Appendix B : Code

2025-10-16

Coding for this section was completed using RStudio 2024.09.0+375 ("Cranberry Hibiscus" Release) and was based on R and MATLAB code provided by Professor Tim Gebbie(STA4028Z).

2.1 Libraries (Gebbie, 2025d)

```
# load required libraries
suppressPackageStartupMessages({
library(openxlsx)
library(timeSeries)
library(xts)
library(zoo)
library(matrixStats)
library(quadprog)
library(quidprog)
library(dplyr)
library(dplyr)
library(tidyr)
library(PerformanceAnalytics)
})
```

2.2 Load data and preprocessing (Gebbie, 2025d)

```
# reading in all 4 sheets into a list
dfS <- list()
for (i in 1:4) {
   dfS[[i]] <- read.xlsx("_raw_data/PT-TAA-JSE-Daily-1994-2017.xlsx", sheet = i, detectData("Sheet", i, "loaded with dimensions:", dim(dfS[[i]]), "\n")
}</pre>
```

```
## Sheet 1 loaded with dimensions: 8439 2
## Sheet 2 loaded with dimensions: 8405 4
## Sheet 3 loaded with dimensions: 8439 28
## Sheet 4 loaded with dimensions: 8439 20
# define entities and which assets to keep
Entities <- c('X1', 'STEFI', 'ALBI', 'J203', 'J500', sprintf("J5%d", seq(10,90,by=10)))</pre>
Items
         <- c('Date', 'TRI', 'Stefi')
#cleaning each sheet
for (i in 1:4) {
  tIO <- sapply(colnames(dfS[[i]]), function(x) any(grepl(paste(Entities, collapse="|")
  tI1 <- sapply(dfS[[i]][2,], function(x) any(grepl(paste(Items, collapse="|"), x)))
  tI <- tIO & tI1
  # remove header rows
  dfS[[i]] \leftarrow dfS[[i]][-c(1,2), tI]
  names(dfS[[i]])[1] <- "Date"</pre>
 newColNames <- strsplit(colnames(dfS[[i]]), ":")</pre>
  for(m in 2:length(newColNames)) names(dfS[[i]])[m] <- newColNames[[m]][1]</pre>
  cat("Sheet", i, "columns after cleaning:", colnames(dfS[[i]]), "\n")
}
## Sheet 1 columns after cleaning: Date ALBI
## Sheet 2 columns after cleaning: Date RATESTEFI
## Sheet 3 columns after cleaning: Date J500 J510 J520 J530 J540 J550 J560 J580 J590
## Sheet 4 columns after cleaning: Date J203
# fixing ALBI column
dfS[[1]][,2] <- as.numeric(dfS[[1]][,2])
dfS[[1]] \leftarrow dfS[[1]][!is.na(dfS[[1]][,2]), ]#removes rows where ALBI is NA
2.3 Merge into single timeSeries object (Gebbie, 2025d)
# converts first sheet to timeSeries
tsTAA <- timeSeries(dfS[[1]][, 2:ncol(dfS[[1]])], as.Date(dfS[[1]][,1]))
```

```
cat("Initial tsTAA dimensions:", dim(tsTAA), "\n")
## Initial tsTAA dimensions: 4324 1
# merges remaining sheets
for (i in 2:4) {
  tsTmp <- timeSeries(dfS[[i]][, 2:ncol(dfS[[i]])], as.Date(dfS[[i]][,1]))
 tsTAA <- cbind(tsTAA, tsTmp)
  cat("After merging sheet", i, "dimensions:", dim(tsTAA), "\n")
}
## After merging sheet 2 dimensions: 8437 2
## After merging sheet 3 dimensions: 8437 11
## After merging sheet 4 dimensions: 8437 12
# renaming indices for clarity
setFinCenter(tsTAA) <- "Johannesburg"</pre>
names(tsTAA)[grep("TS.1.1", names(tsTAA))] <- "ALBI"</pre>
names(tsTAA)[grep("TS.1.2", names(tsTAA))] <- "STEFI"</pre>
names(tsTAA)[grep("TS.1", names(tsTAA))] <- "ALSI"</pre>
cat("Columns after renaming:", colnames(tsTAA), "\n")
## Columns after renaming: ALBI STEFI J500 J510 J520 J530 J540 J550 J560 J580 J590 ALSI
#all numeric columns are numeric
for (j in 1:ncol(tsTAA)) {
  tsTAA[, j] <- as.numeric(tsTAA[, j])
}
#remove rows with all NAs
tsTAA <- tsTAA[rowSums(is.na(tsTAA)) < ncol(tsTAA), ]
# Using timeSeries daily2monthly and ensure tsTAA is valid
tsTAA_monthly <- tryCatch(
  daily2monthly(tsTAA),
  error = function(e) {
    stop("Error in daily2monthly: tsTAA might contain non-timeSeries columns or non-nume
  }
)
```

```
# monthly price index
tsIdx <- index2wealth(tsTAA monthly)</pre>
# geometric monthly returns
tsGRet <- diff(log(tsIdx))</pre>
cat("tsTAA_monthly dimensions:", dim(tsTAA_monthly), "\n")
## tsTAA monthly dimensions: 261 12
cat("tsGRet dimensions:", dim(tsGRet), "\n")
## tsGRet dimensions: 261 12
cat("Columns in tsGRet:\n"); print(colnames(tsGRet))
## Columns in tsGRet:
## [1] "ALBI" "STEFI" "J500" "J510" "J520" "J530" "J540" "J550" "J560"
## [10] "J580" "J590" "ALSI"
2.4 Arithmetic Returns (Gebbie, 2025c)
setFinCenter(tsTAA) <- "Africa/Johannesburg"</pre>
summary(dfS[[1]][,2])
##
     Min. 1st Qu. Median
                             Mean 3rd Qu.
                                              Max.
     173.7 256.9
##
                     343.9
                             357.5
                                   442.0
                                             545.9
# Checks that tsTAA is a proper 'timeSeries' object
tsTAA monthly <- tryCatch(
 daily2monthly(tsTAA),
 error = function(e) {
   message("Error in daily2monthly(): converting tsTAA to xts first")
   xts obj <- as.xts(tsTAA)</pre>
   apply.monthly(xts_obj, colMeans, na.rm=TRUE)
 }
```

```
#geometric returns
tsGRet <- diff(log(tsTAA monthly))</pre>
# fill missing data using LOCF
tsGRet filled <- na.locf(as.xts(tsGRet), na.rm = FALSE)
summary(tsGRet filled[,"ALBI"])
##
        Index
                                           ALBI
## Min.
           :1995-06-30 00:00:00.00
                                      Min.
                                             :-0.06908
    1st Qu.:2000-11-30 00:00:00.00
                                      1st Qu.:-0.00232
## Median :2006-04-30 00:00:00.00
                                      Median : 0.00362
## Mean
           :2006-04-30 22:31:43.45
                                      Mean : 0.00701
   3rd Qu.:2011-09-30 00:00:00.00
                                      3rd Qu.: 0.01581
##
                                      Max.
##
   Max.
           :2017-02-28 00:00:00.00
                                             : 0.16900
##
                                      NA's
                                             :99
any(!is.na(tsGRet_filled[,"ALBI"]))
## [1] TRUE
#checking for columns that are all NA
cols_allNA <- colSums(!is.na(tsGRet_filled)) == 0</pre>
tsGRet filled <- tsGRet filled[, !cols allNA]</pre>
# converting to arithmetic returns
simple mat <- exp(as.matrix(tsGRet filled)) - 1</pre>
rets_xts <- xts(simple_mat, order.by = index(tsGRet_filled))</pre>
colnames(rets xts) <- colnames(tsGRet filled)</pre>
# Excludes cash asset
cash_idx <- grep("STEFI", colnames(rets_xts), ignore.case = TRUE)</pre>
cash_name <- ifelse(length(cash_idx) > 0, colnames(rets_xts)[cash_idx[1]], NA)
rets opt <- if(!is.na(cash name)) rets xts[, -cash idx, drop=FALSE] else rets xts
rets cash <- if(!is.na(cash name)) rets xts[, cash idx, drop=FALSE] else NULL
cat("Assets used for optimisation:\n"); print(colnames(rets opt))
```

```
## Assets used for optimisation:
## [1] "ALBI" "J500" "J510" "J520" "J530" "J540" "J550" "J560" "J580" "J590"
## [11] "ALSI"
if(!is.na(cash_name)) cat("Cash excluded from optimisation:", cash_name, "\n")
## Cash excluded from optimisation: STEFI
```

