#disruptGT: A Mathematical Model for Gifted and Talented Programs in Boulder Valley School District

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Abstract

Gifted and talented (GT) programs are designed to provide advanced learning support to students who are identified as gifted in one or more areas. Colorado is one of fifteen states in the United States with a mandate dictating identification and programming policies for gifted education and providing state funding for such. However, according to a report published by Purdue University's Gifted Education Research and Resource Institute, there are still between 16,859 and 22,174 students who are missed for gifted identification each year, with most missing students coming from Title I schools and historically underserved populations. Furthermore, in many Colorado schools, students of American Indian/Alaskan Native (AIAN), Black, Hispanic/Latinx, and Native Hawaiian/Pacific Islander (NHPI) heritage are severely underrepresented in GT programs, with many schools having 50% or less as many underserved students in GT populations as in the general school population. In this study, we develop a mathematical model for GT enrollment in Boulder Valley School District (BVSD) in Boulder, CO, with the goal of informing better strategies for increasing representation in BVSD and other districts. Specifically, we construct an absorbing Markov chain model, a model relying on conditional probabilities for transitioning between grade levels, for elementary schoolers in BVSD using data from the 2017-18 to 2021-22 academic years. We initialize the Markov chain with GT enrollment values in the 2017-18 year and run simulations for the next four academic years, priming the chain each year with an influx of new kindergartners. For each of the four simulations, students of White, Asian, or two or more racial backgrounds are consistently represented in GT populations at higher rates than in the general district population, whereas students of Latinx/Hispanic, Black, AIAN, and NHPI backgrounds are consistently represented in GT at less than 50% of their representation in the general population. Clearly, there is a need for intervention to increase representation of underserved students in the GT programs in BVSD and other districts. Future directions for this work include model validation and improvement and development of an app to provide schools with a better tool for measuring and improving their representation in GT. This work was supported in part by the Interdisciplinary Quantitative Biology (IQ Biology) program at the BioFrontiers Institute, University of Colorado Boulder.

1 Introduction

According to the National Association for Gifted Children, gifted and talented (GT) students "perform—or have the capability to perform—at higher levels compared to others of the same age, experience, and environment in one or more domains. They require modification(s) to their educational experience(s) to learn and realize their potential" [13]. As early as 1868, U.S. educators began to realize the importance of gifted education [11]. Even so, it took until 1988 for Congress to pass the Jacob Javits Gifted and Talented Students Education Act as part of the Elementary and Secondary Education Act, which was most recently reauthorized through the Every Student Succeeds Act in 2015 [11, 14]. The Javits Act funds scientific and educational research to improve gifted and talented programs in the U.S., including efforts to better serve underrepresented students [14]. However, unlike special education, for which the federal Individuals with Disabilities Education Act mandates that all states must provide a "free and appropriate public education to eligible children with disabilities" and dictates requirements that states must follow, there exists no federal law mandating requirements for gifted education [9]. As such, only 15 states in the country have mandates in place for gifted identification and programming as well as state funding for local GT programs [12], one of which is Colorado.

Under Colorado's Exceptional Children's Educational Act (ECEA), gifted children are defined as: "those persons between the ages of four and twenty-one whose aptitude or competence in abilities, talents, and potential for accomplishment in one or more domains are so exceptional or developmentally advanced that they require special provisions to meet their educational programming needs" [8]. The ECEA specifies these domains to be the following:

- General or specific intellectual ability
- Specific academic aptitude in reading, writing, mathematics, science, social studies, or world language
- Talent aptitudes including visual arts, performing arts, musical, dance, or psychomotor abilities
- Creative or productive thinking
- Leadership abilities [15]

In additional to providing a definition for gifted children in Colorado schools, the ECEA also details GT identification protocols for all schools in Colorado, provides some funding to local GT programs, and provides guidelines for GT programming. GT identification, which we are concerned with here, is discussed in more detail in a later section.

While many people understand the importance of special education in today's world, the significance of gifted education is often misunderstood. However, according to the National Association for Gifted Children, "adverse developmental effects have been noted for gifted students who do not have opportunities for early education or to participate in challenging programs," effects which are experienced even harder for children living in poverty, who are at greater risk for dropping out of school [13]. Furthermore, gifted children require support to develop not just in their area of talent but also socially and emotionally [13]. Indeed, gifted children often experience increased sensitivities, difficulties with social skill development, perfectionism, and low self-esteem compared to their peers, which can lead to problematic behaviors [16]. Interventions like GT programming take a more holistic view of the child and help them to develop in all areas and receive the support that they need, just as special education does for students with disabilities. In fact, about 6% of gifted students are twice exceptional, meaning that they receive both gifted and special education services [1].

Unfortunately, like in many areas of education and society, students of color are underrepresented in GT programs at a national scale as compared to White students. This underrepresentation can be quantified by use of a measure called the representation index (RI), which is defined as the percent of a group that is in the GT population divided by the percent of a group that is in the general population [10]. If an RI is greater than 1, students of a given group are represented more in GT populations than in the general population; likewise, if an RI is less than 1, students of a given group are underrepresented in GT populations as compared to the general population. Researchers at Purdue University led a well-known study in gifted education in 2019, and they found that Black and Latinx students have an RI of less than 0.95 in all U.S. states. Furthermore, the same study found that between 48% and 74% of Black, Latinx, Native Hawaiian / Pacific Islander (NHPI), and American Indiean Alaskan Native (AIAN) students are missed for gifted identification each year, compared to between 20% and 49% for White, Asian, and Two or More Races (TMR) students [10].

