Charge-to-Mass Ratio of the Electron

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1 Introduction

In this experiment we aim to measure the charge-to-mass ratio of the electron, $R_{em} = e/m_e$. First measured by J.J. Thompson in 1987, the significance of this quantity primarily lied on the more ambitious task of calculating the mass of the electron.

2 Setup

We use Helmholtz coils consisting of two circular electromagnets centered in the same (normal) axis, separated by a distance equal to their radius.

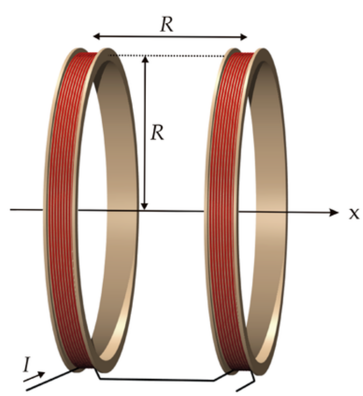


Figure 1: Helmholtz coils[1]

Between these two coils a cathode ray tube is placed inside a spherical glass bulb filled with helium. Electrons are released by thermionic emission from the cathode and are accelerated towards a cylindrical anode in presence of a magnetic field set up by the coils (Figure 2). The velocity v_e of the electrons is controlled by the voltage supplied and the path curvature varies with current. The emission spectrum for helium produced when electrons excite helium atoms in a gas discharge tube. In this experiment, the wavelength of light emitted is around 500nm (green light).

When the electrons are released from the cathode, they follow a circular path in response to the uniform magnetic field at the center, losing potential energy U and gaining kinetic energy K.

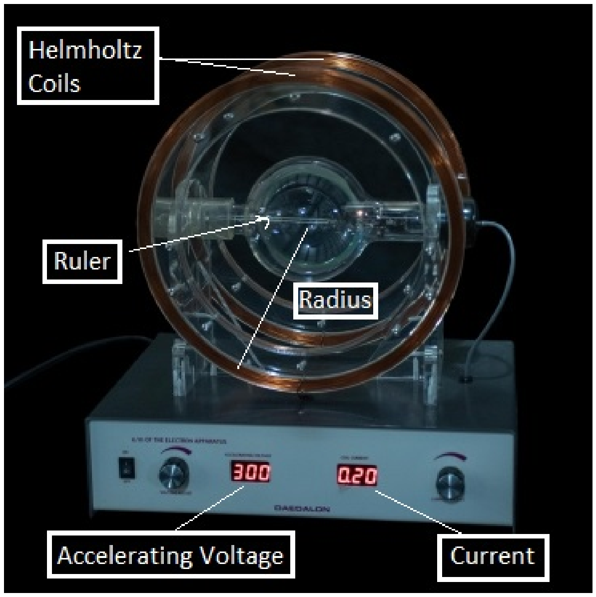


Figure 2: E/M Apparatus, with Power Supply, Daedalon[2]

The potential energy of an electron is given by

$$U = eV$$

And so,

$$\begin{aligned} eV &= K \\ &= \frac{1}{2} m_e v_e^2 \\ v_e &= \sqrt{\frac{2eV}{m_e}} \end{aligned} \tag{1}$$

The magnetic field $\mathbf{B}$ in the middle region is aligned perpendicularly with the velocity vector of the electrons. The magnetic force felt by the electrons is given by

$$\mathbf{F} = e\mathbf{v}_e \times \mathbf{B}$$

and

$$F = ev_eB. \tag{2}$$

This force can also be interpreted as the centripetal force acting on the electrons

$$F = \frac{m_e v_e^2}{r}$$

where r is the radius of curvature. Combining this equation with equation 1 and equation 2 gives

$$R_{em} = \frac{e}{m} = \frac{2V}{B^2 r^2} \tag{3}$$

Using the Biot-Savart law we can find an expression for the magnetic field between the Helmholtz coils (Figure 3). If the apparatus consisted of a single coil with N turns carrying a current I, the field at a distance x along the axis of the coil would be

$$B(x) = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$$

where μ_0 is the permeability of free space. At $x = R/2$,

$$B\left(\frac{R}{2}\right) = \frac{\mu_0 N I R^2}{2(R^2 + (R/2)^2)^{3/2}} = \frac{\mu_0 N I}{2R}\left(\frac{4}{5}\right)^{3/2}$$