2.5 Black-Litterman function (Gebbie, 2025c, 2025d)

```
# Function: blacklitterman
# -----
# BLACKLITTERMAN computes the posterior mean vector, posterior covariance
# matrix, and optimal Black-Litterman portfolio weights.
# Inputs:
   Pi: (n \times 1) vector of equilibrium excess returns (prior mean)
   Sigma: (n \times n) covariance matrix of excess returns
          : (k \times n) matrix defining k linear views on asset returns
          : (k x 1) vector of view returns
#
   Q
#
   Omega: (k x k) diagonal covariance matrix of view uncertainty
   tau : scalar (typically small, e.g. 0.05), controls prior confidence
   gamma : scalar, risk aversion parameter
#
   constrain: logical, if TRUE set negative weights to zero and renormalise
#
# Outputs:
   A list containing:
       $weights - posterior (Black-Litterman) portfolio weights
       $mu post - posterior mean vector E[R | views]
       $Sigma post - posterior covariance matrix Cov[R | views]
blacklitterman <- function(Pi, Sigma, P, Q, Omega, tau = 0.05, gamma = 1, constrain = The
```

```
# Define matrix dimensions and inverses
# -----
n <- length(Pi) # number of assets
Sigma inv <- solve(Sigma) # inverse of prior covariance
Omega inv <- solve(Omega) # inverse of view covariance</pre>
# -----
# Calculate posterior mean (Black-Litterman expected returns)
# Formula: * = [(\Sigma)^{(-1)} + P'\Omega^{(-1)}P]^{(-1)}[(\Sigma)^{(-1)}\Pi + P'\Omega^{(-1)}Q]
# -----
mu_post <- solve(Sigma_inv * tau + t(P) %*% Omega_inv %*% P) %*%</pre>
        (Sigma inv %*% (tau * Pi) + t(P) %*% Omega_inv %*% Q)
# -----
# Calculate posterior covariance matrix
# Formula: \Sigma * = \Sigma + [(\Sigma)^{(-1)} + P'\Omega^{(-1)}P]^{(-1)}
# (Note: equivalent to \Sigma* = [(\Sigma/)^{(-1)} + P'\Omega^{(-1)}P]^{(-1)} under scaling)
# -----
Sigma post <- Sigma + solve(Sigma inv * tau + t(P) %*% Omega inv %*% P)
# ------
# optimal portfolio weights
# Formula: w* = (1/) \Sigma * (-1) *
# -----
w <- solve(gamma * Sigma_post) %*% mu_post
# -----
# non-negativity constraint and normalisation
# -----
if (constrain) {
 w[w < 0] <- 0 # set short positions to zero
 w <- w / sum(w) # re-normalise to full investment
}
# -----
# Assign asset names
```

2.5 a. Test Case

2.6 Rolling Window Experiment (Gebbie, 2025c, 2025d)

```
### Rolling-window Black-Litterman Backtest
train.m <- 60  # 5 yr training period
test.m <- 12  # 1 yr test period
roll_step <- 1  # 1-mthincrements
n_obs <- nrow(rets_opt)
start_idxs <- seq(1, n_obs - train.m - test.m + 1, by=roll_step)
results <- list()</pre>
```

```
prev w <- rep(0, ncol(rets opt)) # initialising previous weights for turnover
# ALSI column for beta calc
benchmark <- tsGRet filled[, "ALSI", drop=FALSE]</pre>
for(i in seq_along(start_idxs)){
  s <- start_idxs[i]</pre>
  train idx <- s:(s+train.m-1)</pre>
  tst.idx <- (s+train.m):(s+train.m+test.m-1)
  train_rets <- rets_opt[train_idx, , drop=FALSE]</pre>
  tst_rets <- rets_opt[tst.idx, , drop=FALSE]</pre>
  bench train <- benchmark[train idx, , drop=FALSE]</pre>
  bench_test <- benchmark[tst.idx, , drop=FALSE]</pre>
  # invalid windows
  if(any(!is.finite(as.matrix(train rets))) | any(!is.finite(as.matrix(tst rets)))) ne
            <- colMeans(train_rets, na.rm=TRUE)</pre>
  Sigma_train <- cov(as.matrix(train_rets), use="complete.obs")</pre>
  # Risk-free
  rf_train <- if(!is.null(rets_cash)) mean(rets_cash[train_idx, ], na.rm=TRUE) else 0</pre>
  rf_test <- if(!is.null(rets_cash)) mean(rets_cash[tst.idx, ], na.rm=TRUE) else 0
  # Black-Litterman views
  P <- matrix(0, nrow=1, ncol=ncol(train_rets))</pre>
  P[1, 1:3] <- 1/3
                      # simple view
  Q <- matrix(0.01, nrow=1) # expected 1% return
  Omega <- matrix(0.0001, nrow=1) # view uncertainty</pre>
  bl_res <- blacklitterman(Pi=mu_train, Sigma=Sigma_train, P=P, Q=Q, Omega=Omega, tau=O
  # Constraint weights: no short, fully invested
  w_hat <- bl_res$weights</pre>
  w hat[w hat < 0] <- 0</pre>
```

```
w_hat <- w_hat / sum(w_hat)</pre>
  turnover <- sum(abs(w hat - prev w))</pre>
  prev_w <- w_hat</pre>
  # Portfolio returns
  port_train <- as.numeric(as.matrix(train_rets) %*% w_hat)</pre>
  port test <- as.numeric(as.matrix(tst rets) %*% w hat)</pre>
  # Tracking Error relative to ALSI
port_diff <- port_test - as.numeric(bench_test)</pre>
t.err <- sd(port_diff, na.rm = TRUE)</pre>
  # Jensen Alpha & Beta relative to ALSI
xcess.p.train <- port_train - rf_train</pre>
xcess.b.train <- as.numeric(bench_train) - rf_train</pre>
CAPM train <- lm(xcess.p.train ~ xcess.b.train)
alpha_IS <- coef(CAPM_train)[1]</pre>
beta_IS <- coef(CAPM_train)[2]</pre>
# 00S excess returns
excess_port_test <- port_test - rf_test</pre>
excess_bench_test <- as.numeric(bench_test) - rf_test</pre>
CAPM_test <- lm(excess_port_test ~ excess_bench_test)</pre>
alpha_00S <- coef(CAPM_test)[1]</pre>
beta OOS <- coef(CAPM test)[2]
  # Cumulative equity curve
eq_curve_test <- cumprod(1 + port_test)
  # Results
  results[[length(results)+1]] <- list(</pre>
  train_period = paste(index(train_rets)[1], index(train_rets)[nrow(train_rets)], sep="
  test_period = paste(index(tst_rets)[1], index(tst_rets)[nrow(tst_rets)], sep=" / "),
  mu IS = mean(port train, na.rm=TRUE),
```

```
var IS = var(port train, na.rm=TRUE),
  SR_IS = (mean(port_train, na.rm=TRUE) - rf_train)/sqrt(var(port_train, na.rm=TRUE)),
  mu OOS = mean(port test, na.rm=TRUE),
  var OOS = var(port test, na.rm=TRUE),
  SR_OOS = (mean(port_test, na.rm=TRUE) - rf_test)/sqrt(var(port_test, na.rm=TRUE)),
  alpha IS = alpha IS,
  beta_IS = beta_IS,
  alpha OOS = alpha OOS,
  beta_00S = beta_00S,
  turnover = turnover,
  t.err = t.err,
  eq_curve_test = eq_curve_test,
  weights = w hat,
  assets = colnames(rets_opt)
}
# makes it easier to plot
summary_df <- do.call(rbind, lapply(results, function(x) data.frame(</pre>
  train=x$train_period, test=x$test_period,
 mu IS=x$mu IS, var IS=x$var IS, SR IS=x$SR IS,
  mu_00S=x$mu_00S, var_00S=x$var_00S, SR_00S=x$SR_00S,
  alpha IS=x$alpha IS, beta IS=x$beta IS,
  alpha_00S=x$alpha_00S, beta_00S=x$beta_00S,
 turnover=x$turnover,
 t.err = x$t.err
)))
knitr::kable(summary df,
  digits = 4,
  caption = "In-sample vs Out-of-sample BL Portfolio Statistics"
)
```

