Part 2

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(a) $x_{t+1} = mx_t + c$

$$x = mx + c$$

$$-c = (m-1)x$$

$$x = -\frac{c}{m-1} \text{ if } m \neq 1$$

(b) $x_{t+1} = mx_t + c$

x_0: initial value

$$x_1 = mx_0 + c$$

$$x_2 = mx_1 + c$$
$$= m(mx_0 + c) + c$$
$$= m^2x_0 + mc + c$$

$$x_3 = mx_2 + c$$
$$= m(m^2x_0 + mc + c) + c$$
$$= m^3x_0 + m^2c + mc + c$$

$$\vdots$$

$$x_n = m^nx_0 + m^{n-1}c + m^{n-2}c + \cdots + m^1c + m^0c$$

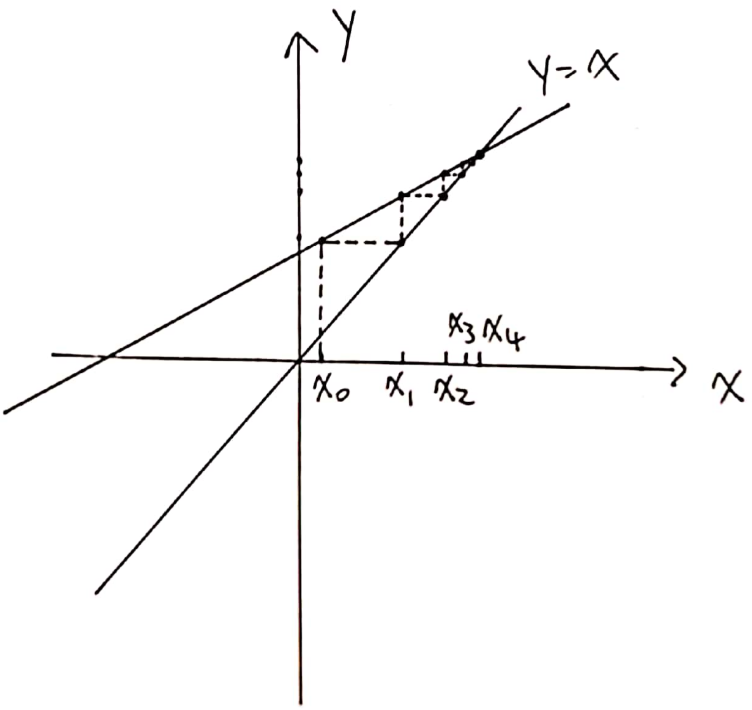
So $f(x_0, t) = m^tx_0 + m^{t-1}c + m^{t-2}c + \cdots + m^1c + m^0c$

$$= m^tx_0 + \sum_{i=0}^{t-1} m^ic$$

(C)

