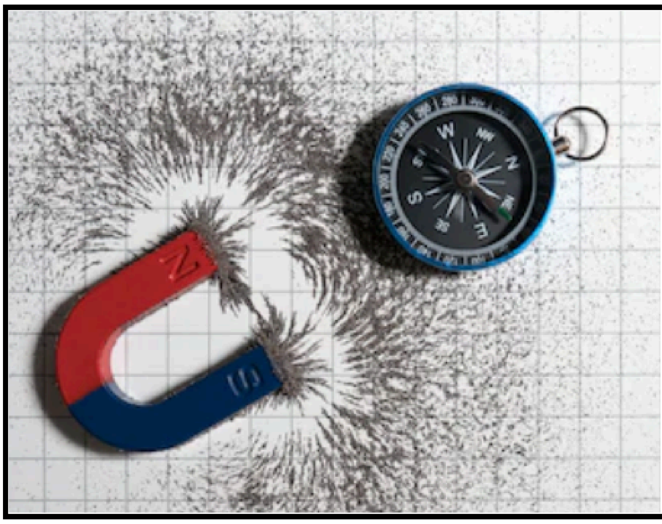
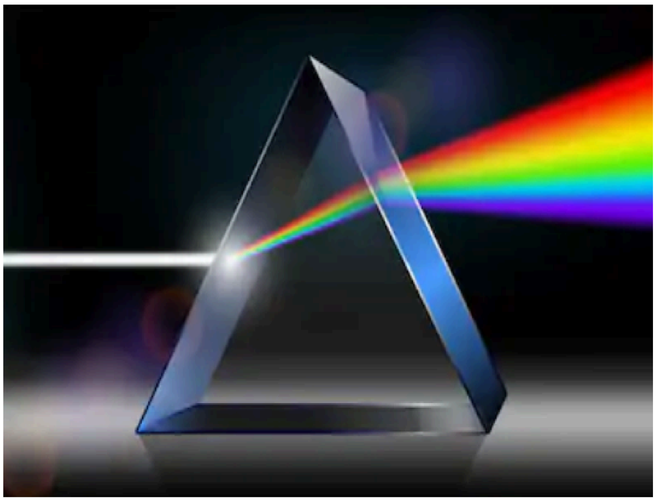
PHYSICS

JEE (MAIN + ADVANCED)

FRICTION

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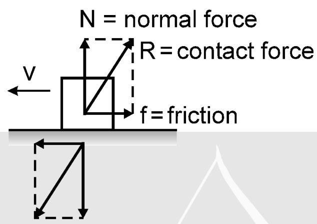
FRICTION

1. FRICTION

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (figure. The perpendicular component is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

Therefore if R is contact force then

$$R = \sqrt{f^2 + N^2}$$

2. REASONS FOR FRICTION

(i) Inter-locking of extended parts of one object into the extended parts of the other object.

(ii) Bonding between the molecules of the two surfaces or objects in contact.

3. FRICTION FORCE IS OF TWO TYPES.

a. Kinetic b. Static

(a) Kinetic Friction Force

Kinetic friction exists between two contact surfaces only when there is **relative motion** between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

DIRECTION OF KINECTIC FRICTION ON AN OBJECT

It is opposite to the relative velocity of the object with respect to the other object in contact considered.

Note that its direction is not opposite to the force applied it is opposite to the relative motion of the body considered which is in contact with the other surface.

MAGNITUDE OF KINETIC FRICTION

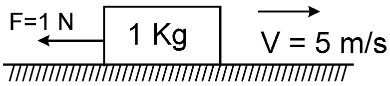
The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

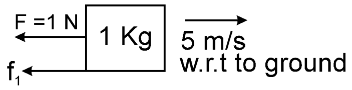
where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.**

Solved Examples

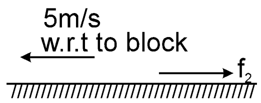
Example 1. Find the direction of kinetic friction force
(a) on the block, exerted by the ground.
(b) on the ground, exerted by the block.



Solution : (a)



(b)

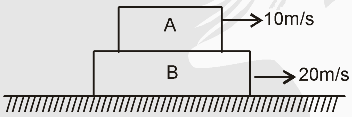


where f_1 and f_2 are the friction forces on the block and ground respectively.

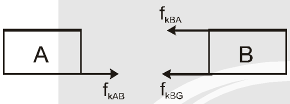
Example 2. In above example correct relation between magnitude of f_1 and f_2 is
(A) $f_1 > f_2$ (B) $f_2 > f_1$
(C) $f_1 = f_2$ (D) not possible to decide due to insufficient data.

Solution : By Newton's third law the above friction forces are action-reaction pair and equal but opposite to each other in direction. Hence (C).
Also note that the direction of kinetic friction has nothing to do with applied force F.

Example 3. All surfaces as shown in the figure are rough. Draw the friction force on A & B



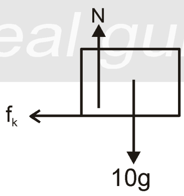
Solution :



Kinetic friction acts in such a way so as to reduce relative motion.

Example 4. Find out the distance travelled by the blocks shown in the figure before it stops.

Solution :



$N - 10\,g = 0$

$N = 100$ N

$f_x = \mu_k N$

$\mu = \mu_s = \mu_k$ when not mentioned

$f_x = 0.5 \times 100 = 50$ N

$F_x = ma$

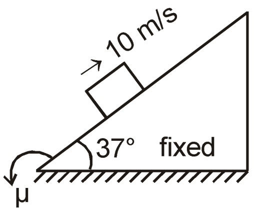
$50 = 10\,a \Rightarrow a = 5$

$\therefore\ v^2 = u^2 + 2as$

$0^2 = 10^2 + 2\,(-5)\,(S)$

$\therefore\ S = 10$ m

Example 5. Find out the distance travelled by the block on incline before it stops. Initial velocity of the block is 10 m/s and coefficient of friction between the block and incline is $\mu = 0.5$.



Solution : $N = mg \cos 37°$

$\therefore$ $mg \sin 37° + \mu N = ma$

$a = 10$ m/s^2 down the incline

Now $v^2 = u^2 + 2as$

$0 = 10^2 + 2(-10)\,S$

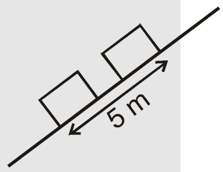
$\therefore$ $S = 5$ m

Example 6. Find the time taken in the above example by the block to reach the initial position.

Solution : $a = g \sin 37° - \mu g \cos 37°$

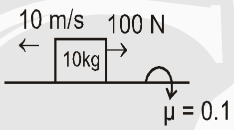
$\therefore$ $a = 2$ m/s^2 down the incline

$\therefore$ $S = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad S = \frac{1}{2} \times 2 \times t^2$



$\therefore$ $t = \sqrt{5}$ sec.

Example 7. A block is given a velocity of 10 m/s and a force of 100 N in addition to friction force is also acting on the block. Find the retardation of the block?



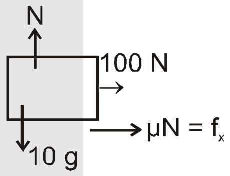
Solution : As there is relative motion

$\therefore$ kinetic friction will act to reduce this relative motion.

