In [196]:

```
# import useful modules
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from PIL import Image
import random
import time
import math
random.seed(time.time())
```

In [197]:

```
def showImage(x, title):
    img = np.array(x, dtype=float)
    plt.figure()
    plt.imshow(img, cmap='gray')
    plt.title(title)
```

In [198]:

```
def showHeat(x, title):
    img = np.array(x, dtype=float)
    plt.figure()
    plt.imshow(img)
    plt.title(title)
    plt.colorbar()
```

Problem generation

In [199]:

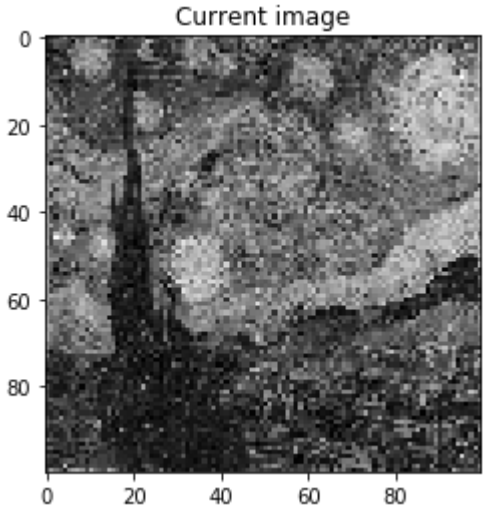
```
# variables
bordersize = 50
imagedim = (100,100)
#filename = "shepplogan.png"
filename = "starrynight.jpg"
```

In [200]:

```
# Read image and create another to match provided requirements
img = Image.open(filename).resize(imagedim).convert('L')
img.save(filename.split(".")[0]+"edited.png")
```

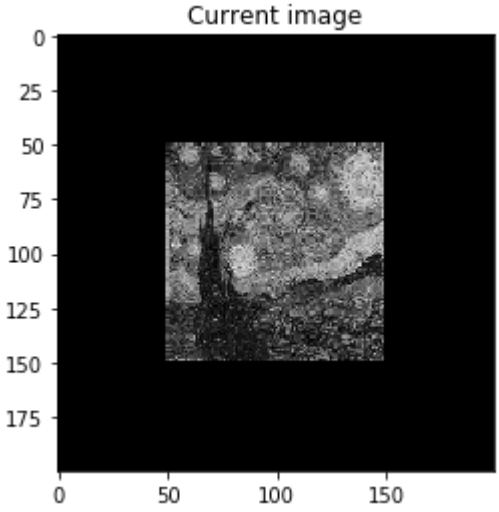
In [201]:

```
# load image into matplotlib and create matrix of numbers
img = mpimg.imread(filename.split(".")[0]+"edited.png")
showImage(img, "Current image")
```



In [202]:

```
img = np.pad(img, pad_width=bordersize, mode='constant',
               constant_values=0)
showImage(img, "Current image")
```



In [203]:

```
# Computer fourier transform/absolute value:
b = np.abs(np.fft.fft2(img, norm='ortho'))
supp = (img != 0).astype(int)
print("Fourier modulus values:")
print(b)
print("Support:")
print(supp)
```

```
Fourier modulus values:
[[17.551176   10.951516    1.4073188  ...  3.6118863   1.4073188
  10.951516  ]
 [12.321084    7.918615    1.129325   ...  1.9249402   0.55950516
   7.5351524 ]
 [ 3.2396545   2.4502225   0.5290358  ...  0.88783807  0.54911816
   1.6544088 ]
 ...
 [ 2.1314697   1.226513    0.7966078  ...  0.85742044  0.5782972
   1.321955  ]
 [ 3.2396545   1.6544088   0.54911816 ...  1.0253432   0.5290358
   2.4502225 ]
 [12.321084    7.5351524   0.55950516 ...  2.9887643   1.129325
   7.918615  ]]
Support:
[[0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]
 ...
 [0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]]
```

Algorithms

In [204]:

```
# Algorithm variables
numiterations = 1000
maxdist = 1e-4
displayperiod = 250
averageover = 100
```

Projections

In [205]:

```
# projection onto image modulus constraint
def proj_c1(x):
    Fx = np.fft.fft2(x, norm='ortho')
    corrected = b * math.e**(1j*np.angle(Fx))
    #corrected = b*Fx/(np.abs(Fx)+1e-12)
    return np.fft.ifft2(corrected, norm='ortho')
```

In [206]:

```
# projection onto real number & non-negativity & support constraint
def proj_c2(x):
    return supp*(np.real(x).clip(min=0))
```

In [207]:

```
# projection onto real number & non-negativity ONLY
def real_nn(x):
    return np.real(x).clip(min=0)
```

Stopping conditions

In [208]:

```
def stop_cond1(cnt):
    return (cnt < numiterations)
```

In [209]:

```
def stop_cond2(dist):
    return (dist > maxdist)
```

Initial point (support)

In [210]:

```
def initialPoint():
    x = supp
    showImage(x, "Initial image")
    return x
```

1) Error reduction algorithm (a.k.a. method of alternating projections)