Table 1: In-sample vs Out-of-sample BL Portfolio Statistics

train	test	mu_tsr_tsr_tsr_tsu_0x0s_0tos_0tos_0tos_terr
(Interce 200) 3-09-	2008-09-	0.0186.0013.3252 - 0.0048 - 0.00140.7444 - 0.7920.0000.024
30 /	30 /	$0.0048 \qquad 0.1914 \qquad \qquad 0.0020$
2008-08-	2009-08-	
31	31	
(Interce 200) \$-10-	2008-10-	0.0167.0015.2488.0038.0036 - 0.0009.7493 - 0.77630.09790.0236
31 /	31 /	0.0830 0.0034
2008-09-	2009-09-	
30	30	
(Interce 200)2 -11-	2008-11-	0.0145.00167.1838.0158.0026.1474.00130.7612 - 0.77960.06490.0209
30 /	30 /	0.0030
2008-10-	2009-10-	
31	31	
(Interce 200)3 -12-	2008-12-	0.0148.0016.1939.0144.0026.1349.00160.7484 - 0.77840.07910.0209
31 /	31 /	0.0045
2008-11-	2009-11-	
30	30	
(Interce 200)4 -01-	2009-01-	0.014 2 .001 6 .181 8 .014 2 .002 6 .138 7 .001 7 0.7373 - 0.773 1 0.030 0 .0206
31 /	31 /	0.0053
2008-12-	2009-12-	
31	31	
(Interce 200)4 -02-	2009-02-	0.013 3. 001 6. 157 9. 014 2. 002 5. 145 0. 001 9. 7363 - 0.780 2. 051 2. 0194 2. 001 10. 001 10
29 /	28 /	0.0059
2009-01-	2010-01-	
31	31	
(Interce 200)4 -03-	2009-03-	0.01 22 .001 7 .12 49 .020 4 .001 8 .323 0 .002 1 0.7345 - 0.854 6 0.037 8).013
31 /	31 /	0.0087
2009-02-	2010-02-	
28	28	
(Interce 200)4 -04-	2009-04-	0.0136.0018.1545.0192.0017.3069.00210.7311 - 0.90280.02610.0123
30 /	30 /	0.0089
2009-03-	2010-03-	
31	31	

train	test	mu_ksr_ksR_ksu_0x0s_0xs_0xs_0xsabeksa_alpha_betost0xxverr
(Interce 200)& -05-	2009-05-	0.0136.0016.1550.0194.0016.3212.00160.7239 - 0.85340.08900.0126
31 /	31 /	0.0064
2009-04-	2010-04-	
30	30	
(Interce 200)4 -06-	2009-06-	0.015 \text{Q} .001\text{Q}.188\text{Q}.008\text{Q}.001\text{Q}.060\text{Q}.002\text{Q}.7285 - 0.772\text{Q}.046\text{Q}.0136
30 /	30 /	0.0058
2009-05-	2010-05-	
31	31	
(Interce 200)110 07-	2009-07-	0.01466.00168.17462.0108.0010.1516.00140.7335 - 0.69940.14670.0148
31 /	31 /	0.0024
2009-06-	2010-06-	
30	30	
(Interce 200)41 08-	2009-08-	0.015 5 .001 9 .194 0 .010 9 .001 0 .149 0 .001 5 0.7236 - 0.754 3).073 5).0113
31 /	31 /	0.0018
2009-07-	2010-07-	
31	31	
(Interce 200)112 09-	2009-09-	0.014 7 .001 8 .17 7 4.00 75 .001 0 .049 6 .001 4 0.7222 - 0.718 9 .064 0 .0139
30 /	30 /	0.0008
2009-08-	2010-08-	
31	31	
(Interce 200)43 10-	2009-10-	0.014
31 /	31 /	
2009-09-	2010-09-	
30	30	
(Interce 200)14 11-	2009-11-	$0.014 \\ 9.001 \\ 8.184 \\ 5.013 \\ 0.001 \\ 2.08 \\ 0.001 \\ 5.718 \\ 0.0013 \\ 0.7167 \\ 0.1383 \\ 0.0145 \\ 0.0013 \\ 0$
30 /	30 /	
2009-10-	2010-10-	
31	31	
(Interce 200)45 12-	2009-12-	$0.012 \\ 9.001 \\ 9.156 \\ 8.011 \\ 2.001 \\ 0.168 \\ 2.0007 \\ 0.638 \\ 7.0014 \\ 0.6704 \\ 0.1492 \\ 0.0168 \\ 2.0014 \\ 0.6704 \\ 0.1492 \\ 0.0168 \\ 2.0007 \\ 0.638 \\ 7.0014 \\ 0.6704 \\ 0.1492 \\ 0.0168 \\ 2.0007 \\ 0.638 \\ 7.0014 \\ 0.6704 \\ 0.1492 \\ 0.0168 \\ 2.0007 \\ 0.638 \\ 3.0007 \\ 3.0007 \\ 0.638 \\ 3.0007 \\ 0.638 \\ 3.0007 \\ 0.638 \\ 3.0007 \\ 0.638 \\ 3.0007 \\ 0.638 \\ 3.0007 \\ 0.638 \\ 3.0007 \\ 0.638 \\ 3.0007 \\ 3.0007 \\ 3.0007 \\ 0.638 \\ 3.0007$
31 /	31 /	
2009-11-	2010-11-	
30	30	