$f_k = \mu N = 0.1 \times 10 \times 10 = 10$ N

$100 + 10 = 10a$

$a = \frac{110}{10} = 11$ m/s^2



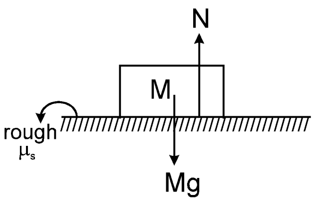
(b) STATIC FRICTION

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

For example consider a bed inside a room ; when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the opposite direction of the applied force.

Solved Example

Example 8. What is value of static friction force on the block?



Solution : In horizontal direction as acceleration is zero.

Therefore $\Sigma F = 0$.

$\therefore\ f = 0$

Direction of static friction force :

The static friction force on an object is opposite to its impending motion relative to the surface.

Following steps should be followed in determining the direction of static friction force on an object.

(i) Draw the free body diagram with respect to the other object on which it is kept.

(ii) Include pseudo force also if contact surface is accelerating.

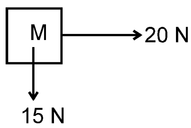
(iii) Decide the resultant force and the component parallel to the surface of this resultant force.

(iv) The direction of static friction is opposite to the above component of resultant force.

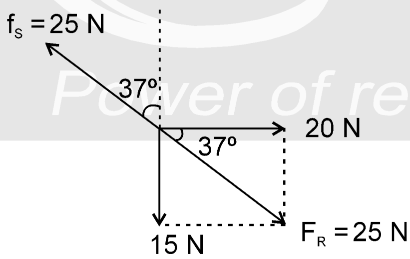
Note : Here once again the static friction is involved when there is no relative motion between two surfaces.

Solved Example

Example 9. In the following figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.



Solution : In the above problem we first draw the free body diagram of find the resultant force.



As the object doe not move this is not a case of limiting friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure by f_s. Its magnitude is equal to 25 N.

4. MAGNITUDE OF KINETIC AND STATIC FRICTION

Kinetic friction :

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.** If the surfaces are smooth μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

Static friction :

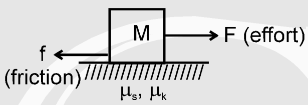
The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. **This means it is a variable and self adjusting force. However it has a maximum value called limiting friction.**

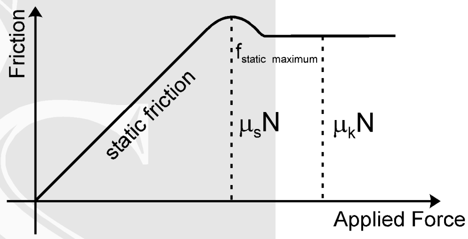
$$f_{max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0 \leq f_s \leq f_{smax}$$

Here μ_s and μ_k are proportionality constants. μ_s is called coefficient of static friction and μ_k is called coefficient of kinetic friction. They are dimensionless quantities independent of shape and area of contact . It is a property of the two contact surfaces. $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned then $\mu_s = \mu_k$ can be taken. Value of μ can be from 0 to ∞.



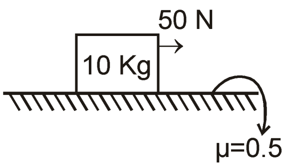


Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

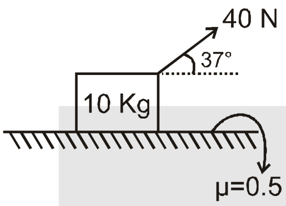
Solved Examples

Example 10. Find acceleration of block. Initially the block is at rest.

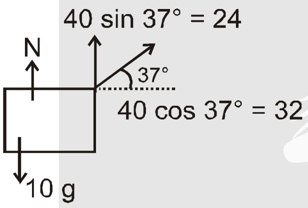


Solution : zero

Example 11. Find out acceleration of the block. Initially the block is at rest.



Solution :



$N + 24 - 100 = 0$ for vertical direction

$\therefore$ $N = 76$ N

Now $0 \le f_s \le \mu_s N$

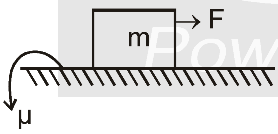
$0 \le f_s \le 76 \times 0.5$

$0 \le f_s \le 38$ N

$\therefore$ $32 < 38$ Hence $f = 32$

$\therefore$ acceleration of block is zero.

Example 12. Find out acceleration of the block for different ranges of F.



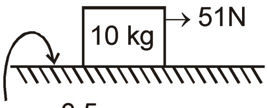
Solution : $0 \le f \le \mu_S N$

$0 \le f \le \mu_S mg$

$a = 0$ if $F \le \mu_S mg$

$$a = \frac{F - \mu Mg}{M} \quad \text{if } F > \mu Mg$$

Example 13. Find out acceleration of the block. Initially the block is at rest.



$\mu_s = 0.5$
$\mu_k = 0.3$

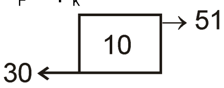
Solution : $0 \le f_s \le \mu_S N$

$0 \le f_s \le 50$

Now $\quad 51 > 50$

$\therefore$ Block will move but if the block starts moving then kinetic friction is involved.

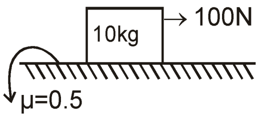
$K_F = \mu_k N = 0.3 \times 100 = 30$ N



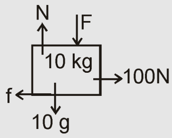
$\therefore\ 51 - 30 = 10\,a$

$\therefore\ a = 2.1$ m/s^2

Example 14. Find out the minimum force that must be applied on the block vertically downwards so that the block doesn't move.



Solution :



$100 - f_s = 0$

$\therefore \quad f_s = 100$ .... (1)

$F + 10\,g = N \Rightarrow N = 100 + F$ .... (2)

Now $\quad 0 \le f_S \le \mu N$

$100 \le 0.5$ N

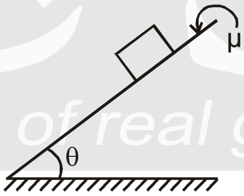
$100 \le 0.5\ [100 + F]$

$200 \le 100 + F$

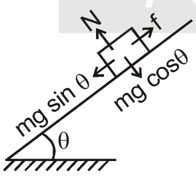
$F \ge 100$ N

$\therefore$ Minimum F = 100 N

Example 15. The angle of inclination is slowly increased. Find out the angle at which the block starts moving.



Solution :



$0 \le f \le \mu_S N$

mg $\sin\theta > f_{smax}$

mg $\sin\theta > \mu N$

mg $\sin\theta > \mu$ mg $\cos\theta$

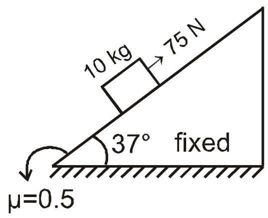
$\therefore \tan\theta > \mu$

$\theta = \tan^{-1}\mu$

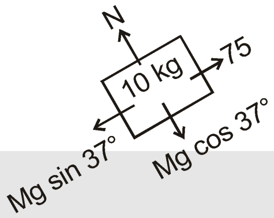
for $\tan\theta \le \mu$ no sliding on inclined plane.

