MIMO Communication: Channel Matrix Asymptotic Analysis

Objective

In this experiment, properties of MIMO channel matrices in 5G wireless communications are studied in the asymptotic number of antennas. In particular, the condition number of a large MIMO matrix, which dictates the performance of spatial multiplexing and subsequently the behaviour of the eigen spectrum of large MIMO matrices, are investigated through simulations setup in NetSim v13.2.

Introduction

A MIMO channel matrix in wireless communications is, in general, a random matrix. Hence, all the parameters obtained out of matrix entries are essentially random variables. For e.g., the eigen values, the condition number are all random variables. In particular, the condition number of a MIMO channel matrix is of significant importance in characterizing the capacity of a MIMO channel. For example, by virtue of Jensen's inequality we have,

$$\sum_i \log\left(1+\frac{P\sigma_i}{N}\right) \leq r\log\left(1+\frac{1}{r}\sum_i \frac{P\sigma_i}{N}\right),$$

with equality holding true iff condition number of the MIMO matrix is 1 and other parameters have usual meanings, and with r being the rank of the MIMO channel matrix. Thus, for a given rank and SNR of the channel, condition number of the matrix determines how close to the upper bound the achievable rate is. In this view, the eigen spectrum and condition number distribution of large MIMO matrices are investigated. By means of theory, it will be shown that the condition number of a large MIMO matrix stabilizes near unity, indicating superior spatial multiplexing capability in the asymptotic number of antennas.

Procedure:

1. Use the following download Link to download a compressed zip folder which contains the workspace: GitHub link
2. Extract the zip folder.
3. The extracted project folder consists of a NetSim workspace file 5G_Advanced_IISC_experiment_v13.2.20.netsimexp.
4. Go to NetSim Home window, go to Your Work and click on Import.

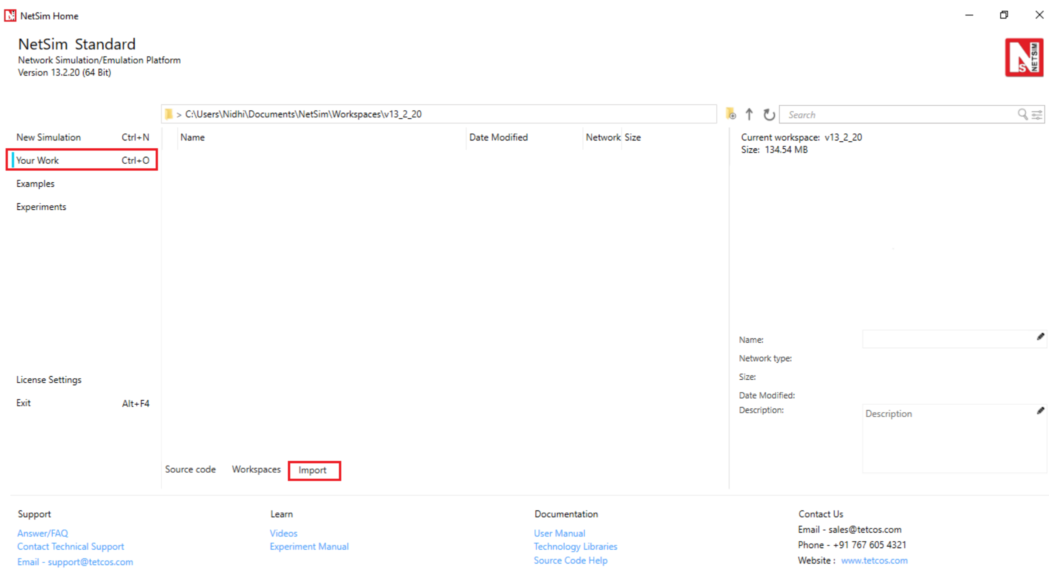


Fig 1: NetSim Home Window

5. In the Import Workspace Window, browse and select the 5G_Advanced_IISC_experiment_v13.2.20.netsimexp file from the extracted directory. Click on create a new workspace option and browse to select a path in your system where you want to set up the workspace folder.
6. Choose a suitable name for the workspace of your choice. Click Import.