train	test	mu_ISr_BR_ISu_@@S_GBS_@PSa_bISa_aI\$ha_b@@St@OSteerr
(Interce 200) 56 01-	2010-01-	0.0130.0014.1599.0138.0012.2345.0006.6418.00210.68860.09180.0167
31 /	31 /	
2009-12-	2010-12-	
31	31	
(Interce 200) 5 702-	2010-02-	$0.012 4.001 4.137 4.014 \overline{v}.001 2.268 9.000 40.647 0.0022 0.685 0.079 60.0164 0.0022 0.0000 1.00000 1.0000 1.0000 1.0000 1.0000 1.0000 1.00000 1.00000 1.00000 1.0000 1.0000 1.000$
28 /	28 /	
2010-01-	2011-01-	
31	31	
(Interce 200) 5 803-	2010-03-	$0.011 \\ 0.001 \\ 0.121 \\ 0.001 \\ 0.001 \\ 0.001 \\ 0.344 \\ 0.000 \\ 0.0650 \\ 0.0034 \\ 0.689 \\ 0.055 \\ 0.0165 \\$
31 /	31 /	
2010-02-	2011-02-	
28	28	
(Interce 200) 5 904-	2010-04-	$0.012 \\ \boxed{7.001} \\ \boxed{3.151} \\ \boxed{6.0129.001} \\ \boxed{0.2428.000} \\ \boxed{6.021} \\ \boxed{0.0032} \\ 0.670 \\ \boxed{6.097} \\ \boxed{0.0164}$
30 /	30 /	
2010-03-	2011-03-	
31	31	
(Interce 200)2 905-	2010-05-	0.013 \$.001 4.178 \$.013 5.001 0.261 0.000 \$.642 \$.0025 0.685 \$.132 \$.0159
31 /	31 /	
2010-04-	2011-04-	
30	30	
(Interce 200)2 406-	2010-06-	0.012 \$.001 \$.141 \$.016 \$.000 \$.372 \$.001 \$.707 \$.002 \$.002 \$.730 \$.154 \$.0130 \$.1
30 /	30 /	
2010-05-	2011-05-	
31	31	
(Interce 200)232 07-	2010-07-	$0.011 \\ \textbf{@}.001 \\ \textbf{@}.119 \\ \textbf{@}.016 \\ \textbf{@}.000 \\ \textbf{@}.401 \\ \textbf{@}.001 \\ \textbf{@}.699 \\ \textbf{@}.0022 \\ \textbf{@}.7265 \\ \textbf{@}.0301 \\ \textbf{D}.0128 \\ \textbf{@}.001 \\ \textbf{@}.001$
31 /	31 /	
2010-06-	2011-06-	
30	30	
(Interce 200)23 08-	2010-08-	0.012 22.0016.1261.0108.0008.2080.00120.7098.00190.74800.03220.011400.00120.0019
31 /	31 /	
2010-07-	2011-07-	
31	31	

train	test	mu_tsr_tsr_tsu_oos_ors_ors_opsabtsa_al\$ha_beto_stoosteerr
(Interce 200)2 409-	2010-09-	0.01202.00108.11906.0120.00008.27809.00100.7360.00120.81040.10440.0086
30 /	30 /	
2010-08-	2011-08-	
31	31	
(Interce 200)2 510-	2010-10-	0.011 \$.001 7.112 2.005 \$.000 7.013 \$.001 40.733 \$.00200.898 \$.032 \$.0057
31 /	31 /	
2010-09-	2011-09-	
30	30	
(Interce 200)2 611-	2010-11-	$0.013 \\ 0.001 \\ 8.137 \\ 8.008 \\ 9.001 \\ 0.108 \\ 9.001 \\ 80.759 \\ 2.0012 \\ 0.882 \\ 80.0804 \\ 0.0062$
30 /	30 /	
2010-10-	2011-10-	
31	31	
(Interce 200)27 12-	2010-12-	$0.0125.001 \\ \textbf{9}.124 \\ \textbf{0}.010 \\ \textbf{8}.001 \\ \textbf{0}.185 \\ \textbf{0}.001 \\ \textbf{0}.763 \\ \textbf{7}.0021 \\ \textbf{0}.885 \\ \textbf{2} \textbf{0}.018 \\ \textbf{9}.0062 \\$
31 /	31 /	
2010-11-	2011-11-	
30	30	
(Interce 200)18 801-	2011-01-	0.0129.0020.1293.0039.0010 - 0.0020.8022.00150.90770.09860.0053
31 /	31 /	0.0249
2010-12-	2011-12-	
31	31	
(Interce 200)29 02-	2011-02-	0.011
28 /	28 /	
2011-01-	2012-01-	
31	31	
(Interce 200)80 03-	2011-03-	0.01 29 .00 20 .12 80 .006 6 .001 0 .060 8 .002 6).8184 - 0.902 4).054 1).0067
31 /	31 /	0.0007
2011-02-	2012-02-	
28	29	
(Interce 200)6 104-	2011-04-	0.01109.00200.10903.00319.0012 - 0.00250.8127 - 0.92640.03970.0088
30 /	30 /	$0.0374 \qquad \qquad 0.0026$
2011-03-	2012-03-	
	31	

train	test	mu_I&r_BR_I&u_00S_OBS_OPSab4&a_l\$ha_beoStoo&teerr
(Interce 200)82 05-	2011-05-	0.0113.0019.0959.0040.0012 - 0.00220.8034 - 0.92640.02260.0088
31 /	31 /	0.0161 0.0023
2011-04-	2012-04-	
30	30	
(Interce 200)83 06-	2011-06-	0.011 0 .001 8 .095 4 .002 5 .0013 - 0.001 8).7804 - 0.891 7).072 6).0091
30 /	30 /	$0.0604 \qquad \qquad 0.0017$
2011-05-	2012-05-	
31	31	
(Interce 200)8 407-	2011-07-	0.010 2 .001 6 .07 72 .005 0 .001 0 .012 0 .001 6 .7426 - 0.856 5 .113 D .0100
31 /	31 /	0.0019
2011-06-	2012-06-	
30	30	
(Interce 200)8 508-	2011-08-	0.010 2 .001 6 .080 5 .008 0 .001 0 .105 6 .001 8).7275 - 0.838 4).043 8).0105
31 /	31 /	0.0021
2011-07-	2012-07-	
31	31	
(Interce 200)86 09-	2011-09-	0.009 8 .001 5 .073 2 .008 9 .001 0 .135 0 .002 1 0.7044 - 0.827 6 0.097 6 0.0103
30 /	30 /	0.0033
2011-08-	2012-08-	
31	31	
(Interce 200)87 10-	2011-10-	0.009@.0015.0523.0134.0009.2945.002@.7125 - 0.84810.03070.0106
31 /	31 /	0.0027
2011-09-	2012-09-	
30	30	
(Interce 200)8 811-	2011-11-	0.009\&0.001\&0.073\&0.009\&0.000\&0.216\&0.002\&0.7158 - 0.804\&0.030\&0.0104
30 /	30 /	0.0027
2011-10-	2012-10-	
31	31	
(Interce 200)6 912-	2011-12-	0.010 0 .001 6 .077 7 .008 9 .000 5 .163 6 .002 6 .7158 - 0.787 3).018 6).0101
31 /	31 /	0.0046
2011-11-	2012-11-	
30	30	