This method is used for finding out the value of μ practically.

Example 16. Find out the acceleration of the block. If the block is initially at rest.



Solution : (FBD of the block excluding friction)

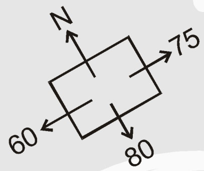


$N = 10\,g \cos 37° = 80$ N

Now $0 \le f_s \le \mu N$

$0 \le f_s \le 0.5 \times 80$

$\therefore$ $f_s \le 40$ N



We will put value of f in the last i.e. in the direction opposite to resultant of other forces. f acts down the incline and its value is of = 75 – 60 = 15 N

So acceleration is zero

Example 17. In the above problem how much force should be added to 75 N force so that block starts to move up the incline.

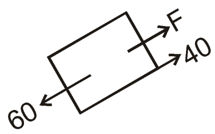
Solution : $\therefore$ $60 + 40 = 75 + f_{extra}$

$\therefore$ $f_s = 25$ N

Example 18. In the above problem what is the minimum force by which 75 N force should be replaced with so that the block does not move.

Solution : In this case the block has a tendency to move downwards.

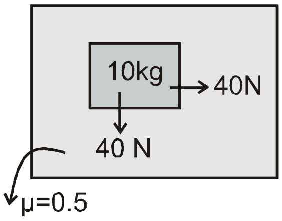
Hence friction acts upwards.



$\therefore$ F + 40 = 60

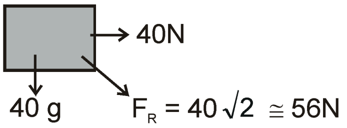
$\therefore$ F = 20 N

Example 19. Top view of a block on a table is shown (g = 10 m/s^2) .



Find out the acceleration of the block.

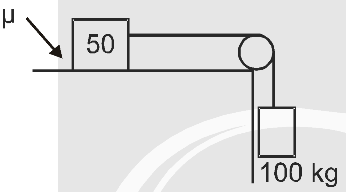
Solution :



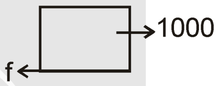
Now $f_s \le \mu N$
$\therefore\ f_s \le 50$
$F_R > f_{smax}$
Hence the block will move.

$$a = \frac{40\sqrt{2} - 50}{10} = (4\sqrt{2} - 5)\,m/s^2$$

Example 20. Find minimum μ so that the blocks remain stationary.

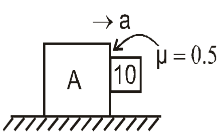


Solution : T = 100 g = 1000 N
$\therefore$ f = 1000 to keep the block stationary
Now f_{max} = 1000

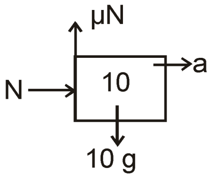


μN = 1000
μ = 2
Can μ be greater than 1 ?
Yes $0 < \mu \le \infty$

Example 21. Find out minimum acceleration of block A so that the 10 kg block doesn't fall.



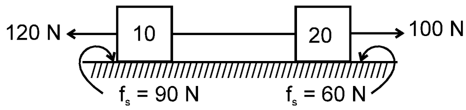
Solution :



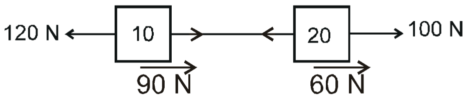
Applying NL in horizontal direction
N = 10 a .... (1)
Applying NL in vertical direction
10 g = μ N ....(2)
10 g = μ 10 a from (1) & (2)

$$\therefore\ a = \frac{g}{\mu} = 20\ m/s^2$$

Example 22. Find the tension in the string in situation as shown in the figure below. Forces 120 N and 100 N start acting when the system is at rest and the maximum value of static friction on 10 kg is 90 N and that on 20 kg is 60N?

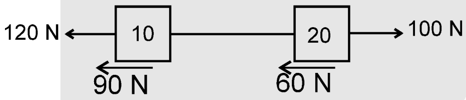


Solution : (i) Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore the assumption is not valid.



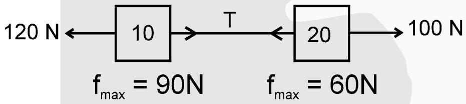
Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

(ii) Similarly let us assume that system moves towards right.

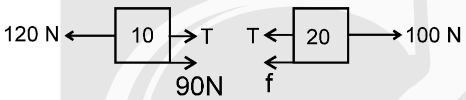


Above assumption is also not possible as net force on the system is towards left in this situation. Hence assumption is again not valid.

Therefore it can be concluded that the system is stationary.



Assuming that the 10 kg block reaches limiting friction first then using FBD's.



$120 = T + 90 \quad \Rightarrow \quad T = 30$ N

Also $\quad T + f = 100$

$\therefore\ 30 + f = 100 \quad \Rightarrow$

f = 70 N which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have

$T + 60 = 100$ N $\quad \Rightarrow \quad T = 40$ N

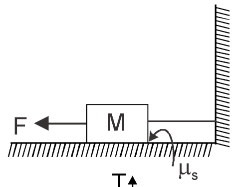
Also $\quad f + T = 120$ N $\quad \Rightarrow \quad f = 80$ N

This is acceptable as static friction at this surface should be less than 90 N.

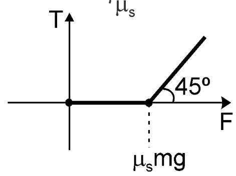
Hence the tension in the string is

$T = 40$ N.

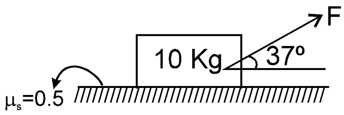
Example 23. In the following figure force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s.



Solution : As the external force F is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s mg$, the further increase in F is transferred to the string.



Example 24. Force F is gradually increased from zero. Determine whether the block will first slide or lift up?

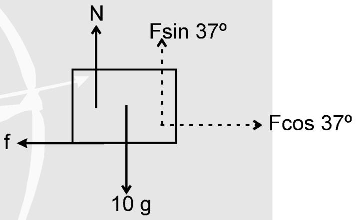


Solution : There are minimum magnitude of forces required both in horizontal and vertical direction either to slide on lift up the block. The block will first slide on lift up will depend upon which minimum magnitude of force is lesser.

For vertical direction to start lifting up

F sin 37° + N – Mg ≥ 0.

N becomes zero just lifting condition.



$$F_{lift} \geq \frac{10g}{3/5}$$

$$\therefore \quad F_{lift} \geq \frac{500}{3} N$$

For horizontal direction to start sliding

$F \cos 37 \geq \mu_s N$

$F \cos 37° > 0.5\,[\,10g - F \sin 37°]$ $\qquad (\because\ N = 10\,g - F \sin 37°)$

Hence $F_{slide} > \dfrac{50}{\cos 37° + 0.5 \sin 37°}$

$$F_{slide} > \frac{500}{11} N$$

$$F_{lift} > \frac{500}{3} N.$$

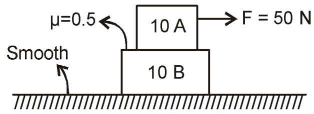
$\Rightarrow \quad F_{slide} < F_{lift}$

Therefore the block will begin to slide before lifting.