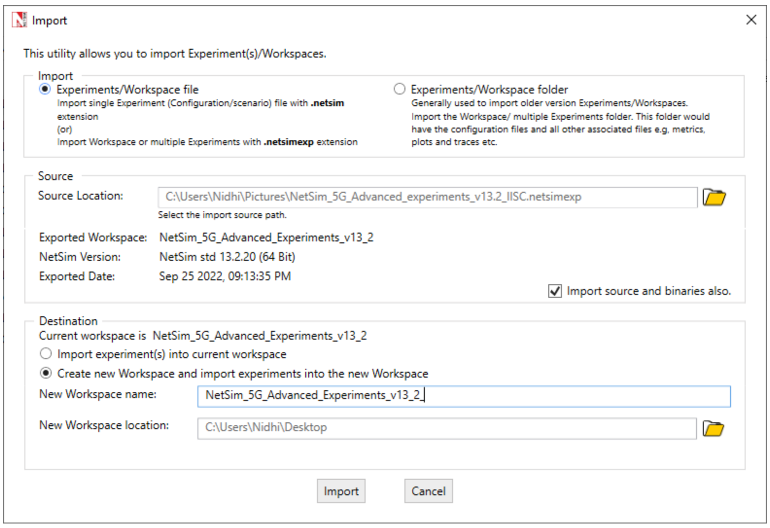


Fig 2: NetSim Import workspace window

7. The Imported Project workspace will automatically be set as the current workspace.
8. The list of experiments is now loaded onto the selected workspace.

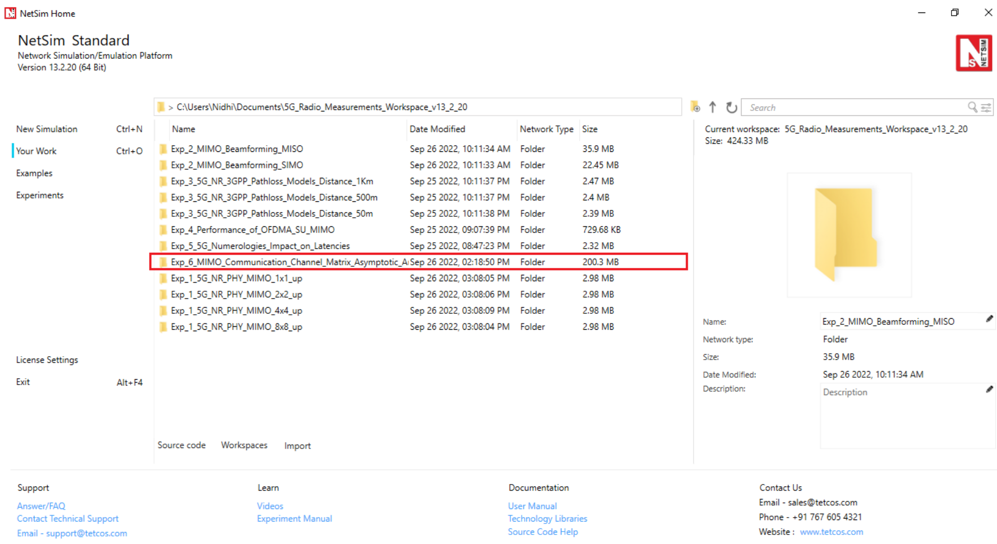


Fig 3: NetSim Your Work Window with the experiment folders inside the workspace

Network Scenario

NetSim UI would display the network topology shown in the screenshot below when you open the example configuration file.

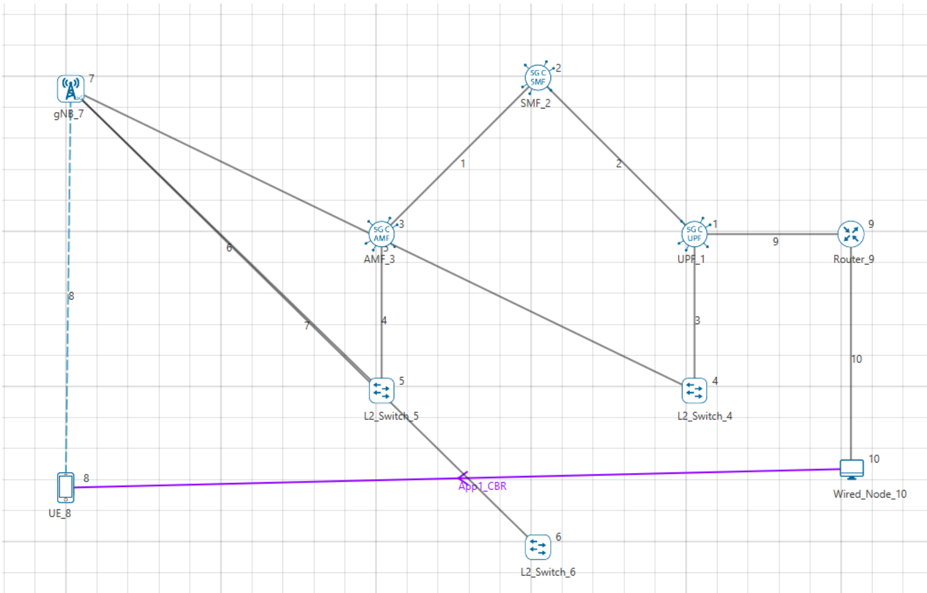


Fig 4: Network topology in this experiment

Settings:

The following parameters were configured in the network setup:

1. The gNB- Interface 5G_RAN were set with the following properties:

gNB- Interface 5G_RAN Parameters	
gNB Height	10m
Tx Power	40 dBm

Duplex Mode	TDD
CA Type	SINGLE BAND
CA Configuration	n78
DL: UL Ratio	4:1
Numerology	0
Channel Bandwidth (MHz)	10
Tx Antenna Count	Varied from 16 to 128
Rx Antenna Count	16
MCS Table	QAM64LOWES
CQI Table	TABLE1
Pathloss Model	3GPPTR38.901-7.4.1
Outdoor Scenario	Rural Macro
LOS NLOS Selection	User Defined
LOS Probability	1 (LOS)
Shadow Fading Model	LOG_NORMAL
Fading and Beam Forming	RAYLEIGH with EIGEN Beamforming
Coherence Time (ms)	10
O2I Penetration Model	LOW_LOSS_MODEL
Additional Loss Model	None

Table 1: gNB properties

2. The UE properties were configured with the following parameters:

UE Interface 5G RAN	
Tx Power	23 dBm
UE Height	1.5m
Tx Antenna Count	16
Rx Antenna Count	16

Table 2: UE properties

3. A downlink CBR application was configured from wired node to UE with Transport protocol as UDP and Packet Size of 1460 Bytes and Inter Arrival time of 2000 µs and the Start Time was set to 1s[1].
4. Run simulation for 10s.

[1] The application end time value of 10,000s is not changed. In NetSim the application runs for $min(AppEndTime,\ SimulationTime)$. Since the simulation is run for 10s, the application runs for only 10s.

After the simulation, note down the average linear Beamforming Gain (eigen value) obtained for the DL application from the log file generated and then condition number can be obtained.

Part 1: Asymptotic Condition Number Mean

Steps to calculate the condition number through Eigen value (BeamForming Gain Linear):

1. In the results window, expand the Log Files option in the left panel and select 5G_LTE_Parameter_Log.csv file.

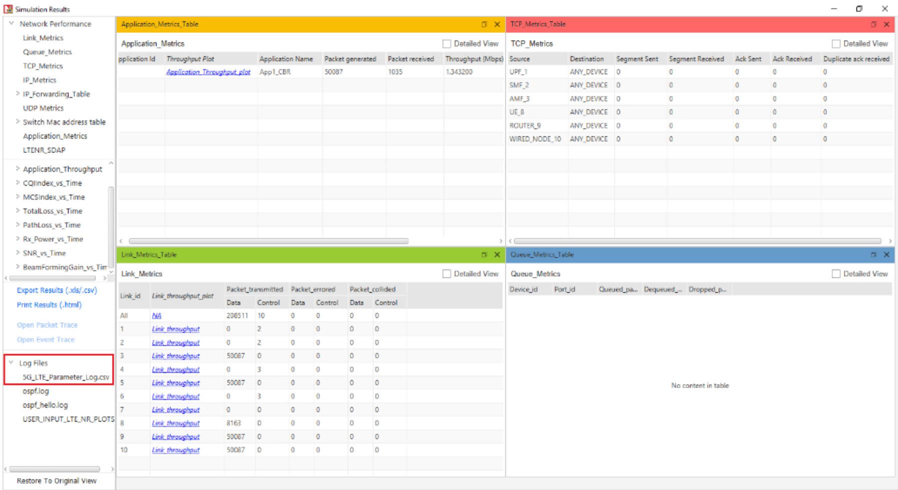


Fig 5: NetSim Results window showing access to log file generated

2. This will open a csv file which logs the parameters beamforming gain, over time as shown below.

	Channel	LAYER_ID	Tx_Power(dBm)	TotalLoss(dB)	PathLoss(dB)	ShadowFad	Rx_Power(dBm)	SNR(dB)	SINR(dB)	Interferenc	BeamFormi	CQI_Index	MCS_Index
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A
1	SSB	N/A	40	90.633637	92.383716	-1.750078	-26.551238	77.276719	N/A	N/A	24.0824	N/A	N/A

Fig 6: 5G Parameter log file created after simulation.