Adding another coil at a distance R from the first one aligned in the same axis with current flowing in the same direction, the field strength in the middle doubles, yielding the desired formula

$$B = \frac{\mu_0 N I}{R}\left(\frac{4}{5}\right)^{3/2} \tag{4}$$

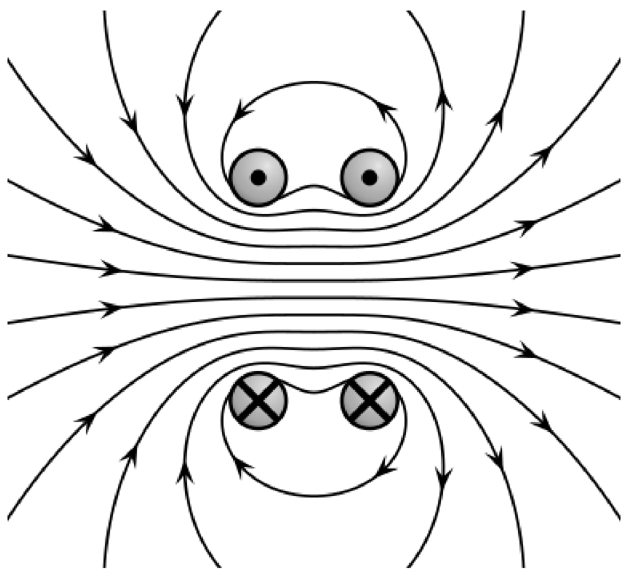


Figure 3: Magnetic field at the center of the Helmholtz Coils. The current through the coils flows into the page at the bottom and out of the page at the top. Note that the field is roughly uniform at the center point.[3]

3 Data

We fix voltages of 150. V, 200. V, 300. V, and 400. V and make 10 measurements of varying current for each voltage value. For each current I we record the diameter of curvature of the electrons' path D and its uncertainty σ_D.

V = 150. Volts

I (A)	D (cm)	σ_D (cm)
0.90	11.04	0.20
1.00	9.90	0.13
1.10	9.02	0.15
1.20	8.31	0.20
1.30	7.68	0.20
1.40	7.10	0.15
1.50	6.60	0.15
1.60	6.13	0.13
1.70	5.78	0.15
1.80	5.50	0.12

V = 200. Volts

I (A)	D (cm)	σ_D (cm)
1.00	11.36	0.20
1.13	10.15	0.15
1.26	9.07	0.15
1.39	8.20	0.13
1.52	7.56	0.15
1.65	6.97	0.17
1.78	6.40	0.13
1.91	6.02	0.15
2.04	5.65	0.16
2.17	5.25	0.13

V = 300. Volts

I (A)	D (cm)	σ_D (cm)
1.30	10.60	0.20
1.45	9.64	0.15
1.60	8.75	0.17
1.75	8.00	0.15
1.90	7.40	0.15
2.05	6.86	0.17
2.20	6.45	0.18
2.35	5.95	0.15
2.50	5.60	0.15
2.65	5.32	0.17

V = 400. Volts

I (A)	D (cm)	σ_D (cm)
1.50	10.80	0.15
1.65	9.84	0.17
1.80	8.98	0.17
1.95	8.31	0.20
2.10	7.75	0.18
2.25	7.25	0.15
2.40	6.75	0.15
2.55	6.37	0.16
2.70	6.00	0.16
2.85	5.65	0.15

Other Measured Quantities

Name	Variable	Value
Diameter of Helmholtz coils	D_c	28.5 cm
Uncertainty in D_c	σ_{D_c}	0.02 cm
Number of turns of each coil	N	132 turns
Uncertainty in the current	σ_I	0.01 A

4 Analysis

4.1 Mean, Error, and Uncertainty

Using Equation 4 we compute the magnetic field for each data point as well as its uncertainty, σ_B. This uncertainty is given by

$$\sigma_B = \sqrt{\left(\frac{\partial B}{\partial N}\sigma_N\right)^2 + \left(\frac{\partial B}{\partial I}\sigma_I\right)^2 + \left(\frac{\partial B}{\partial R}\sigma_R\right)^2}.$$

