Performance of OFDMA SU-MIMO in 5G

The 5G cellular system utilizes OFDMA MIMO technology at the physical layer. This technology permits degrees of freedom in frequency, time, and “space” for multiplexing the data of multiple users. Let us consider the down-link direction, i.e., from the base-station (gNB) to the users (UE). The system bandwidth (e.g., 100 MHz), has several OFDM carriers, separated by a carrier spacing (e.g., 60 KHz, yielding 132 physical resource blocks (PRBs) or carriers). Each PRB has 12 consecutive sub carriers. These carriers (set of usable PRBs) have time-division framing (e.g., 0.25 ms), each frame carrying 14 symbols of which 18% on average is modeled as overheads in NetSim. When the above system is used to carry data to multiple users, it is called OFDMA. In addition, MIMO technology exploits spatial multiplexing, thereby effectively carrying multiple (spatially multiplexed) symbols for each time-symbol in the OFDM carrier. In MIMO all the spatially multiplexed symbols for an OFDM symbol can be destined to one user, in which case it is called Single User MIMO, or SU-MIMO.

In this experiment we study the performance of OFDMA along with SU-MIMO to carry downlink data to multiple UEs. Since we have SU-MIMO, multiple users are multiplexed by OFDMA, and SU-MIMO is used to obtain spatial multiplexing gain, depending on the number of antennas available at the UE. In Multi User MIMO (MU-MIMO) different spatial layers within the same resource block, can be allotted to different UEs.

Objective:

Simulate a maximum data transmission rate[1], using OFDMA and SU-MIMO, for the following 4 cases

- 1 gNB with 8 Tx antennas transmitting to 1 UE with 8 Rx antennas using all the 132 resource blocks
- 1 gNB with 8 Tx antennas transmitting to 2 UEs each with 4 Rx antennas, each using $\frac{132}{2}$ resource blocks on average
- 1 gNB with 8 Tx antennas transmitting to 4 UEs each with 2 Rx antennas, each using $\frac{132}{4}$ resource blocks on average,
- 1 gNB with 8 Tx antennas transmitting to 8 UEs each with 1 Rx antennas, each using $\frac{132}{8}$ resource blocks on average

Repeat the above cases with the number of UEs now set to 1 for each case, while the antenna counts remain the same. Show that there is no difference in maximum capacity between a single user and a multi-user transmission, when using SU-MIMO. And finally, explain the results obtained for different number of receive antennas by using matrix theory to compute the eigen values of the associated Gram matrices.

Introduction:

We begin with a description of the channel model. Consider a transmitter with N_t transmit antennas, and a receiver with N_r receive antennas. The channel can be represented by the

[1] We mean the saturation or full buffer case. There is always a packet in the gNB queue to transmit; the queue is never empty.

$N_r \times N_t$ matrix $\boldsymbol{H}$ of channel gains h_{ij} representing the gain from transmit antenna j to receive antenna i. The $N_r \times 1$ received signal $\boldsymbol{y}$ is equal to

$$\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{n}$$

The channel state information is the channel matrix $\boldsymbol{H}$ and/or its distribution.

Under rich scattering conditions the MIMO channel can be decomposed into parallel non-interfering channels. The number of such parallel streams is known as the layer count and is equal to $Min(N_t, N_r)$. These parallel channels are commonly referred to as the *eigenmodes* of the channel because the singular values of $\boldsymbol{H}$ are equal to the square root of the eigenvalues of the Wishart[2] matrix $W = H\,H^{\dagger}$ (for $N_t \geq N_r$).

Since each layer is reduced to a flat fading SISO channel, i.e., for layer $j, 1 \leq j \leq LayerCount$,

$$y_j = \sqrt{\lambda_j}\, x_j + w_j$$

where, x_j is the symbol transmitted, λ_j is the corresponding eigenvalue of the Wishart matrix obtained as in the previous section, w_j is circular symmetric complex Gaussian noise, and y_j is the complex valued baseband received symbol.