In Colorado alone, the Purdue study indicated that between $17,000$ and $22,000$ students are missed for gifted identification every year, with most of those students coming from Title I schools and historically underserved populations. The study also evaluated students from across 60 different locales and Title I / non-Title I designations for representation in GT programs. In 51 of the 60 categories, students of Black, Latinx, NHPI, and AIAN backgrounds had a representation index of 0.8 or less, which is the RI threshold that the researchers assigned to a failing score for equity. Furthermore, of those 60 categories, 29 of them had an RI of 0.5 or less for Black, Latinx, NHPI, and AIAN students [10].

Given the demonstrated importance of gifted education programming for gifted children's intellectual, social, emotional, and overall development, the lack of equity in GT programming at both the national and state scale is jarring and demands correction.

1.1 Factors Influencing Underrepresentation

Scientists in interdisciplinary fields seek to identify factors that can lead to underrepresentation in a number of contexts, including employment and education. Of these, bias (conscious and unconscious perceptions by others) and homophily (the tendency to seek out others like oneself) are dominant in the literature [2, 19]. Beverly Daniel Tatum says it best in her "Complexity of Identity," where she explains that the subordinate parts of one's identity, rather than the dominant parts, are the ones that people notice about themselves because those are the aspects that are reflected back towards them in society [23]. Futhermore, the theory of possible selves, a concept taken from psychology that describes a person's conceptions about who they might become (e.g. their hopes, dreams, fears, and goals), has a clear link to one's identity, and studies have shown that academic possible selves are linked to achievement gaps experienced by underserved students [18, 20]. All of these factors, among others, likely play an important role in understanding underrepresentation in GT.

1.2 Boulder Valley School District

Boulder Valley School District (BVSD) is located in Boulder, Colorado and serves more than $31,000$ students in 11 communities in the greater Boulder Valley, covering an area of over 500 miles and containing 56 schools [3]. In the 2021-2022 academic year, the student body composition for the district was 66.3% White, 20.1% Latinx, 5.7% Asian, 1.0% Black, 0.3% AIAN, 0.1% NHPI, and 6.5% TMR, with 14% of students qualifying for the free and reduced meal program and 10.5% of students being English language learners (ELL) [6, 21]. Given its close proximity to our institution and history of collaborating with researchers at our institution, BVSD is a convenient model system for our study. However, we would like to note that underrepresentation in GT is a nationwide problem, and eventually, we would like to incorporate data from other school districts as well.

1.3 GT Programs in Colorado

Because there is no federal law that mandates strict requirements for gifted education, there is significant variation in gifted policy from state to state. However, gifted identification necessarily shares some steps between all states, such as referral, testing, and identification. The details of these steps, in contrast, may vary widely.

In Colorado, GT programming may begin as early as kindergarten and, once a student is identified, such programming typically continues until the student graduates from high school. However, students as young as 3 who are classified as Highly Advanced Gifted Children can begin to receive early access GT services and may be enrolled in kindergarten or first grade [15]. Per ECEA, GT identification begins with a referral. Gifted students may be referred for formal GT identification based on a number of factors including test data, classroom observations, interviews, and performances, with such referrals being made by the student, a parent, teacher, or other adult familiar with the students' skills and talents. Most of these referrals come from parents or other family members [7]. Furthermore, some school districts conduct universal screening processes wherein students are identified by qualitative and/or quantitative testing across the entire grade level [15]. This varies by district, but in BVSD, this happens in second grade.

After students have been referred for gifted identification, a committee will meet to review the referrals and determine if students will undergo formal evaluation. If a student is selected to proceed with the identification process, the student, their legal guardians, and their teachers will be tasked with collecting a body of evidence to support the student's GT identification. Under the ECEA, the body of evidence must be comprehensive, including, at a minimum, "the identification assessment results, parental input and multiple types of measures and data sources" [15].

Identification assessments vary depending on the category in which the student is referred. To measure intellectual ability, students undergo cognitive testing to assess skills such as fluid reasoning (i.e. analogies and patterns) and crystalized abilities (i.e. reading comprehension, math problems). While the body of evidence generally requires that students demonstrate giftedness across several assessments, students who score in the 95th percentile or higher on cognitive tests may receive a gifted identification, a practice that helps in identifying those students who lack motivation or otherwise are not performing at their optimal level academically [15].

To be identified as gifted in other categories, the body of evidence must be held to a more rigorous standard in that multiple measures and sources of data must be used, including quantitative measures like norm- or criterion-referenced tests and qualitative measures like observations and rubrics. From a quantitative perspective, to be categorized as gifted, students must score at or above the 95th percentile on "standardized, nationally-normed test[s] or observation tool[s]," with the exception of performance assessments, on which students must demonstrate advanced talent compared to their age mates through means such as juried performances, contests/competitions, portfolios, or classroom performances [15]. Such tests or tools include creativity tests, achievement tests, and behavior observation scales, and many students are evaluated based on a combination of these and cognitive tests. Students who test below the 95th percentile on cognitive tests but above the 95th percentile on an achievement test in a specific academic aptitude area are often monitored across grade levels before being identified as gifted. In general, quantitative and qualitative results are used together to evaluate the student for GT identification, and, with the exception of cognitive testing, no one score or data point will determine the student's identification. Rather, the body of evidence is considered holistically [15].

