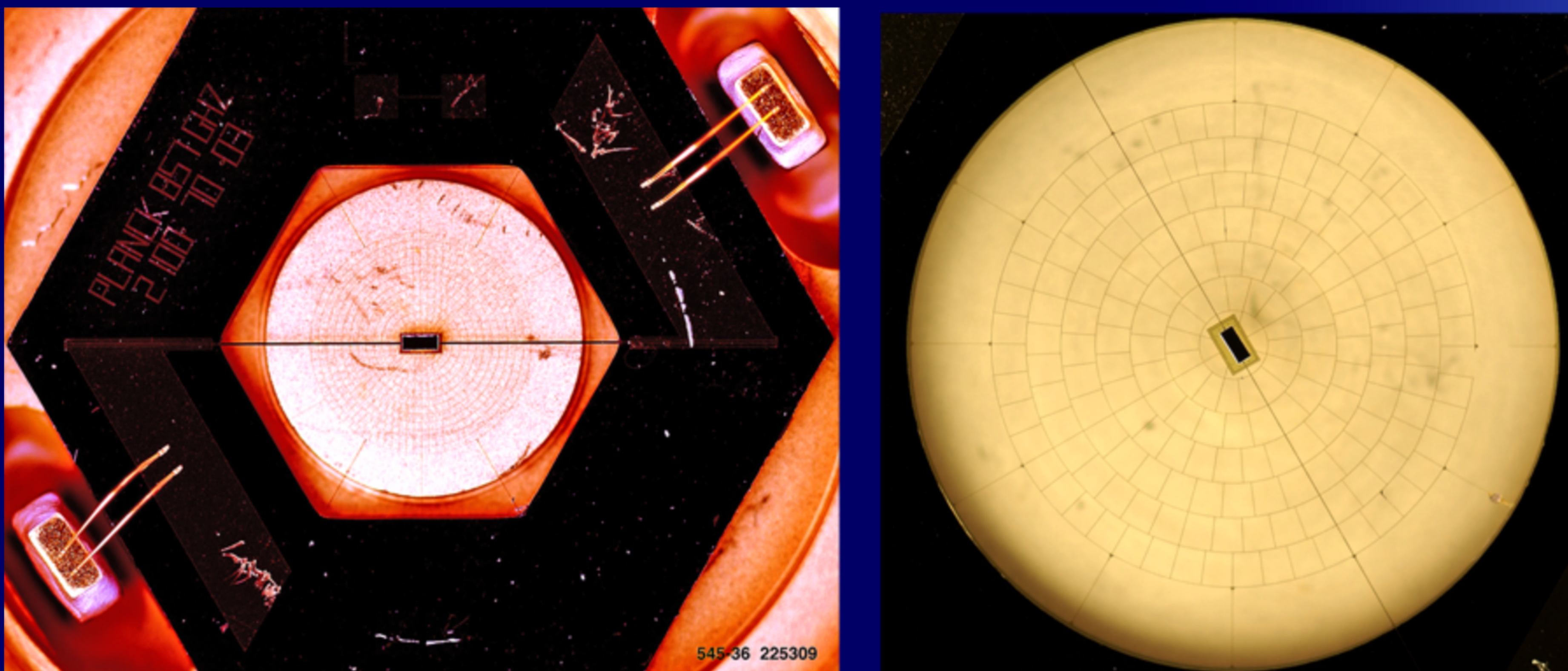
UNIVERSITÀ DEGLI STUDI DI TRENTO

Exercise on Bolometer

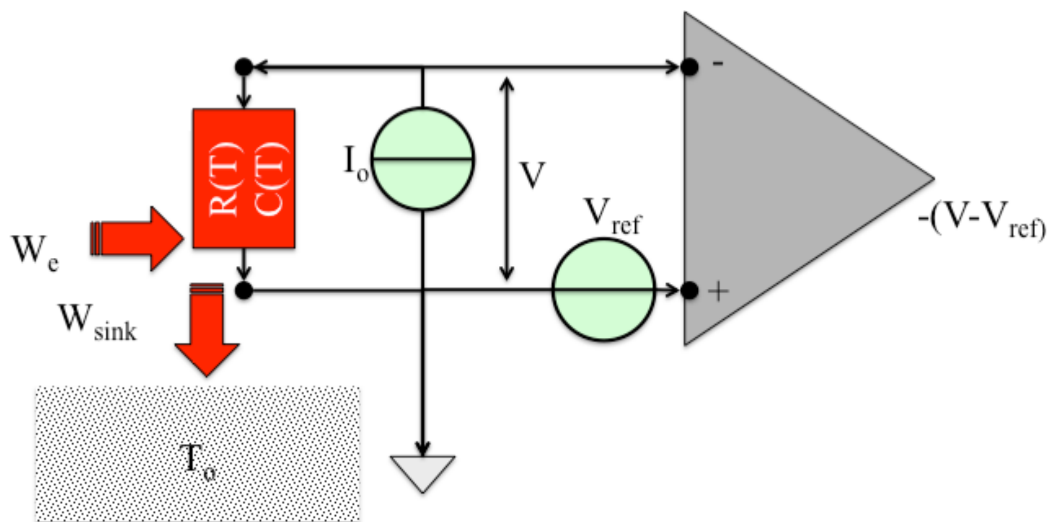
- A modern bolometer is miniature version of the calorimeter
- The calorimeter body behaves like a resistor with temperature dependent resistivity
- Thus the variations of the temperature T of the calorimeter can be measured by measuring the corresponding variation of its electrical resistance R(T)



Jet Propulsion Laboratory 545 GHz (left) and 100 GHz (right) spider-web bolometers for HFI. (from Holmes et al. (2008)). The web absorbs the light and the thermistor (small rectangle in the center of each web) measures the resulting increase in temperature. (from Holmes et al. (2008))

