ASSIGNMENT

Deadline: March 30, 2020. By 18:00.
Number of students per submission: At most 2 students.

- You are not to consult or discuss in any way shape or form anything related to this assignment with any person that is not your sole partner for this specific assignment.
- Any references you find and use in the literature or the web you are obligated to report in full following the style of the course notes.

The goal of the first problem is to verify that you understand the material of the first practical session and in particular the proof of Gallai's theorem.

PROBLEM 1. Gallai's theorem (see Theorem 1.2 in the lecture notes) defines an algorithm for finding a minimum edge-cover (also referred to as a minimum line cover) in a graph.

1. Write out the algorithm implicit in Gallai's theorem explicitly using a style that conforms with that seen in [1] or in Algorithm 1.4.1 in the course lecture notes.
2. Search the literature for the best running time known for finding a maximum matching in a general graph. A good place to start your search would be the course notes and focusing on the so called Edmonds blossom algorithm (which the notes do not cover but do give several references to various results concerning Edmonds algorithm and improvements thereof).
3. After finding out the best running time known for finding a maximum matching in a graph, provide an run time analysis for the Gallai's theorem based algorithm you stated above.

The next problem is designed to have you review Berge's theorem as well as Tutte's theorem.

PROBLEM 2. Recall that for a graph G, we write

$$\delta(G) := \min_{v \in V(G)} \deg_G(v)$$

to denote the minimum degree of G. In this problem, you are asked to provide two distinct proofs for the following statement:

Let G be an n-vertex graph with n even and satisfying $\delta(G) \geq n/2$. Then, G has a 1-factor.

1. Prove the aforementioned statement using Berge's theorem. To that end, consider the following approach.
   (A) Let M be a matching in G. Prove that if $xy \in M$ and $u, v \in V(G) \setminus V(M)$ are vertices such that x, y, u, v are all distinct vertices, then if $e_G(\{x, y\}, \{u, v\}) \geq 3$, then M admits an M-augmenting path.
   (B) Let M be a maximum matching in G; i.e., $|M| = \nu(G)$. We seek to prove that $V(M) = V(G)$, i.e., that M is perfect. Deduce this from the fact that $\delta(G) \geq n/2$, the claim appearing in (A) (in companion with Berge's theorem).
2. Prove the aforementioned claim using Tutte's theorem. Start by examining whether a graph as in the premise of the claim can satisfy Tutte's condition.

Bibliography

[1] Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein, *Introduction to algorithms*, third ed., MIT Press, Cambridge, MA, 2009. MR 2572804