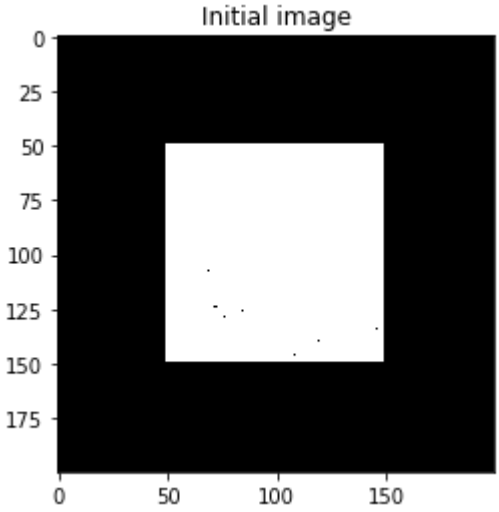
In [211]:

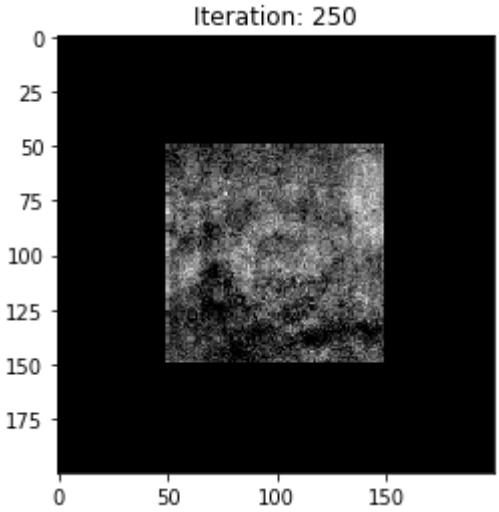
```
cnt = 0
dist = 1e12
finalX = []
x = initialPoint()
err = []
while(stop_cond1(cnt)):
    #print(dist)
    #print(cnt)
    xinit = x
    x = proj_c1(x)
    x = proj_c2(x)
    dist = np.linalg.norm(x-xinit)
    err.append(dist)
    cnt += 1
    if(cnt%displayperiod == 0):
        showImage(x, "Iteration: " + str(cnt))
    if(cnt >= averageover):
        finalX.append(x)
average= np.mean(finalX,axis=0)
showImage(x, "Final Image")
showImage(average, "Averaged final image")
showImage(img, "Desired Image")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(x)))),"Final Image Fourier Transform
Modulus (shifted)")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(average)))),"Averaged Final Image Fo
urier Transform Modulus (shifted)")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(img)))),"Desired Image Fourier Trans
form Modulus (shifted)")
print("Number of iterations:", cnt, "Final distance:", dist)
plt.figure()
plt.plot(np.log(err))
plt.title("Log of error between terms of the sequence")
```

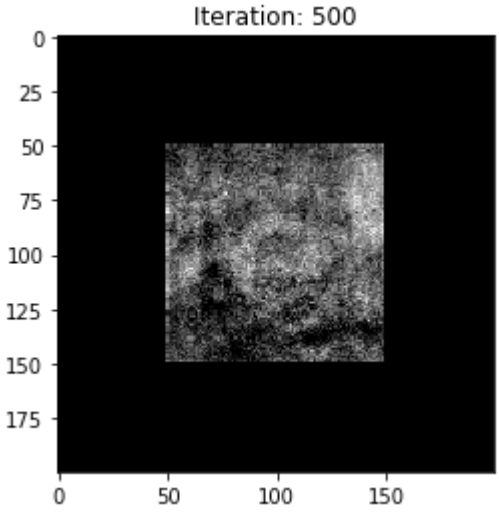
```
Number of iterations: 1000 Final distance: 0.005038171794781778
```

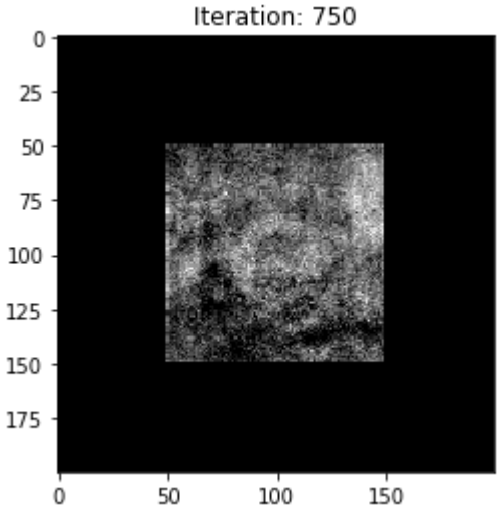
Out[211]:

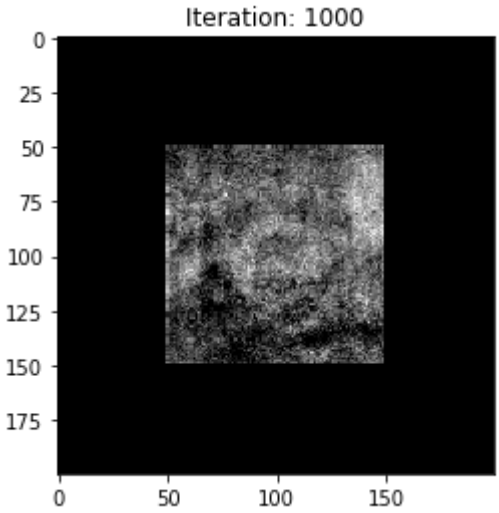
```
Text(0.5, 1.0, 'Log of error between terms of the sequence')
```