Similarly we compute the charge-to-mass ratio, R_{em}, and its uncertainty $\sigma_{R_{em}}$ (Equation 3),

$$\sigma_{R_{em}} = \sqrt{\left(\frac{\partial R_{em}}{\partial V}\sigma_V\right)^2 + \left(\frac{\partial R_{em}}{\partial B}\sigma_B\right)^2 + \left(\frac{\partial R_{em}}{\partial r}\sigma_r\right)^2}$$

where $r = D/2$ and $\sigma_r = \sigma_D/2$. It is important to note that the terms $\partial B/\partial N$ and $\partial R_{em}/\partial V$ both vanish to zero since they are constants throughout each set of 10 trials in the experiments. The partial derivatives of B are

$$\frac{\partial B}{\partial N} = 0 \qquad \frac{\partial B}{\partial I} = \frac{\mu_0 N}{R}\left(\frac{4}{5}\right)^{3/2} \qquad \frac{\partial B}{\partial R} = -\frac{\mu_0 N I}{R^2}\left(\frac{4}{5}\right)^{3/2}$$

and the partial derivatives of R_{em} are

$$\frac{\partial R_{em}}{\partial V} = 0 \qquad \frac{\partial R_{em}}{\partial B} = -\frac{4V}{B^3 r^2} \qquad \frac{\partial R_{em}}{\partial B} = -\frac{4V}{B^2 r^3}$$

resulting in the equations

$$\sigma_B = \left(\frac{4}{5}\right)^{3/2}\mu_0\sqrt{\left(\frac{N}{R}\right)^2\sigma_I^2 + \left(\frac{IN}{R^2}\right)^2\sigma_R^2} \tag{5}$$

$$\sigma_{R_{em}} = \sqrt{\left(\frac{4V}{B^3 r^2}\right)^2\sigma_B^2 + \left(\frac{4V}{B^2 r^3}\right)^2\sigma_r^2}. \tag{6}$$

The weighted average of the charge-to-mass ratio $\bar{R}_{em}$ and its error $\sigma_{\bar{R}_{em}}$ are determined by

$$\bar{R}_{em} = \frac{\sum R_{em_i}/\sigma^2_{R_{em_i}}}{\sum 1/\sigma^2_{R_{em_i}}} \tag{7}$$

$$\sigma_{\bar{R}_{em}} = \sqrt{\frac{1}{\sum 1/\sigma^2_{R_{em_i}}}}. \tag{8}$$

4.2 Computations

Using the previous information we compute the values of B, σ_B, R_{em}, and $\sigma_{R_{em}}$ summarized in the following tables and graphs.

$V = 150.$ Volts

B (T)	σ_B (T)	R_{em} (C/kg)	$\sigma_{R_{em}}$ (C/kg)
0.000750	9.85E-06	1.75E+11	7.84E+09
0.000833	1.02E-05	1.76E+11	6.33E+09
0.000916	1.05E-05	1.76E+11	7.10E+09
0.000999	1.09E-05	1.74E+11	9.19E+09
0.00108	1.13E-05	1.74E+11	9.73E+09
0.00117	1.17E-05	1.75E+11	8.19E+09
0.00125	1.21E-05	1.76E+11	8.72E+09
0.00133	1.25E-05	1.80E+11	8.34E+09
0.00142	1.30E-05	1.79E+11	9.86E+09
0.00150	1.34E-05	1.76E+11	8.32E+09

$V = 200.$ Volts

B (T)	σ_B (T)	R_{em} (C/kg)	$\sigma_{R_{em}}$ (C/kg)
0.000833	1.02E-05	1.79E+11	7.66E+09
0.000941	1.06E-05	1.75E+11	6.52E+09
0.00105	1.11E-05	1.77E+11	6.94E+09
0.00116	1.16E-05	1.78E+11	6.66E+09
0.00127	1.22E-05	1.75E+11	7.70E+09
0.00137	1.27E-05	1.74E+11	9.10E+09
0.00148	1.33E-05	1.78E+11	7.89E+09
0.00159	1.39E-05	1.74E+11	9.21E+09
0.00170	1.45E-05	1.74E+11	1.03E+10
0.00181	1.52E-05	1.78E+11	9.29E+09