TWO BLOCK PROBLEMS

Solved Examples

Example 25. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution : **Method of solving**

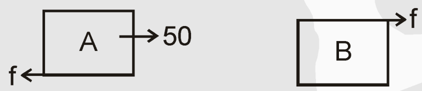
Step 1 : Make force diagram.
Step 2 : Show static friction force by f because value of friction is not known.
Step 3 : Calculate separately for two cases.

Case 1 : Move together

Step 4 : Calculate acceleration.
Step 5 : Check value of friction for above case.
Step 6 : If required friction is less than available it means they will move together else move separately.
Step 7: (a) above acceleration will be common acceleration for both

Case 2 : Move separately

Step 7(b) If they move separately then kinetic friction is involved. whose value is μN.
Step 8 : Calculate acceleration for above case.



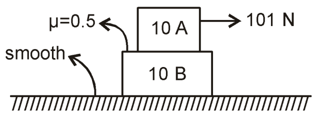
$f_{max} = \mu N$
$\therefore\ f \leq 50$ N (available friction)

Move together	**Move separately**
(i) $a = \dfrac{50}{10+10} = 2.5$ m/s^2	No need to calculate

(ii) Check friction for B :
f = 10 × 2.5 = 25

25 N is required which is less than available friction hence they will move together.
and $a_A = a_B = 2.5$ m/s^2

Example 26. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?

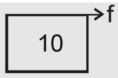


Solution : $f_{max} = 50$ N

$\therefore$ $f \leq 50$ N

(i) If they move together $a = \dfrac{101}{20} = 5.05$ m/s^2

(ii) Check friction on B

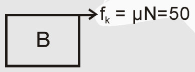


f = 10 × 5.05 = 50.5 (required)

50.5 > 50 (therefore required > available)

Hence they will <u>not</u> move together.

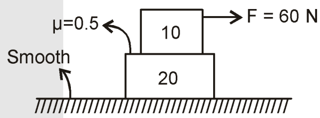
(iii) Hence they move separately so kinetic friction is involved.



$\therefore$ for $a_A = \dfrac{101-50}{10} = 5.1$ m/s^2 $\Rightarrow$ $a_B = \dfrac{50}{10} = 5$ m/s^2

Also $a_A > a_B$ as force is applied on A.

Example 27. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution :

<u>Move Together</u>

$a = \dfrac{60}{30} = 2$ m/s^2

Check friction on 20 kg.

f = 20 x 2

f = 40 (which is required)

40 < 50 (therefore required < available)

$\therefore$ will move together.

<u>Move Separately</u>

No need to calculate.

Example 28. In above example find maximum F for which two blocks will move together.

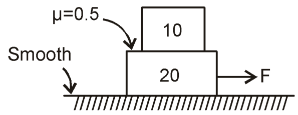
Solution : Observing the critical situation where friction becomes limiting.

$\therefore$ $F - f_{max} = 10\,a$ .... (1)

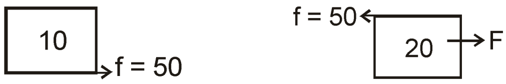
$f_{max} = 20\,a$ .... (2)

$\therefore$ F = 75 N

Example 29. Initially the system is at rest. find out minimum value of F for which sliding starts between the two blocks.



Solution : At just sliding condition limiting friction is acting.



$F - 50 = 20a$ .... (1)

$f = 10a$ .... (2)

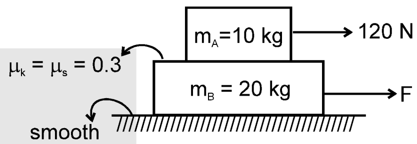
$50 = 10a$

$\therefore a = 5 \text{ m/s}^2$

hence $F = 50 + 20 \times 5 = 150$ N

$\therefore F_{min} = 150$ N

Example 30. In the figure given below force F applied horizontally on lower block, is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Take g = 10 m/s²).



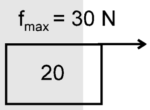
Solution : In the above situation we see that the maximum possible value of friction between the blocks is $\mu_s m_A g = 0.3 \times 10 \times 10 = 30$ N.

Case (i) When F = O.

Considering that there is no slipping between the blocks the acceleration of system will be

$$a = \frac{120}{20+10} = 4 \text{ m/s}^2$$

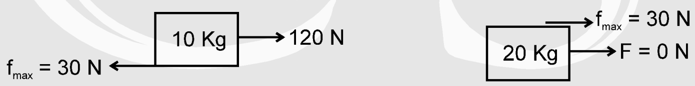
But the maximum acceleration of B can be obtained by the following force diagram.



$a_B = \dfrac{30}{20} = 1.5 \text{ m/s}^2$ ($\because$ only friction force by block A is responsible for producing acceleration in block B)

Because 4 > 1.5 m/s² we can conclude that the blocks do not move together.

Now drawing the F.B.D. of each block, for finding out individual accelerations.

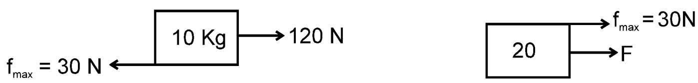


$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2 \text{ towards right}$$

$$a_B = \frac{30}{20} = 1.5 \text{ m/s}^2 \text{ towards right.}$$

Case (ii) F is increased from zero till the two blocks just start moving together.

As the two blocks move together the friction is static in nature and its value is limiting. FBD in this case will be



$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2 \quad \Rightarrow \quad a_B = \frac{F+30}{20} = a_A \quad \Rightarrow \quad \frac{F+30}{20} = 9$$

$\therefore \quad F = 150$ N

Hence when 0 < F < 150 N the blocks do not move together and the friction is kinetic. As F increases acceleration of block B increases from 1.5 m/s^2.

At F = 150 N limiting static friction start acting and the two blocks start moving together.

Case (iii) When F is increased above 150 N.

In this scenario the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case the FBD is as follows

$$a_A = a_B = \frac{120 - f}{10} = \frac{F + f}{20}$$

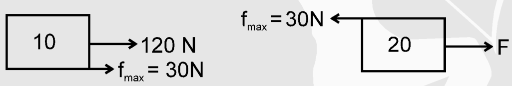
$\therefore$ when friction force f gets reduced to zero the above accelerations become

$$a_A = \frac{120}{10} = 12 \text{ m/s}^2 \quad \Rightarrow \quad a_B = \frac{F}{20} = a_A = 12 \text{ m/s}^2$$

$\therefore \quad F = 240$ N

Hence when $150 \le F \le 240$ N the static friction force continuously decreases from maximum to zero at F = 240 N. The accelerations of the blocks increase from 9 m/s^2 to 12 m/s^2 during the change of force F.