train	test	mu_ISr_ISR_ISu_@@S_GRS_@QSab4Sa_al\$ha_b@@S@QSt@Oo\$teerr
(Interce 200)47 901-	2012-01-	0.009 3 .001 5 .061 9 .010 5 .000 4 .285 5 .002 7 0.6953 - 0.798 9 .085 3).0097
31 /	31 /	0.0061
2011-12-	2012-12-	
31	31	
(Interce 200)4 402-	2012-02-	0.0099.0016.0786.0090.0004.2267.0030.7078 - 0.8239.04380.0098
28 /	29 /	0.0064
2012-01-	2013-01-	
31	31	
(Interce 200)472 03-	2012-03-	$0.00999.001 \ \ 3.081 \ \ 9.007 \ \ 6.000 \ \ 4.156 \ \ 8.003 \ \ 0.6801 - 0.7573 \ \ 0.0967 \ \ 0.0107 \ \ 0.0000 \ \ $
31 /	31 /	0.0046
2012-02-	2013-02-	
29	28	
(Interce 200)43 04-	2012-04-	$0.008 \\ 7.001 \\ 3.055 \\ 3.012 \\ 8.000 \\ 2.603 \\ 6.002 \\ 5.627 \\ 6.0015 \\ 0.554 \\ 40.360 \\ 5.0118$
30 /	30 /	
2012-03-	2013-03-	
31	31	
(Interce 200)47 405-	2012-05-	0.008 \$.001 \$.059 \$.010 5.000 2.421 9.0027 0.627 9.0019 0.517 \$.0321 0.0137
31 /	31 /	
2012-04-	2013-04-	
30	30	
(Interce 200)47 506-	2012-06-	$0.008 \\ 0.001 \\ 2.039 \\ 3.017 \\ 6.000 \\ 0.885 \\ 6.002 \\ 5.625 \\ 0.004 \\ 20.500 \\ 6.256 \\ 9.0149$
30 /	30 /	
2012-05-	2013-05-	
31	31	
(Interce 200)47 607-	2012-07-	0.008 2.0012.0467.0142.0004.4740.00240.6168.00350.55470.19150.0173
31 /	31 /	
2012-06-	2013-06-	
30	30	
(Interce 2010)47 708-	2012-08-	0.008
31 /	31 /	
2012-07-	2013-07-	
31	31	

train	test	mu_tsr_tsr_tsu_oos_obs_opsabtsa_al\$habeostoosværr
(Interce 201)/4 809-	2012-09-	0.008
30 /	30 /	
2012-08-	2013-08-	
31	31	
(Interce 200)47 910-	2012-10-	0.008 4.001 3.055 4.014 8.000 6.443 6.0027 0.631 5.0011 0.603 9.204 9.0163 6.001 6.00
31 /	31 /	
2012-09-	2013-09-	
30	30	
(Interce 200)5 011-	2012-11-	$0.007 \\ 9.001 \\ 3.041 \\ 3.016 \\ 0.000 \\ 6.467 \\ 7.002 \\ 20.634 \\ 0.002 \\ 00.648 \\ 70.057 \\ 30.0149 \\$
30 /	30 /	
2012-10-	2013-10-	
31	31	
(Interce 200)5 412-	2012-12-	$0.008 \underline{4.001} \underline{2.058} \underline{3.014} \underline{0.000} \overline{0.379} \underline{6.002} \underline{0.636} \underline{6.002} \underline{0.656} \underline{0.179} \underline{0.0149} \underline{0.008} $
31 /	31 /	
2012-11-	2013-11-	
30	30	
(Interce 200)82 01-	2013-01-	$0.008 \\ 9.0012.0735.0142.0007.3749.0018.640 \\ 7.00190.6703.046 \\ 70.0141$
31 /	31 /	
2012-12-	2013-12-	
31	31	
(Interce 200)83 02-	2013-02-	$0.009 \$.001 \$.100 \$.011 \$.000 \$.268 \\ 0.001 \$.658 \$.0025 \\ 0.686 \$.0621 \\ 0.0139$
29 /	28 /	
2013-01-	2014-01-	
31	31	
(Interce 200)8 403-	2013-03-	$0.008 \\ \textbf{@}.001 \\ \textbf{@}.073 \\ \textbf{@}.015 \\ \textbf{@}.000 \\ \textbf{@}.403 \\ \textbf{0}.0021 \\ \textbf{0}.650 \\ \textbf{@}.0025 \\ \textbf{0}.673 \\ \textbf{@}.0843 \\ \textbf{0}.0146 \\ \textbf{0}.0021 \\ \textbf{0}.650 \\ \textbf{@}.0025 \\ \textbf{0}.673 \\ \textbf{@}.0843 \\ \textbf{0}.0146 \\ \textbf{0}.0021 \\ \textbf{0}.650 \\ \textbf{0}.0025 \\ \textbf{0}.673 \\ \textbf{@}.0025 \\ \textbf{0}.673 \\ \textbf{0}.0025 \\ \textbf{0}.002$
31 /	31 /	
2013-02-	2014-02-	
28	28	
(Interce 200)85 04-	2013-04-	0.009 4.001 2.100 4.016 2.000 8.426 8.002 5.649 8.0029 0.675 6.0076 20.0148 6.0029 6.0020 6.0020 6.0020 6.0020 6.0020 6.00
30 /	30 /	
2013-03-	2014-03-	
31	31	