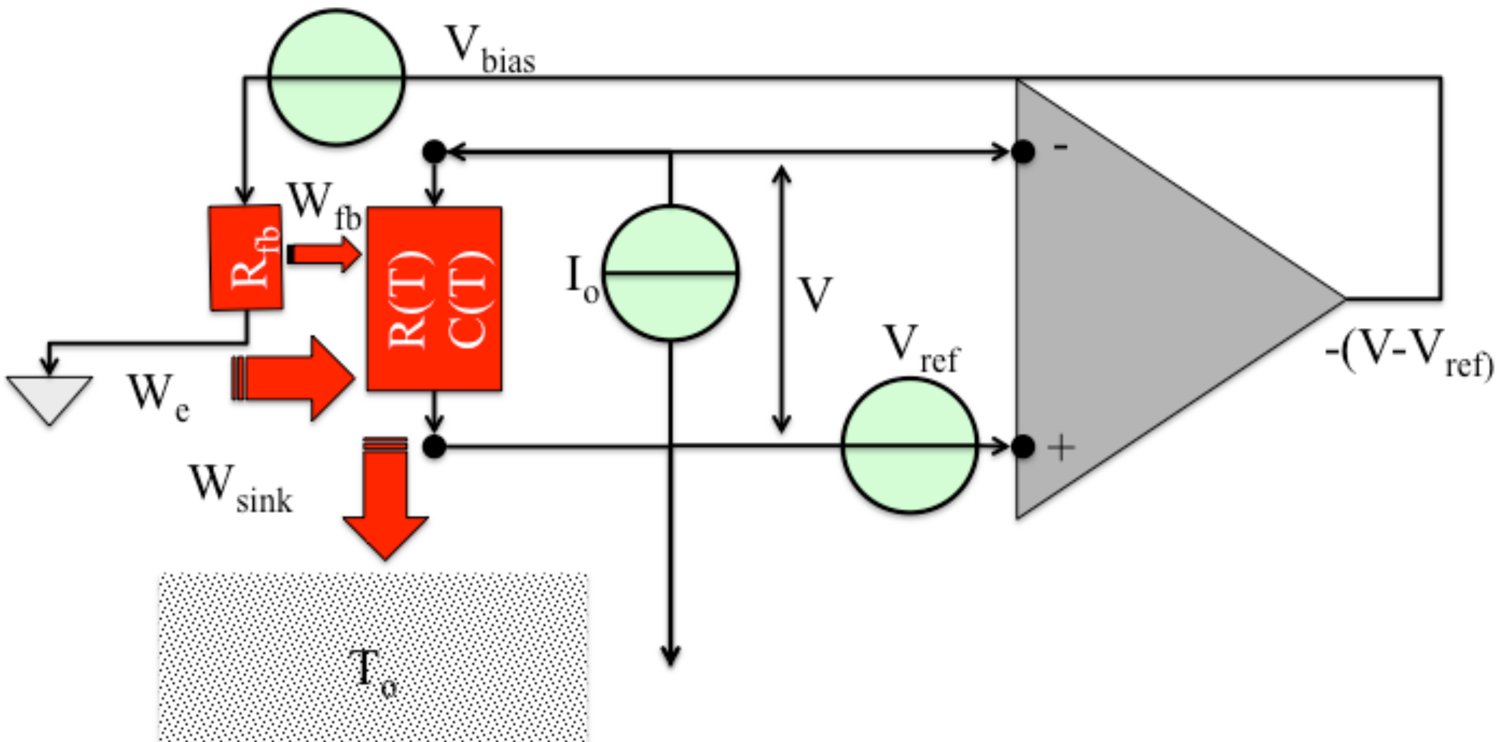
Model for a bolometer

- A constant bias current I_o, generated by a dc current generator, passing through the bolometer converts the change of resistance into a change of voltage V across the resistor. This is the final output of the instrument.
- The bolometer is in conductive thermal contact with a thermal sink at temperature T_o, with a fairly temperature independent conductance κ. Thus $W_{sink} = \kappa(T - T_o)$
- As for all solids at low enough temperatures, which is where bolometers are used nowadays, the heat capacity depends on temperature as $C(T) = \frac{C_o}{T_o^3} T^3$, with C_o a constant.
- The voltage V across the bolometer is measured by an ideal, infinite input impedance differential amplifier with gain G=1.
- The way the amplifier is biased, produces an output $-(V - V_{ref})$, with $V_{ref} = I_o R(T_o)$ so that the amplifier output is zero when $T = T_o$



A possible feedback loop

- The picture shows a possible feedback loop.
  - the output $-(V - V_{ref})$ of the amplifier, is superimposed to a positive dc voltage V_{bias} (the role of this will be clarified later). The sum of the two voltages makes a current $I_{fb} = \left(V_{bias} - (V - V_{ref})\right)/R_{fb}$ flow through a resistor, which is perfectly thermally coupled to the bolometer.
  - The heat W_{fb}, generated by I_{fb} via Joule effect in the resistor, is used to close the thermal feedback loop



Further notes

- In order for the bolometer to work properly the $\delta T = T - T_o$ must be so small that the all relevant equations might be linearized. The feedback loop is used to achieve this task.