Notably, some testing measures are more effective at identifying gifted students in traditionally underrepresented populations than others. These include universal screening, monitoring students over time across grade levels in conjunction with the response to intervention approach, use of local norms on nationally-norm-referenced tests, and use of qualitative as well as quantitative identification measures [15]. These assessment methods may be critical in understanding underrepresentation in GT programs in BVSD and beyond.

After the body of evidence is collected, a review team meets to evaluate it. Each review team must consist of at least one teacher trained in gifted education, and they must work with parents/legal guardians and teachers working with the student in question. The review team follows state guidelines to make one of the following determinations:

- Gifted identification
- No gifted identification at this time
- More data is required to make a determination
- Student would benefit from the talent pool
- Student would benefit from special education services as well as gifted education services

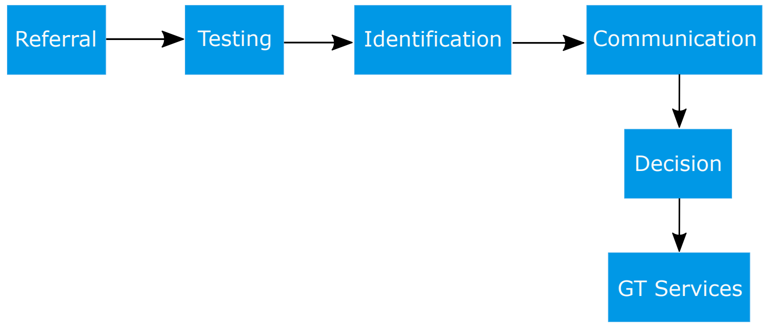

Figure 1: Summary of the steps of the gifted identification process under the ECEA.

Students who are not identified as gifted at the time of evaluation may be re-evaluated again in the future (1 year or more after the first identification process), particularly if they are identified as needing additional monitoring or for talent pool placement. Students who are placed into the talent pool demonstrate advanced or exceptional abilities but do not qualify for gifted identification at the time of evaluation. However, students in the talent pool may enter the GT program at a later time, and in the meantime, they receive advanced or exceptional programming to support their advanced needs [7, 15]. Not all school districts have talent pools; while BVSD is hoping to create one in the future, they do not yet have one.

If students are identified as gifted, the decision will be communicated to the student, parents/legal guardians, and teachers, and an advanced learning plan (ALP) will be developed based on the student profile built during the collection of the body of evidence. The ALP will include achievement and affective goals for the student, the student's progress towards these goals, the programming needs and advanced curriculum and services that will be made available to the student, progress reports, and personnel involved in the development and maintenance of the ALP. If students are identified as gifted in high school, an Individual Career and Academic Plan (ICAP) will be developed in lieu of an ALP [15]. The ECEA GT identification process is summarized in Figure 1.

Another important feature of the ECEA is portability between districts. Portability means that gifted determinations are portable between school districts within Colorado as long as students' records are transferred from the previous district. Students moving from outside of Colorado are evaluated based on their records from the previous district, while students in military families who have been identified as gifted are allowed to maintain that designation under the Military Compact Agreement [15].

Outside of gifted identification policies, which are universal throughout Colorado, other GT policies are subject to differences between school districts and even individual schools. In BVSD, once a student begins receiving GT services, they continue to receive GT services until they graduate or opt out of services, although opting out is relatively rare. If a student does choose to opt out of services, they may do so at any time over the course of the academic year and may choose to opt back in at any time, although they will need to be re-evaluated before receiving services again [7]. BVSD evaluates students for GT identification through the process described above twice a year in October and April, with incoming kindergartners being evaluated starting in October of their kindergarten year. Upon evaluation, students typically begin receiving services by the end of the semester in which they were evaluated (fall or spring) [4, 7]. We note that students may be evaluated in any year of their education, although it is more common for them to be identified in elementary school. With that being said, students of color and students identified as being talented in the arts or sports may be more likely to be identified in middle or high school [7, 15].

1.4 Proposal

To address the issue of underrepresentation in GT programs, we construct an absorbing Markov model for progression through elementary school for both students receiving GT services and students not receiving GT services. We train the model with data from BVSD. In the rest of this paper, we describe our methods for data acquistion, our model and its assumptions and limitations, our preliminary results, and our future directions for this work.

2 Methods

Here, we describe our methods for data acquisition and our model.

2.1 Data Acquisition

Educational data is often difficult to obtain without Institutional Review Board (IRB) approval due to privacy concerns. Fortunately, BVSD maintains a Tableau profile containing several Strategic Plan Metrics in areas where they are attempting to improve, one of which is representation in their GT program [6]. From this dataset, we obtained annual GT enrollment data by race, grade, and academic year from 2017-18 to 2021-22, as well as BVSD student body composition by race, grade, and year. We used this in conjunction with BVSD's publicly available student counts by grade and year to find the number of students in GT and not in GT in each grade and racial group for each year [5]. In the last academic year, 2021-22, across all grades, the GT student body composition was 74% White, 9.7% Latinx, 7.3% Asian, 0.4% Black, 0.2% AIAN, 0.1% NHPI, and 8.3% TMR. In contrast, the general student body composition was 66.3% White, 20.1% Latinx, 5.7% Asian, 1.0% Black, 0.3% AIAN, 0.1% NHPI, and 6.5% TMR [6]. This suggests that there are more White, Asian, and TMR students in the GT population than the general population, while Latinx, Black, and AIAN students are underrepresented, and NHPI students are equally represented in both populations. As we will see, this trend is consistent over the past four academic years.