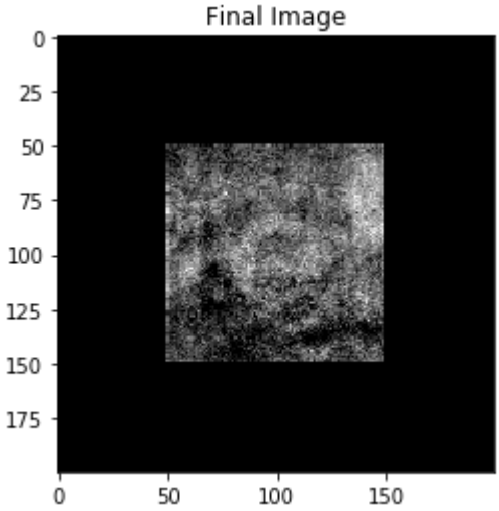


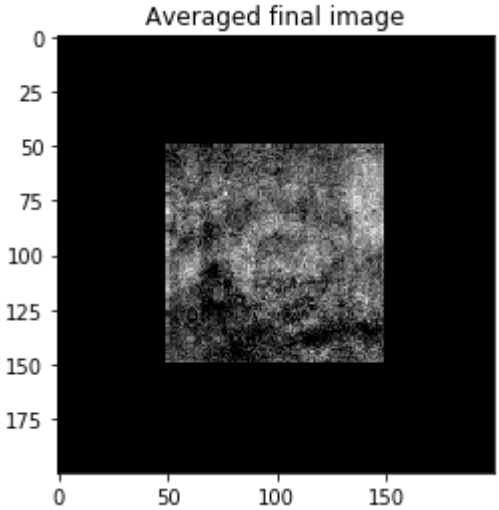


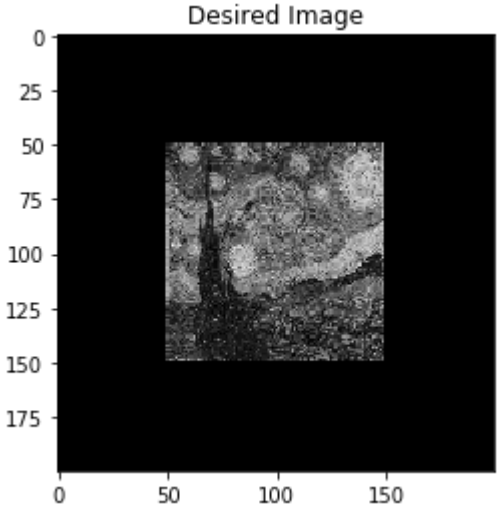

Iteration: 500
0
25
50
75
100
125
150
175
0
50
100
150

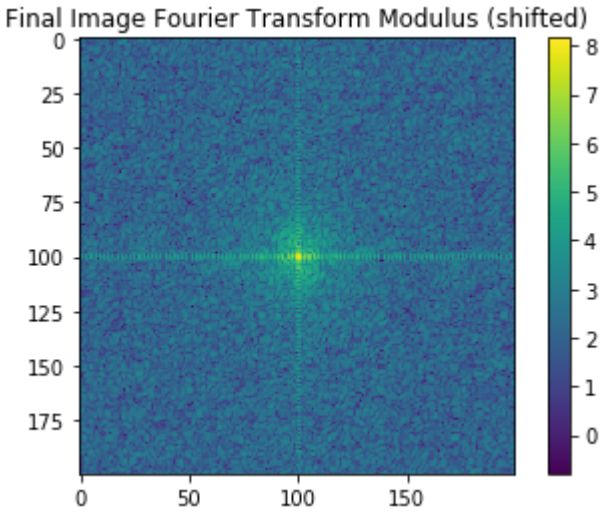

Iteration: 750
0
25
50
75
100
125
150
175
0
50
100
150

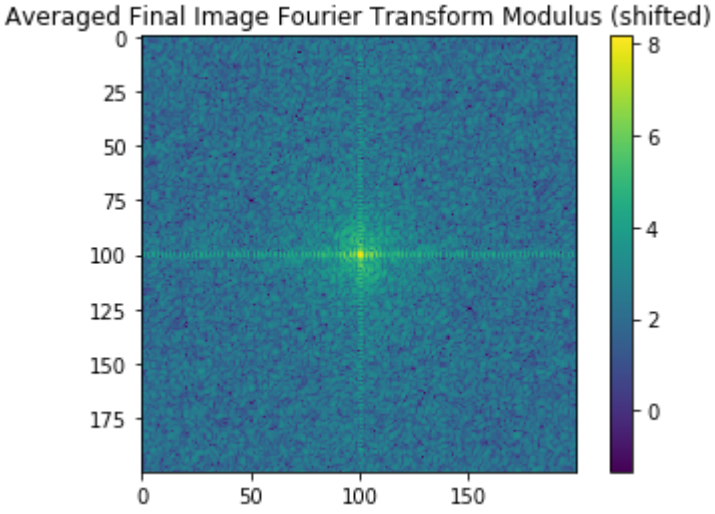

Iteration: 1000
0
25
50
75
100
125
150
175
0
50
100
150

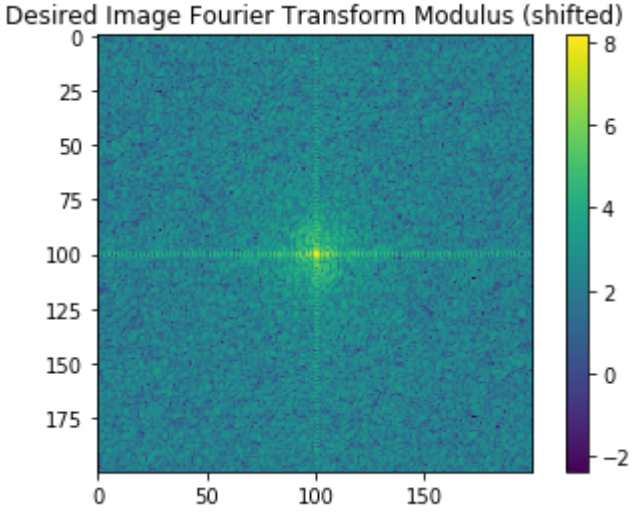

Final Image
0
25
50
75
100
125
150
175
0
50
100
150

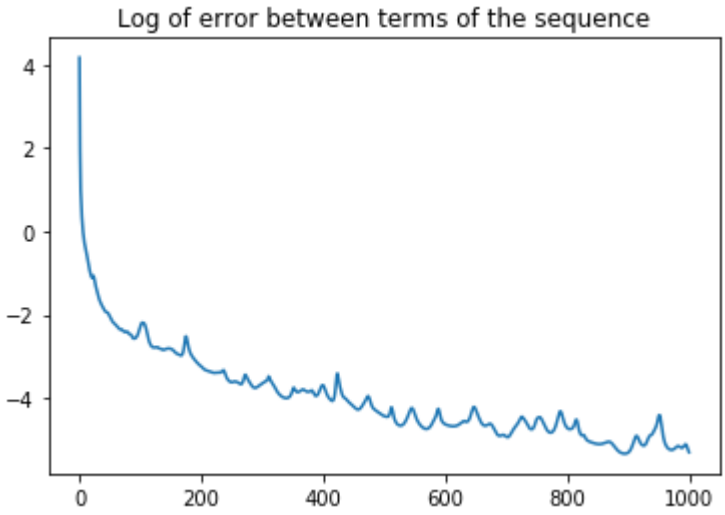

Averaged final image
0
25
50
75
100
125
150
175
0
50
100
150


Desired Image
0
25
50
75
100
125
150
175
0
50
100
150


Final Image Fourier Transform Modulus (shifted)
0
25
50
75
100
125
150
175
0
50
100
150
8
7
6
5
4
3
2
1
0


Averaged Final Image Fourier Transform Modulus (shifted)
0
25
50
75
100
125
150
175
0
50
100
150
8
6
4
2
0


Desired Image Fourier Transform Modulus (shifted)
0
25
50
75
100
125
150
175
0
50
100
150
8
6
4
2
0
−2



2) Fienup's Basic Input-Output Algorithm (a.k.a. Dykstra's algorithm)

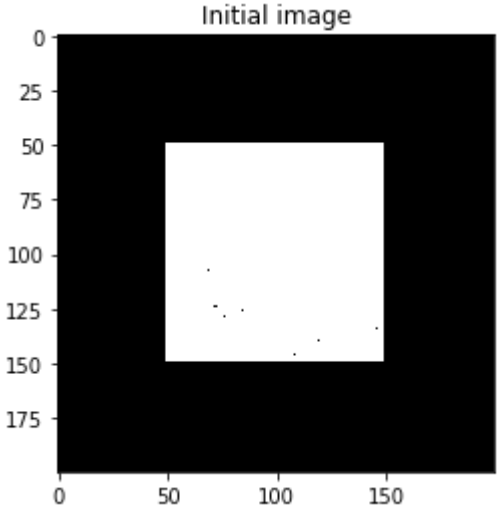
In [212]:

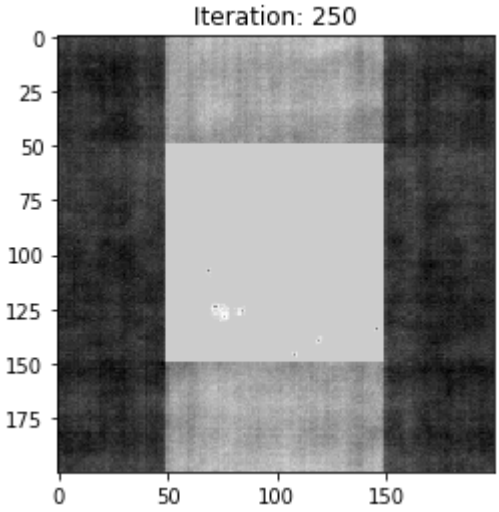
```
cnt = 0
dist = 1e12
finalX = []
err = []
x = initialPoint()
while(stop_cond1(cnt)):
    #print(dist)
    #print(cnt)
    xinit = x
    PmX = proj_c1(x)
    x = proj_c2(PmX) + x - PmX
    #x = real_nn(x)
    dist = np.linalg.norm(x-xinit)
    err.append(dist)
    cnt += 1
    if(cnt%displayperiod == 0):
        showImage(x, "Iteration: " + str(cnt))
    if(cnt >= averageover):
        finalX.append(x)
average= np.mean(finalX,axis=0)
showImage(x, "Final Image")
showImage(average, "Averaged final image")
showImage(img, "Desired Image")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(x)))),"Final Image Fourier Transform
Modulus (shifted)")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(average)))),"Averaged Final Image Fo
urier Transform Modulus (shifted)")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(img)))),"Desired Image Fourier Trans
form Modulus (shifted)")
print("Number of iterations:", cnt, "Final distance:", dist)
plt.figure()
plt.plot(np.log(err))
plt.title("Log of error between terms of the sequence")
```