- In the absence of external applied heath W_e, the bolometer will reach an equilibrium temperature $T_e = T_o + \Delta T$. Again, for the magnetometer to work properly, ΔT but be small.
- V_{bias} is needed to obtain a double-sided feedback: as the Joule effect is quadratic in the voltage, you need to apply some heat $W_{fbo} = V_{bias}^2/R_{fb}$ also for zero feedback signal. Then

$$\delta W_{fb} = W_{fb} - W_{fbo} = \frac{\left(V_{bias} - \left(V - V_{ref}\right)\right)^2}{R_{fb}} - \frac{V_{bias}^2}{R_{fb}}$$

  can take any sign, and warm up or cool down the bolometer relative to T_e
- In the whole operating range of the experiment

$$R(T) = R_o + \rho\,(T - T_o)$$

Numerical values

- $R_o = 1\ \Omega$
- $T_o = 10\ mK$
- $C_o = 150\ pJ/K$
- $\kappa = 10\ nW/K$
- $\rho = 2\ k\Omega/K$
- $I_o = 1\ \mu A$
- $R_{fb} = 100\Omega$
- $V_{bias} = 50\ \mu V$

Exercise

- Linearize the thermodynamic equations for δT and calculate the equilibrium value ΔT, that is the value for $W_e = 0$
- Calculate the voltage at amplifier output at equilibrium
$$V_e = V(T_o + \Delta T) - V_{ref}$$
- Calculate the equilibrium value for W_{fb}, W_{fbe}
- Consider W_e as a small input signal, and find the impulse and frequency responses $W_e \rightarrow \delta T' = \delta T - \Delta T$ and $W_e \rightarrow \delta V = V - V_{ref} - V_e$
- What would those be if $V_{bias} = 0$, that is, open loop?
- Find the impulse and frequency response $W_e \rightarrow \delta W_{fb} = W_{fb} - W_{fbe}$ and discuss how well $-\delta W_{fb}$ estimates W_e
- If $W_e = W_o Sin(2\pi f t)$ what is the maximum value that $|W_o|$ can take if one wants to keep $|\delta T'| \leq 1\ \mu K$.