Speed of Light

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October 14th, 2019

1 Introduction

The speed of light, c, is one of top fundamental constants across the history of physics. Initially, it was discussed that light traveled instantly with infinite speed until Danish astronomer Ole Römer proved otherwise in 1676. Studying Jupiter's moons, he noted that eclipses occurred sooner than expected when the Earth was closest to Jupiter and much later when the Earth was farthest from the planet. He hypothesized the cause of this enigma was that light took more time to travel longer distances, implying the speed of light was finite[1]. Then on, more accurate experiments settled the discussion, which we will replicate to estimate the speed of light.

2 Setup

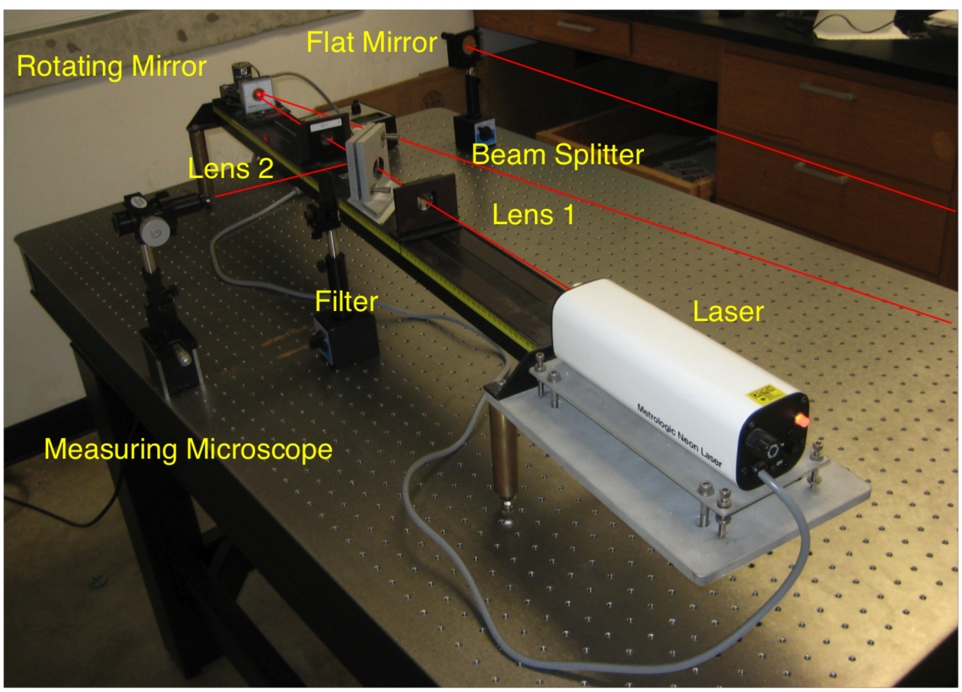

Figure 1: Experimental equipment.

Our setup is a redesign of Léon Foucault's apparatus to measure the speed of light. A laser delivers a focused beam of light that passes through a set of thin lenses and bounces off a rotating mirror and a pair of flat mirrors.

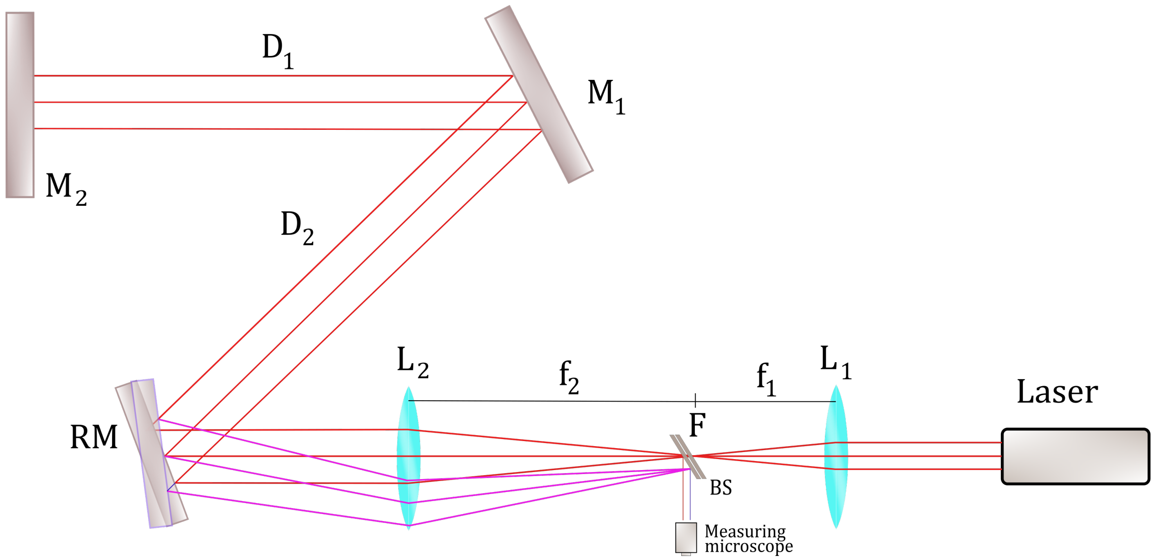

Figure 2: Light ray diagram.