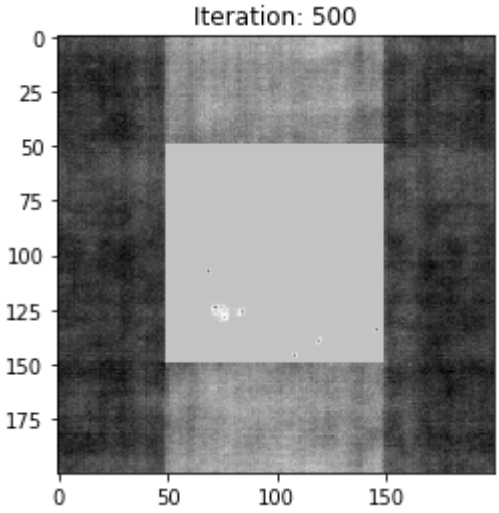
```
C:\Users\Netanya\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: Comp
lexWarning: Casting complex values to real discards the imaginary part


Number of iterations: 1000 Final distance: 17.45241350888513
```

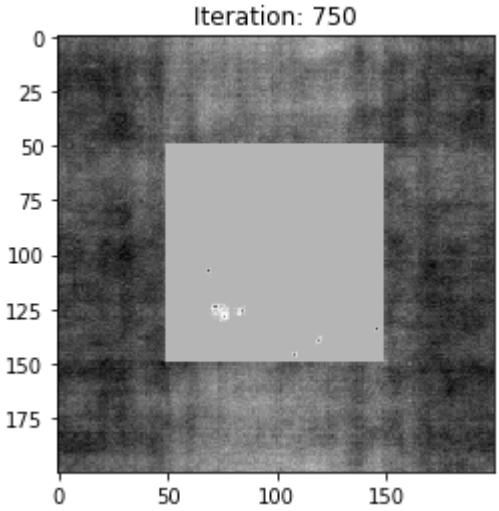
Out[212]:

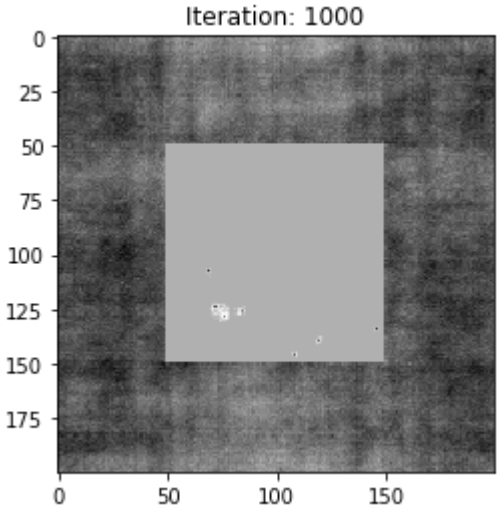
```
Text(0.5, 1.0, 'Log of error between terms of the sequence')
```