$V = 300.$ Volts

B (T)	σ_B (T)	R_{em} (C/kg)	$\sigma_{R_{em}}$ (C/kg)
0.00108	1.13E-05	1.82E+11	7.85E+09
0.00121	1.19E-05	1.77E+11	6.52E+09
0.00133	1.25E-05	1.77E+11	7.62E+09
0.00146	1.32E-05	1.77E+11	7.35E+09
0.00158	1.39E-05	1.75E+11	7.73E+09
0.00171	1.46E-05	1.75E+11	9.17E+09
0.00183	1.53E-05	1.72E+11	1.00E+10
0.00196	1.61E-05	1.77E+11	9.38E+09
0.00208	1.68E-05	1.77E+11	9.88E+09
0.00221	1.76E-05	1.74E+11	1.15E+10

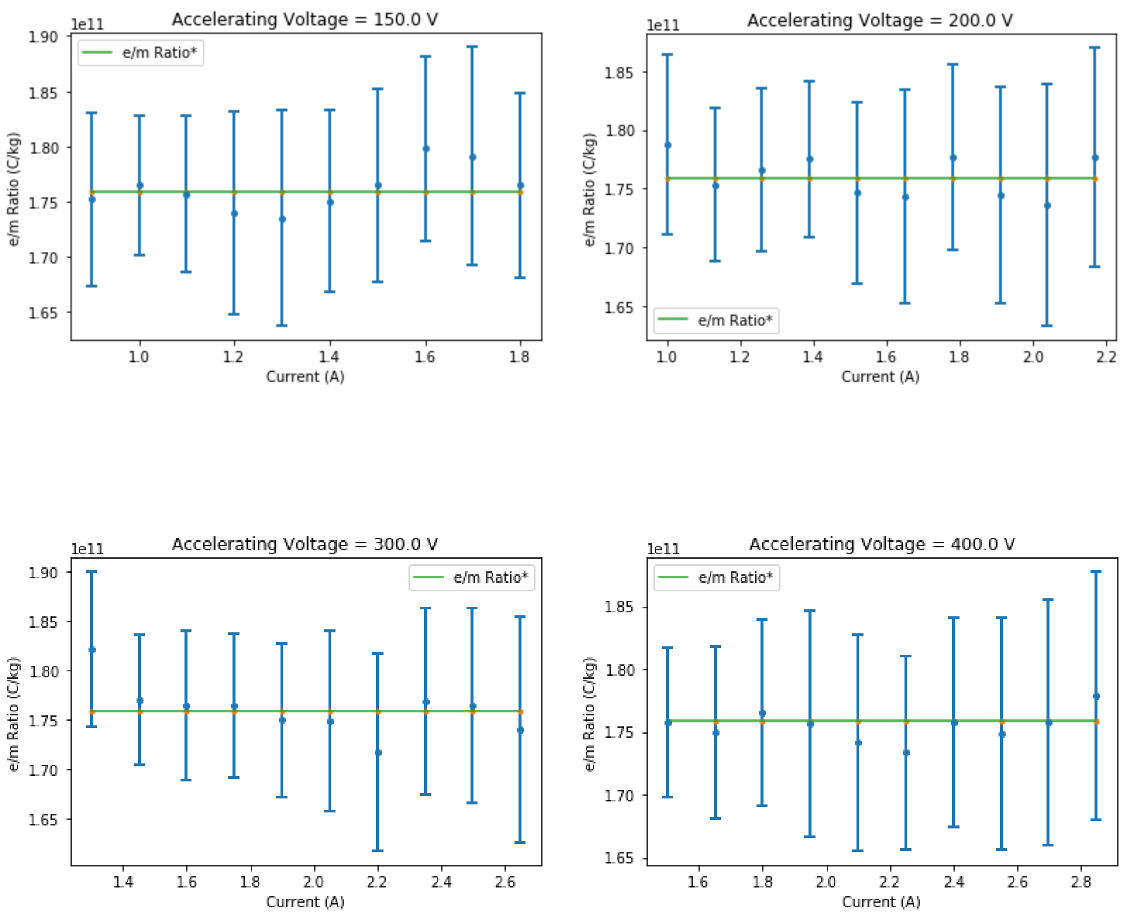
$V = 400.$ Volts

B (T)	σ_B (T)	R_{em} (C/kg)	$\sigma_{R_{em}}$ (C/kg)
0.00125	1.21E-05	1.76E+11	5.95E+09
0.00137	1.27E-05	1.75E+11	6.86E+09
0.00150	1.34E-05	1.77E+11	7.39E+09
0.00162	1.41E-05	1.76E+11	8.99E+09
0.00175	1.48E-05	1.74E+11	8.61E+09
0.00187	1.56E-05	1.73E+11	7.73E+09
0.00199	1.63E-05	1.76E+11	8.32E+09
0.00212	1.71E-05	1.75E+11	9.22E+09
0.00225	1.78E-05	1.76E+11	9.78E+09
0.00237	1.86E-05	1.78E+11	9.85E+09

4.3 Results

The published value for the charge-to-mass ratio of the electron is

$$R^*_{em} = 1.75882 \times 10^{11} \text{ C/kg}$$



Values of R_{em} for each current are plotted with their respective uncertainties. The straight orange line indicates the published value of R^*_{em}.