Case (iv) When F is increased again from 240 N the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. FBD at this juncture will be



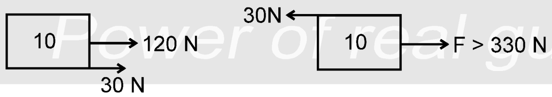
The blocks move together therefore.

$$a_A = \frac{120 + 30}{10} = 15 \text{ m/s}^2 \quad \Rightarrow \quad a_B = \frac{F - 30}{20} = a_A = 15 \text{ m/s}^2$$

$$\therefore \quad \frac{F - 30}{20} = 15 \text{ m/s}^2$$

Hence F = 330 N.

Case (v) When F is increased beyond 330 N. In this case the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).



$\therefore \quad a_A = 15$ m/s^2 which is constant

$$a_B = \frac{F - 30}{20} \text{ m/s}^2 \quad \text{where } F > 330 \text{ N.}$$

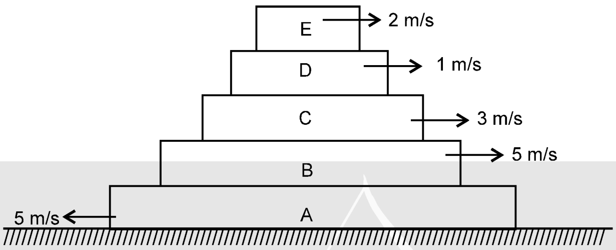
EXERCISE-1

PART - I : SUBJECTIVE QUESTIONS

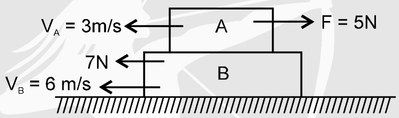
* **Marked Questions are having more than one correct option.**

SECTION (A) : KINETIC FRICTION

A-1. In the given diagram find the direction of friction forces on each block and the ground (Assume all surfaces are rough and all velocities are with respect to ground).



A-2. In the following figure, find the direction of friction on the blocks and ground .

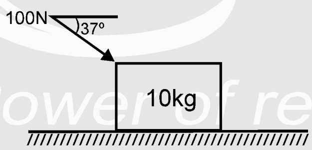


A-3. A block is shot with an initial velocity $5ms^{-1}$ on a rough horizontal plane. Find the distance covered by the block till it comes to rest. The coefficient of kinetic friction between the block and plane is 0.1.

SECTION (B) : STATIC FRICTION

B-1. The angle between the resultant contact force and the normal force exerted by a body on the other is called the angle of friction. Show that, if λ be the angle of friction and μ the coefficient of static friction, $\lambda \leq \tan^{-1} \mu$

B-2. In the figure shown calculate the angle of friction. The block does not slide. Take g = 10 m/s^2.



B-3. What is the minimum value of force (in following two cases) required to pull a block of mass M on a horizontal surface having coefficient of friction μ? Also find the angle this force makes with the horizontal.
(a) If force is parallel to horizontal surface
(b) If force is in any direction (Also find the angle this force makes with the horizontal.)

SECTION (C) : MISCELLANEOUS QUESTIONS

C-1. A body of mass 5 kg is kept on a rough horizontal surface. It is found that the body does not slide if a horizontal force less than 30 N is applied to it. Also it is found that it takes 5 seconds to slide throughout the first 10 m if a horizontal force of 30 N is applied and the body is gently pushed to start the motion. Taking g= 10 m/s^2, calculate the coefficients of static and kinetic friction between the block and the surface.

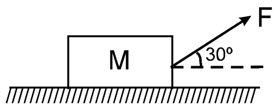
PART - II : OBJECTIVE QUESTIONS

SECTION (A) : KINETIC FRICTION

A-1. Starting from rest a body slides down a 45º inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The co-efficient of friction between the body and the inclined plane is:

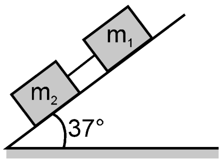
(A) 0.75 (B) 0.33 (C) 0.25 (D) 0.80

A-2. A block of mass M = 5 kg is resting on a rough horizontal surface for which the coefficient of friction is 0.2. When a force F = 40 N is applied as shown in figure the acceleration of the block will be (g = 10 m/s^2) :



(A) 5.73 m/sec^2 (B) 8.0 m/sec^2
(C) 3.17 m/sec^2 (D) 10.0 m/sec^2

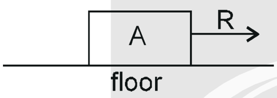
A-3. Two blocks m_1 = 4kg and m_2 = 2kg, connected by a weightless rod on a plane having inclination of 37^0 as shown in figure. The coefficients of dynamic friction of m_1 and m_2 with the inclined plane are μ = 0.25. Then the common acceleration of the two blocks and the tension in the rod are:

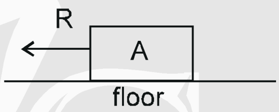


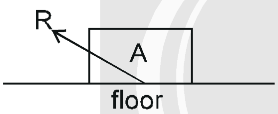
(A) 4 m/s^2, T = 0 (B) 2 m/s^2, T = 5 N
(C) 10 m/s^2 ,T = 10 N (D) 15 m/s^2, T = 9N

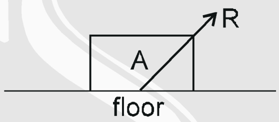
SECTION (B) : STATIC FRICTION

B-1. A box 'A' is lying on the horizontal floor of the compartment of a train running along horizontal rails from left to right. At time 't', it decelerates. Then the resultant contact force R by the floor on the box is given best by :

(A) 

(B) 

(C) 

(D) 

B-2. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6 If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is :

(A) 5 N (B) 6 N (C) 10 N (D) 15 N

B-3. A block of mass 2 kg rests on a rough inclined plane making an angle of 30^0 with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is :

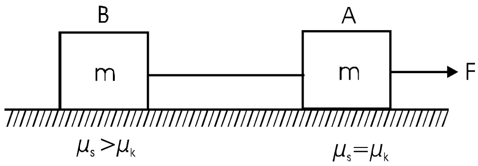
(A) 9.8 N (B) 0.7 × 9.8 $\sqrt{3}$ N (C) 9.8 × 7 N (D) 0.8 × 9.8 N

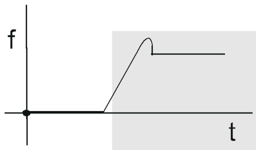
SECTION (C) : MISCELLANEOUS QUESTIONS

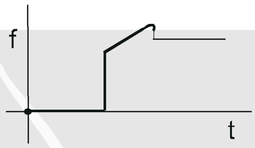
C-1. A 60 kg body is pushed horizontally with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficient of static friction and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is :

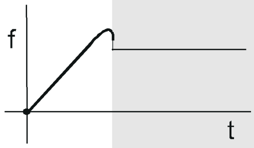
(A) 6 m/s^2 (B) 4.9 m/s^2 (C) 3.92 m/s^2 (D) 1 m/s^2

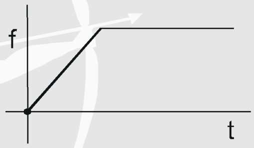
C-2. A force F = t is applied to block A as shown in figure. The force is applied at t = 0 seconds when the system was at rest and string is just straight without tension. Which of the following graphs gives the friction force between B and horizontal surface as a function of time 't'.