If fast fading with eigen-beamforming is enabled in NetSim's GUI, then the MIMO link is modelled by parallel SISO channels with the symbol level beamforming gain derived from the eigenvalues[3] of the Wishart matrix.

$$BeamFormingGain\ (dB) = 10 \log_{10}(\lambda)$$

Three assumptions made in NetSim are:

A1. Perfect CSIT and CSIR: The channel matrix $\boldsymbol{H}$ is assumed to be known perfectly, at the start of each frame, at the transmitter and receiver, respectively. With perfect CSIT the transmitter can adapt its transmission rate (MCS) relative to the instantaneous channel state (SNR).
A2. No channel errors.
A3. The transmit power is equally split between all layers transmitted. The justification lies in the fact that at a high SNR, (iterative) water-filling will lead to nearly equal power allocation across all subcarriers and all layers.

Note that the *LOS probability* parameter in NetSim is solely used to compute the large scale pathloss per the 3GPP 38.901 standard. This parameter is not used in the channel rank (MIMO layers) computations. The *Fading and Beam Forming* parameter is used to determine (i) the number of MIMO layers and (ii) the gains in each layer, as shown in the table below.

Parameter drop down option	No. of MIMO layers	Beamforming Gain
No fading MIMO unit gain	Min (N_t, N_r)	Unity (0 dB)
No fading MIMO array gain	Min (N_t, N_r)	Max (N_t, N_r)
Rayleigh with Eigen Beamforming	Min (N_t, N_r)	Eigen values of the Wishart Matrix

[2] Explanation on the Wishart Matrix, $W = H\,H^{\dagger}$ and it's eigen values is provided in the experiment titled "MIMO Beamforming in 5G: A start with MISO and SIMO.

[3] Note that the eigenvectors are not required as they are only a part of the receive and transmit signal processing; NetSim only needs to work with the equivalent symbol-by-symbol flat fading SISO channels.

Procedure:

1. Use the following download Link to download a compressed zip folder which contains the workspace: [GitHub link]
2. Extract the zip folder.
3. The extracted project folder consists of a NetSim workspace file 5G_advanced_experiments_with_NetSim.netsimexp
4. Go to NetSim Home window, go to Your Work and click on Import.

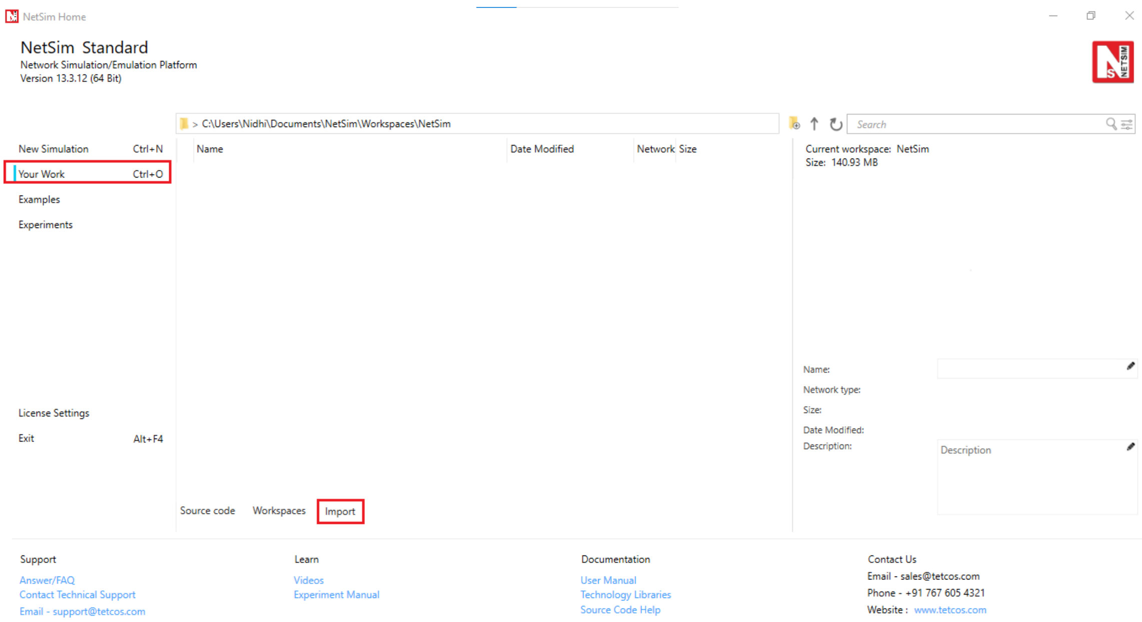

Fig 1: NetSim Home Window

5. In the Import Workspace Window, browse and select 5G_advanced_experiments_with_NetSim.netsimexp file from the extracted directory. Click on create a new workspace option and browse to select a path in your system where you want to set up the workspace folder
6. Choose a suitable name for the workspace of your choice. Click Import.

