Counting Statistics

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1 Introduction

In this experiment we aim to determine the activity of a ^{90}Sr source. We make extensive use of probability distributions as part of our analysis to investigate the random electron emission process of the source, first quantitatively explained by Marie Curie, together with her husband Pierre Curie, in their study of spontaneous radiation in the early 20th century.

2 Setup

We implement the use of a Strontium-90 disk as the radioactive source and a Geiger-Muller detector tube. The detector is placed right above the source fixed with a support stand (Figure 2).

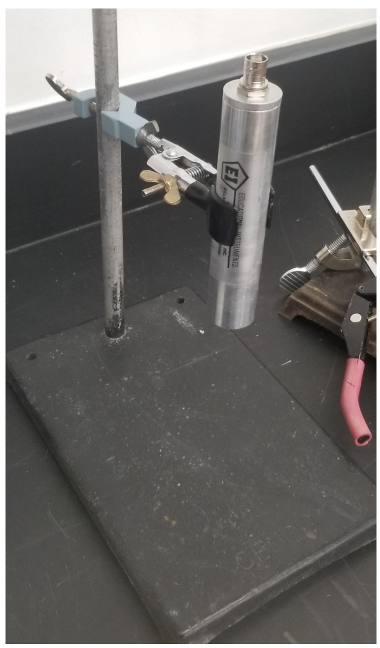

Figure 1: Geiger-Muller detector.

Figure 2: Counter apparatus.

The ^{90}Sr source constantly undergoes β^- decay where a neutron turns into a proton, converting Sr nuclei into Yttrium-90 nuclei and releasing electrons and electron antineutrinos. This process is known as nuclear transmutation.

$$^{90}Sr \longrightarrow ^{90}Y + e^- + \bar{v}_e$$

Each of the electrons possesses an energy of about 0.546 MeV [1]. Before electrons reach the detector, they interact with the atmosphere which decreases their energy through ionization and inelastic scattering with air molecules. The Geiger-Muller tube does not detect any decay electrons yet. As electrons reach the entrance window of the detector, a wire at some electric potential recollects the electrons at the center of the tube, where they collide with a gaseous detecting material. Electrons must possess sufficient energy to ionize the gas and produce electrical discharge pulses since the Geiger-Muller detector functions through readings of such electrical signals. If the counter apparatus does record any events at some voltage, we must increase the potential of the wire until electrons ionize the gas adequately to produce discharges. In order to operate a Geiger counter properly, the voltage source must be set in the *plateau* region, where a similar count output is consistently obtained for all charged particles traversing the counter. If the voltage is further increased, spontaneous discharges occur, making the detector less efficient. Another important consideration for Geiger counters is the *quenching* of the discharge initiated by the traversal of a charged particle. Until the gas is returned to its neutral state, the passage of a charged particle will not produce an output pulse; this is the period of time during which the counter is *dead*. To avoid this, we stay in the low-voltage plateau region of the detector.

The decay process is fundamentally probabilistic and therefore we cannot determine *exactly* when a decay event will occur. However, we can estimate the probability of this process happening within a time interval, which is best done by determining its probability distributions. We will attempt modeling the probability density functions with Gaussian and Poisson distribution. The Gaussian distribution

$$P_G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \tag{1}$$

has count mean μ and standard deviation σ, where for large counts (high-count statistics)

$$\mu_{exp} = \bar{N} = \frac{1}{N}\sum_{i=1}^{N} n_i \tag{2}$$

becomes an good estimator of the true mean μ, and

$$\sigma^2_{exp} = \mu_{exp}. \tag{3}$$

The Poisson distribution is defined by

$$P_P(x) = \frac{\mu^x e^{-\mu}}{x!} \tag{4}$$

where its mean and standard deviation are estimated by Equations (2) and (3) under the same conditions. We note that the Poisson distribution is a more compact version of the Binomial distribution when the event probabilities as quite small (low-count statistics).

For this experiment we assume that the ^{90}Sr source emits electrons uniformly in 4π steradians. If the entrance window of the detector has a diameter of d, then at a distance r the detector receives the fraction

$$f = \frac{\pi d^2}{4}\frac{1}{4\pi r^2} = \frac{d^2}{16r^2} \tag{5}$$

of all emitted electrons per second. The count rate $\dot{N}$ is then related to the distance r by

$$\dot{N} = S_{Sr} \cdot f \cdot (1\ Ci) = S_{Sr}\,\frac{d^2}{16r^2}\,(3.7\times 10^{10}) \tag{6}$$

where S_{Sr} is the total activity of the Strontium-90 source and 1 Ci is a standard unit of activity equivalent to 3.7×10^{10} decays per second.