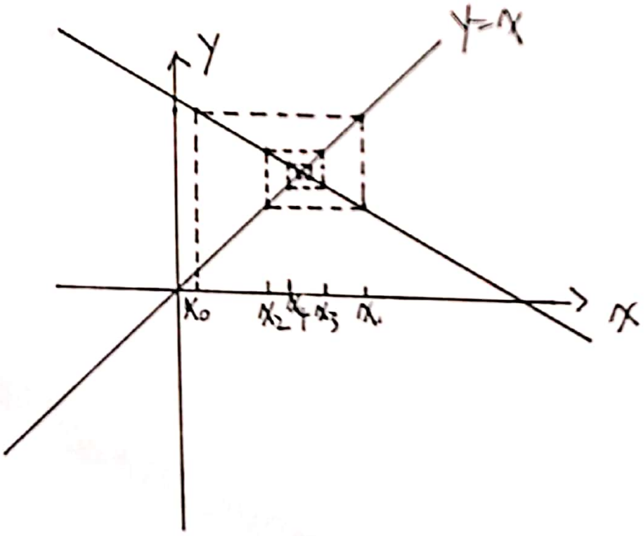
Case 1: $|m|<1$ $\quad -1<m<1$

① $0<m<1$



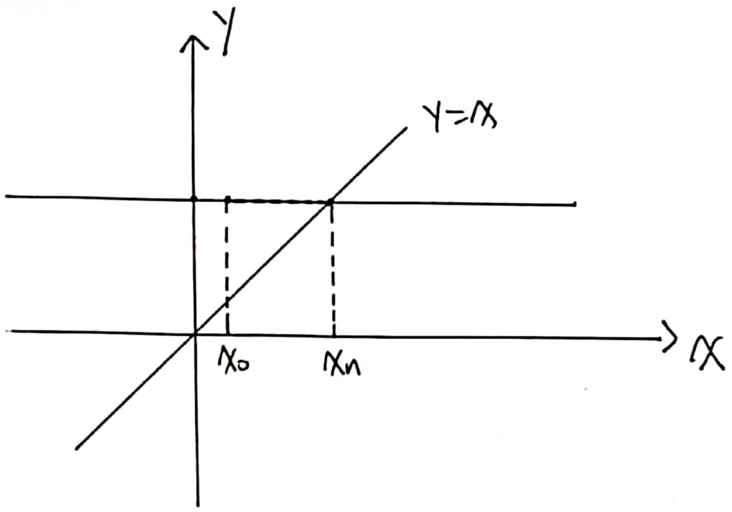
System will converge to a fixed point after n iterations.

② $-1<m<0$



System will converge to a fixed point after n iterations.

