ASSIGNMENT

INSTRUCTIONS

Deadline: May 10, 2020, by 18:00.
Number of students per submission: At most 2 students.

- You are not to consult or discuss in any way shape or form anything related to this assignment with any person that is not your sole partner for this specific assignment.

- Any references you find and use in the literature or the web you are obligated to report in full following the style of the course notes.

PROBLEMS

The following problem shows that the task of determining the chromatic number of a graph can be reduced to determining the chromatic number of its blocks.

PROBLEM 1. Let SEARCH-3-COL denote the problem that upon being given a graph G calls for *finding* a 3-colouring of G if one exists. Prove that SEARCH-3-COL is self-reducible.

PROBLEM 2. For a graph G and a legal vertex-colouring $\psi : V(G) \to \mathbb{N}$ of G, write

$$\sigma_\psi(G) := \sum_{v \in V(G)} \psi(v),$$

and set

$$\sigma(G) := \min_\psi \sigma_\psi(G),$$

where the minimum ranges over all valid vertex colourings of G. Prove that

$$\{(G, k) : \sigma(G) \leq k\} \in \text{NPC}.$$

PROBLEM 3. For a graph G, let $\omega(G)$ denote the size of a largest complete subgraph in G. Prove that

$$\left\{ G : \omega(G) \geq \sqrt{v(G)} \right\} \in \text{NPC}.$$

PROBLEM 4. Prove that

$$\{\varphi : \ \varphi \text{ is a CNF-formula admitting at least two satisfying assignments}\} \in \text{NPC}.$$