3 Data and Analysis

To find the detector plateau, we started at a voltage of 500. V and increased by 20. V until we obtained a high number of counts with the detector close to the Strontium-90 source (Figure 3).

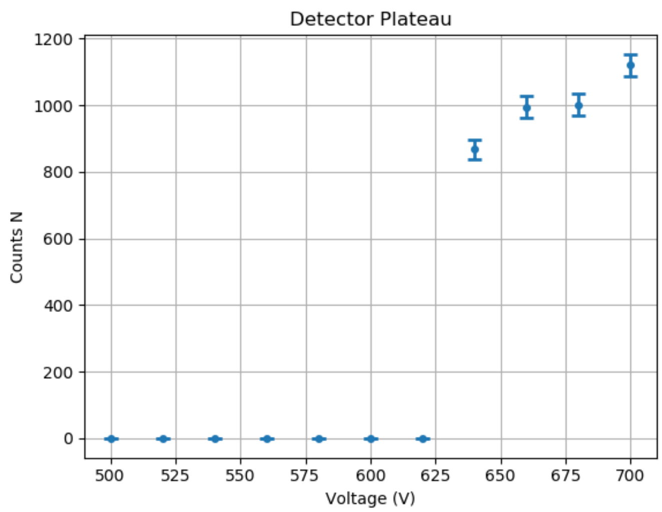

Figure 3: Detector plateau. Plateau region is around 660. - 680. V.

For low-count statistics, we set the counter at such a distance from the source that we record about 1-2 events in the shortest time period possible at a voltage of 670 V. These data are shown in the plot below, taken at a distance of (46.4 ± 0.5) cm.

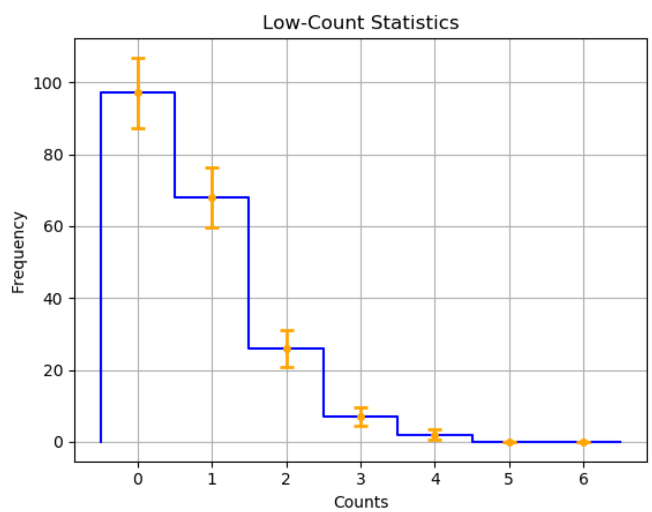

Figure 4: Low-count statistics plot of 200 trials. Uncertainties are taken to be $\sigma = \sqrt{N}$.

Using Equations 2 and 3, we estimate the mean and standard deviation of the low-count statistics plot

