Experimental Methods
Instructions for the exam

Dec 4, 2020

Final examination

- You are requested to take a written test. The test consists of one exercise. The time available for this test is the full morning or the full afternoon.
- While working to the test you are allowed to search the web and check your notes, but you are not allowed to have any interaction with third parties (email, chat..).
- At the end you are requested to write a short paper and send it over to me, as a pdf file, by email.
- In a few days you will get a feedback from me with a commented copy of your test paper and a tentative range for the final mark.
- You are then requested to attend the "oral" exam session. During the oral exam we will shortly discuss your paper. At the end I will give you the final mark
- In the following you may find:
  - a synopsis of the subjects you should have studied
  - a list of exercises, questions on which may be included in the test
  - The test may also make reference to the past homework

List of key subjects

- **SIGNALS**
  - Fourier transforms of continuous signals: main properties.
  - Principle of uncertainty
  - Effect of signal truncation in the time domain.
  - Sampling and sampling theorem.
  - Discrete Time Fourier Transform
  - Discrete Fourier Transforms: properties and the FFT algorithm
  - Narrow band-signals
- **SYSTEMS**
  - Physical systems and models. inputs and outputs.
  - Linear systems and small signals linearization.
  - Impulse response, frequency response and transfer function.
  - Series of systems
  - Feedback loop. Input -output relations, feedback signal
  - Linearization by feedback.
  - Causality and Kramers-Krönig relations.
  - Two-ports systems.
  - Up-conversion and down conversion of signals. Phase sensitive detector.

List of key subjects

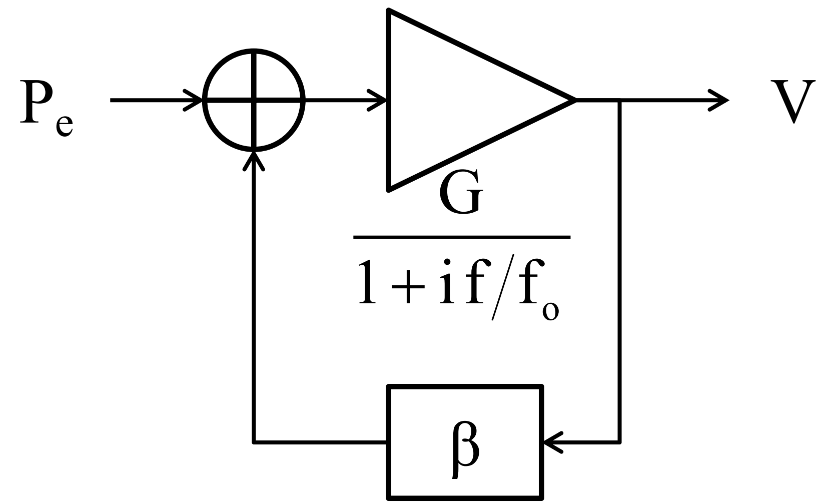
- **NOISE**
  - Stochastic processes.
  - Mean value, autocorrelation, auto-covariance, cross-correlation/covariance
  - Poisson processes. Shot noise. Normal processes. White noise
  - Stationary processes.
  - Power spectral density and cross-power spectral density
  - Noise through linear systems. Output auto-correlation. Input-output cross-correlation. Output spectral density. Input-output cross-spectral density.
  - Noise through phase-sensitive detectors.
  - Spectral estimation and radiometric law.
  - Spectral estimation by FFT. Aliasing and spectral leak
- **NOISE IN PHYSICAL SYSTEMS**
  - Shot noise in quantized processes
  - Thermal noise and fluctuation-dissipation theorem.
  - Nyquist noise. Brownian motion.
  - Noise in two port devices. Noise energy and impedance. Noise temperature and noise figure
  - Quantum limit to noise energy

List of key subjects

- **SIGNAL DETECTION**
  - Wiener-Kolmogorov optimal filter
  - Signal to noise ratio.
  - Measurements limits and minimum detectable energy.
  - Pulse detection.
  - Narrow band signal detection by phase-sensitive detectors

Exercise 1

- A system consists of a pressure transducer with an operating range of ±1 Pa. Within this operating range, the transducer behaves like a simple low pass, with a gain of G=100 V/Pa, and a roll-off frequency of f_o= 10 Hz.
- Outside the operating range the transducer saturates at ±100 V.
- To operate the transducer, a feedback loop sends back its output to a piezoelectric crystal, able to exert a pressure of β=1Pa/V on the transducer



- Calculate:
  - The transfer function of the closed loop system.
  - The maximum peak value allowed for a sinusoidal external pressure signal, in order to avoid saturation. Give the answer as a function of frequency.

