Foreword

The final product of the exam must be a short report containing the answers to the various questions (Q) listed in the following section, together with a brief justification for those answers. The report must be in the form of a .pdf file and should be emailed, at the end of the exam, to stefano.vitale@unitn.it, with a copy to mario.scotoni@unitn.it, from your 'studenti' account. Please put 'EM_yourname' in the subject line. You are allowed, during the test, to use notes, books, and lecture slides. You are also allowed to consult Wikipedia and other databases. The use of computing tools like Mathematica, Matlab, or any other tool of your choice is warmly encouraged. You are allowed to use whatever method you find fit to answer the questions: analytical calculations, numerical calculations, simulations etc. You are allowed to approximate and neglect terms at your will, provided such approximations are physically sound and justified. You are not required to answer all the questions to obtain full credit. However, you are supposed to grasp the essence of the exercise. It is absolutely forbidden to exchange any information with third parties, either fellow students or external experts. Good luck!

Test

Please note: all needed numerical values are listed in table I on page 4.

The experiment

In many fields of modern physics, ranging from quantum communication to astrophysics, optical systems use InGaAs diodes as final, high sensitivity photodetectors [1]. Such diodes are operated in the so called photoconductive mode described in fig. 1, panel a. The diode is reverse biased, and the light collected by the diode causes a current $I_p = A \times W$, to flow through it, with W the value of the total collected optical power, and A a constant. In the standard feedback scheme of the figure, such current flows through the feedback branch, as

