Date : 19 November 2020

Experimental Methods

Exercise on radiometric measurements

In radio-wave detection systems, noise is often described by a *noise temperature* $T_n(\omega)$ with ω the frequency of the radio wave. This is not the definition of noise temperature we discussed in connection with amplifiers, but is defined by assigning to each source of noise an equivalent input power (per unit frequency) hitting the antenna, equal to that from a black body at temperature T_n.

Remember that black-body radiation at these frequencies, correspond to a fluctuating electric field along any direction and at any point with Power Spectral Density (PSD) given by:

$$PSD_E(\omega) = \frac{4}{3}\frac{k_B T}{\varepsilon_o c^3}\omega^2 \qquad 1$$

Here c is the speed of light, k_B is the Boltzmann constant, and ε_o is the vacuum permittivity. This power will be transmitted to the final receiver and will produce a voltage noise whose PSD proportional to that of the electric field.

Figure 1 reports the noise temperature of various sources in the sky. The most important of them, for the sake of the present exercise, are those marked with B, F and E(90°). Superimposed to the original picture, taken from Recommendation ITU-R P.372-11, 09/2013 of the International Telecommunication Union, is a putative noise temperature curve (in red) of a radio-telescope *receiver*, including thermal noise, amplifier noise etc.

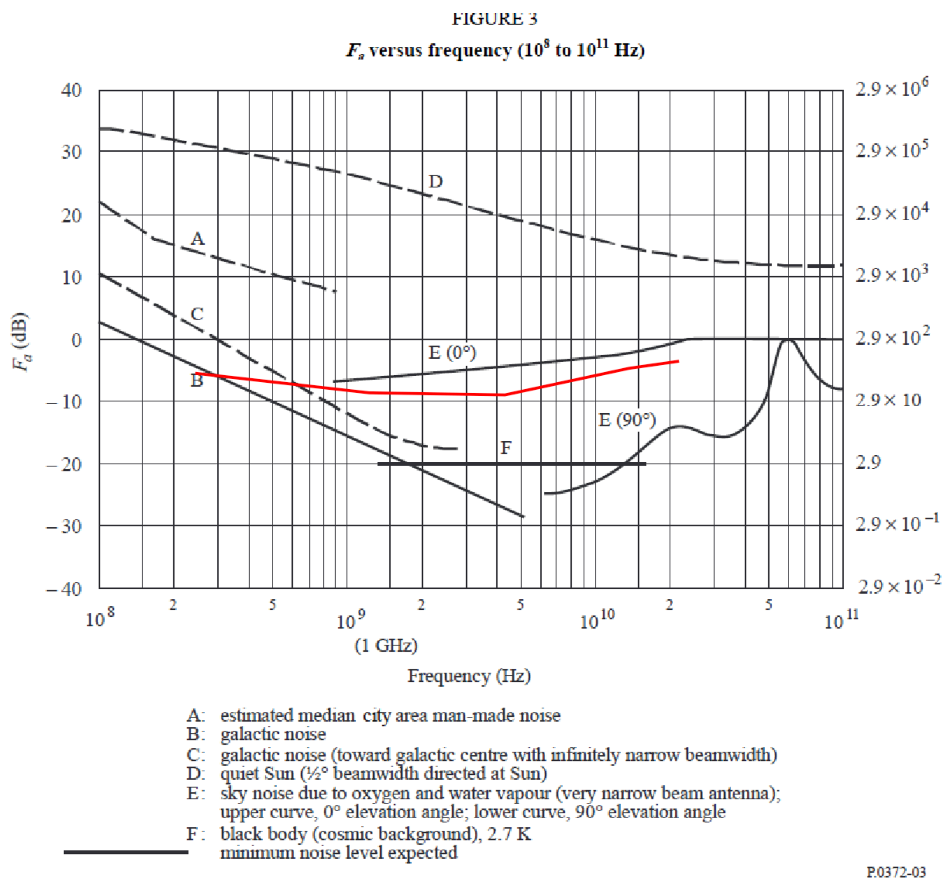

Figure 1 Noise temperature of various sources in the sky. The red line represents the contribution of a hypothetical receiver amplifier.