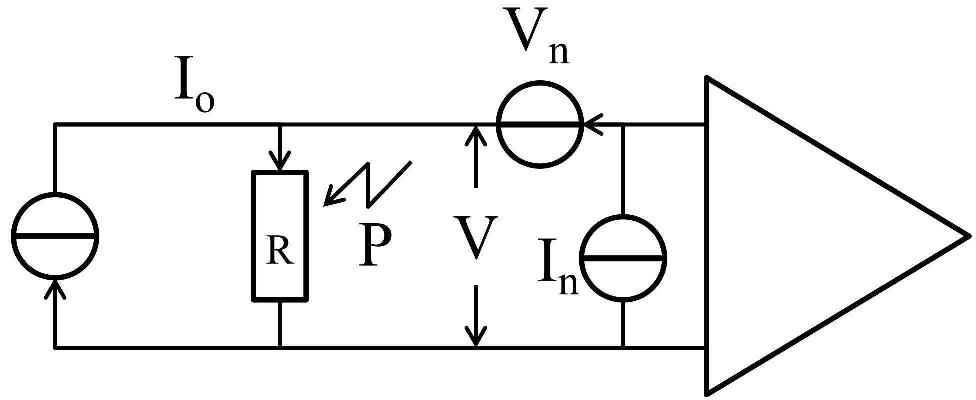
Exercise 2

- An infrared laser beam of wavelength λ=1μm and total power P=10 pW hits a photodiode.
- The beam is very stable so that any power fluctuation due to external causes (electronics, temperature etc.,) is totally negligible.
- The response of the photodiode to each photon is a voltage signal, the Fourier transform of which is:

$$s(\omega) = V_o \frac{\tau_2 - \tau_1}{(1 + i\omega\tau_1)(1 + i\omega\tau_2)}$$

- Here τ_1=100 μs, τ_2=1 ms, and V_o=1 mV
- Calculate the mean value of the voltage output of the photodiode.
- Calculate its power spectral density.
- Calculate its autocorrelation.
- Is the process approximately Gaussian?

Exercise 3



- The bolometer of the previous exercise, is connected to a voltage amplifier with frequency independent noise temperature T_n , noise resistance R_n and zero noise correlation
- Calculate the total noise spectral density at the amplifier input, including the thermal noise of the bolometer. *Assume that the bolometer is operated without the feedback loop*
- Assume that P(t) is a short pulse. What is the minimum deposited energy that can be detected?
- Assume that P(t) is a sinusoidal signal at frequency f_o and with duration T. What is the minimum detectable signal amplitude?

Exercise 4: the piezo transducer

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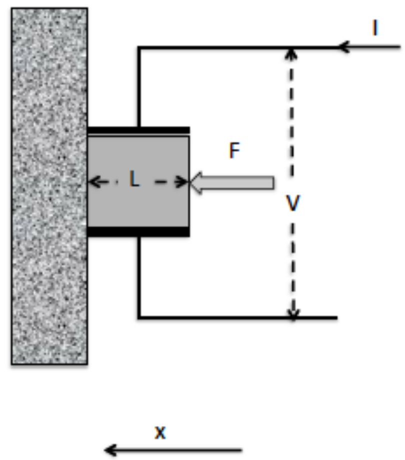
Experimental methods

Piezo transducer

A piezo electric crystal obeys the following equations in the Fourier domain:

$$\begin{aligned} \delta L &= -\frac{F}{k} + d\,V \\ I &= -d\,s\,F + s\,C\,V \end{aligned} \qquad (1)$$

L is the length of the piezo along the x axis and δL is its small signal variation, F is the component of the force applied to the piezo along the same axis, V is the voltage across the piezo along a direction normal to x and I is the corresponding current (see picture). All the quantities must be taken as small signal variations, so that the device is indeed linear



Furthermore s=$i\,\omega$ is the Laplace angular frequency, k is the mechanical stiffness of the piezo, C its total capacitance, and *d* the piezoelctric coefficient.
Mechanical losses give a small imaginary part to mechanical stiffness, so that:

$$k = k_o\,(1 + i\,\delta) \tag{2}$$

with all quantities in principle frequency dependent. In addition electrical losses within the device are well represented by a parallel output resistor of resistance R .
To the x-face of the piezo is rigidly attached a body of mass M. Within the chosen geometry, the coordinate x of the body is x= -δL and the force that the piezo exerts on the body is -F. Finally an

external force F_e may also be applied to the body. The final scheme is then that of the following figure