train	test	mu_ISr_BR_ISu_QQS_GBS_QQSabISabISa_all\$habQQStQQSteerr
(Interce 201)86 05-	2013-05-	0.008 7 .001 2 .081 5 .017 4 .000 7 .480 8 .002 5 .650 8 .00060.710 9 .098 8).0124
31 /	31 /	
2013-04-	2014-04-	
30	30	
(Interce 200)87 06-	2013-06-	$0.009 4.001 2.101 4.013 \overline{\textbf{0}}.000 6.384 \overline{\textbf{0}}.002 80.657 \underline{\textbf{0}}.0006 0.729 80.080 90.0107 60.000 20.000 20.000 20.000 20.000 20.000 20.000 20.000 20.0000 20.0000 20.0000 20.0000 20.0000 20.0000 20.0000 20.0000 20.00000 20.00000 20.00000 20.00000 20.00000 20.00000 20.00000 20.000000 20.00000 20.00000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.000000 20.0000000 20.0000000 20.0000000 20.00000000$
30 /	30 /	
2013-05-	2014-05-	
31	31	
(Interce 200)8 807-	2013-07-	$0.010 \\ 0.001 \\ 0.001 \\ 0.001 \\ 0.0019 \\ 0.0009 \\ 0.849 \\ 0.0035 \\ 0.638 \\ 0.0011 \\ 0.722 \\ 0.3023 \\ 0.0097 \\ 0.0035 \\ 0.0011 \\ 0.0023 \\ 0.0011 \\ 0.0023 \\ 0.0023 \\ 0.0097 \\ 0.0035 \\ 0.0011 \\ 0.0023 \\ 0.0035 \\ 0.0011 \\ 0.0023 \\ 0.0035 \\ 0.0011 \\ 0.0023 \\ 0.0035 \\ 0.0011 \\ 0.0023 \\ 0.0035 \\ 0.0011 \\ 0.0023 \\ 0.0035 \\ 0.0011 \\ 0.0023 \\ 0.0035 \\ 0.0011 \\ 0.0035 \\$
31 /	31 /	
2013-06-	2014-06-	
30	30	
(Interce 200)8 908-	2013-08-	$0.010 \\ 9.0016.131 \\ 3.0195.000 \\ 8.460.002 \\ 60.780 \\ 2.0024 0.777 \\ 10.5329.0077 \\ 2.0024 0.777 \\ 10.5329.0077 \\ 2.0024 0.002$
31 /	31 /	
2013-07-	2014-07-	
31	31	
(Interce 200)6 009-	2013-09-	$0.011 \\ 2.001 \\ 7.135 \\ 6.017 \\ 5.000 \\ 4.686 \\ 5.002 \\ 5.811 \\ 0.002 \\ 0.798 \\ 5.1047 \\ 0.0072 \\$
30 /	30 /	
2013-08-	2014-08-	
31	31	
(Interce 200)61 10-	2013-10-	$0.013 \ 3.001 \ 4.206 \ 0.012 \ 4.000 \ 4.381 \ 9.002 \ 10.803 \ 4.0017 \ 0.826 \ 40.093 \ 80.0072 \ 0.001 \$
31 /	31 /	
2013-09-	2014-09-	
30	30	
(Interce 200)62 11-	2013-11-	0.01 5 0.0010.28 5 6.00 79 .000 4 .166 0 .002 1 0.7693 - 0.756 3 0.196 4 0.0110
30 /	30 /	0.0007
2013-10-	2014-10-	
31	31	
(Interce 200)63 12-	2013-12-	0.014 0 .001 0 .260 8 .006 6 .000 5 .090 0 .001 5 0.7657 - 0.750 6).023 4).0149
31 /	31 /	0.0030
2013-11-	2014-11-	
30	30	

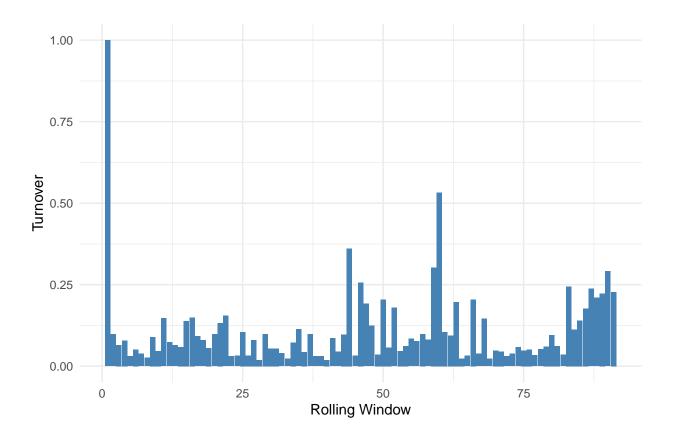
train	test	mu_ksr_ksR_ksu_0x0s_0xs_0xs_0xsabeksa_alpha_betost0xxverr
(Interce 200)94 01-	2014-01-	0.0140.0010.2603.0030.0005 - 0.00120.7611 - 0.78780.03200.015
31 /	31 /	0.0803 0.0048
2013-12-	2014-12-	
31	31	
(Interce 200)05 02-	2014-02-	$0.014 \times 5.001 \times 260 \times 9.006 \times 0.000 $
28 /	28 /	0.0074
2014-01-	2015-01-	
31	31	
(Interce 200)06 03-	2014-03-	0.016 3 .001 2 .331 2 .005 2 .000 5 .014 9 .00040.8392 - 1.035 2).038 0 .0123
31 /	31 /	0.0075
2014-02-	2015-02-	
28	28	
(Interce 200)97 04-	2014-04-	0.016 2 .001 0 .334 0 .002 0 .0006 - 0.001 1 0.8682 - 1.095 9 0.145 2 0.0101
30 /	30 /	0.1247 0.0083
2014-03-	2015-03-	
31	31	
(Interce 200)98 05-	2014-05-	0.0163.0010.3388.0036.0007 - 0.00120.8656 - 1.10780.02270.0105
31 /	31 /	0.0556 0.0086
2014-04-	2015-04-	
30	30	
(Interce 200)09 06-	2014-06-	0.01466.00100.30700.00286.0007 - 0.00080.8531 - 0.78190.04770.0175
30 /	30 /	0.1045 0.0041
2014-05-	2015-05-	
31	31	
(Interce 200)9 007-	2014-07-	$0.015 \\ 5.000 \\ 9.346 \\ 4.013 \\ 6.003 \\ 0.153 \\ 9.001 \\ 0.838 \\ 9.009 \\ 00.439 \\ 4.045 \\ 30.055 \\ 8.001 \\ 10.003 \\ 10.$
31 /	31 /	
2014-06-	2015-06-	
30	30	
(Interce 201) 91 08-	2014-08-	$0.014 (5.000) 9.321 (6.020) 2.003 \\ \textbf{5}.255 \\ \textbf{2}.001 \\ \textbf{10}.855 \\ \textbf{8}.0158 \\ 0.485 \\ \textbf{0}.030 \\ \textbf{3} \\ 0.059 \\ \textbf{2}.001 \\ \textbf{3} \\ \textbf{3}$
31 /	31 /	
2014-07-	2015-07-	
31	31	

train	test	mu_ISr_ISR_ISu_000S_CRS_00PSab4Sa_al\$ha_b00_St000Steerr
(Interce 201) 72 09-	2014-09-	0.014@.0009.3074.0157.0040.1662.001D.8512.01360.72520.03790.060
30 /	30 /	
2014-08-	2015-08-	
31	31	
(Interce 200) 9 310-	2014-10-	$0.013 \\ 7.000 \\ 9.296 \\ 0.018 \\ 0.004 \\ 0.197 \\ 0.001 \\ 30.840 \\ 9.0137 \\ 0.6097 \\ 0.0581 \\ 0.0641 \\$
31 /	31 /	
2014-09-	2015-09-	
30	30	
(Interce 200) 9 411-	2014-11-	$0.012 \\ \boxed{7.0009.271} \\ \boxed{7.025} \\ \boxed{2.004} \\ \boxed{4.3006.001} \\ \boxed{10.8278.0175} \\ 0.620 \\ \boxed{30.0478}.064$
30 /	30 /	
2014-10-	2015-10-	
31	31	
(Interce 200) 9 512-	2014-12-	$0.012 \\ 22.000 \\ 9.251 \\ 7.021 \\ 8.004 \\ 8.239 \\ 2.000 \\ 8.836 \\ 7.0164 \\ 0.813 \\ 8.0515 \\ 0.062$
31 /	31 /	
2014-11-	2015-11-	
30	30	
(Interce 201) 76 01-	2015-01-	$0.011 \\ 2.000 \\ 9.221 \\ 5.020 \\ 2.005 \\ 4.204 \\ 9.000 \\ 4.843 \\ 7.015 \\ 90.872 \\ 9.033 \\ 30.065 \\$
31 /	31 /	
2014-12-	2015-12-	
31	31	
(Interce 201) 07 02-	2015-02-	$0.012 {3.000} {9.256} {3.015} {0.0058.128} {2.00040.8398.0155} {0.9367} {0.05190.067}$
28 /	28 /	
2015-01-	2016-01-	
31	31	
Interce 201)7 803-	2015-03-	0.0129.0009.2707.0122.0054.0945.0003.8528.01611.0206.0605.064
31 /	31 /	
2015-02-	2016-02-	
28	29	
Interce 201) 79 04-	2015-04-	0.010 8 .000 9 .204 6 .019 0 .006 9 .1732 - 0.856 2 .01580.744 8).09540.074
30 /	30 /	0.0005
2015-03-	2016-03-	
31	31	