Unfortunately, the data availability does not coincide well with the complexity of the GT identification process, and at this time, we are unable to obtain data on referrals, testing, and decisions to opt in/out of GT services. Additionally, while GT identification in BVSD happens on a semesterly basis, data is only compiled on the Tableau profile on an annual basis, which limits our modeling ability as well. Finally, there is insufficient data regarding the number of students who enter and leave BVSD every year, which makes it difficult to account for these students in the model. We are hopeful to obtain such data in the future to improve our model, and we discuss the benefits of incorporating such data later in this paper.

2.2 Markov Chains

Markov chains are a type of discrete mathematical model often used to represent transitions between different states. In particular, Markov chains track the probability of changing between the different states available in the system. Such probabilities are known as transition probabilities, and these are expressed as conditional probabilities where the probability of entering a given state is dependent on the previous state (i.e. $P(\text{State A during iteration } i \mid \text{State B during iteration } i-1$ is the probability of being in State A during iteration i, given that the system was in State B during iteration $i-1$). If the initial probabilities of being in each state are known, then the transition probabilities can be arranged into a transition matrix, where the previous states at iteration $i-1$ represent the rows and the next states at iteration i represent the columns. Then element (B_{i-1}, A_i) of the transition matrix is the probability of transitioning from state B to state A. Performing matrix multiplication on the vector of initial probabilities (also called the initial state vector), we obtain the probabilities of being in each state after one iteration. Iterating many times eventually produces stable values for the probabilities of being in each state. Additionally, the initial state vector need not be comprised of strictly initial probabilities; rather, if the vector contains the number of elements in each state, then one iteration of matrix multiplication will give the predicted number of elements in each state. If the number of iterations are allowed to increase without a limit, the expected number of elements in each state will eventually stabilize as well. The only exception to this stabilization is if new elements are continually introduced to the system [22].

In general, Markov chains are useful for discrete systems where system elements may transition between states in an iterative fashion. For example, they are widely used in the field of bioinformatics and genomics for genome annotation, including predicting genomic features based on preceding nucleotide bases [24]. They are also useful in sociological contexts like we consider here. One such example is a 2021 study on undergraduate student body racial/ethnic demographics over the course of undergraduate education from application to graduation [17]. In this study, researchers developed an absorbing Markov chain model for student transitions from state to state (i.e. freshman fall semester to freshman spring semester). An absorbing Markov chain differs from an ordinary Markov chain in that there are transient and absorbing states. Elements (students in this case) will move between transient states with certain transition probabilities, but there is a 100% chance that students will end up in one or more absorbing states. Once in an absorbing state, students have a 0% probability of moving to another state in the system [22]. In this example, there are two absorbing states: did not finish, which can be reached directly from any of the transient states, and graduation, which can only be reached through a path of transient states that imply program completion [17]. Given its similar context to our model system in that students must progress through different "grades," we draw from this model to construct our own, which we present now.

2.3 Model

Similarly to our reference study, we also develop an absorbing Markov chain model for student demographics in BVSD's GT programs. Specifically, we develop 15 transition states, 13 of which are transient and 2 of which are absorbing. Since the majority of students are identified for GT in elementary school, we only model transition states from grades K-6, and we assume that middle school (defined as entry of grade 6) is an absorbing state. That is, we assume that students have a 100% chance of staying in GT in middle school if they enter that state as incoming sixth graders and that students have a 100% chance of staying in non-GT in middle school if they enter that state as incoming sixth graders. This defines our two absorbing states, MS_{GT} and MS_{NGT}. Our transient states, then, are the grade levels and GT status of students in a given year; for example, 1_{GT} is a transient state for students in first grade and receiving GT services. A more complete list of transition states is provided in Table 1.

Due to the data acquisition difficulties we encountered, we make several assumptions to correct for lack of data, although we hope to obtain more data in the future to make our model more realistic. We also make additional simplifying assumptions that may be loosened in future versions of the model. In particular, we assume that:

1. students do not get held back, skip grades, or drop out
2. that there is no net transfer of students in or out of the district, i.e. we assume that students enter the school district at a rate of 50% and leave at a rate of 50% across all racial/ethnic groups.
3. that if students opt out of GT, they may restart the GT identification process and opt back into GT services in a subsequent academic year. Academic years are defined as the period from August to May.