$$\mu_L = 0.746 \text{ counts and } \sigma_L = 0.863 \text{ counts} \tag{7}$$

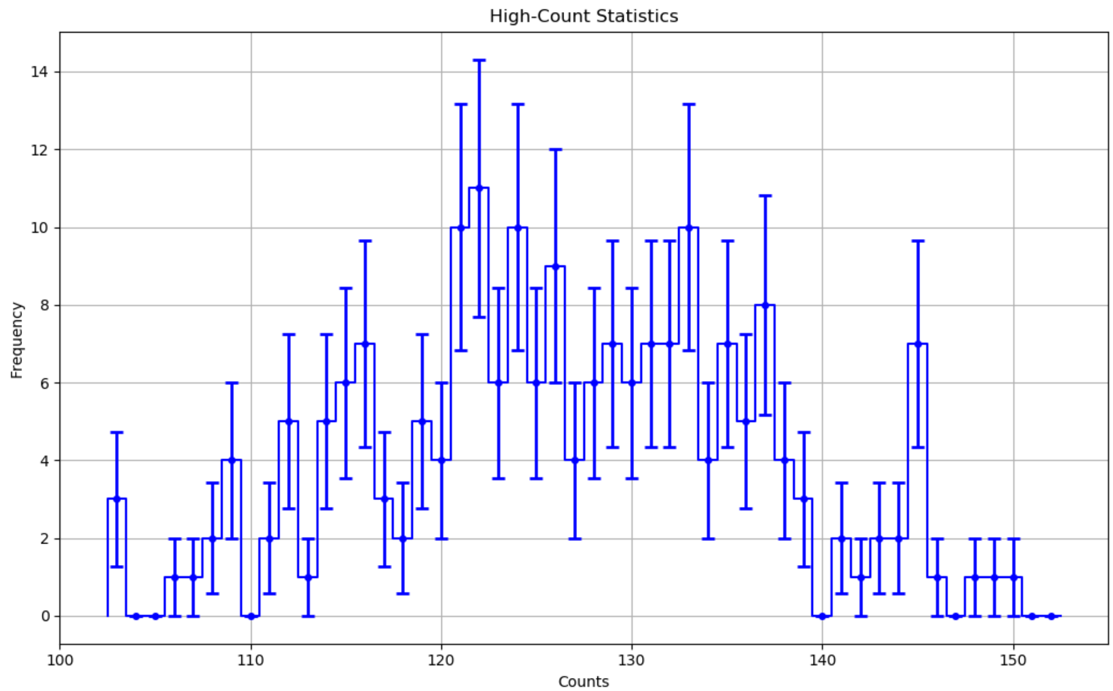

Figure 5: High-count statistics plot of 200 trials. Uncertainties are taken to be $\sigma = \sqrt{N}$.

whereas for the high-count statistics plot,

$$\mu_H = 126.7 \text{ counts and } \sigma_H = 11.25 \text{ counts.} \tag{8}$$

Next, we fit a Gaussian and Poisson curves to the low-count statistics plot (Figure 6) and compare the calculated means and standard deviations. The Gaussian fit gives

$$\mu_G = 0.327 \pm 0.095 \text{ counts and } \sigma_G = 0.827 \pm 0.078 \text{ counts} \tag{9}$$

and the Poisson fit gives

$$\mu_P = 0.723 \pm 0.009 \text{ counts and } \sigma_G = 0.850 \pm 0.097 \text{ counts} \tag{10}$$

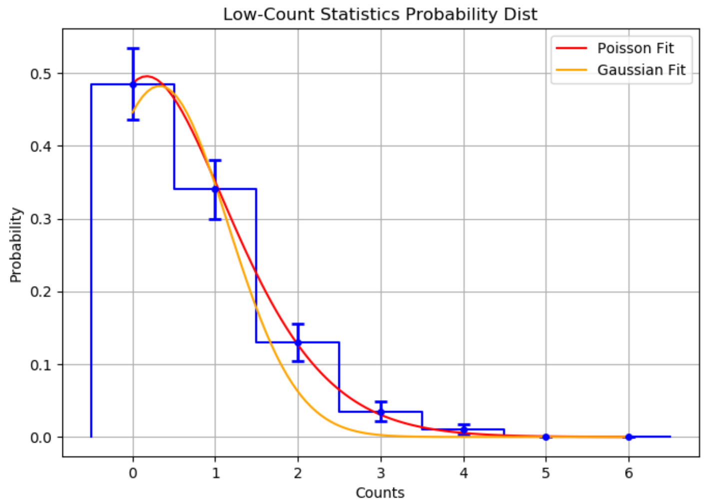

Figure 6: Gaussian and Poisson probability distribution fits for low-count statistics.

Similarly, for high-count statistics we attempt to plot Poisson and Gaussian fits. However, since the Poisson distribution involves the factorial function, it is not possible to compute $N!$ for very large counts, so we only plot a Gaussian distribution.

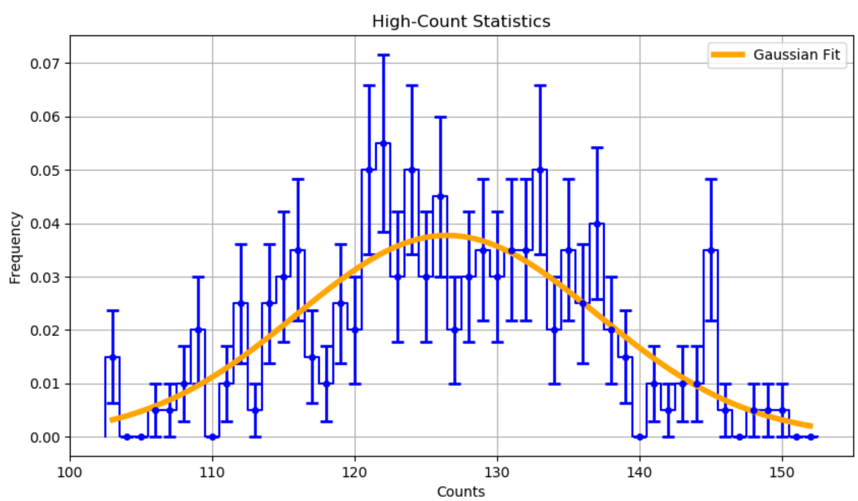

Figure 7: Gaussian probability distribution fit for high-count statistics.