Light rays (red) leave the source fairly parallel to the horizontal platform and when they reach the first lens L_1, the rays converge at the focal point F, which is also the focal point of the second lens L_2 (Figure 2). Before light reaches the second lens it passes through a beam splitter, BS (which we can momentarily ignore). Light rays emerge parallel from the second lens, encountering a rotating mirror (RM) with angular frequency ω and reflecting off of it. The reflected rays then reach a pair of flat mirrors M_1 and M_2. The second mirror M_2 is placed such that light reflects back at a straight angle, where the rays re-encounter the first flat mirror and the rotating mirror (pink rays), thus traveling a distance of approximately $2(D_1 + D_2)$.

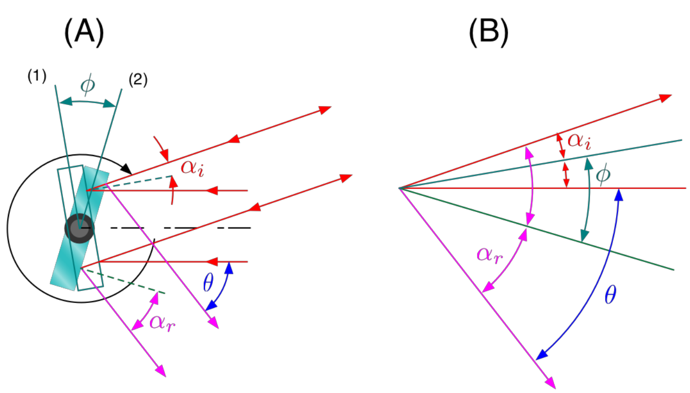

Figure 3: Ray diagram close-up around rotating mirror. Angles α_i and α_r are the angles of incidence and reflection, respectively, as measured from the mirror surface normals.

On the way back, the RM has rotated an angle ϕ and the light rays are reflected at an angle θ (Figure 3). The beam splitter creates a focused image point located a small distance Δx off the focal point F which we measure with a microscope. Using Figure 3B,

$$\alpha_r = \alpha_i + \phi$$

and

$$\theta = 2\alpha_r - 2\alpha_i = 2\phi \tag{1}$$

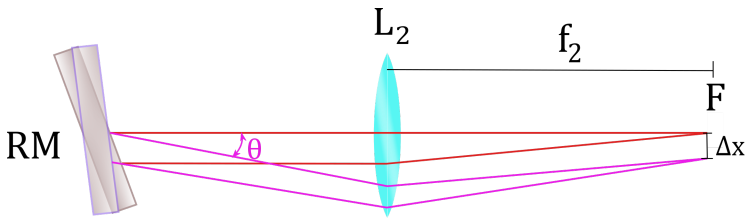

Figure 4: Angle of the image point at Δx.

For very small angles θ, Δx can be obtained by

$$\Delta x = f_2\theta. \tag{2}$$

At an angular frequency of ω, light travels a distance $2(D_1 + D_2)$ during a time interval Δt. We get the next expression for the angle of **clockwise** rotation ϕ of the RM

$$\phi = 2\pi\omega\Delta t \tag{3}$$

and

$$\Delta t = \frac{2(D_1 + D_2)}{c/n_{\text{air}}}. \tag{4}$$

Where n_{air} is the index of refraction of air. Combining Equations 1, 2, 3, and 4, we obtain

$$\Delta x = 8\pi\omega f_2 \frac{D_1 + D_2}{c} n_{\text{air}} \tag{5}$$

We note that $\Delta x = x - x_0$ is measured relative to a displacement x_0 obtained at an initial angular frequency ω_0. These same relations hold when the RM is set to rotate **counterclockwise**. We summarize our data in the next section.