[1] See, for instance, https://doi.org/10.1515/nanoph-2015-0012

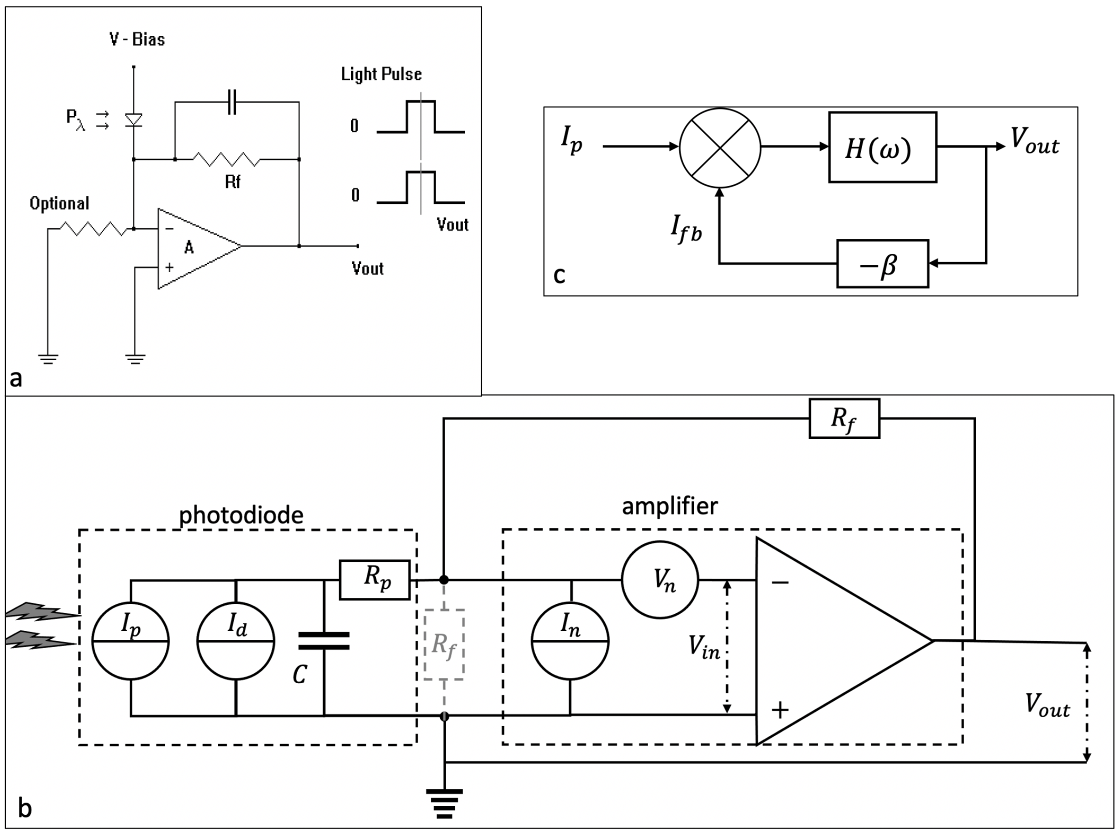

FIG. 1. Panel a: Schematic of a photodiode operated in the so called "photoconductive mode". Panel b: equivalent circuit of the photodetector of panel a. Panel c: Ideal feedback scheme used in the discussion. See text for the meaning of the symbols

the input impedance of the amplifier can be considered infinite. Notice that the diode is connected to the inverting input of the operational amplifier.

We will represent such photodetector arrangement with the equivalent circuit of panel b of fig. 1 (taken from the web). Notice that we have removed both the optional resistor of panel a, and the capacitor in parallel to the feedback resistor, and that we use R_f, instead of Rf, for the resistance of this one.

The circuit includes the I_p photocurrent generator, and that of the dark current I_d, permanently flowing through the photodiode, even at zero illumination, because of the reverse bias. The electrical properties of the diode are represented by a parallel capacitance $\mathcal{C}$ and a series resistance R_p, while the parallel resistance, for such kind of diodes, can be considered infinite.

The panel also shows the noise generators associated to the amplifier, which we assume to be uncorrelated, with a noise temperature $T_n\left(1+f_o/f\right)$, with f the frequency and T_n a constant, and a frequency independent noise resistance R_n. The photodiode is operated at thermal equilibrium, and we call T_{th} its thermodynamic temperature.

We assume that the amplifier has frequency independent gain: $V_{out}=-GV_{in}$.

As you probably know from electronics courses, a little care is needed when comparing the resistive feedback of panel a or b, with the ideal closed loop scheme of panel c. As in closed loop actual physical currents are flowing through R_f, to calculate the equivalent open loop transfer function $H(\omega)$ of the photodetector, it is not sufficient to just remove the resistive feedback branch, but this must also be replaced by a resistor, with same resistance R_f, across the input of the amplifier, as represented in panel b by the grey dashed resistor.

Just to make reference to some practical application, let us suppose that the light collected by one of the 8 meter diameter telescopes of the European Southern Observatory, is entirely focused on such photodetector, and the that the telescope is used to observe a star, of apparent (bolometric) magnitude m, during the time ΔT during which it remains within the useful telescope field of view. With such a telescope diameter, the power collected by the photodetector, from a star of apparent magnitude m, is $W=W_t10^{-0.4m}$ with W_t given in table I.

Questions

- Q1 List all sources of noise that affect the photodetector and give their respective Power Spectral Densities (PSD).
- Q2 Calculate, as a function of frequency, the total noise PSD of the detector due to the effect of sources above. Write it as the PSD of an equivalent input optical power W

Q3 Calculate the maximum magnitude (minimum power) which is observable, with a relative precision of 1, from the square wave signal, mentioned above, of amplitude $W_t 10^{-0.4m}$ and duration ΔT , due to the transit of the star through the field of view of the telescope

Q4 Calculate again the maximum observable magnitude, this time by assuming that the light from the telescope is sent to the photodetector through a fast light chopper (see for instance lecture 25), that multiplies it by the square wave modulation $\frac{1}{2}\left(1 + Sign\left(Sin(2\pi f_h t)\right)\right)$

Parameter	Value	Units	Parameter	Value	Units
A	0.7	V^{-1}	R_p	10	Ω
$\mathcal{C}$	2.5	pF	R_f	50	kΩ
T_n	100	K	R_n	1.7	kΩ
f_o	100	Hz	T_{th}	293	K
I_d	100	nA	ΔT	1	min
W_t	1.3	μW	f_h	500	Hz
G	10				

TABLE I. List of parameter values. We give a value for all parameters mentioned in the text. That does not mean that all values are needed for the calculations.