# Assignment 1 solution

Netanel Albert

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Problem 1:

1) An algorithm for finding a minimum edge-cover

Input: $G$ s.t. $0 < \delta(G)$.
Output: $Q$ as a minimum edge-cover.

1. : Let $Q := (V(G), \phi)$.
2. : **for** every $v \in V(G)$.
3. :     Mark $v$ as VISITABLE.
4. : **end-for**
5. : **for** every $v \in V(G)$ sort by $deg_G(v)$.
6. :     **if** $deg_Q(v) = 0$.
7. :         Let $v \in N_G(v)$ s.t. $u$ is VISITABLE and $deg_Q(u)$ is smallest.
8. :         **if** $0 < deg_Q(u)$ : mark $v$ as UNVISITABLE.
9. :         **if** $1 = deg_Q(u)$ : mark $u$'s neighbor as UNVISITABLE.
10. :         Add $uv$ to $Q$.
11. :     **end-if**
12. : **end-for**
13. : **Return** $Q$.

2) The best running time knonwn is $O(e(G) * \sqrt{v(G)})$ by Micali and Vazirani [1].

3) Run time analyzing for 1) algorithm (line by line):

1. $O(1)$
2. $O(v(G))$
3. $O(1)$
4. -
5. $O(v(G) * log(v(G)))$ for the sorting, $O(v(G))$ loop iterations.
6. $O(1)$
7. $O(v(G))$ to find minimum
8. $O(1)$

9. $O(1)$
10. $O(1)$
11. -
12. -
13. $O(1)$

**Toatl:** $O(v(G)) + O(v(G) * log(v(G))) + O(v(G) * v(G)) = O(v(G)^2)$

Problem 2:

Prove that: $\frac{n}{2} \leq \delta(G) \Rightarrow G$ has $1 - factor$

1. (A)
   $3 \leq e_G(\{x, y\}, \{u, v\}) \Rightarrow (ux, vy \in M) \vee (uy, vx \in M)$.
   $WLOG ux, vy \in M \Rightarrow (u, x, y, v)$ is a M-augmenting path.

   (B)
   Assuming that M isn't perfect, $\Rightarrow |M| < \frac{n}{2} \Rightarrow \exists u, v : uv \notin M$.
   $uv \notin G$, otherwise M isn't maximal.
   $e_G(\{u, v\}, G \setminus \{u, v\}) \geq n$
   According to pigeonhole principle, $\exists x, y : xy \in M \wedge e_G(\{x, y\}, \{u, v\}) \geq 3 \Rightarrow$ according to (A) M has an augmenting path $\Rightarrow$ according to Berges theorem M isn't maximum. Contradiction. $\blacksquare$

2. Let $G$ be a graph as in the premis. Let $S \subseteq V(G)$ be a subgraph. We will show that $C_o(G \setminus S) \leq |S|$. Lets start to count the connected components. First, lets take a look at 2 components: Eche one has at least $\frac{n}{2} - |s| + 1$ vertexes. So, in $G \setminus S$ we left with $n - |s| - 2 * (\frac{n}{2} - |s| + 1) = |s| - 2$ So it isn't possible to get more then $|s| - 2$ components here. When we will add the 2 components we counted earlyer, we get at most $|S|$ components in $G \setminus S$. in particular there is at most $|S|$ odd components there. So, according to Tutte theorem, there is a $1 - factor$ in $G \blacksquare$

Bibliography

[1] https://link.springer.com/article/10.1007/BF01762129