The summary of this fit model is

$$\mu_G = 126.50 \pm 0.92 \text{ counts and } \sigma_G = 10.57 \pm 0.76 \text{ counts} \tag{11}$$

To confirm the validity of the relation presented in Equation 6, we place the Geiger-Muller detector at varying heights until we obtain about 200 counts, recording the time for each setting. We obtain the data in the table below

Height r (cm)	σ_r (cm)	Counts N	Time Elapsed t (s)
6.0	0.05	213	15
8.2	0.05	210	26
13.7	0.03	204	71
18.7	0.04	202	119
22.0	0.05	202	168
25.2	0.05	204	194

Next we determine the count rates $\dot{N} = N/t$ and plot them against the heights r in Figure 8.

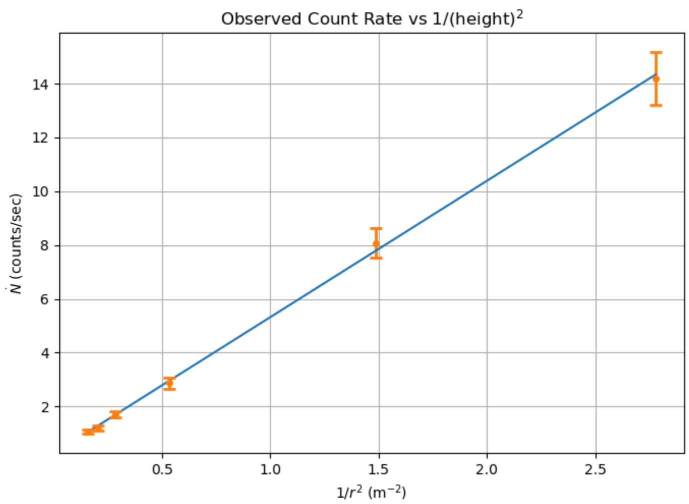

Figure 8: Count rate plot.

where we took the error in the count rate to be

$$\sigma_{\dot{N}} = \frac{\sqrt{N}}{t}. \tag{12}$$

The linear regression for the last graph above returns a slope value of $m = 5.072 \pm 0.073$. From Equation 6, m corresponds to

$$m = S_{Sr}\frac{d^2}{16}\left(3.7 \times 10^{10}\right) \implies S_{Sr} = \frac{16m}{d^2(3.7 \times 10^{10})}$$

where we measured the detector diameter $d = (2.60 \pm 0.03)$ cm. Therefore,

$$\boxed{S_{Sr} = 3.24 \times 10^{-6} \text{ decays/s}}$$

and

$$\boxed{\sigma_{S_{Sr}} = \sqrt{\left(\frac{\partial S_{Sr}}{\partial m}\sigma_m\right)^2 + \left(\frac{\partial S_{Sr}}{\partial d}\sigma_d\right)^2} = 8.82 \times 10^{-8} \text{ decays/s}}$$

4 Conclusion

Comparing the means $\mu_L = 0.746$, $\mu_G = 0.327$, and $\mu_P = 0.723$ for the low-count statistics section, it is clear that the Poisson fit was a better model as its mean is much more closer to the experimental mean μ_L than the Gaussian fit; a similar pattern holds for their standard deviations. Visually, the Poisson distribution lies, on average, closer to the bin centers than the Gaussian distribution. This is due to the fact that Poisson distribution is more adequate for modeling data that is right- or left-skewed like the low-count statistics distribution (Figure 6), whereas Gaussian fits are more suitable for symmetric distributions such as high-count statistics (Figure 7), supported by the close values of $\mu_H = 126.7$ and $\mu_G = 126.5$. We determined the strength of the Strontium-90 source to be $(3.24 \pm 0.09) \times 10^{-6}$ decays per second per Strontium nuclei. This means that for a 4-gram $^{(}$90)Sr source, there are about 9.0×10^{16} emitted electrons per second, which is a reasonable value considering the atmospheric conditions (22°C) of the experiment.

5 References

[1] http://nucleardata.nuclear.lu.se/toi/nuclide.asp?iZA=380090