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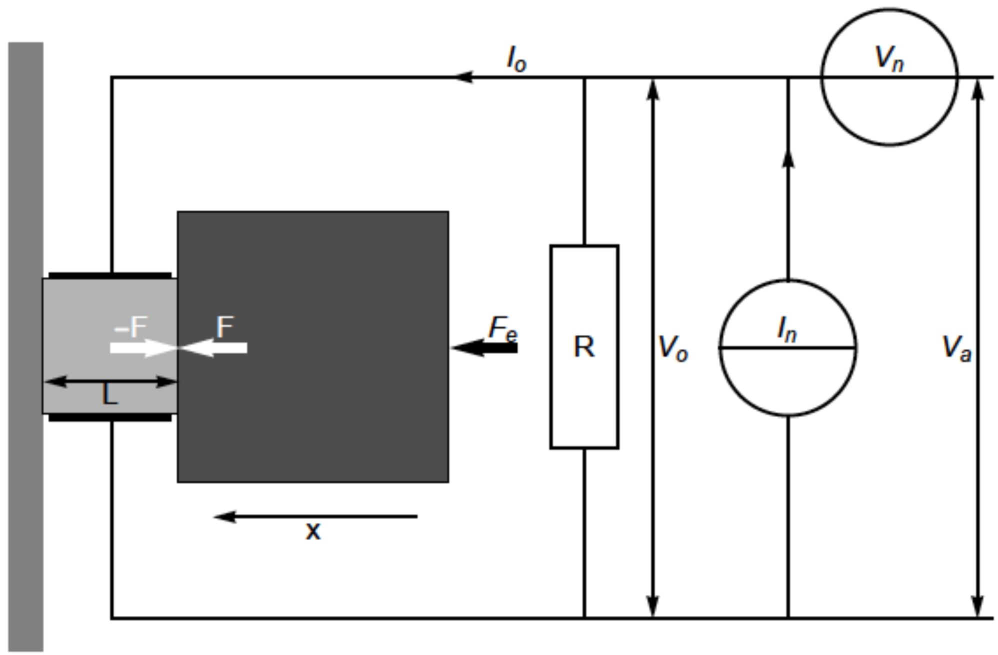

Numerical figures. Take the following numbers

$$
\begin{gathered}
C = 0.4\ \text{pF} \\
k = 8 \times 10^{8}\ \text{kg}/\text{s}^{2} \\
M = 0.1\ \text{kg} \\
d = -3\ \text{pm}/\text{V} \\
R = 10\ \text{M}\Omega \\
\delta = 10^{-3} \\
T = 20°\text{C}
\end{gathered}
\tag{3}
$$

Questions

1) give an equivalent block scheme of the device as a two port system taking F_e, and the coordinate x of the body as the relevant quantities for the input port , and $V_o$and I_o as the output ones
2) Write down input force and output voltage as a function of x and I_o
3) Write down the power spectrum of the sources of thermal noise connected to the resistor R and to the imaginary stiffness k'' respectively.
4) Write down the total power spectrum of x, due to the above sources of thermal noise. Assume that no device is attached neither to the output nor to the input.

Assume that the output port is attached to the amplifier with frequency independent noise temperature $T_n = 5\,K$ and resistance $R_n = 3 \times 10^5\,\Omega$. Assume that a short pulse of force, approximated as

$$F(t) = P\delta(t) \tag{4}$$

is applied to the system.

5) Calculate the voltage signal at the input V_a of the amplifier

6) Calculate the power spectrum of total equivalent noise at the input of the amplifier.

7) Calculate the spectrum of the equivalent input force that would generate the noise of question2.

8) Calculate the minimum possible uncertainty on the measurement of P.

9) Calculate the filter function required to achieve the resolution of question 1

For the sake of the calculations above, neglect the imaginary part of k

Exercise 5 the photodetector

A photo-detector consists of a photodiode connected to an amplifier (Figure 1).

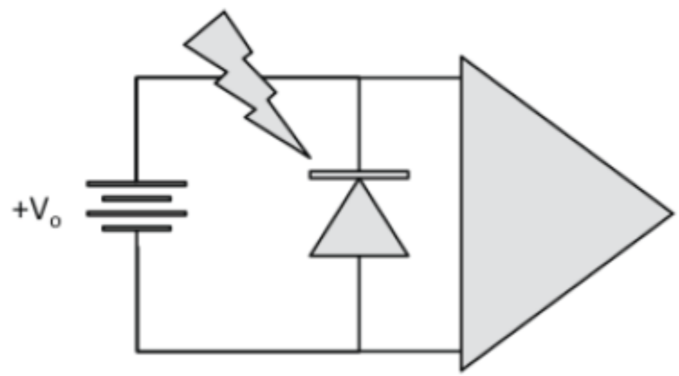

Figure 1 The photo-detector

The photodiode is kept in the so-called photoconductive mode, i.e. it is reverse biased. When light hits the photodiode, the diode behaves as a current generator that produces a photocurrent I, which is a function of the collected power W according to:

$$I = I(W) = I_{ph}(W) + I_{dark} \approx q\,W\frac{e\lambda}{hc} + I_{dark} \qquad 1$$