Noise from the sky can be discriminated from that due to the receiver itself, by switching the input of this to a cold resistor R_{cold}, say at T_{cold}=2 K (Figure 2). This can be shown to be equivalent to pointing the antenna to a black body at temperature T_{cold}.

The signal coming from the final amplifier is sent to a mixer and multiplied by a sinusoid of adjustable frequency f_{lo} generated by a local oscillator (Figure 2). The mixed signal is low-passed as in Figure 2.

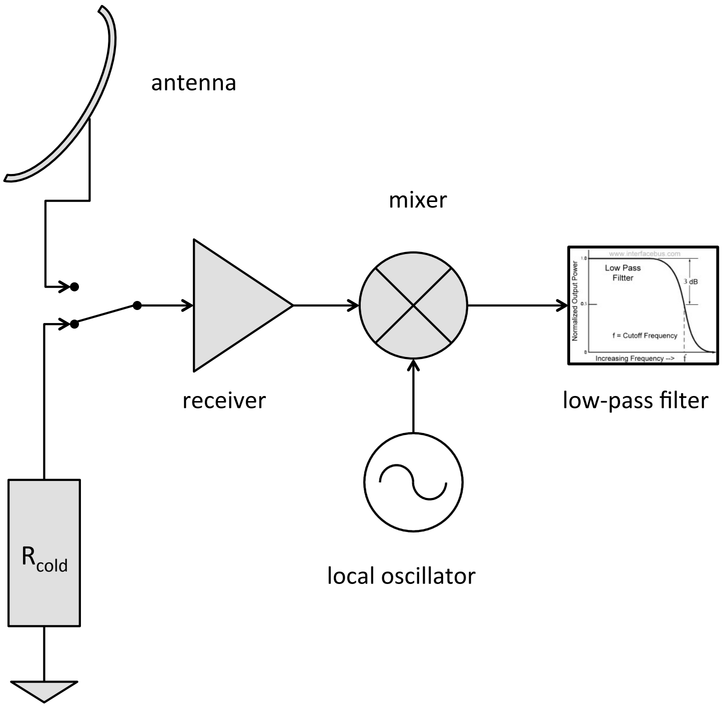

Figure 2 Schematics of a radio telescope.

The figure represents only one phase of such a phase sensitive detector, but the in-quadrature channel is usually also available.

Assume that the low-pass filter is a standard, "single pole" filter with transfer function:

$$h(\omega)=\frac{1}{1+i\omega/(2\pi f_o)} \qquad 2$$

Take f_o 10 MHz. For the rest of the exercise assume also that the measurements are done overnight with 90° elevation of the antenna. With this assumption the sky noise is just the superposition of lines B, F, and E(90°) of Figure 1

Now the questions.

1. If f_{lo}=700 MHz,
   1.1. Calculate the PSD of the noise at the output of the low-pass filter when the receiver is switched to the antenna. Express it as an equivalent input electric field PSD.
   1.2. Same when the receiver is connected to the cold resistor.
2. To measure the noise levels, the output of the low pass filter is squared and then integrated over a time t. Two measurement of equal duration are made, one with receiver connected to the antenna, and one with the receiver connected to the cold resistor. How long must t be to measure, by difference, the temperature of the noise coming from the antenna with better than 5% resolution?
3. Repeat all of the above for f_{lo} = 2 GHz and f_{lo} = 30 GHz.
4. A satellite sends to the antenna a signal

$$E(t) = E_o e^{-\frac{(t-t_o)^2}{2\Delta t^2}} Sin(2\pi f_s t) \qquad 3$$

Here f_s = 4 Ghz and Δt= 2 s. Here $E(t)$ is the electric field amplitude received by the antenna. What is the minimum achievable error σ_{E_o} in the measurement of E_o given the overall background noise?

5. Assume the signal has amplitude equal to σ_{E_o}. Write down the Fourier transform of the power deposited by the signal onto the antenna. To do that, remember that the intensity (power per unit surface) carried by a plane wave is connected to the electrical field by:

$$I = \frac{1}{2}\varepsilon_o c E^2 \qquad 4$$

Assume an antenna with an effective area of A=9 m^2.