In a given academic year, the Markov chain necessarily begins with incoming kindergartners, which are represented by the state K_B. During the first semester of kindergarten, students may be identified for GT at a rate of β_0, which is the probability that students are identified for GT in kindergarten given that they entered kindergarten that year. These students will begin services during the second semester of the year and enter the state K_{GT}. Otherwise, students

Transition State	Description
K_B	Number of new kindergartners
K_{GT}	Number of GT kindergartners
K_{NGT}	Number of non-GT kindergartners
1_{GT}	Number of GT 1st-graders
1_{NGT}	Number of non-GT 1st-graders
2_{GT}	Number of GT 2nd-graders
2_{NGT}	Number of non-GT 2nd-graders
3_{GT}	Number of GT 3rd-graders
3_{NGT}	Number of non-GT 3rd-graders
4_{GT}	Number of GT 4th-graders
4_{NGT}	Number of non-GT 4th-graders
5_{GT}	Number of GT 5th-graders
5_{NGT}	Number of non-GT 5th-graders
MS_{GT}	Number of GT middle-schoolers
MS_{NGT}	Number of non-GT middle-schoolers

Table 1: Transition states for a given academic year. All states are transient except MS_{GT} and MS_{NGT}, which are absorbing.

will not receive GT services during kindergarten and will enter in the state K_{NGT} at a rate of $1 - \beta_0$. These transitions mirror the true path that students take upon entering GT or non-GT populations in kindergarten. However, since we only have access to annual data as opposed to semesterly data, the rest of the model rests upon the assumption that students who qualify for GT in academic year $i - 1$ will start receiving GT services in academic year i. Similarly, we assume that students can opt out of GT between years only. Thus, for all additional transition states in our model, the student's GT status at the beginning of a given year is the same as their status at the end of the year and will not change until the next academic year. Notably, this assumption begins with kindergartners who are identified for GT in the second semester of their kindergarten year, and such students will not begin receiving GT services until first grade.

To make the transition from kindergarten to first grade, four processes may occur under our model:

1. Those students who were identified for GT in the second half of kindergarten will begin receiving GT services at a rate of ϵ_0, which is the probability that students will opt into GT in first grade given that they were in non-GT in kindergarten.
2. Those students who were identified for GT in the first half of kindergarten will continue receiving GT services at a rate of β_1, or the probability that students will continue receiving GT services in first grade given that they received services in kindergarten.
3. Those students who wish to opt out of GT services will opt out at a rate of $1 - \beta_1$, or the probability of opting out of GT services in first grade given that the student received them in kindergarten.
4. Those students who were not identified for GT in the second half of kindergarten will not receive GT services in first grade at a rate of $1 - \epsilon_0$, or the probability of not receiving GT services in first grade given that the student did not receive services in kindergarten.

These four options persist for the transitions from first to second grade, second to third grade, third to fourth grade, fourth to fifth grade, and fifth to sixth grade, with the only difference being that students who are identified for GT in a given grade in either semester will not receive services until the following year. The transition probabilities for these four possibilities continue

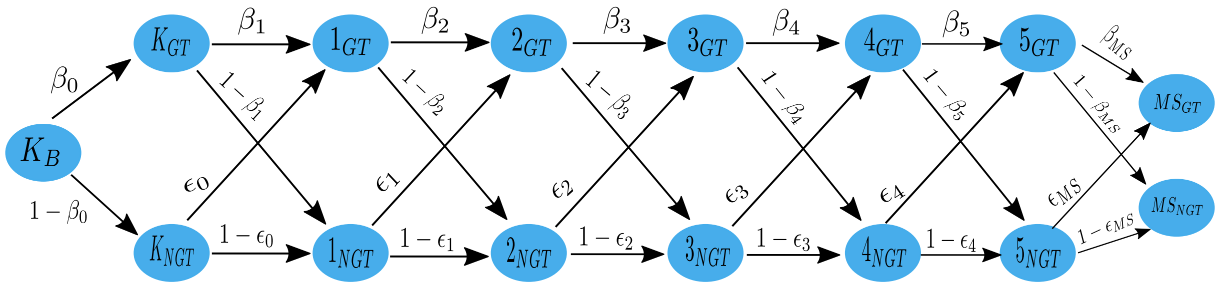

Figure 2: A directed graph for movement of students through GT and non-GT populations in elementary school. Refer to Tables 1–2 for more details on the transition states and probabilities.

to be represented by increasingly higher iterations of β_n and ϵ_n and are summarized in Table 2. Taking the transition states and probabilities together, the model is visualized as a directed graph in Figure 2.

Transition Probability	Description
β_0	P(Entering GT in grade K | Entered grade K)
β_1	P(Staying in GT in grade 1 | In GT in grade K)
ϵ_0	P(Joining GT in grade 1 | Not in GT in grade K)
β_2	P(Staying in GT in grade 2 | In GT in grade 1)
ϵ_1	P(Joining GT in grade 2 | Not in GT in grade 1)
β_3	P(Staying in GT in grade 3 | In GT in grade 2)
ϵ_2	P(Joining GT in grade 3 | Not in GT in grade 2)
β_4	P(Staying in GT in grade 4 | In GT in grade 3)
ϵ_3	P(Joining GT in grade 4 | Not in GT in grade 3)
β_5	P(Staying in GT in grade 5 | In GT in grade 4)
ϵ_4	P(Joining GT in grade 5 | Not in GT in grade 4)
β_{MS}	P(Staying in GT in middle school | In GT in grade 5)
ϵ_{MS}	P(Joining GT in middle school | Not in GT in grade 5)

Table 2: Transition probability parameters for a given academic year.