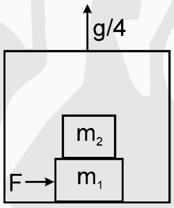
(A) 

(B) 

(C) 

(D) 

C-3. A plank of mass m_1 = 8 kg with a bar of mass m_2 = 2 kg placed on its rough surface, lie on a smooth floor of elevator ascending with an acceleration g/4. The coefficient of friction is μ = 1/5 between m_1 and m_2. A horizontal force F = 30 N is applied to the plank. Then the acceleration of bar and the plank in the reference frame of elevator are:



(A) 3.5 m/s^2, 5 m/s^2 (B) 5 m/s^2, $\frac{50}{8}$ m/s^2

(C) 2.5 m/s^2, $\frac{25}{8}$ m/s^2 (D)4.5 m/s^2, 4.5m/s^2

EXERCISE-2

PART - I : OBJECTIVE QUESTIONS

Single choice type

1. A body is projected up along the rough inclined plane from the bottom with some velocity. It travels up the incline and then returns back. If the time of ascent is t_a and time of descent is t_d, then

(A) $t_a = t_d$ (B) $t_a > t_d$ (C) $t_a < t_d$ (D) data insufficient

2. The upper portion of an inclined plane of inclination α is smooth and the lower portion is rough. A particle slides down from rest from the top and just comes to rest at the foot. If the ratio of the smooth length to rough length is m : n, the coefficient of friction is :

(A) $\left[\frac{m+n}{n}\right]\tan\alpha$ (B) $\left(\frac{m+n}{n}\right)\cot\alpha$ (C) $\left(\frac{m-n}{n}\right)\cot\alpha$ (D) $\frac{1}{2}$

3. A 1.5 kg box is initially at rest on a horizontal surface when at t = 0 a horizontal force $\vec{F} = (1.8t)\hat{i}\,N$ (with t in seconds), is applied to the box. The acceleration of the box as a function of time t is given by : (g = 10m/s²)

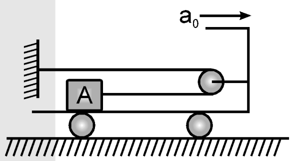
$\vec{a} = 0$ for $0 \le t \le 2.85$

$\vec{a} = (1.2t - 2.4)\hat{i}$ m/s² for $t > 2.85$

The coefficient of kinetic friction between the box and the surface is :

(A) 0.12 (B) 0.24 (C) 0.36 (D) 0.48

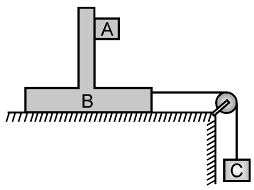
4. Starting from rest, A flat car is given a constant acceleration a_o = 2 m/s². A cable is connected to a crate A of mass 50 kg as shown. Neglect the friction between floor and car wheels and mass of pulley. The cofficient of friction between crate & floor of the car is μ = 0.3. The tension in cable is -



(A) 700 N (B) 350 N

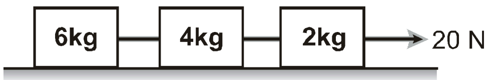
(C) 175 N (D) 0

5. In the arrangement shown in the figure mass of the block B and A are 2 m, 8 m respectively. Surface between B and floor is smooth. The block B is connected to block C by means of a pulley. If the whole system is released then the minimum value of mass of the block C so that the block A remains stationary with respect to B is : (Co-efficient of friction between A and B is μ and pulley is ideal)



(A) $\frac{m}{\mu}$ (B) $\frac{2m}{\mu+1}$ (C) $\frac{10\,m}{1-\mu}$ (D) $\frac{10m}{\mu-1}$

6. **(i)** In the arrangement shown tension in the string connecting 4kg and 6kg masses is



(A) 8N (B) 12N (C) 6N (D) 4N

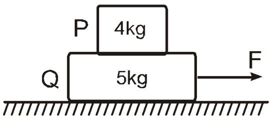
(ii) Friction force on 4 kg block is

(A) 4N (B) 6 N (C) 12 N (D) 8 N

(iii) Friction force on 6 kg block is

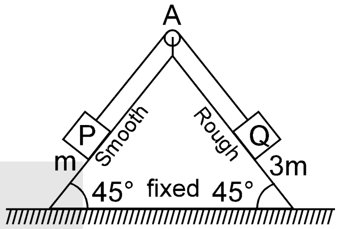
(A) 12 N (B) 8 N (C) 6 N (D) 4 N

7. In the given figure the coefficient of friction between 4kg and 5 kg blocks is 0.2 and between 5 kg block and ground is 0.1 respectively. Choose the correct statements



(A) Minimum force needed to cause system to move is 17 N
(B) When force is 4N static friction at all surfaces is 4N to keep system at rest
(C) Maximum acceleration of 4kg block is 2m/s^2
(D) Slipping between 4kg and 5 kg blocks start when F is > 17N

8. A fixed wedge with both surface inclined at 45^0 to the horizontal as shown in the figure. A particle P of mass m is held on the smooth plane by a light string which passes over a smooth pulley A and attached to a particle Q of mass 3m which rests on the rough plane. The system is released from rest. Given that the acceleration of each particle is of magnitude $\frac{g}{5\sqrt{2}}$ then



(i) the tension in the string is :

(A) mg (B) $\frac{6mg}{5\sqrt{2}}$ (C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

(ii) In the above question the coefficient of friction between Q and the rough plane is :

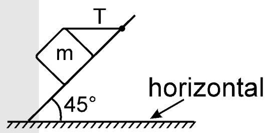
(A) $\frac{4}{5}$ (B) $\frac{1}{5}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

(iii) In the above question the magnitude and direction of the force exerted by the string on the pulley is :

(A) $\frac{6mg}{5}$ downward (B) $\frac{6mg}{5}$ upward (C) $\frac{mg}{5}$downward (D) $\frac{mg}{4}$downward

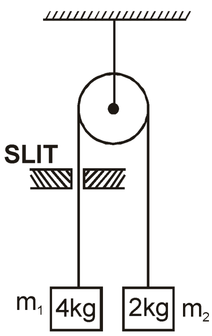
More than one choice type

9. A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact is (g = 10 m/s^2)



(A) 1/2 (B) 2/3
(C) 3/4 (D) 1/4

10. Two masses m_1 = 4 kg and m_2 = 2kg are connected with an inextensible, massless string that passes over a frictionless pulley and through a slit, as shown. The string is vertical on both sides and the string on the left is acted upon by a constant friction force 10 N by the slit as it moves. (use g = 10 m/s^2)



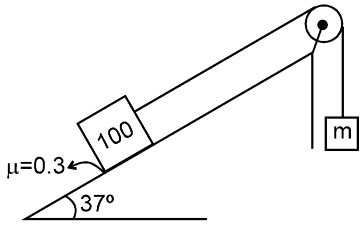
(A) Acceleration of mass m_1 is $\frac{5}{3}$ m/s^2, downwards.

(B) Tension in the string is same throughout.

(C) Force exerted by the string on mass m_2 is $\frac{70}{3}$ N.

(D) If positions of both the masses are interchanged, then 2kg mass moves up with an acceleration $\frac{10}{3}$ m/s^2.