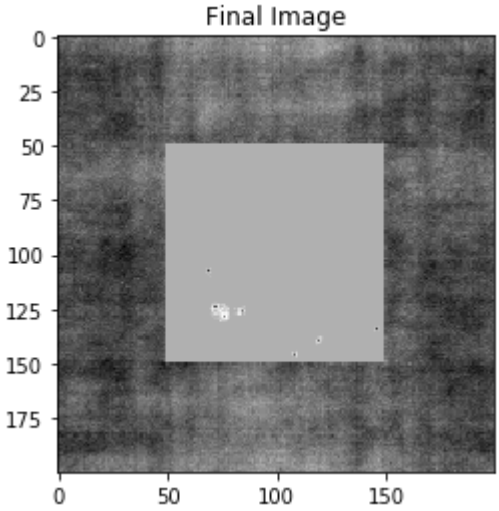


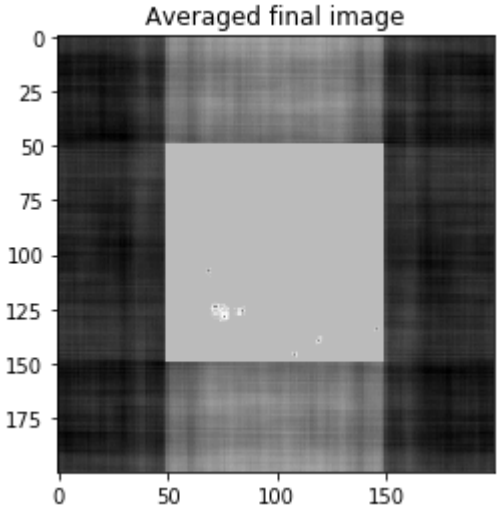


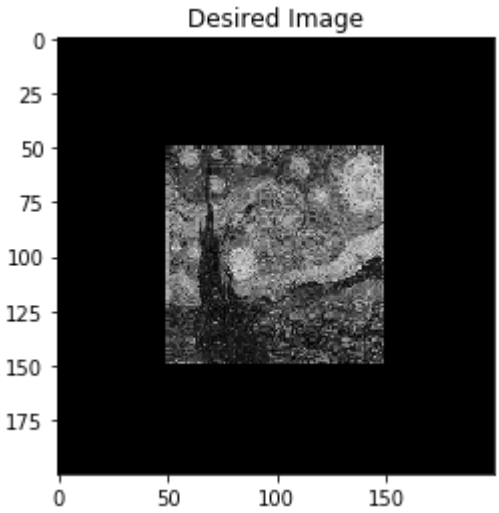


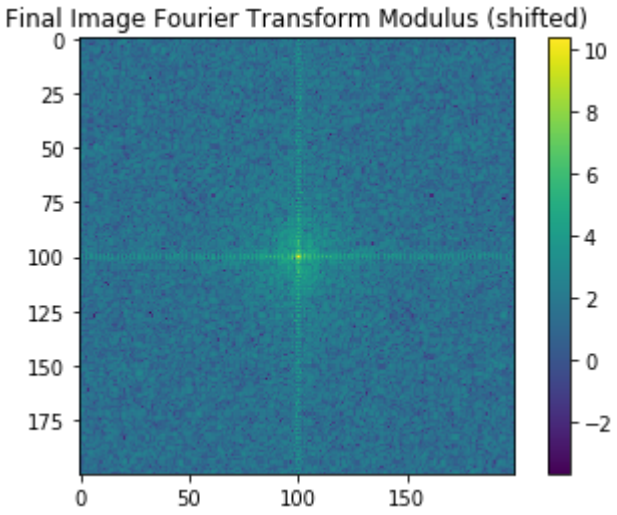

Iteration: 750
0
25
50
75
100
125
150
175
0
50
100
150

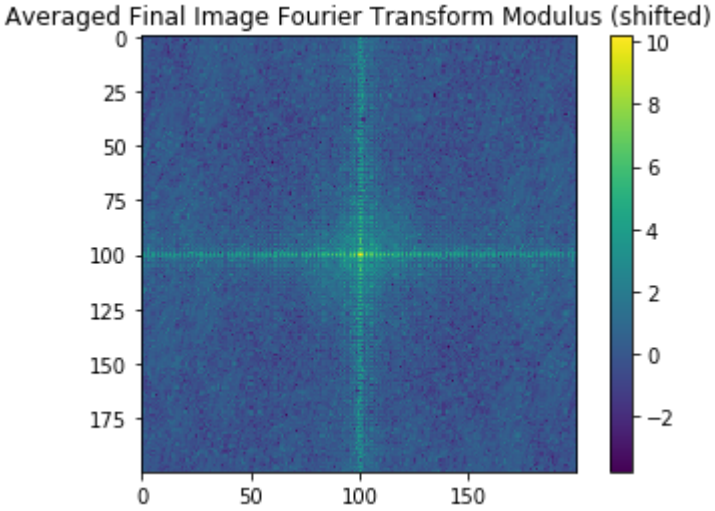

Iteration: 1000
0
25
50
75
100
125
150
175
0
50
100
150

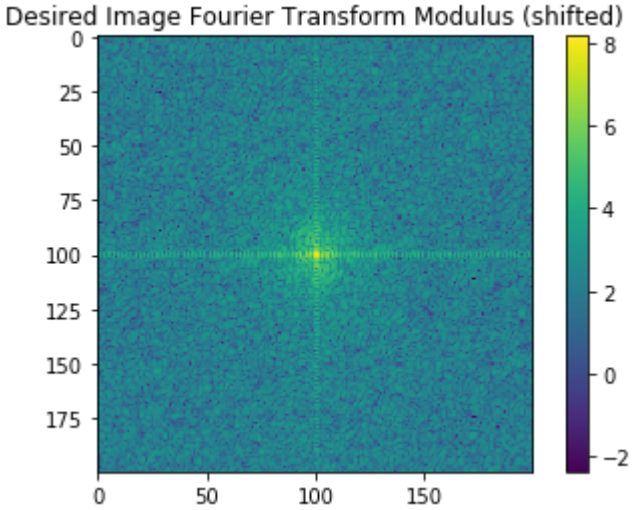

Final Image
0
25
50
75
100
125
150
175
0
50
100
150

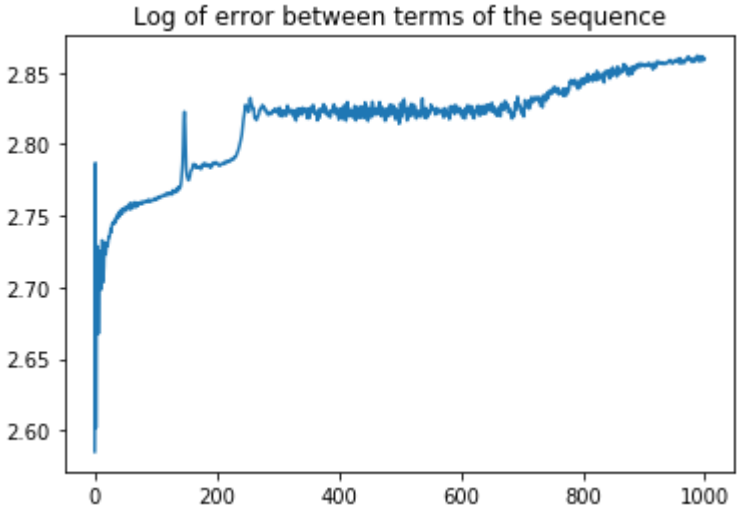

Averaged final image
0
25
50
75
100
125
150
175
0
50
100
150


Desired Image
0
25
50
75
100
125
150
175
0
50
100
150


Final Image Fourier Transform Modulus (shifted)
0
25
50
75
100
125
150
175
0
50
100
150
10
8
6
4
2
0
−2







3) Fineup's Hybrid Input-Output Algorithm (a.k.a. Douglas-Rachford Algorithm)