Where q is the so-called quantum efficiency, λ is the wavelength of light, and I_{dark} is a dc current that flows through the diode independently of W. To help the reader, I report the values of the fundamental constants appearing in eq. 1: $e = 1.6\times10^{-19}$ C; $h = 6.6\times10^{-34}$ J s; c = 3.0×10^{8} m/s. For future reference, I also report the value of the Boltzmann constant $k_B = 1.4\times10^{-23}$ J/K

The equivalent circuit in Figure 2, describes the losses, the dynamical behavior of the diode junction, and the noise properties of the amplifier.

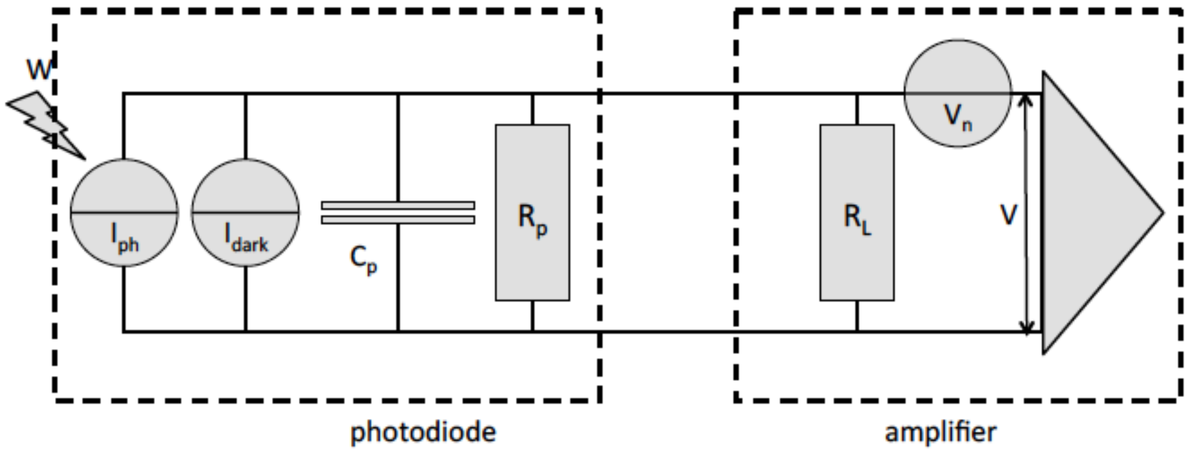

Figure 2 Schematics of photo-detector. I_{ph} and I_{dark} are defined in eq. 1. V_n is a white voltage generator, with power spectral density $S_{V_nV_n}$, that includes the effect of all noise sources in the amplifier. The (noiseless) input impedance of the amplifier has the purely resistive value R_L. The amplifier gain (frequency response) is just a constant, so all calculations can be performed at the amplifier input V. In the figure sources of noise within the diode are not explicitly represented.

A laser beam of wavelength λ and perfect stability (except for shot noise), illuminates the diode. Its amplitude varies in time according to the formula

$$W(t) = W_{dc} + \delta W(t) \qquad 2$$

Where:

$$|\delta W(t)| << W_{dc} \qquad 3$$

The dc current $q\, W_{dc}(e\lambda/hc) + I_{dark}$ causes a corresponding dc voltage V_{dc} at the output.

Considering $\delta W(t)$ as the input and $\delta V(t) = V(t) - V_{dc}$ as the output, calculate:

1. The frequency and the impulse response of the photo-detector (for the sake of answering this question, neglect the condition in eq. 3)
2. The total voltage noise PSD at the input of the amplifier. In the model of Figure 2, this due to
    a. Shot current noise due to dark current
    b. Shot noise in the laser beam
    c. Nyquist noise in the photodiode
    d. Amplifier noise
3. Express the noise as an equivalent input power noise, and calculate its PSD.
4. Calculate the minimum error of the estimate of W_o if $\delta W(t) = W_o \mathrm{Sinc}(2\,\pi\, f_o t)$
5. Calculate the "template" $h_W(t)$ by which one needs to multiply the data and integrate to obtain the best estimate in point 4.

Assume the numerical values in Table 1

Table 1. Numerical values for the exercise

Quantity	Value	Quantity	Value
R_L	1 MΩ	q	0.6
$S^{1/2}_{V_n V_n}$	2nV/√Hz	λ	1 μm
R_p	100 MΩ	T	293 K
C	1 pF	W_{dc}	0.1mW
I_{dark}	50 nA	f_o	3 Hz