We tabulate the weighted means and uncertainties of R_{em} for all accelerating voltages using Equations 7 and 8:

Voltage (V)	$\bar{R}_{em}$ (C/kg)	$\sigma_{\bar{R}_{em}}$ (C/kg)
150.	1.76E+11	2.57E+9
200.	1.76E+11	2.58E+9
300.	1.76E+11	2.64E+9
400.	1.75E+11	2.52E+9

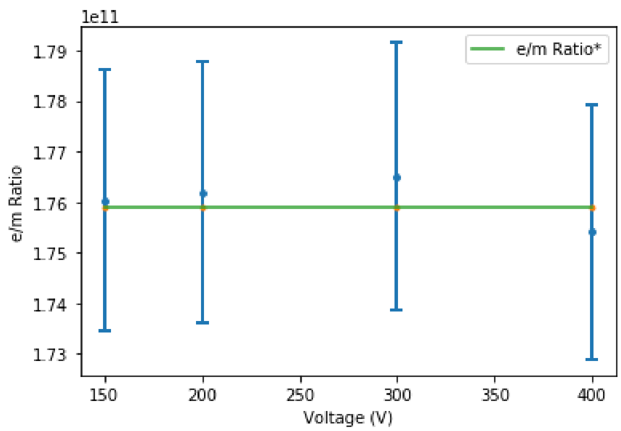


Figure 4: Final results for the charge-to-mass ratio for each voltage.

Lastly, we calculate the final weighted mean of $\bar{R}_{em}$ and its uncertainty using Equations 7 and 8 once again.

$$\bar{\bar{R}}_{em} = 1.76\text{E+11 C/kg} \qquad \sigma_{\bar{\bar{R}}_{em}} = 1.29\text{E+9 C/kg}$$

5 Conclusion

Our results show that the charge-to-mass ratio of the electron is $e/m = (176 \pm 1) \times 10^9$ C/kg, compared to the published value of 175.882×10^9 C/kg. Our best estimates originate at lower voltages and our deviations fluctuated with increasing voltage. However, the e/m ratio varied more at lower voltages with increasing current. From the latter we can conclude that the charge-to-mass ratio measurements behave more sensitively toward changing current, which can be seen through the proportionality between these quantities: R_{em} is inversely proportional to B^2, and the magnetic field is directly proportional to the current. Since the magnetic field values at the lower voltages (150. V and 200. V) are relatively closer to 0, the cause of deviation is more clear, as the function $1/B^2$ is much more responsive to such lower values.

Another source of deviation comes from the radius of curvature. As the electrons leave the cathode they collide inelastically with the Helium atoms, lowering their theoretical speed. The radius of curvature is directly proportional to the velocity, and similarly the charge-to-mass ratio is inversely proportional to the radius squared. This indicates that our measurements are sensitive at small values of curvature, which is more seen clearly at the lower pair of accelerating voltages.

Although our least precise results came from the higher pair of accelerating voltages (Figure 4), we generally obtained more precise values and less fluctuations for R_{em} as the current increased; this is evidenced by the lowest uncertainty value at 400 V, $\sigma_{\bar{R}_{em}}$ = 2.52E+9 C/kg. Therefore, we would obtain better readings at higher voltages (and currents) if we were to repeat this experiment. The radius of curvature of the electrons' path would be smaller and their speed would be higher, meaning that the effect of the inelastic collisions would decrease, and the electrons would stay closer to the area where the magnetic field is more uniform, yielding better measurements.

6 References

[1] https://en.wikipedia.org/wiki/Helmholtz-coil

[2] https://shop.sciencefirst.com/science-first/7502-em-apparatus-with-power-supply.html

[3] https://en.wikipedia.org/wiki/Helmholtz-coil