3 Data

As described earlier, we take sets of measurements for clockwise and counterclockwise rotations of the rotating mirror which we tabulate below:

Clockwise Rotation		Counterclockwise Rotation	
ω (Hz)	Δx (μm)	ω (Hz)	Δx (μm)
340	12	193	8
428	17	335	13
530	22	454	18
681	27	566	23
798	32	673	28
913	37	782	33
1037	42	876	36
1150	47	931	38
		1048	43
		1120	47

Table 1: Angular frequency and image distance of clockwise and counterclockwise rotations.

Other Quantities

Name	Variable	Value
Uncertainty in CW and CCW frequencies	σ_ω	15 Hz
Uncertainty in image distance	$\sigma_{\Delta x}$	4 μm
Focal length of second lens	f_2	25 cm
Uncertainty in f_2	σ_{f_2}	2 cm
Distance of RM to $M1$	D_1	9.60 m
Distance of $M1$ to $M2$	D_2	10.03 m
Total distance traveled by light	D	19.63 m
Uncertainty in D	σ_D	25 cm
Relative point of measurement for CW rotation	$x_{0,\text{cw}}$	75 μm
Relative point of measurement for CCW rotation	$x_{0,\text{ccw}}$	16 μm
Index of refraction of air	n_{air}	1.0003

Table 2: Additional values required to calculate c and its uncertainty.

4 Analysis

4.1 Mean and Uncertainty

Now we proceed to evaluate an expression for the uncertainty in the speed of light σ_c from Equation 5.

$$c = \beta \omega f_2 \frac{D}{\Delta x}$$

where $\beta = 8\pi n_{\mathrm{air}}$ and $D = D_1 + D_2$.

$$\sigma_c = \sqrt{\left(\sigma_\omega \frac{\partial c}{\partial \omega}\right)^2 + \left(\sigma_{f_2} \frac{\partial c}{\partial f_2}\right)^2 + \left(\sigma_D \frac{\partial c}{\partial D}\right)^2 + \left(\sigma_{\Delta x} \frac{\partial c}{\partial \Delta x}\right)^2}. \tag{6}$$

Taking each of these partial derivatives, we obtain

$$\sigma_c = \frac{\beta}{\Delta x} \sqrt{(f_2 D \sigma_\omega)^2 + (\omega D \sigma_{f_2})^2 + (\omega f_2 \sigma_D)^2 + \left(\omega f_2 \frac{D}{\Delta x^2} \sigma_{\Delta x}\right)^2}. \tag{7}$$

Additionally, we compute the weighted average of the speed of light $\bar{c}$ and its uncertainty $\sigma_{\bar{c}}$ through the following equations

$$\bar{c} = \frac{\sum c_i / \sigma_{c_i}^2}{\sum 1/\sigma_{c_i}^2} \tag{8}$$

$$\sigma_{\bar{c}} = \sqrt{\frac{1}{\sum 1/\sigma_{c_i}^2}} \tag{9}$$

where $c_i = \beta D f_2 \omega_i / \Delta x_i$.

4.2 Results

We plot the image displacement Δx against the angular frequency ω with a linear fit in the following graph.

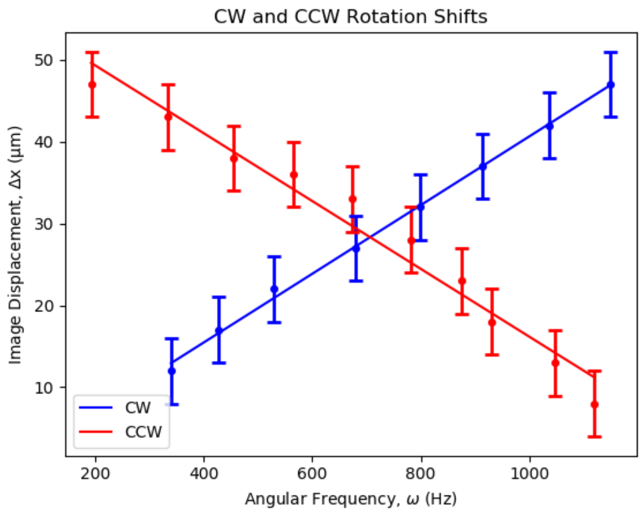

Figure 5: Data plots of image shift Δx vs. angular frequency ω of the rotating mirror.

As we increased the angular frequency, the image points moved in opposite directions depending on a clockwise or counterclockwise setup. The fit with positive slope indicates our clockwise RM rotation measurements. Similarly, the fit with negative gradient represents the counterclockwise RM rotation data. Each gradient m represents the magnitude of the quantity $8\pi n_{\text{air}} f_2 D/c$.