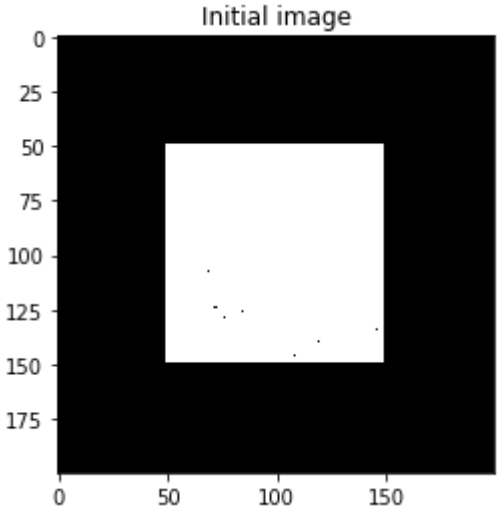
In [213]:

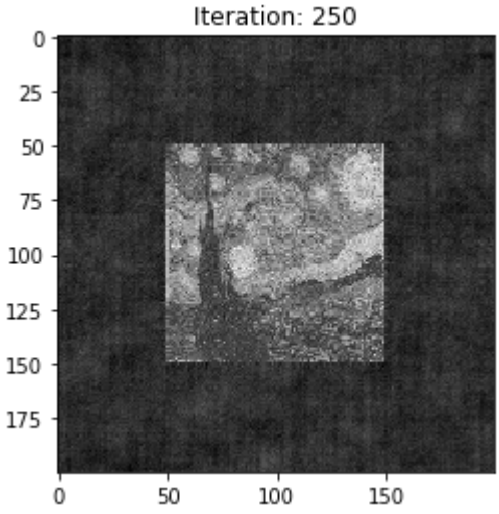
```
cnt = 0
dist = 1e12
finalX = []
err = []
x = initialPoint()

beta = 1

while(stop_cond1(cnt)):
    #print(dist)
    #print(cnt)
    xinit = x
    PmX = proj_c1(x)
    x = proj_c2(PmX) + x - beta*PmX - proj_c2(x) + beta*proj_c2(PmX)
    #x = real_nn(x)
    dist = np.linalg.norm(x-xinit)
    err.append(dist)
    cnt += 1
    if(cnt%displayperiod == 0):
        showImage(x, "Iteration: " + str(cnt))
    if(cnt >= averageover):
        finalX.append(x)
average= np.mean(finalX,axis=0)
showImage(x, "Final Image")
showImage(average, "Averaged final image")
showImage(img, "Desired Image")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(x)))),"Final Image Fourier Transform
Modulus (shifted)")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(average)))),"Averaged Final Image Fo
urier Transform Modulus (shifted)")
showHeat(np.fft.fftshift(np.log(np.abs(np.fft.fft2(img)))),"Desired Image Fourier Trans
form Modulus (shifted)")
print("Number of iterations:", cnt, "Final distance:", dist)
plt.figure()
plt.plot(np.log(err))
plt.title("Log of error between terms of the sequence")
```

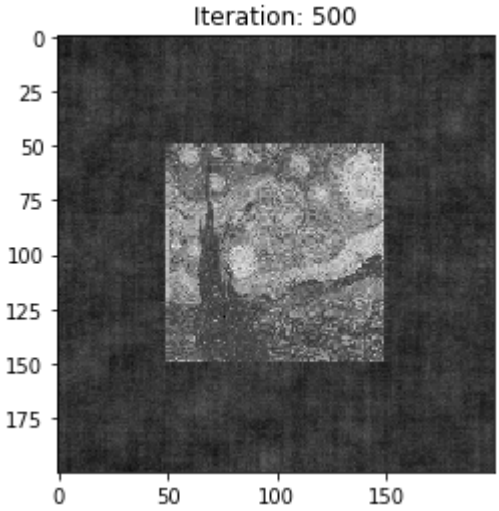
```
C:\Users\Netanya\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: Comp
lexWarning: Casting complex values to real discards the imaginary part


Number of iterations: 1000 Final distance: 0.007547102007817316
```

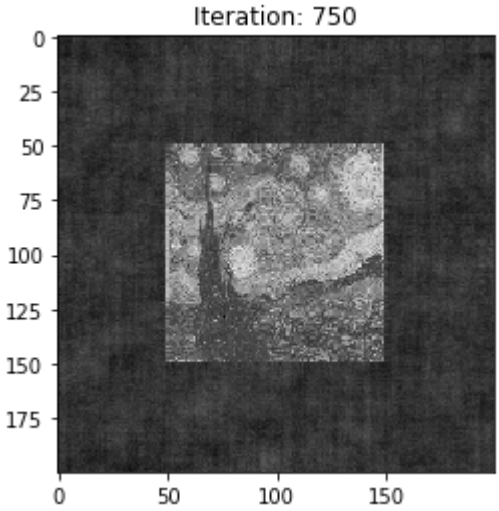
Out[213]:

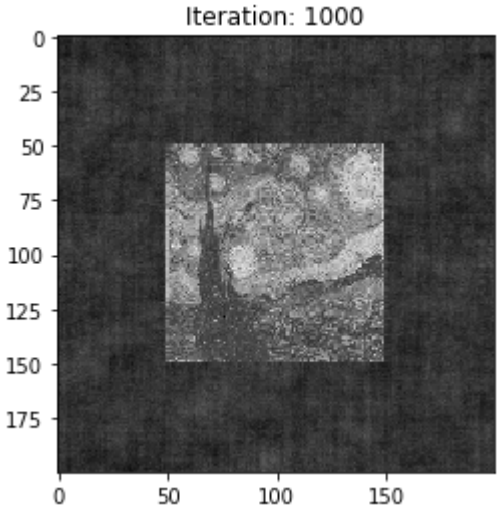
```
Text(0.5, 1.0, 'Log of error between terms of the sequence')
```