3. To change the beamforming gain from dB scale to linear, the following method is used:

$$Eigen\ Value\ (Beam\ Forming\ Gain, Linear) = 10^{\left(\frac{BeamFormingGain_{dB}}{10}\right)}$$

In a new column, enter the following function to calculate the linear Beamforming gain:

$$= POWER(10, [@[BeamFormingGain(dB)]]/10)$$

4. Now goto Insert, select Pivot table option, and then select new sheet option and click ok.

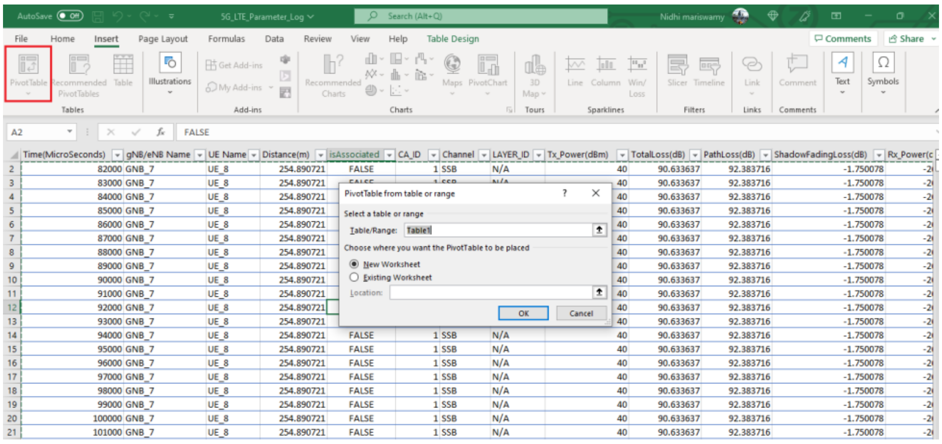


Fig 7: Create pivot tale

5. Drag and drop the Channel and LAYER_ID field to filter block. Filter the Channel to only PDSCH since we have considered a DL application from server to UE. Similarly, drag and drop the linear beamforming gain to values field and Time to Rows field.

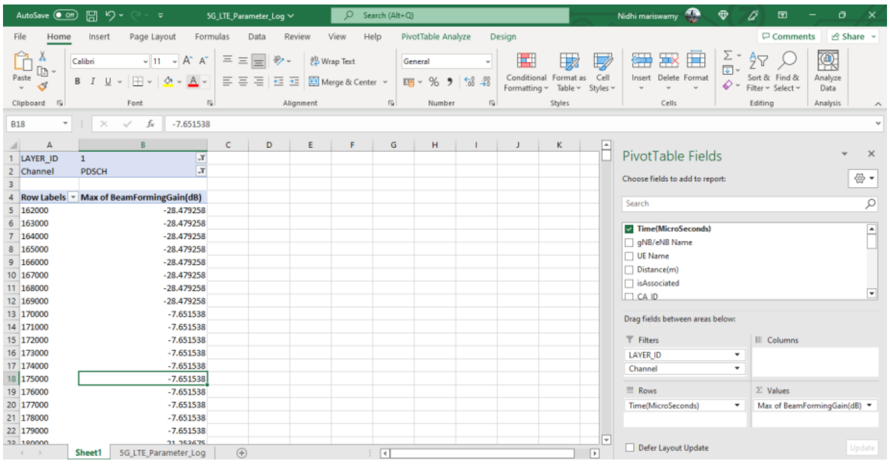
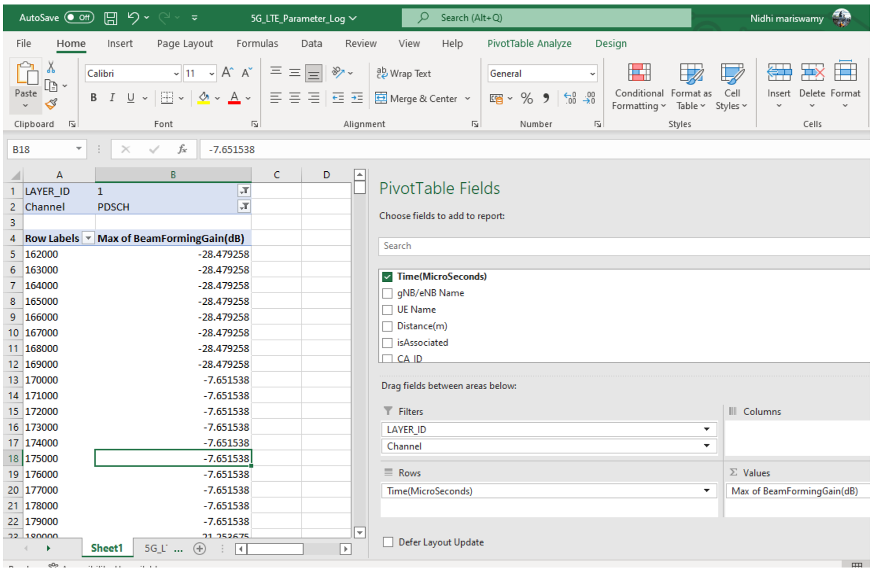


