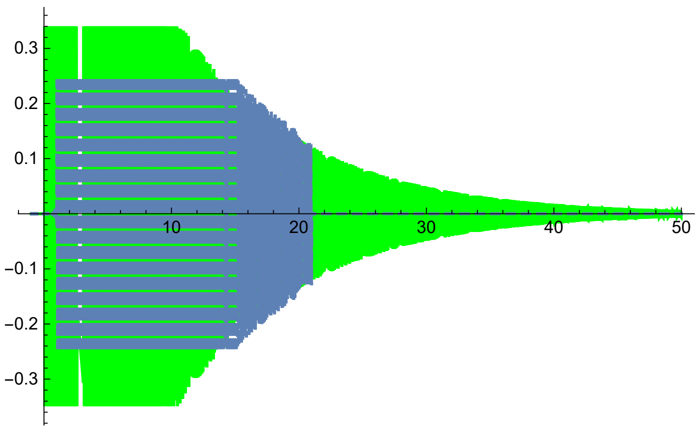
Out[346]= 0.101286

In[347]:=
```
(* νs=100 *)
points = Range[-1, 20, 1/νs[[4]] ];
samples = Map[s, points];
Show[ {Plot[ s[t],                                {t, -1, 20}, PlotStyle → Green ],
       Plot[ srec[t, samples, 1/νs[[4]]], {t, -1, 20}, PlotStyle → Dashed]},
 PlotRange → All]
N[Sum[ (s[t] - srec[t, samples, 1/νs[[4]]])^2, {t, -1, 20} ] , 6]
```

Out[349]=


Out[350]= 1.52861

In[351]:=
```
points = Range[-1, 20, 1/νs[[4]] ];
samples = Map[s, points];
Show[ {Plot[ s[t],                                {t, -1, 50}, PlotStyle → Green ],
       Plot[ srec[t, samples, 1/νs[[4]]], {t, -1, 50}, PlotStyle → Dashed]},
 PlotRange → All]
N[Sum[ (s[t] - srec[t, samples, 1/νs[[4]]])^2, {t, -1, 50} ] , 6]
```

Out[353]=


Out[354]= 1.53495