11. In the given figure the value(s) of mass m for which the 100 kg block remains in static equilibrium is $(g = 10\ m/s^2)$



(A) 35 kg

(B) 37 kg

(C) 83 kg

(D) 85 kg

12.* The force F_1 parallel to inclined plane that is necessary to move a body up an inclined plane is double the force F_2 that is necessary to just prevent it from sliding down, then :

(A) $F_2 = w \sin(\theta - \phi) \sec\phi$ (B) $F_1 = w \sin(\theta - \phi) \sec\phi$

(C) $\tan\phi = 3\tan\theta$ (D) $\tan\theta = 3\tan\phi$

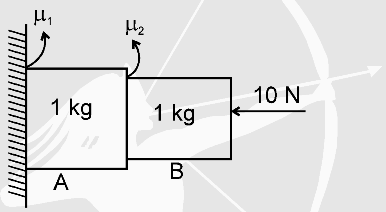
Where ϕ = Limiting angle of repose

θ = angle of inclined plane

w = weight of the body

PART - II : MATCH THE COLUMN

1. In the given figure find the accelerations of blocks A and B for the following cases $(g = 10\ m/s^2)$

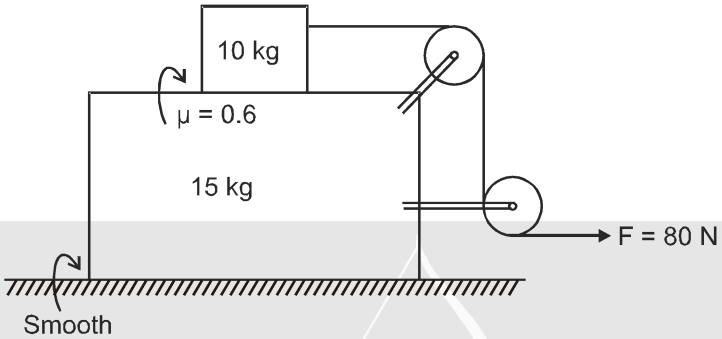


Column - I	Column - II
(A) $\mu_1 = 0$ and $\mu_2 = 0.1$	(p) $a_A = a_B = 9.5\ m/s^2$
(B) $\mu_2 = 0$ and $\mu_1 = 0.1$	(q) $a_A = 9\ m/s^2$, $a_B = 10\ m/s^2$
(C) $\mu_1 = 0.1$ and $\mu_2 = 1.0$	(r) $a_A = a_B = g = 10\ m/s^2$
(D) $\mu_1 = 1.0$ and $\mu_2 = 0.1$	(s) $a_A = 1$, $a_B = 9\ m/s^2$

PART - III : COMPREHENSION

Comprehension - 1

A block of mass 15 kg is placed over a frictionless horizontal surface. Another block of mass 10 kg is placed over it, that is connected with a light string passing over two pulleys fastened to the 15 kg block. A force F = 80 N is applied horizontally to the free end of the string. Friction coefficient between two blocks is 0.6. The portion of the string between 10 kg block and the upper pulley is horizontal as shown in figure Pulley string & connecting rods are massless. (Take $g = 10\ m/s^2$)

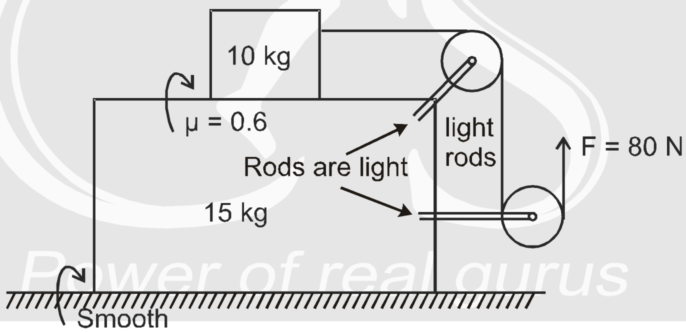


1. The magnitude of acceleration of the 10 kg block is :
(A) $3.2\ m/s^2$ (B) $2.0\ m/s^2$ (C) $1.6\ m/s^2$ (D) $0.8\ m/s^2$

2. The magnitude of acceleration of the 15 kg block is :
(A) $4.2\ m/s^2$ (B) $3.2\ m/s^2$ (C) $16/3\ m/s^2$ (D) $2.0\ m/s^2$

3. If applied force F = 120 N, then magnitude of acceleration of 15 kg block will be :
(A) $8\ m/s^2$ (B) $4\ m/s^2$ (C) $3.2\ m/s^2$ (D) $4.8\ m/s^2$

4. Continuing with the situation, if the force F = 80 N is directed vertically as shown, the acceleration of the 10 kg block will be :



(A) $2\ m/s^2$, towards right (B) $2\ m/s^2$, towards left
(C) $6\ m/s^2$, towards left (D) $16/5\ m/s^2$, towards right

5. In the situation of the previous question, acceleration of the 15 kg block will be :
(A) $4\ m/s^2$, towards right (B) $16/5\ m/s^2$, towards right
(C) $2/3\ m/s^2$, towards right (D) $4/3\ m/s^2$, towards left

PART - IV : ASSERTION / REASONING

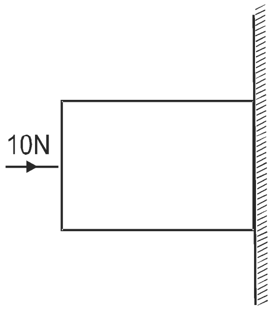
1. **STATEMENT-1 :** A body is lying at rest on a rough horizontal surface. A person accelerating with acceleration $a\hat{i}$ (where a is positive constant and $\hat{i}$ is a unit vector in horizontal direction) observes the body. With respect to him, the block experiences a kinetic friction.
STATEMENT-2 : Whenever there is relative motion between two rough contact surfaces then kinetic friction acts.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

2. **STATEMENT-1 :** A block of mass m = 10 kg lies on rough horizontal surface. The coefficient of friction between block and horizontal surface is $\mu = \frac{3}{4}$. Initially only forces acting on block are its weight and normal reaction due to horizontal surface. An additional force of magnitude 70 N can move the block on horizontal surface.
STATEMENT-2 : The magnitude of minimum force required to move a block of mass m placed on rough horizontal surface is μN. (Where μ is co-efficient of friction and N is normal reaction acting on the block).
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

EXERCISE-3

PART - I : JEE-MAIN PROBLEMS (PREVIOUS YEARS)

1. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is : **[AIEEE-2003]**



(1) 20 N (2) 50 N
(3) 100 N (4) 2 N

2. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10s. Then the coefficient of friction is : **[AIEEE-2003]**
(1) 0.02 (2) 0.03 (3) 0.06 (4) 0.01

3. A block rests on a rough inclined plane making an angle of 30º with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take g = 10 m/s^2) : **[AIEEE-2004]**
(1) 2.0 (2) 4.0 (3) 1.6 (4) 2.5

4. A smooth block is released at rest on a 45º incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is- **[AIEEE-2005]**
(1) $\mu_s = 1 - \frac{1}{n^2}$ (2) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$ (3) $\mu_k = 1 - \frac{1}{n^2}$ (4) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