In practice, we develop five Markov chains, one for each of the five major racial/ethnic groups in BVSD. These are White, Latinx, Asian, TMR, and Other. There are several important points that we wish to make about these groupings. First, we must address the “Other” category. We note that this category name is not meant to be an act of erasure or othering of the racial/ethnic groups contained within. Rather, the Other category contains Black, AIAN, and NHPI students, whose numbers are so low in both BVSD and BVSD’s GT programs that the model output would not be statistically significant without grouping them into the same category. However, we want to note that we acknowledge and recognize each of these groups. Additionally, we are aware that there is currently disagreement among academic and cultural communities on the use of the terms “Hispanic” versus “Latinx.” Here, we choose to use “Latinx” to include non-binary and non-gender-conforming individuals in the Latinx community, but we recognize and respect both names. We would also like to note that here, the name “Asian” is inclusive of both Asian immigrant and Asian American students and recognize that most of the students in this category are Asian American. Finally, while we assume that all students of more than one racial/ethnic background are contained in the TMR category, such that students of other racial backgrounds are strictly of one background, we do not seek to erase the multiple cultures and identities of students in the TMR category and will seek to identify trends based on TMR students' diverse identities if the data becomes available. Furthermore, if any of the names we use for the racial/ethnic groups in this study are inappropriate in any way, we invite readers to reach out and correct us, and we apologize for any mistakes.

To construct each chain, we use the data we obtained from BVSD to compute our transition probabilities and the initial number of students in each state for each race. For our initial state vector, we use the number of students of each race that are in each state at the end of the 2017-18 academic year, using the number of incoming kindergartners from the 2018-19 data for the initial K_B values. Since we only have data on an annual basis, the iterations for our Markov chain are defined as the period of one academic year. At the end of each year, we save the output of the Markov chain, so that we can use it as the new initial state vector for the next iteration. At the beginning of the next year, we must prime the system with the new K_B values, update the transition matrix to the current year, and find the number of students in each state for the new year. We repeat this process for the four academic years from 2018-19 to 2021-22.

Now, we share some concluding comments about the model construction process. First, in completing the probability and initial calculations, we round down to the nearest student, which will introduce some rounding error into our data. Furthermore, since we only have yearly data, we assume that all students who were in GT in the previous academic year stay in GT in the current academic year for a given racial group unless the numbers for that group drop between years. Then the difference is the number of students who opted out of GT between years. If the numbers for that group increase between years, then the increase in students is the number of students opting into GT services between years. Additionally, we acknowledge that the calculations for the transition probabilities were computed by hand, since the data could not be exported from BVSD's Tableau profile. As a result, there may be errors in the computations, which will require validation at a later time. Finally, we cannot guarantee that the data we used was updated at the same time every year, since dates for data collection were not provided for GT enrollment.

3 Preliminary Results and Conclusions

So far, we have trained our model on the BVSD dataset but have yet to perform predictions for future years. However, we have observed several trends in the BVSD dataset that we will share now.

Most notably, over the four years of study, there is no significant change in representation in BVSD's GT programs. White students, who typically make up about 67% of the general population, make up over 70% of the GT population every year, while the other four groups make up about 10% or less of the GT population each year. While Asian, TMR, and Other students make up less than 10% of the general population, Latinx students make up about 20% of the general population and are therefore only half as represented in GT as they are in the school district. These trends are visualized in Figure 3.

Perhaps more illustrative are the trends we observed in RI, which supplement the findings in Figure 3. In particular, we found that Asian, TMR, and White students consistently have an RI greater than 1, while Latinx and Other students consistently have an RI less than 1 across the four years of study, trends which are depicted in Figures 4–5. Overall, the current trends in BVSD tell a story of consistent underrepresentation of Latinx and Other students over the past four years. While data is not presently available for the years prior to 2017-18, it appears that BVSD's current GT policies are not having a significant impact on improving representation, and they may benefit from considering more intensive or alternative approaches.

Additionally, while we expected to see underrepresentation in Latinx and Other students and overrepresentation of White and Asian students in BVSD's GT population, we were surprised by the consistent overrepresentation of TMR students. Since the largest demographic groups in BVSD are White and Latinx, we suspect that many students in the TMR group have one White parent and one Latinx parent, which may confer some White advantage to these students. On a related note, students with one White parent and one Latinx parent likely have a paler skin