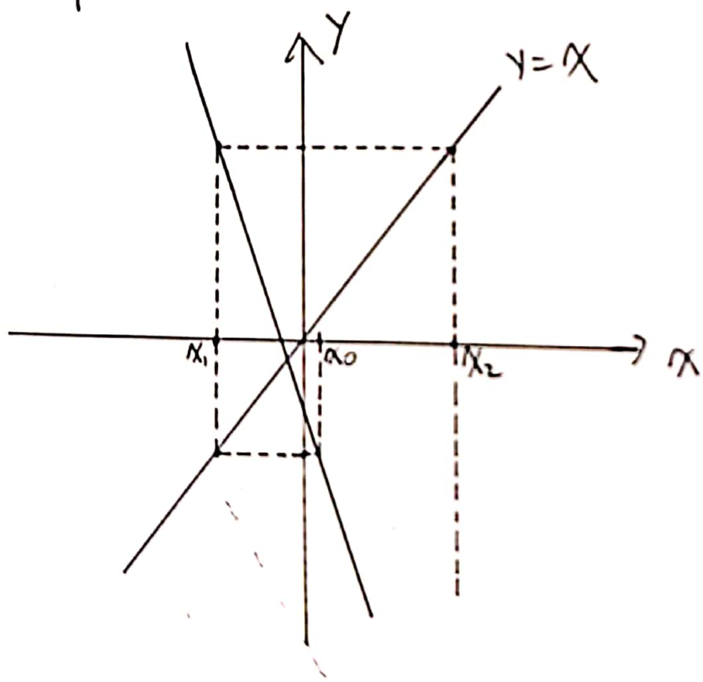
③ $m=0$



System will converge to a fixed point after n iterations

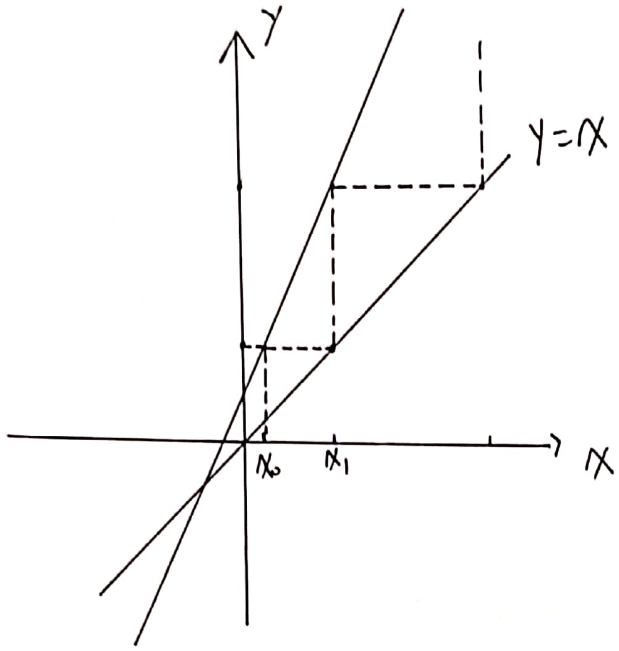
Case 2: $|m|>1$

① $m<-1$



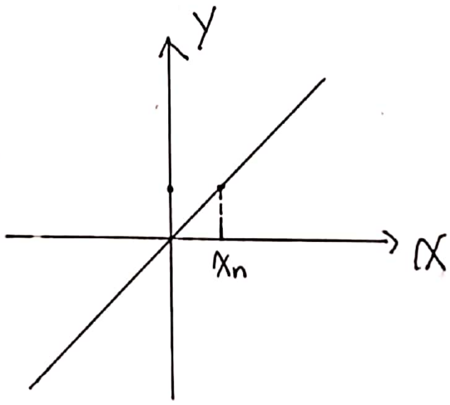
System will not converge, remain unstable.

② $m>1$



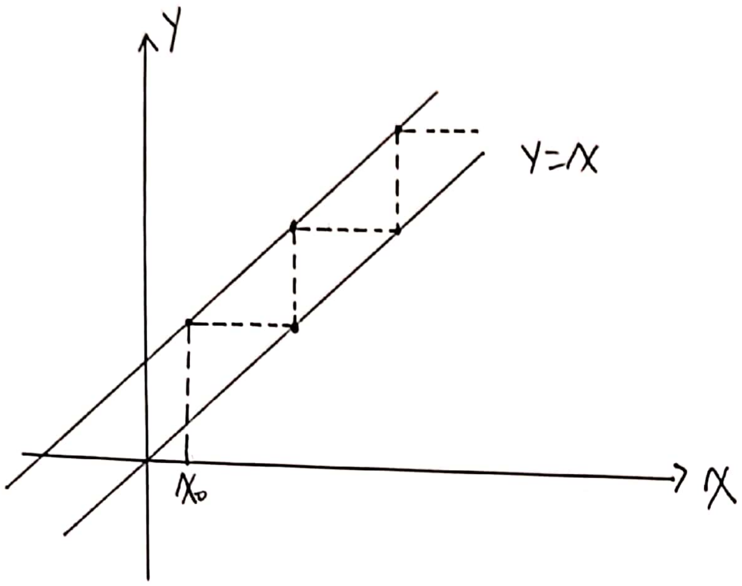
System will not converge, remain unstable.

Case 3: $m=1, c=0$



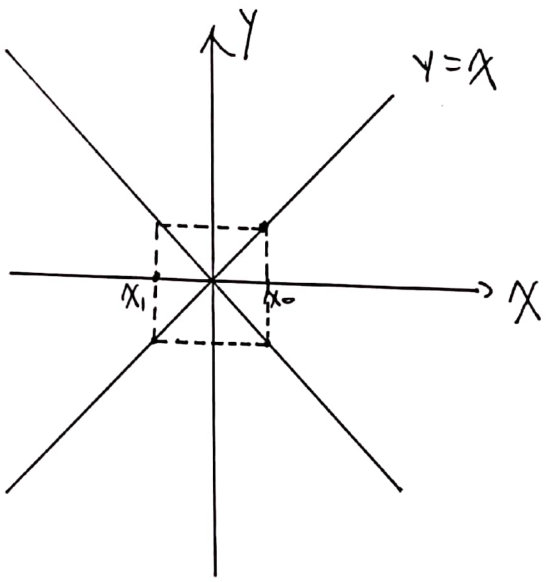
All points are same, system is stable.

Case 4: $m=1, c \neq 0$



System will not converge, remain unstable.

Case 5: $m=-1$



Divergent oscillation, system remain unstable.

(d)

$x_{t+1} = f(x_t)$

$x_{t+1} = mx_t + c$

So $f(x_t) = mx_t + c$

Perform a Taylor series expansion on $f(x_t)$

$f(x_t) \approx f(x^*) + f'(x^*)(x_t - x^*)$

$x_{t+1} = f(x^*) + f'(x^*)(x_t - x^*)$

$x_{t+1} = f'(x^*)x_t + (f(x^*) - f'(x^*)x^*)$

$x_{t+1} = mx_t + c$

where $m = f'(x^*)$ and $c = f(x^*) - f'(x^*)x^*$

from our finding of Part 2c, when $|m|<1$ and $m=1, c\neq 0$,

our system is stable.

Since f is a non-linear function, f is said to be locally stable if $|f'(x^*)<1|$