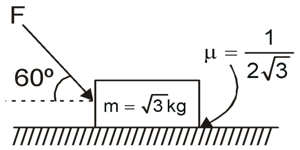
5. The upper half of an incline plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the the bottom if the coefficient of friction for the lower half is given by **[AIEEE-2005]**
(1) 2 tan ϕ (2) tan ϕ (3) 2 sin ϕ (4) 2 cos ϕ

6. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which car can be stopped is [μ_k = 0.5] **[AIEEE-2005]**
(1) 100 m (2) 400 m (3) 800 m (4) 1000 m

7. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that tan θ = 2μ then the ratio $\frac{F_1}{F_2}$ is : **[AIEEE-2011]**
(1) 1 (2) 2 (3) 3 (4) 4

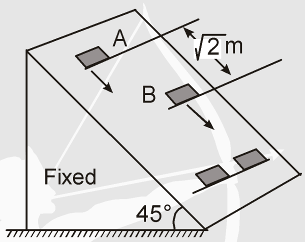
PART - II : JEE-ADVANCED PROBLEMS (PREVIOUS YEARS)

1. What is the maximum value of the force F such that the block shown in the arrangement, does not move : **[JEE 2003]**



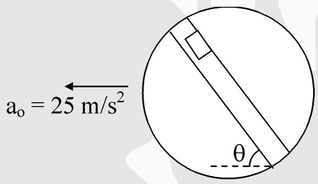
(A) 20 N (B) 10 N (C) 12 N (D) 15 N

2. Two blocks A and B of equal masses are sliding down along straight parallel lines on an inclined plane of 45°. Their coefficients of kinetic friction are $\mu_A = 0.2$ and $\mu_B = 0.3$ respectively. At t = 0, both the blocks are at rest and block A is $\sqrt{2}$ meter behind block B. The time and distance from the initial position where the front faces of the blocks come in line on the inclined plane as shown in figure. (Use g = 10 ms^{-2}.) **[JEE 2004]**



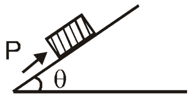
(A) 2s, $8\sqrt{2}$ m (B) $\sqrt{2}$ s, 7m (C) $\sqrt{2}$ s, $7\sqrt{2}$ m (D) 2s, $7/\sqrt{2}$ m

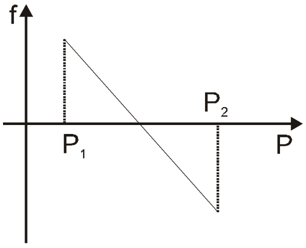
3. A disc is kept on a smooth horizontal plane with its plane parallel to horizontal plane. A groove is made in the disc as shown in the figure. The coefficient of friction between mass m and surface of the groove is 2/5 and sin θ = 3/5. Find the acceleration of mass with respect to the frame of reference of the disc. **[JEE 2006]**

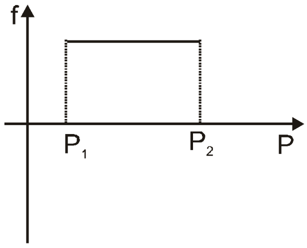


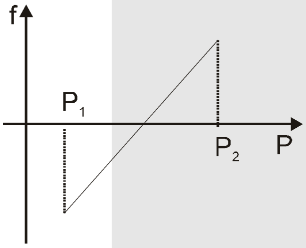
4. **STATEMENT -1**
It is easier to pull a heavy object than to push it on a level ground. **[JEE 2008]**
STATEMENT -2
The magnitude of frictional force depends on the nature of the two surfaces in contact.
(A) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 **is** a correct explanation for STATEMENT -1
(B) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 **is NOT** a correct explanation for STATEMENT -1
(C) STATEMENT -1 is True, STATEMENT -2 is False
(D) STATEMENT -1 is False, STATEMENT -2 is True.

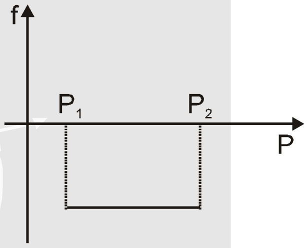
5. A block of mass m is on inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like : **[JEE 2010]**



(A)


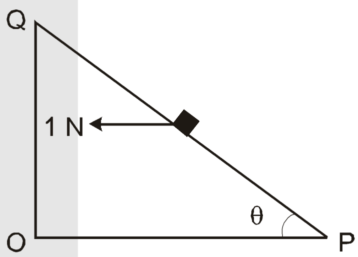
(B)


(C)


(D)


6. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\ \mu$, then N is **[JEE 2011]**

7. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N on the block through its center of mass as shown in the figure. The block remains stationary if (take $g = 10\ m/s^2$) **[IIT-JEE-2012]**



(A) $\theta = 45°$
(B) $\theta > 45°$ and a frictional force acts on the block towards P.
(C) $\theta > 45°$ and a frictional force acts on the block towards Q.
(D) $\theta < 45°$ and a frictional force acts on the block towards Q.

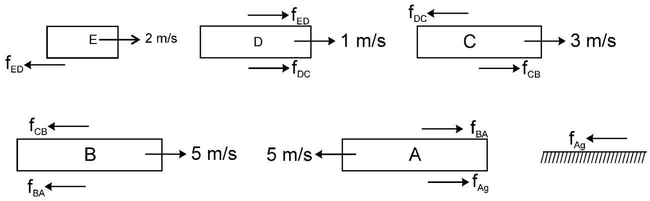
ANSWER KEY

EXERCISE - 1

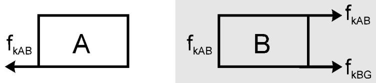
PART-I

SECTION (A)

A-1.



A-2.



A-3. 12.5 m

SECTION (B)

B-1. $\lambda \le \tan^{-1}\mu$ **B-2.** $\theta = \tan^{-1}\frac{1}{2}$ **B-3.** (a) μMg (b) $\frac{\mu Mg}{\sqrt{1+\mu^2}}$, $\tan^{-1}\mu$.

SECTION (C)

C-1. $\mu_s = 0.60$, $\mu_k = 0.52$

PART-II

SECTION (A)

A-1. (A) **A-2.** (A) **A-3.** (A)

SECTION (B)

B-1. (C) **B-2.** (A) **B-3.** (A)

SECTION (C)

C-1. (D) **C-2.** (A) **C-3.** (C)

EXERCISE - 2

PART-I

1. (C) **2.** (A) **3.** (B) **4.** (B) **5.** (D)

6. **(i)** (A) **(ii)** (D) **(iii)** (B) **7.** (C)

8. (i) (B) (ii) (D) (iii) (A)

9. (ABC) **10.** (AC) **11.** (BC) **12.** (AD)

PART-II

1. (A) r, (B) q, (C) p , (D) s

PART-III

1. (A) **2.** (B) **3.** (B) **4.** (A) **5.** (D)

PART-IV

1. (D) **2.** (C)

EXERCISE - 3

PART-I

1. (4) **2.** (3) **3.** (1) **4.** (3) **5.** (1) **6.** (4) **7.** (3)

PART-II

1. (A) **2.** (A) **3.** 10 m/s^2 **4.** (B) **5.** (A) **6.** N = 5
7. (AC)