train	test	mu_ISr_BR_ISu_@@S_GBS_@PBab4Sa_aI\$hab@@S@D&veerr
(Interce 201)89 05-	2015-05-	0.0116.0009.2257.0156.0058.1348 - 0.8685.01430.77400.06140.0708
31 /	31 /	0.0005
2015-04-	2016-04-	
30	30	
(Interce 20) 8 406-	2015-06-	$0.012 \ 3.000 \ 9.258 \ 7.013 \ 8.006 \ 2.098 \ 7.000 \ 30.834 \ 8.0081 \ 0.7791 \ 0.0347 \ 0.0741$
30 /	30 /	
2015-05-	2016-05-	
31	31	
(Interce 201)82 07-	2015-07-	0.0167.00100.3723 - 0.0023 - 0.00520.7442 - 0.89410.24420.0344
31 /	31 /	$0.0034 \qquad 0.1877 \qquad 0.0068$
2015-06-	2016-06-	
30	30	
(Interce 201)83 08-	2015-08-	0.016 9 .000 9 .3975 - 0.0012 - 0.006 5).7239 - 0.857 6).112 6).0155
31 /	31 /	$0.0103 \qquad 0.4544 \qquad \qquad 0.0142$
2015-07-	2016-07-	
31	31	
(Interce 201)84 09-	2015-09-	0.016 5 .001 0 .3594 - 0.0012 - 0.005 6).7841 - 0.897 0).140 1).0142
30 /	30 /	$0.0077 \qquad 0.3927 \qquad \qquad 0.0145$
2015-08-	2016-08-	
31	31	
(Interce 201)8 510-	2015-10-	0.01507.00102.3179 - 0.0011 - 0.00600.7468 - 0.84790.17700.0141
31 /	31 /	$0.0108 \qquad 0.5100 \qquad 0.0162$
2015-09-	2016-09-	
30	30	
(Interce 201)8 611-	2015-11-	0.01666.0010.3541 - 0.0006 - 0.00590.8112 - 0.80100.23750.0111
30 /	30 /	$0.0172 \qquad 0.9282 \qquad \qquad 0.0160$
2015-10-	2016-10-	
31	31	
(Interce 201)87 12-	2015-12-	0.0169.0014.3270 - 0.0004 - 0.00700.7622 - 0.68500.20990.0119
31 /	31 /	0.0190 1.2514 0.0208
2015-11-	2016-11-	
30	30	

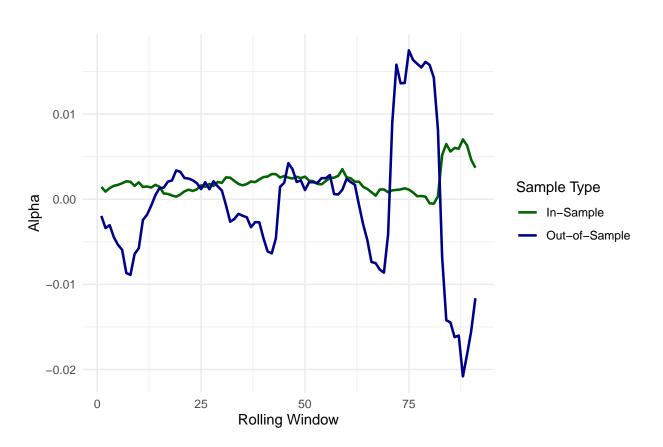
train	test	mu_lsr_lsr_lsu_@@S_GRS_@DSa_blsa_alsha_b@@St@@St@@Steerr
(Interce 201)8 801-	2016-01-	0.0150.0014.2953 - 0.0004 - 0.00630.8521 - 0.76410.22330.0083
31 /	31 /	0.0152 1.0300 0.0182
2015-12-	2016-12-	
31	31	
(Interce 201)8 902-	2016-02-	0.0137.0013.2562 - 0.0003 - 0.0046.8288 - 0.6596.29080.0112
28 /	29 /	0.0080 0.7585 0.0155
2016-01-	2017-01-	
31	31	
(Interce 201)9 003-	2016-03-	0.0128.0010.2412 - 0.0005 - 0.00370.8953 - 0.78000.22720.0089
31 /	31 /	$0.0015 \qquad 0.3367 \qquad 0.0116$
2016-02-	2017-02-	
29	28	

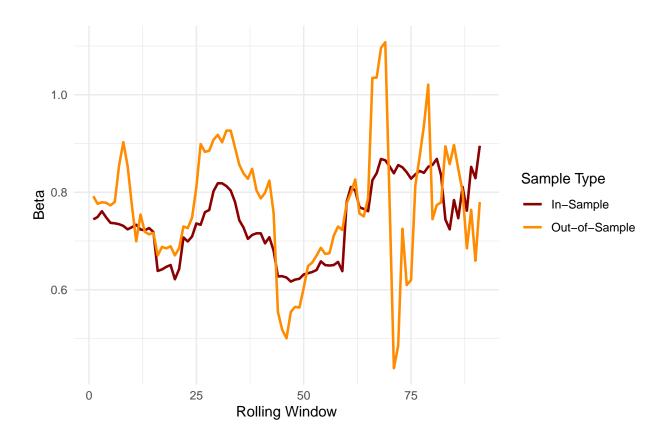
2.7 Portfolio Turnover



2.8 Jensens alpha and beta

```
theme_minimal() +
scale_color_manual(values = c("In-Sample" = "darkgreen", "Out-of-Sample" = "darkblue")
```