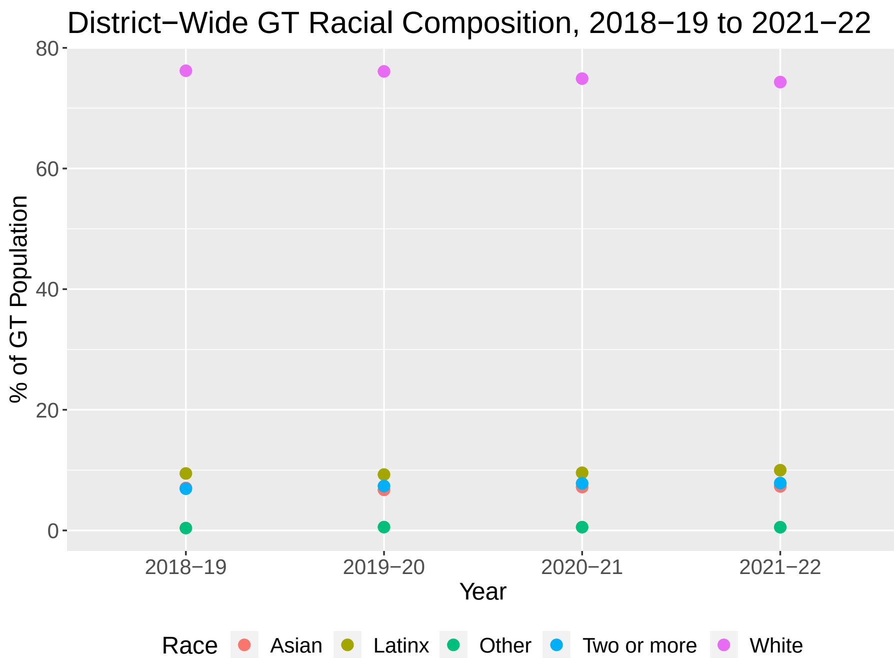

Figure 3: BVSD GT student body composition between 2018-19 and 2021-22.

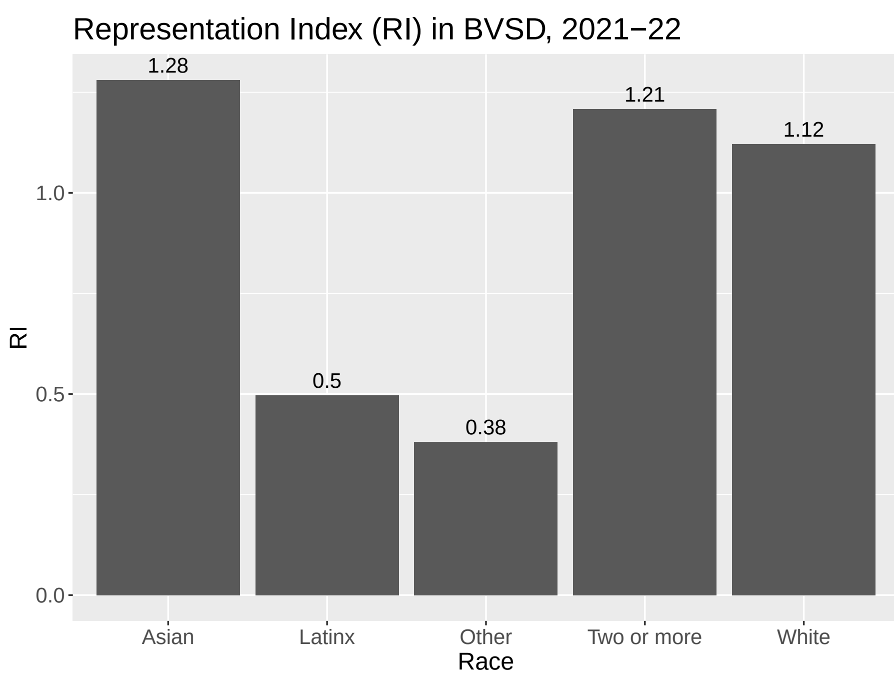

Figure 4: BVSD representation indices for the five racial/ethnic groups in 2021-22.

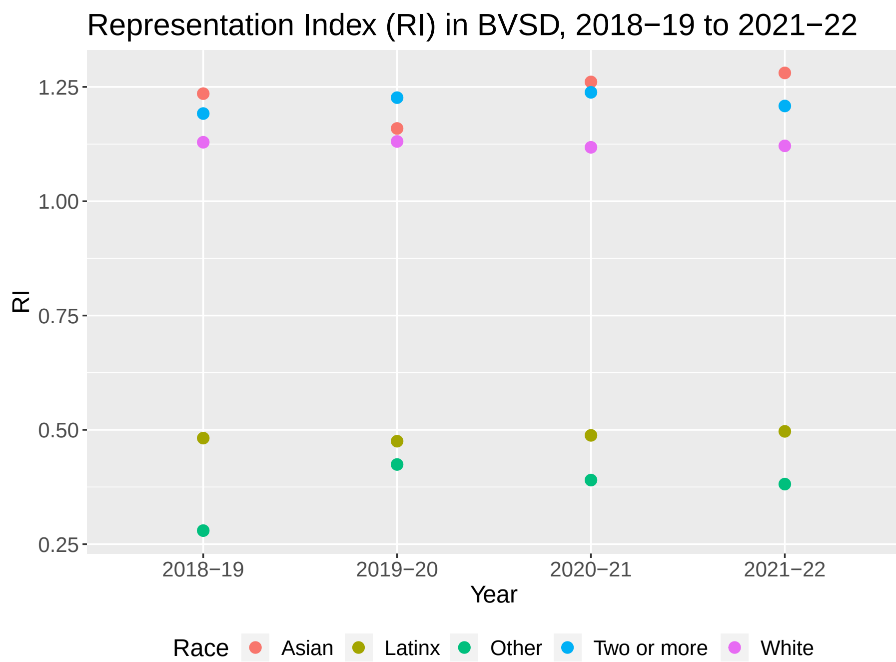

Figure 5: BVSD representation indices for the five racial/ethnic groups between 2018-19 and 2021-22.

tone compared to students with two Latinx parents; therefore, colorism may play a role in the higher representation of TMR students in the GT population as well.

