CH 5
5.1.4

4. Let P(n) be the statement that 13 + 23 + ⋯ + n3 = (n(n + 1)∕2)2 for the positive integer n. a) What is the statement P(1)? b) Show that P(1) is true, completing the basis step of the proof of P(n) for all positive integers n. c) What is the inductive hypothesis of a proof that P(n) is true for all positive integers n? d) What do you need to prove in the inductive step of a proof that P(n) is true for all positive integers n? e) Complete the inductive step of a proof that P(n) is true for all positive integers n, identifying where you use the inductive hypothesis. f) Explain why these steps show that this formula is true whenever n is a positive integer.

a. P(1) = $\left[\frac{1(1+1)}{2}\right]^2$

b. $1^3 =$ $\left[\frac{1(1+1)}{2}\right]^2$ $\left[\frac{1(1+1)}{2}\right]^2$ = $(\frac{2}{2})^2$ = 1 $1^3 = 1$ 1 = 1

c. P(k) = $\left[\frac{k(k+1)}{2}\right]^2$

d. Need to prove P(k) → P(k+1)

e. Need to show that $\left[\frac{k(k+1)}{2}\right]^2$ + $(k+1)^3$ = $\left[\frac{(k+1)(k+2)}{2}\right]^2$

$\left[\frac{(k+1)(k+2)}{2}\right]^2$ + $(k+1)^3$ = $\frac{1}{4}k^2(k+1)^2+(k+1)^3$ = $\frac{1}{4}(k+1)^2+(k^2+4(k+1))$ =
$\frac{1}{4}(k+1)^2+(k^2+4k+4)$ = $\frac{1}{4}(k+1)^2+(k+2)^2$ = $\left[\frac{(k+1)(k+2)}{2}\right]^2$

f. We have shown by proving the basis step and the inductive step that P(1) is true and that P(k) → P(k+1) so we have shown that it is true for every positive n by the rules of mathematical induction.

5.1.56

Suppose that A = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, where a and b are real numbers. Show that A^n = $\begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$ for every positive integer n

Basis: P(1) is true A^1= $\begin{bmatrix} a^1 & 0 \\ 0 & b^1 \end{bmatrix}$

Inductive step: Show that $A^k \rightarrow A^{k+1}$

$A^{k+1} = \begin{bmatrix} a^{k+1} & 0 \\ 0 & b^{k+1} \end{bmatrix}$

Show that $A^{k+1} = A * A^k$

$A * A^k = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} a^k & 0 \\ 0 & b^k \end{bmatrix} = \begin{bmatrix} a^k * a + 0 * 0 & 0 * 0 + 0 * 0 \\ 0 * 0 + 0 * 0 & b^k * b + 0 * 0 \end{bmatrix} = \begin{bmatrix} a^{k+1} & 0 \\ 0 & b^{k+1} \end{bmatrix}$ Therefore P(k+1) is true

5.2.2
Use strong induction to show that all dominoes fall in an infinite arrangement of dominoes if you know that the first three dominoes fall, and that when a domino falls, the domino three farther down in the arrangement also falls.

With strong induction we assume that the statement holds true for all values preceding k.
Basis step: P(1), P(2), and P(3) are true given the question
Inductive: Prove P(k) → P(k+1) , if P(k) falls we can assume all values preceding k are true which means P(k-2) is true. The problem also states that if a domino falls than the domino three down will fall so P(k-2+3) is P(k+1) and is true.

5.3.18

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ Show that $A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$ when n is a positive integer

Base step P(1) is true $A^1 = \begin{bmatrix} f_2 & f_1 \\ f_1 & f_0 \end{bmatrix}$

Inductive Step: P(k) is true $A^k = \begin{bmatrix} f_{k+1} & f_k \\ f_k & f_{k-1} \end{bmatrix}$ show that P(k+1) is true $A^{k+1} = \begin{bmatrix} f_{k+2} & f_{k+1} \\ f_{k+1} & f_k \end{bmatrix}$

$A * A^k = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} f_{k+1} & f_k \\ f_k & f_{k-1} \end{bmatrix} = \begin{bmatrix} f_{k+1} * 1 + 1 * f_k & 1 * f_k + 1 * f_{k-1} \\ f_k * 0 + 1 * f_{k+1} & f_{k-1} * 0 + 1 * f_k \end{bmatrix} = \begin{bmatrix} f_{k+2} & f_{k+1} \\ f_{k+1} & f_k \end{bmatrix}$

5.4.32

Devise a recursive algorithm to find the nth term of the sequence defined by $a_0 = 1$, $a_1 = 2$, $a_2 = 3$, and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, for n = 3, 4, 5, …

```
Recursive(n)
if n ==0
        return 1
if n == 1
        return 2
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```
if n == 2
        return 3
else
        return Recursive(n-1) + Recursive(n-2) + Recursive(n-3)
```

5.5.6

Use the rule of inference developed in Exercise 5 to verify that the program

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if x < 0 then
y := −2|x|/x
else if x > 0 then
y := 2|x|/x
else if x = 0 then
y := 2
```

is correct with respect to the initial assertion T and the final assertion y = 2.

When the initial assertion x < 0 is true the statement y := −2|x|/x is executed and the final assertion y :=2 is true (i.e. x=-3; y=-2|-3|/-3 = 2) when the initial assertion is false the x is either greater than or equal to zero and will execute either y := 2|x|/x(i.e. x=-3; y=2|3|/3 = 2) or y:= 2 in both cases the final assertion y = 2 is true, hence using the rule of inference for program segments of this type the program is correct with respect to the initial and final assertions.