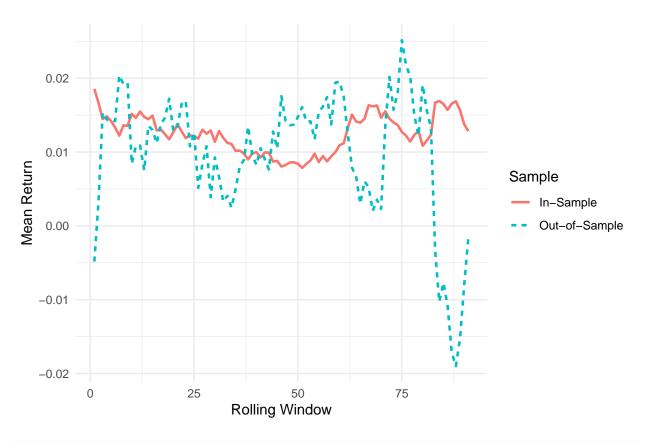


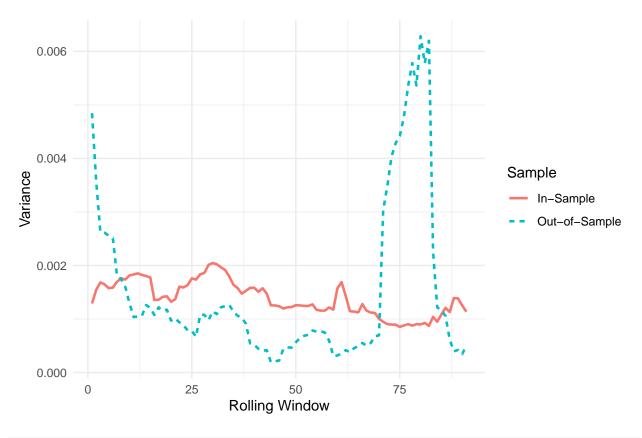


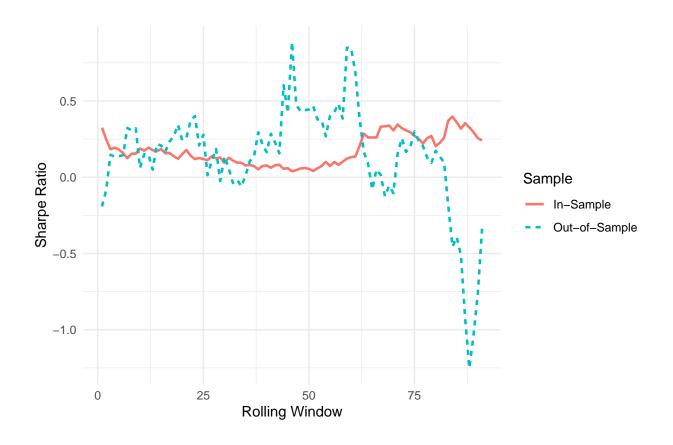
2.9 Plotting In Sample vs Out Of Sample Statistics

```
library(tidyr)
summary_df <- do.call(rbind, lapply(seq_along(results), function(i) {
    x <- results[[i]]
    data.frame(
        Window = i,
        train = x$train_period,
        test = x$test_period,
        mu_IS = x$mu_IS, var_IS = x$var_IS, SR_IS = x$SR_IS,
        mu_00S = x$mu_00S, var_00S = x$var_00S, SR_00S = x$SR_00S,
        alpha_IS = x$alpha_IS, beta_IS = x$beta_IS,
        alpha_00S = x$alpha_00S, beta_00S = x$beta_00S,
        turnover = x$turnover,
        t.err = x$t.err
    )
})</pre>
```

```
sum 1 <- summary df %>%
 pivot_longer(
    cols = c(mu IS, var IS, SR IS, mu OOS, var OOS, SR OOS),
   names_to = "Metric",
   values_to = "Value"
 ) %>%
 mutate(
   Sample = ifelse(grepl("_IS", Metric), "In-Sample", "Out-of-Sample"),
   Metric = gsub("_(IS|OOS)", "", Metric),
   Metric = case_when(
     Metric == "mu" ~ "Mean",
     Metric == "var" ~ "Variance",
     Metric == "SR" ~ "Sharpe",
     TRUE ~ Metric
 )
# Mean
ggplot(subset(sum_l, Metric == "Mean"),
       aes(x = Window, y = Value, color = Sample, linetype = Sample)) +
 geom_line(linewidth = 0.9) +
 labs(
   title = "",
   x = "Rolling Window",
   y = "Mean Return"
 theme_minimal()
```





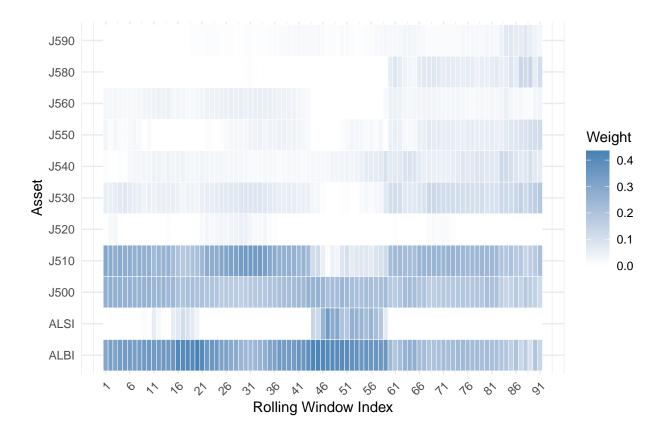


2.10 Portfolio Weights

```
# Combine all rolling window weights into a single data frame
weights_df <- do.call(rbind, lapply(seq_along(results), function(i) {
    data.frame(
        Window = i, #index
        Asset = results[[i]]$assets,# asset names
        Weight = results[[i]]$weights,# weights
        stringsAsFactors = FALSE
    )
}))

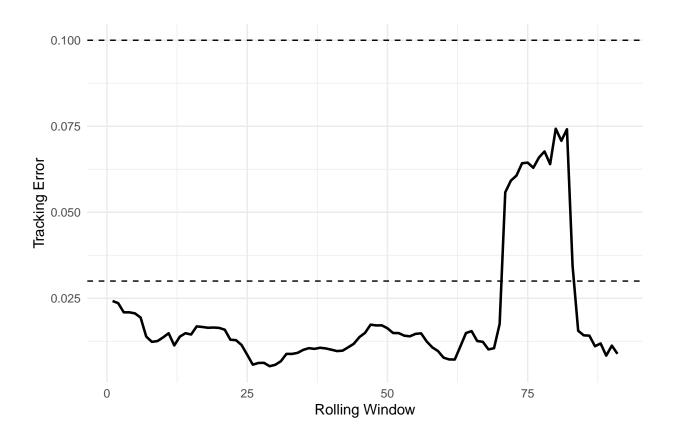
# Evolution of Black-Litterman Portfolio Weights
ggplot(weights_df, aes(x = Window, y = Asset, fill = Weight)) +
    geom_tile(color = "white") +
    scale_fill_gradient(low = "white", high = "steelblue") +
    scale_x_continuous(</pre>
```

```
breaks = seq(min(weights_df$Window), max(weights_df$Window), by = 5)
) +
labs(
   title = "",
   x = "Rolling Window Index",
   y = "Asset",
   fill = "Weight"
) +
theme_minimal() +
theme(axis.text.x = element_text(angle = 45, hjust = 1))
```



2.11 Tracking Error

```
### Tracking Error Plot
t.err_df <- data.frame(
  Window = 1:length(results),
  TrackingError = sapply(results, function(x) x$t.err)</pre>
```



References

Gebbie, T. (2025a). Portfolio Theory-backtest-001.r. Unpublished teaching material.

Gebbie, T. (2025b). Portfolio Theory Lecture 001. mlx. Unpublished teaching material.

Gebbie, T. (2025c). Portfolio Theory Lecture 003. pdf. Unpublished teaching material.

Gebbie, T. (2025d). Portfolio Theory-PrepareData-000.r. Unpublished teaching material.