Fig 8: 5G Parameter log file pivot table showing the filtering process of DL/UL column

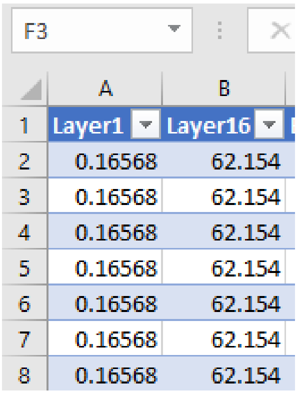
6. Now, filter the Layer_Id to layer 1. Now, copy the beamforming values and paste the values in new sheet with column name as Layer1.



Row Labels	Max of BeamFormingGain(dB)
162000	-28.479258
163000	-28.479258
164000	-28.479258
165000	-28.479258
166000	-28.479258
167000	-28.479258
168000	-28.479258
169000	-28.479258
170000	-7.651538
171000	-7.651538
172000	-7.651538
173000	-7.651538
174000	-7.651538
175000	-7.651538
176000	-7.651538
177000	-7.651538
178000	-7.651538
179000	-7.651538

Fig 9: 5G Parameter log file showing eigen value obtained for layer1.

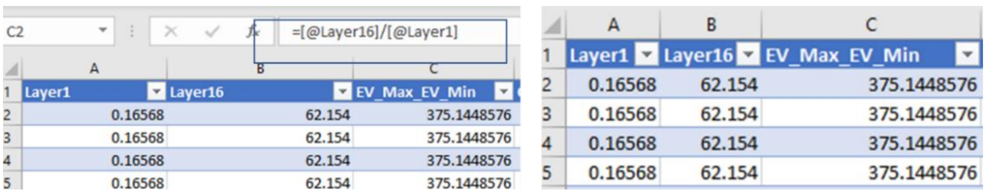
7. Similarly, filter the LAYER_ID as 16 and copy all the eigen values and paste in new sheet with column name Layer16 as below.



Layer1	Layer16
0.16568	62.154
0.16568	62.154
0.16568	62.154
0.16568	62.154
0.16568	62.154
0.16568	62.154
0.16568	62.154

Fig 10: Layer 1 and layer 16 eigen value in new table

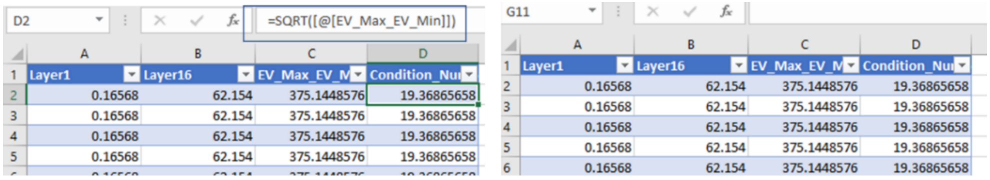
8. Now in the next column, enter the formula, $=[@Layer16]/[@Layer1]$ to calculate EV_max/EV_min. Note that the eigenvalues of Layer 16 are λ_{max} while the eigenvalue of Layer1 is λ_{min}. Rename the column suitably.



Layer1	Layer16	EV_Max_EV_Min
0.16568	62.154	375.1448576
0.16568	62.154	375.1448576
0.16568	62.154	375.1448576
0.16568	62.154	375.1448576

Figure 11: Showing $\sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$ obtained

9. Now in the next column, enter the formula, $= SQRT([@[EV\_Max\_EV\_Min]])$. This will calculate square root of the ratio $\sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}$ which is known as the condition number.



D2 =SQRT([@[EV_Max_EV_Min]])

	Layer1	Layer16	EV_Max_EV_M	Condition_Nu
2	0.16568	62.154	375.1448576	19.36865658
3	0.16568	62.154	375.1448576	19.36865658
4	0.16568	62.154	375.1448576	19.36865658
5	0.16568	62.154	375.1448576	19.36865658

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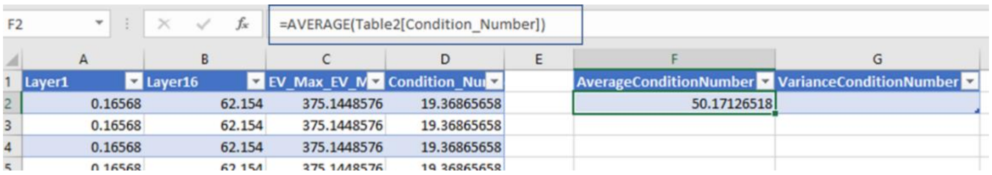
	Layer1	Layer16	EV_Max_EV_M	Condition_Nu
2	0.16568	62.154	375.1448576	19.36865658
3	0.16568	62.154	375.1448576	19.36865658
4	0.16568	62.154	375.1448576	19.36865658
5	0.16568	62.154	375.1448576	19.36865658
6	0.16568	62.154	375.1448576	19.36865658

Figure 12: Condition Number obtained

10. In a new cell, enter the formula $= AVERAGE(Table2[Condition\_Number])$ to calculate the average of condition number.