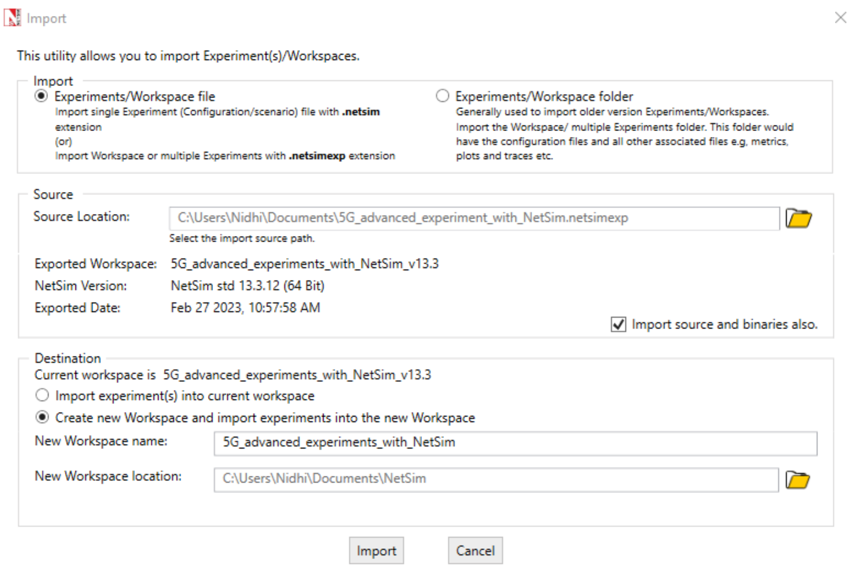

Fig 2: NetSim Import workspace window

7. The Imported Project workspace will automatically be set as the current workspace.
8. The list of experiments is now loaded onto the selected workspace.

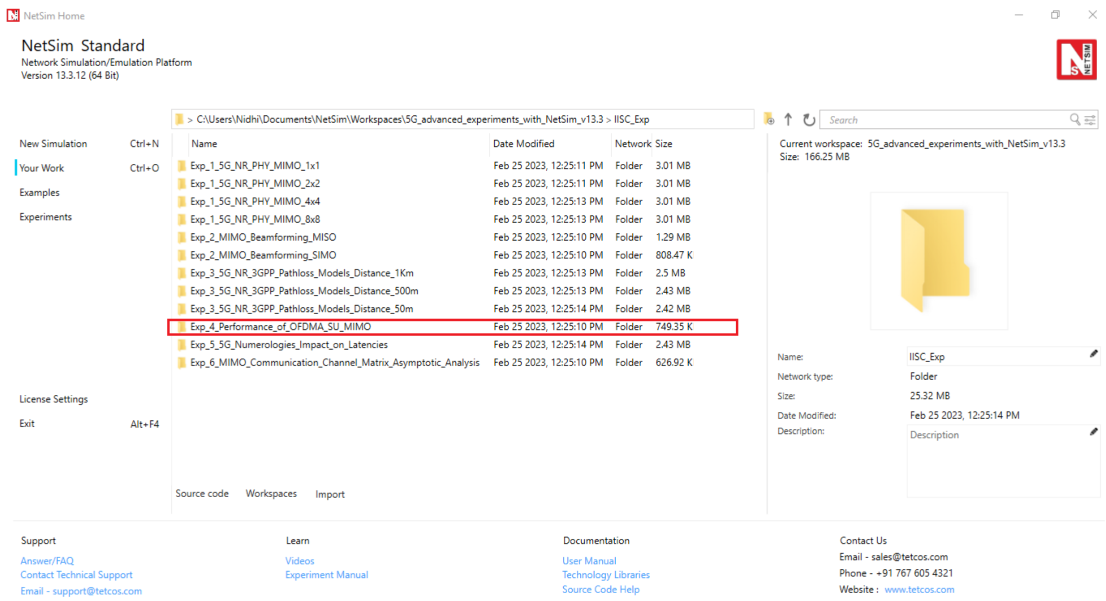

Fig 3: NetSim Your Work Window with the experiment folders inside the workspace

NetSim Settings:

1. The following parameters were configured in Interface 5G RAN- Physical Layer of gNB and UE:

gNB- Interface 5G_RAN Parameters	
gNB Height	10m
Tx Power	40 dBm
Duplex Mode	TDD

CA Type	SINGLE BAND
CA Configuration	n78
DL: UL Ratio	4:1
Numerology	2
Channel Bandwidth (MHz)	100
Tx Antenna Count	8
Rx Antenna Count	1
MCS Table	QAM256
CQI Table	TABLE2
Pathloss Model	3GPPTR38.901-7.4.1
Outdoor Scenario	Urban Macro
LOS NLOS Selection	User Defined
LOS Probability	1
Shadow Fading Model	None
Fading and Beam Forming	RAYLEIGH with EIGEN Beamforming
Coherence Time (ms)	10
Additional Loss Model	None
UE Interface 5G RAN	
Tx Power	23 dBm
UE Height	1.5m
Tx Antenna Count	1
Rx Antenna Count	<varied>

Table 1: gNB and UE properties

2. The following parameters were configured in the wired link properties:

Wired Link Parameters	
Wired Link Speed	10 Gbps
Wired Link BER	0
Wired Link Propagation Delay	5 µs

Table 2: Wired link properties

3. Run simulation for 1.1s.

Case 1: 1 gNB - 8 Tx antennas, 1 UE - 8 Rx antennas

Network Scenario:

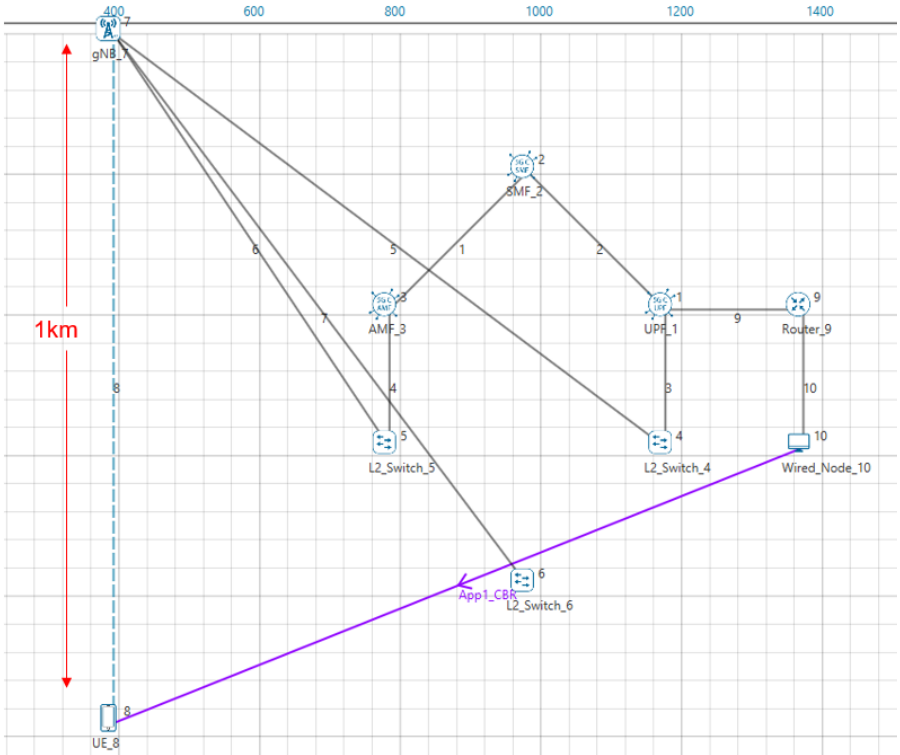

Fig 4: Network topology in this experiment

Additional Settings:

1. The Tx Antenna Count was set to 1 and Rx Antenna Count was set to 8 in Interface 5G RAN- Physical Layer in the UE
2. The following parameters were set in Application Properties:

Application Parameters	
Application	CBR
Packet Size	1460
Inter Packet Arrival Time (µs)	3.33
Start Time	1
Transport Protocol	UDP

Table 3: Application properties

Result:

Application	Throughput (Mbps)
App_1_CBR	1909.09

Table 4: Throughput obtained per UE

Case 2: 1 gNB- 8 Tx antennas, 2 UEs with 4 Rx antennas each

Network Scenario:

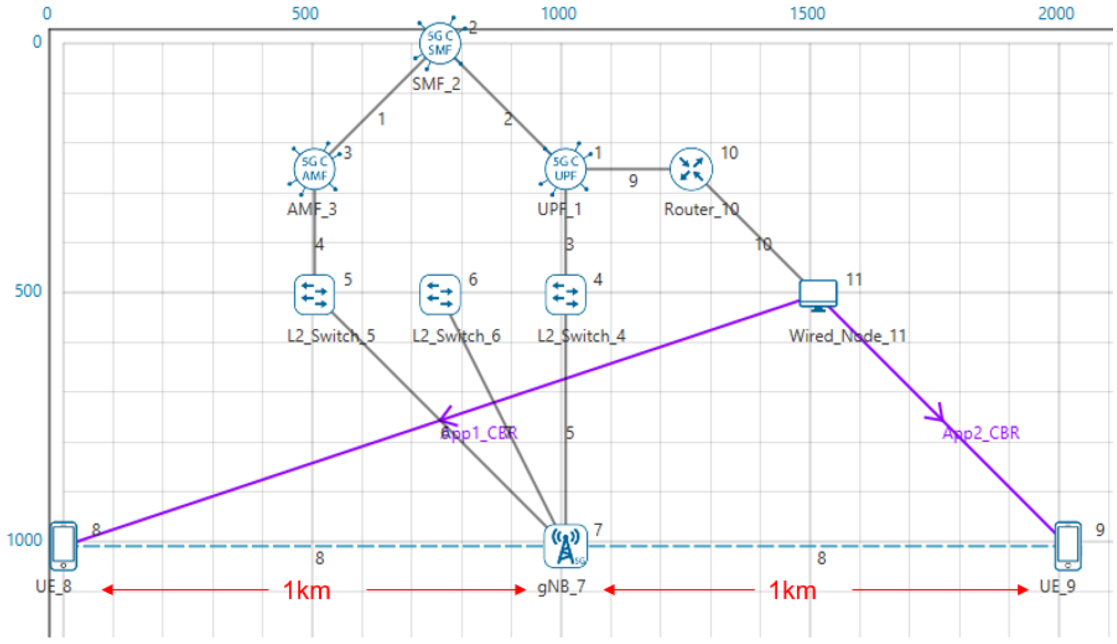

Fig 5: Network topology in this experiment

Additional Settings:

1. The Tx Antenna Count was set to 1 and Rx Antenna Count was set to 4 in Interface 5G RAN- Physical Layer in both the UEs.
2. The following parameters were set in Application Properties:

Application Parameters		
	Wired Node- UE_8	Wired Node- UE_9
Application	CBR	CBR
Packet Size	1460	1460
Inter Packet Arrival Time (µs)	12.97	12.97
Start Time	1	1
Transport Protocol	UDP	UDP

Table 5: Application properties

Result:

Throughput Obtained (Mbps)		
UE_8	UE_9	Aggregate Throughput (Mbps)
666.92	673.35	1340.28

Table 6: Throughput obtained per uE

Case 3: 1 gNB 8- Tx antennas, 4 UEs with 2 Rx antennas each

Network scenario:

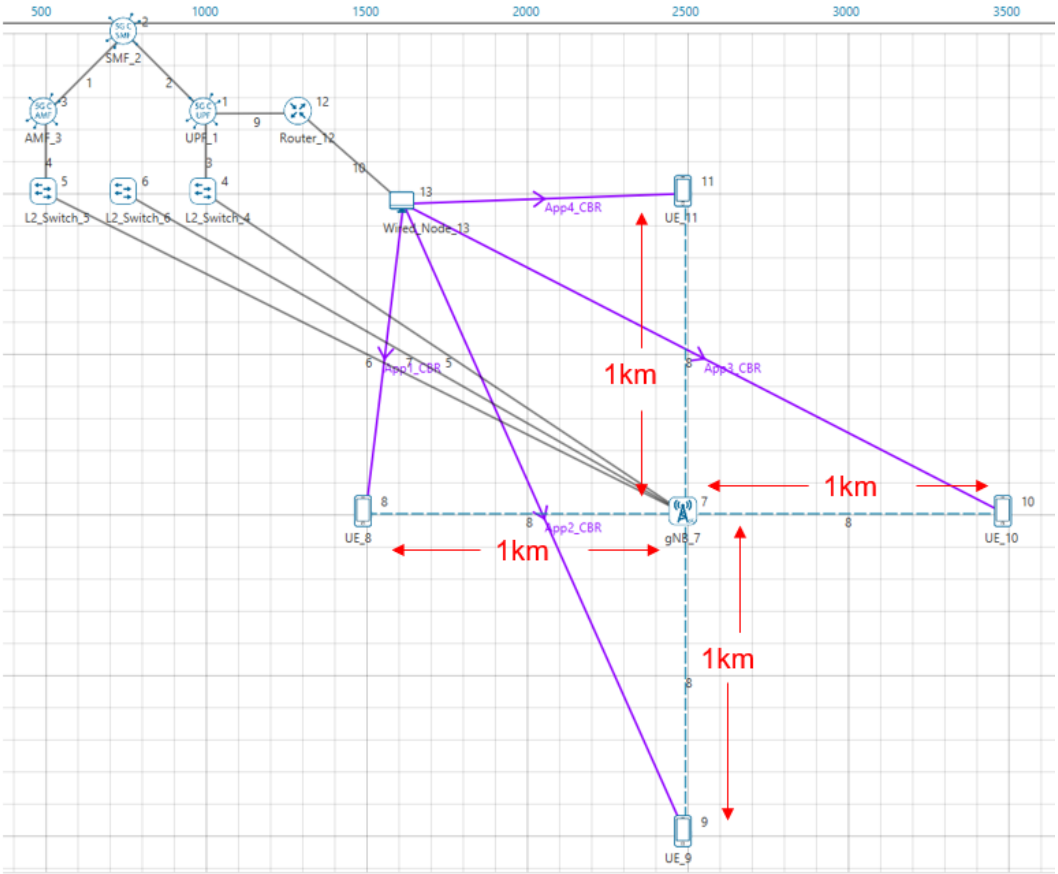

Fig 6: Network topology in this experiment

Additional Settings:

1. The Tx Antenna Count was set to 1 and Rx Antenna Count was set to 2 in Interface 5G RAN- Physical Layer in all the UEs.
2. The following parameters were set in Application Properties:

Application Parameters				
	Wired Node-UE_8	Wired Node-UE_9	Wired Node-UE_10	Wired Node-UE_11
Application	CBR	CBR	CBR	CBR
Packet Size	1460	1460	1460	1460
Inter Packet Arrival Time (µs)	53.09	53.09	53.09	53.09
Start Time	1	1	1	1
Transport Protocol	UDP	UDP	UDP	UDP

Table 7: Application properties

Result:

Throughput (Mbps)				
UE_8	UE_9	UE_10	UE_11	Aggregate Throughput (Mbps)
187.81	196.10	190.26	197.27	771.464

Table 8: Throughputs obtained per UE

Case 4: 1 gNB 8 tx antennas, 8 UEs with 1 Rx antennas each

Network Scenario:

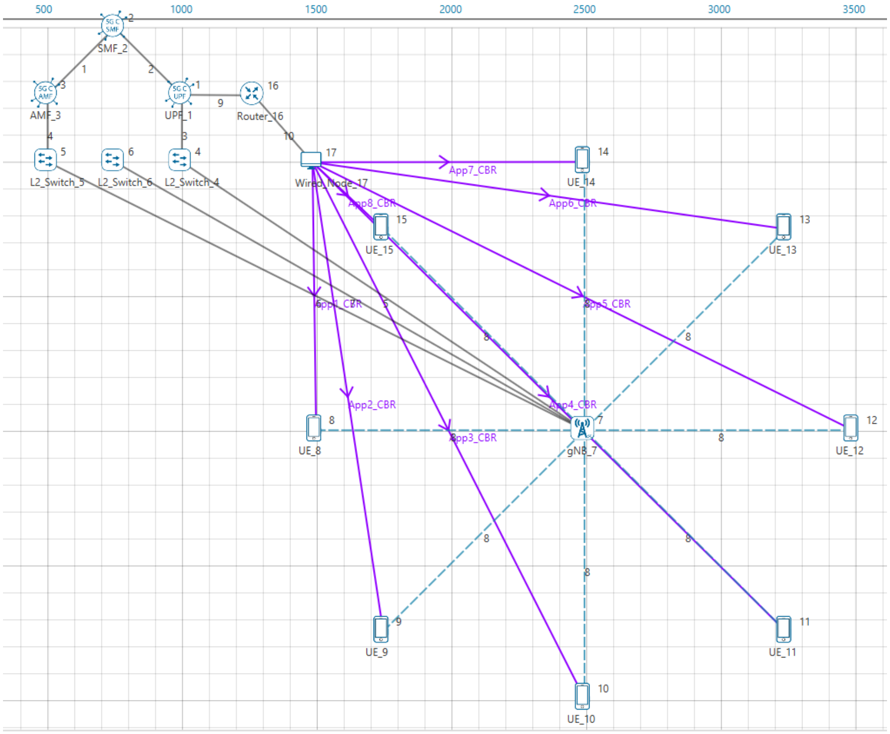

Fig 7: Network topology in this experiment

Additional Settings:

1. The Tx Antenna Count was set to 1 and Rx Antenna Count was set to 1 in Interface 5G RAN- Physical Layer in all the UEs.
2. The following parameters were set in Application Properties:

Application Parameters								
	Wired Node-UE_8	Wired Node-UE_9	Wired Node-UE_10	Wired Node-UE_11	Wired Node-UE_12	Wired Node-UE_13	Wired Node-UE_14	Wired Node-UE_15
Application	CBR	CBR	CBR	CBR	CBR	CBR	CBR	CBR
Packet Size	1460	1460	1460	1460	1460	1460	1460	1460
Inter Arrival Time (µs)	212.36	212.36	212.36	212.36	212.36	212.36	212.36	212.36
Start Time	1	1	1	1	1	1	1	1
Transport Protocol	UDP	UDP	UDP	UDP	UDP	UDP	UDP	UDP

Table 9: Application properties

Results:

Throughput Obtained (Mbps)								
UE_8	UE_9	UE_10	UE_11	UE_12	UE_13	UE_14	UE_15	Aggregate Throughput (Mbps)
52.20	50.69	52.20	52.09	51.50	50.92	51.39	50.69	411.72

Table 10: Throughputs obtained per UE

Discussion:

We combine the results of the four cases and present it in the table below.

Rx Antenna Count per UE	Throughput (Mbps)								Aggregate Throughput (Mbps)
	UE_1	UE_2	UE_3	UE_4	UE_5	UE_6	UE_7	UE_8	
8	1909.09	-	-	-	-	-	-	-	1909.09
4	666.92	673.35	-	-	-	-	-	-	1340.28
2	187.81	196.10	190.26	197.27	-	-	-	-	771.464
1	52.20	50.69	52.20	52.09	51.50	50.92	51.39	50.69	411.72

Table 11: Throughput comparison table for different Rx Antenna Counts

We compare the aggregate throughputs (from Table *11*) with single UE peak throughputs.

UE Rx Antenna Count per UE	Aggregate Throughput (Mbps)	Single UE Peak Throughput (Mbps)
8	1909.09 (1 UE 8 Rx Antennas)	1909.09 (1 UE 8 Rx Antennas)
4	1340.28 (2 UEs with 4 Rx Antennas)	1330.46 (1 UE 4 Rx Antennas)
2	771.464 (4 UEs with 2 Rx Antennas)	754.52 (1 UE 2 Rx Antennas)
1	411.72 (8 UEs with 1 Rx Antennas)	414.75 (1 UE 1 Rx Antennas)

Table 12: Throughputs obtained for different Rx Antenna counts in multi and single UE cases.

From Table *12* it becomes clear that the bandwidth is shared across the UEs by OFDMA. It is just like having one UE use the entire bandwidth.

Theoretical analysis:

8*8 SU-MIMO

- 2^k carriers in the OFDMA system
- S: symbol rate per carrier (assuming the same numerology for all carriers)
- L: is the large-scale pathloss, including shadowing (not including the fading). Note that, L is modelled as $c\left(\frac{d}{d_0}\right)^{-\eta}$ where η is the pathloss coefficient.