4 Future Work

As we consider our body of research so far, we note that there are many different directions in which this project can go. Considering the training data alone, it would be interesting to compare overall BVSD GT demographics from year to year with demographics in one class of BVSD GT students as they progress through different grade levels. In particular, this type of analysis could reveal whether interventions that happen in different grade levels (such as universal screening in second grade) have a significant impact on representation of different racial groups. Furthermore, the current dataset is unique in that it contains two pre-pandemic years and two pandemic-affected years. This presents the opportunity to quantify the impact of the pandemic and virtual education on GT identification and retention across different racial/ethnic groups.

At present, our model has not been sufficiently validated, and this is a necessary next step to move forward with the research. Currently, we do not have sufficient data to rigorously test our model. However, we could do a preliminary validation by training the model on just the first two years of data and using the trained model to predict the next two years of data. If the model's predictions sufficiently match the data we have, then we will have some reassurance that the model is valid, although more testing will be required before the model can be used to make predictions for future years. With a validated model, we will be able to not only make predictions for future years but also modify parameter values and see the impact of those changes on GT representation, which can help in identifying optimal strategies for achieving equal representation in GT.

Currently, our model is based entirely on the data from one school district, which is not a sufficient sample for GT programs on a state or nationwide scale. It would be ideal to get data from other school districts in Colorado and other states to identify optimal GT policies for all GT programs and to see how differences in district and state GT policies and funding impact representation in GT and quality of GT programs. In addition, it would be useful to compare GT representation and program quality in schools/districts with different racial/ethnic diversity levels and with different percentages of ELL students.

Furthermore, a compelling future direction is to fill some of the gaps in the data and build a more realistic model. Ideally, we would like to incorporate critical steps in the identification process such as referral, testing, and decision to opt into GT services. We believe that these could provide valuable insights into underrepresentation in GT programs nationwide. In particular, it would be helpful to know what types of testing measures are used in different schools in BVSD for GT identification, as not all schools use the same measures to inform their GT identifications and this may help to uncover biases that are inherent in the tests (such as not having home language alternatives for ELL students) or their interpretation. Since certain measures like universal screening and use of local norms have been shown to be useful in identifying more students of color, it would be helpful to see the effects of such measures in BVSD as compared to other identifying tests.

While less important than incorporating referral and testing steps into our model, it would also be valuable to obtain data on a semesterly basis to more accurately represent how students may be identified in either semester of the academic year. We can easily iterate by semesters instead of by academic year by adding more transition states to our model. We can further increase the model's complexity and make our model system more realistic by incorporating transfer in and out of the district, which would add more transition states and transition probabilities to the system. However, we cannot move forward with any of these steps until we have the data to validate our model. While we do not expect that they will have a significant effect on the model output, we could also modify the model to allow for students being held back, students skipping grades, and students dropping out.

If we are unable to obtain the data required to pursue these options, one avenue that might be beneficial to explore is to turn our model into a hidden Markov model. In such a model, the transition states are unknown, and only the emission states are known [24]. In the context of our model, we know the student's GT status (the emissions), but we do not know which students were referred, who referred them, which students were tested, or how they were tested. These could all potentially represent hidden states. Therefore, a hidden Markov model may help to resolve some of the data gaps that we are currently experiencing.

Additional ways to expand the model exist as well. Perhaps most obvious among them are including middle- and high-schoolers in the model, which may be particularly important for understanding underrepresentation in GT, as students of color are sometimes identified later than White students [15]. It may also be helpful to consider in our model students who are monitored for later GT identification and/or placed in the talent pool, if it exists in other districts, since underrepresented students may not be selected for GT as early as overrepresented students.

Another route to investigate is introducing additional Markov chains for different combinations of race/ethnicity and gender. This intersectional approach could produce interesting trends for students of underrepresented gender and racial identities. Furthermore, we could also introduce additional chains for different GT categories (i.e. talent aptitude versus specific academic aptitude), as this may reveal race-driven biases in GT identification based on the category in which students are identified.

Some final ways to expand the model are quantifying parental influence on and familiarity with the GT identification process. This may prove to be of particular importance in families where the parent/legal guardian does not speak English or comes from an underrepresented group. Since the majority of referrals come from family members, this may be important for underrepresented students. More generally, we would like to incorporate bias, homophily, and possible selves into our model as well, perhaps by introducing parameters to represent each of these concepts and evaluating how they affect the GT identification process for different racial/ethnic groups.

In terms of improving our methods in conducting this research, it would be fruitful for others to consider using packages such as the markovchain package in R for streamlined and efficient results. Furthermore, computing data by hand as we did is not ideal, since it is temporally expensive and error-prone. We recommend that other researchers consider webscraping alternatives for data that is not readily exportable.

Ultimately, we wish to use our model to develop an app for school districts to easily quantify and find strategies to improve their GT representation. For example, if a school district provides their current GT demographics, the app would ideally compute their current RI, make predictions for RI a few years into the future, and provide strategies for increasing representation of the groups being underrepresented, such as recommending particular testing or screening measures. If we incorporate data from other districts, it may also be helpful to build a database of GT metrics, particularly if the project eventually reaches a state and/or national scale. Overall, we intend to share our results with as wide an audience as possible and are hopeful that our model and its applications will help to make GT programs equitable for all, in BVSD and beyond.

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