Gradient (μm/Hz)	Uncertainty in m (μm/Hz)
m_{cw} = 0.04198666	$\sigma_{m_{\text{cw}}}$ = 8.07974E-4
m_{ccw} = -0.04145682	$\sigma_{m_{\text{ccw}}}$ = 25.7459E-4

Table 3: Summary of linear fits.

This leads to $c = 8\pi n_{\text{air}} f_2 D/|m|$, and we obtain the following for our experimental estimates of the speed of light

Speed of Light (m/s)	Uncertainty in c (m/s)
c_{cw} = 2.94E+8	σ_{cw} = 5.67E+6
c_{ccw} = 2.98E+8	σ_{ccw} = 1.85E+6

Table 4: Final results for the value of c.

We use Equations 8 and 9 to obtain a weighted average of c based on the results of Table 4.

$$\boxed{\bar{c} = (2.96 \pm 0.13) \times 10^8 \text{ m/s}}$$

Lastly, we use an alternative method to estimate c. That is, obtaining each c through the last expression introduced in Section 4.1

$$c_i = 8\pi n_{\mathrm{air}} D f_2 \frac{\omega_i}{\Delta x_i}.$$

We plot these values in the following graph. Note that the uncertainties are now computed using Equation 7.

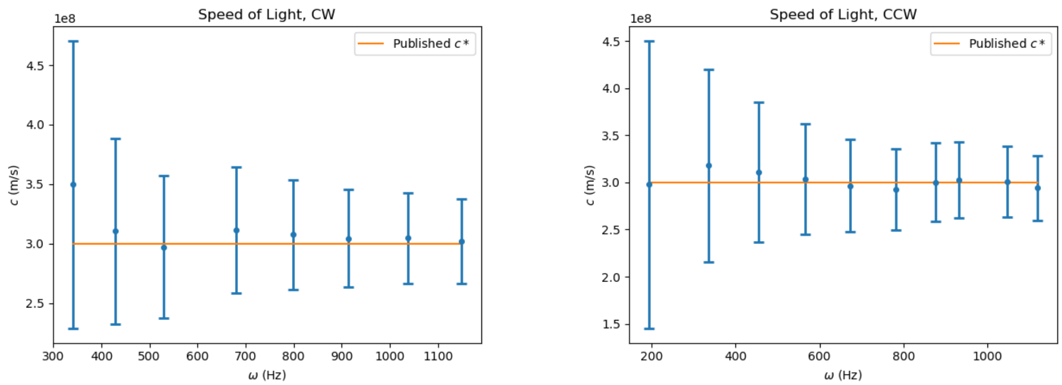

Figure 6: Singular data points from clockwise and counterclockwise rotations as estimate for c. The orange line represents the published value of the speed of light.

The published value for the speed of light is[2]

$$c^* = 299,792,458 \text{ m/s}.$$

The weighted averages of c for each rotation configuration are

Speed of Light (m/s)	Uncertainty in c (m/s)
$\bar{c}_{\mathrm{cw}} = 3.06\mathrm{E}{+}8$	$\sigma_{\mathrm{cw}} = 1.17\mathrm{E}{+}7$
$\bar{c}_{\mathrm{ccw}} = 2.99\mathrm{E}{+}8$	$\sigma_{\mathrm{ccw}} = 1.52\mathrm{E}{+}7$

Table 5: Results for the average values of c through the alternative method.

We use Equations 8 and 9 one last time to obtain a weighted average of c based on the results of Table 5.

$$\boxed{\bar{c} = (3.02 \pm 0.11) \times 10^8 \text{ m/s}}$$

5 Conclusion

Through our original method, we obtained the numerical estimate of $(2.96 \pm 0.13) \times 10^8$ m/s for the speed of light, 0.292 standard deviations below the published value. The alternative method gave us the value $(3.02 \pm 0.11) \times 10^8$ m/s for the speed of light, which is 0.201 standard deviations above c^* and thus indicating the alternative method rendered more accurate estimates for c. It is evident that the counterclockwise rotation setup of the rotating mirror gave us better results in both methods, which is perhaps consequence of a finer equipment alignment and more data points. Of course, we could enhance the precision of our data if we were to include the extra distance the light rays travel on their way back from the rotating mirror to the beam splitter. In Equation 2, we assumed that the reflection angles θ were small enough to neglect higher order terms in the Taylor expansion of $\tan(\Delta x/f_2)$. However, our readings would not have been significantly affected by such neglections and our final results would not have deviated appreciably either.

6 References

[1] https://www.history.com/news/who-determined-the-speed-of-light

[2] https://physics.nist.gov/cgi-bin/cuu/Value?c