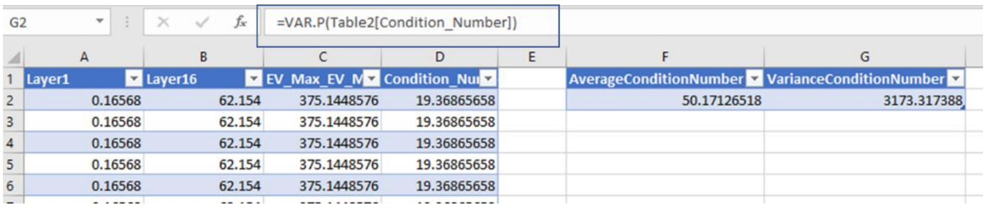


F2 =AVERAGE(Table2[Condition_Number])

	Layer1	Layer16	EV_Max_EV_M	Condition_Nu		AverageConditionNumber	VarianceConditionNumber
2	0.16568	62.154	375.1448576	19.36865658		50.17126518	
3	0.16568	62.154	375.1448576	19.36865658			
4	0.16568	62.154	375.1448576	19.36865658			

Figure 13: Average eigen value obtained

11. Similarly, enter the formula, $= VAR.P(Table2[Condition\_Number])$ in a new cell, to calculate the variance of condition number.



G2 =VAR.P(Table2[Condition_Number])

	Layer1	Layer16	EV_Max_EV_M	Condition_Nu		AverageConditionNumber	VarianceConditionNumber
2	0.16568	62.154	375.1448576	19.36865658		50.17126518	3173.317388
3	0.16568	62.154	375.1448576	19.36865658			
4	0.16568	62.154	375.1448576	19.36865658			
5	0.16568	62.154	375.1448576	19.36865658			
6	0.16568	62.154	375.1448576	19.36865658			

Figure 14: Variance of eigen value obtained

12. Repeat the steps 1 to 10 with varying Tx antenna count in gNB as 32, 64, and 128. Note down the mean and variance of condition number.

Results

Tx, Rx	Condition Number Mean	Condition Number Variance
16, 16	50.118	3168.400
32, 16	4.423	0.259669
64, 16	2.561	0.026435
128,16	1.878	0.005593

Table 3: Showing Mean and Variance of Condition Number with varying Tx

a) Case1: gNB Tx = 32, UE Rx = 16

From theory $K = \frac{\left(1+\sqrt{\frac{16}{32}}\right)}{1-\sqrt{\frac{16}{32}}} = 5.82$. NetSim result = 4.423.

$$\text{Difference} = \frac{5.82 - 4.423}{4.423} = 31.58\ \%$$

b) Case2: gNB Tx = 64, UE Rx = 16

From theory $K = \frac{\left(1+\sqrt{\frac{16}{64}}\right)}{1-\sqrt{\frac{16}{64}}} = 3.0$. NetSim result = 2.561.

$$\text{Difference} = \frac{3 - 2.561}{2.561} = 17.1\ \%$$

c) Case3: gNB Tx = 128, UE Rx = 16

From theory $K = \frac{\left(1+\sqrt{\frac{16}{128}}\right)}{1-\sqrt{\frac{16}{128}}} = 2.09$. NetSim result = 1.879.

$$\text{Difference} = \frac{2.09 - 1.878}{1.878} = 11.28\%$$

Since theory is for the asymptotic mean, we will not get an exact match. The results show the trend that as N increases simulation outputs approach theoretical predictions.

Part 2: Asymptotic Condition Number Distribution

Theory

Consider a i.i.d $N_r \times N_t$ complex Gaussian random matrix $\mathbf{H}$. Define the Wishart matrix $\boldsymbol{W} = \boldsymbol{H} \times \boldsymbol{H}^{\dagger}$ with parameters $n = \min(N_t, N_r)$ and $N = \max(N_t, N_r)$, and eigenvalues as $\lambda_{max} = \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{min} \geq 0$.