Then the expected rate to the single SU-MIMO user is given by

$$\mathbb{E}(R_1) = 2^k \times S \times \mathbb{E}\left(\sum_{j=1}^{8} log\left(1 + \frac{P_j \times L \times \lambda_j}{\sigma^2}\right)\right)$$

where P_j is the power allotted to the j^{th} equivalent channel, with $\sum_{j=1}^{8} P_j = P$, the total transmit power. We assume equal power allocation, so that $P_j = \frac{P}{8}$, so that

$$\mathbb{E}(R_1) = 2^k \times S \times \mathbb{E}\left(\sum_{j=1}^{8} log\left(1 + \frac{P \times L \times \lambda_j}{8\sigma^2}\right)\right)$$

8*1 SU-MIMO, with 8 such UEs

- 2^{k-3} OFDMA carriers per UE

Then, for the same OFDMA symbol rate, $S,$ and large-scale pathloss, $L,$ the total expected rate for the 8 UEs is given by:

$$\mathbb{E}(R_2) = 2^{k-3} \times S \times \left(\sum_{j=1}^{8} \mathbb{E}\, log\left(1 + \frac{P \times L \times \|h_j^2\|}{\sigma^2}\right)\right) = 2^k \times S \times \mathbb{E} \log\left(1 + \frac{P \times L}{\sigma^2} \times \|h_1^2\|\right)$$

From matrix theory we know that the sum of the eigenvalues of a matrix is equal to its trace. Therefore

$$\sum_{i=1}^{8} \lambda_i = \|h_1^2\| + \|h_2^2\| \ldots + \|h_8^2\|$$

We know that when $a, \lambda_1, \lambda_2 > 0$

$$1 + a(\lambda_1 + \lambda_2) \leq (1 + a\lambda_1)(1 + a\lambda_2)$$

and by monotonicity of log function, it follows that

$$log\big(1 + a(\lambda_1 + \lambda_2)\big) \leq log(1 + a\lambda_1) + log(1 + a\lambda_2)$$

Extending this inequality recursively, we get

$$log\big(1 + a(\lambda_1 + \lambda_2 \ldots \lambda_8)\big) \leq log(1 + a\lambda_1) + log(1 + a\lambda_2) \ldots + log\ (1 + a\lambda_8)$$

Using the above expressions, we can compare the expected total downlink throughputs for one 8x8 SU-MIMO UE, with eight 8x1 SU-MIMO UEs, as follows:

$$\mathbb{E}R_1 = 2^k \times S \times \mathbb{E}\left(\sum_{j=1}^{8} log\left(1 + \frac{P \times L \times \lambda_j}{8\sigma^2}\right)\right) \geq 2^k \times S \times \mathbb{E}\, log\left(1 + \frac{P \times L}{8\sigma^2} \times \sum_{j=1}^{8} \lambda_j \right)$$

$$= 2^k \times S \times \mathbb{E} \log\left(1 + \frac{P \times L}{8\sigma^2} \times \sum_{j=1}^{8} \|h_j^2\|\right)$$

$$\geq 2^k \times S \times \sum_{j=1}^{8} \frac{1}{8} \mathbb{E} \log\left(1 + \frac{P \times L}{\sigma^2} \times \|h_j^2\|\right)$$

$$= 2^k \times S \times \mathbb{E} \log\left(1 + \frac{P \times L}{\sigma^2} \times \|h_1^2\|\right) = \mathbb{E}\, R_2$$

where the first inequality arises from the argument given earlier, and the second equality follows from the equality between the trace of a matrix and the sum of its eigenvalues. Physically, the first inequality captures the performance gain obtained by splitting the power over the different spatial degrees of freedom. The second inequality follows from the fact the fact that the logarithm function is concave and by an application of Jensen's inequality. We see from Table *11* that $\mathbb{E}R_1 = 1942.26\ Mbps$, whereas $\mathbb{E}R_2 = 418.46\ Mbps$, showing the large effect of the two inequalities in the above argument.

Exercises:

1. Carefully explain your observations. Also, place the UEs at different distances from the gNB and see how the throughput changes. Again, explain your observations.
2. Vary the height of UEs (as this also effects pathloss) and see how the throughput changes. Again, explain your observations.