The condition number of $\mathbf{H}$ is defined as

$$K(\boldsymbol{H}) = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$$

From [1], if $n = N$, and $N \rightarrow \infty$ then $\frac{K(\boldsymbol{H})}{n}$ converges in distribution to a random variable whose PDF is given by

$$f(x) = \left(\frac{8}{x^3}\right) \times e^{-\left(\frac{4}{x^2}\right)}$$

We simulate the case $N_r = N_t = 16$, since the antenna count is limited to 16, in the UEs in NetSim. Fig 15 is a comparison of the normalized histogram of $\frac{K}{N}$ from NetSim vs. the asymptotic pdf equation. At $N_r = N_t = 16$ itself, there seems to be a reasonable fit.

Comparison of NetSim Results with asymptotic function

Steps to plot histogram of condition number:

1. Calculate the Condition Number using the 5G_Parameter_Log file.
2. In a new column, divide the Condition_Number by 16 using the following excel function:

$$= [@Condition\_Number]/16)$$

3. Click on Insert-> Pivot Table, drag and drop Column1 to Rows field.
4. Copy the values in Row Labels column.
5. In MATLAB create a new file, create an array **Condition_Number_array**, paste the values to it as

```
Condition_Number_array = [c1
                          c2
                          c3
                           ….
                          cn];
```

6. Now use below MATLB code to plot the normalized histogram plot with the function

$$f(x) = \left(\frac{8}{x^3}\right) \times e^{-\frac{4}{x^2}}$$

```
hold on;
c = histogram(Condition_Number_array,'Normalization','probability');
x = 0:0.1:50;  %x varies from 0 to 50 in steps of 0.2.
y = (8./x.^3).*(exp(-4./x.^2));
plot(x,y,'r');
hold off;

% For CDF plot use below MATLB code
cdfplot(Condition_Number_array);
```

Program 1: MATLAB code for plotting condition number from NetSim and comparing against the asymptotic PDF from analysis.

Histogram Plot for 16 Tx Layer Count (gNB) and 16 Rx Layer Count (UE)

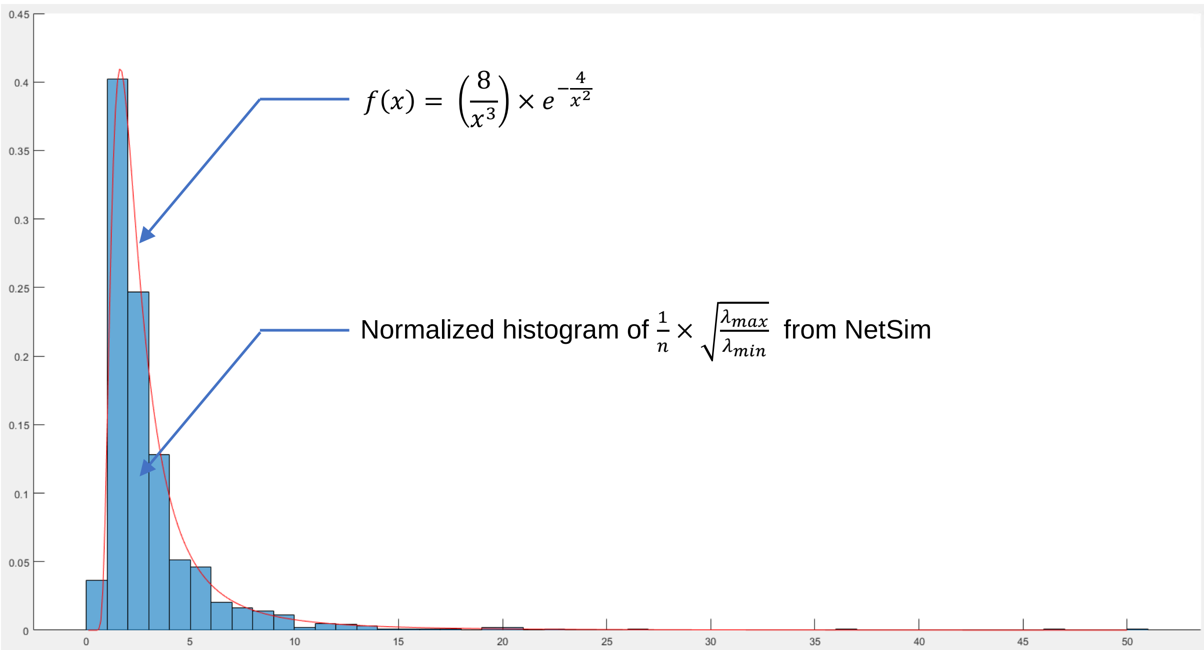


Fig 15: The normalized histogram $\frac{1}{n} \times \sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$ for $N_t = N_r = 16$ itself fits well with the asymptotic distribution of $\frac{K(H)}{n}$

Part 3: Marchenko-Pasteur Distribution

Theory

The Marchenko Pasteur distribution for $\frac{N_r}{N_t} = y$ with $N_t \to \infty$ is

$$f(x) = \frac{1}{2\pi xy} \times \sqrt{(b-x)(x-a)}$$

where $b = \left(1+\sqrt{y}\right)^2$ and $a = \left(1-\sqrt{y}\right)^2$.

Let us consider the case where, the number of transmit antennas N_t and the number of receive antennas N_r, are related as $\frac{N_r}{N_t} = \frac{1}{8} = y$. Substituting for y we get the MP distribution as

$$f(x) = \begin{cases} \frac{4}{\pi x}\sqrt{\left(\frac{1}{2}\right) - \left(x - \frac{9}{8}\right)^2}, & \left(1 - \frac{1}{2\sqrt{2}}\right)^2 \leq x \leq \left(1 + \frac{1}{2\sqrt{2}}\right)^2 \\ 0, & \text{All other } x \end{cases}$$

Comparison of NetSim Results with Marchenko- Pasteur function

Steps to plot the histogram:

1. Create a scenario with Tx antenna count as 128 and Rx antenna count as 16.
2. Now open the file 5G_LTE_Parameter_Log.csv file.
3. Filter the DL/UL to DL.
4. Now, in the 5G parameter log file we have Eigen values from Layer Id 1 to Layer Id 16.
5. Select the eigen values in Beamforming Gain column and copy all the values.
6. Create a new file in MATLAB. Create an array Eigen_value_array and paste the copied values from the 5G parameter log csv file as shown.

   ```
   Eigen_value_array = [ev1
                        ev2
                        …
                        evN];
   ```
7. Use below MATLAB code to plot the MP distribution function along with the normalized histogram.

   For the MP distribution function, the x varies from 0.418 to 1.832 in steps of 0.001.

```
x = 0.418:0.001:1.832;
y = (1.27324./x).*sqrt(0.5 - (x-9/8).^2);
hold on;
plot(x,y);
histogram(Eigen_value_array /128,'Normalization','pdf');
hold off;
```

Program 2: MATLAB code for plotting the MP distribution for $y = \frac{1}{8}$ and pooled eigenvalues histogram, from NetSim simulation results

Result:

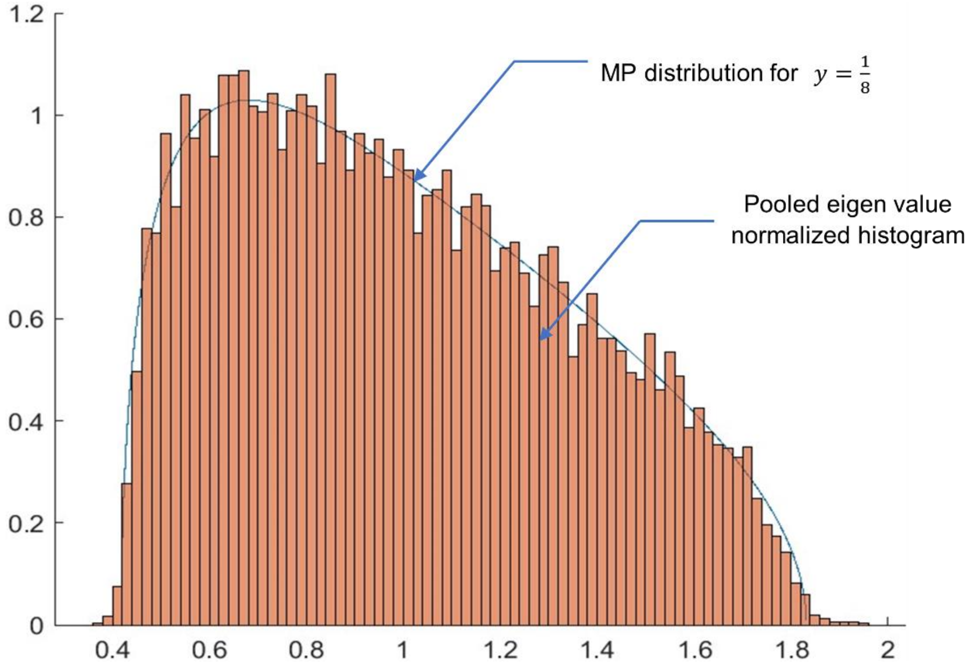


Fig 16: NetSim Results vs. Marchenko-Pasteur distribution for $N_r = 16$ and $N_t = 128$

References

1. Edelman, A. (1988). Eigenvalues and Condition Numbers of Random Matrices. *SIAM Journal on Matrix Analysis and Applications*, 543-560.