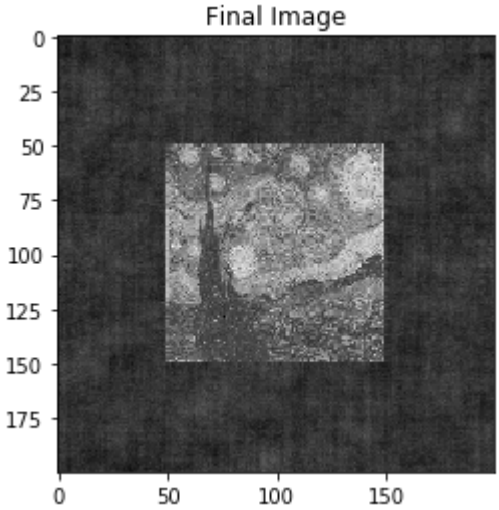


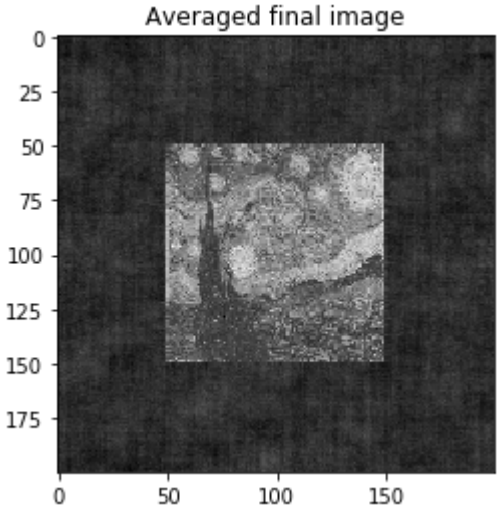



Iteration: 500
0
25
50
75
100
125
150
175
0
50
100
150


Iteration: 750
0
25
50
75
100
125
150
175
0
50
100
150


Iteration: 1000
0
25
50
75
100
125
150
175
0
50
100
150


Final Image
0
25
50
75
100
125
150
175
0
50
100
150


Averaged final image
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50
75
100
125
150
175
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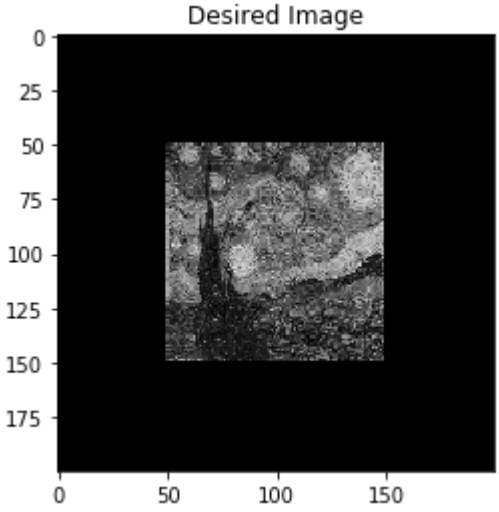
50
100
150


Desired Image
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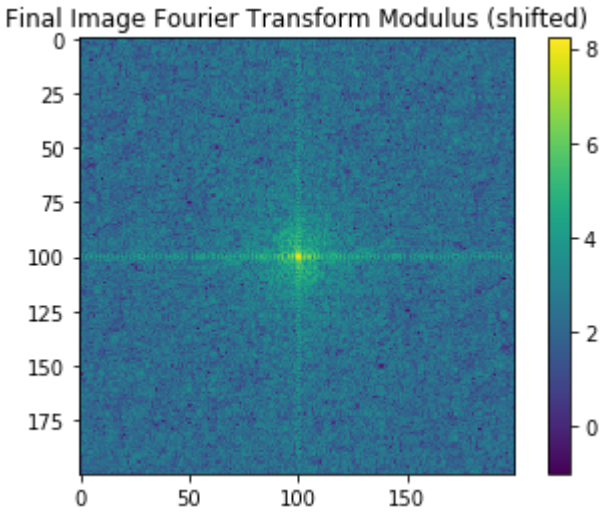
Final Image Fourier Transform Modulus (shifted)
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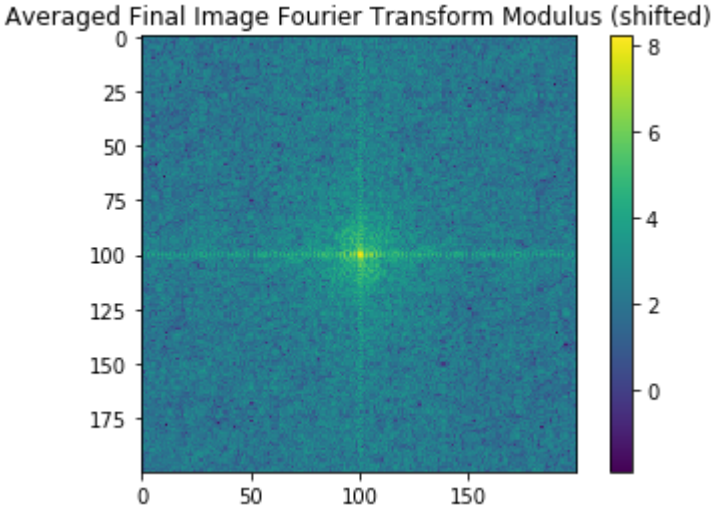
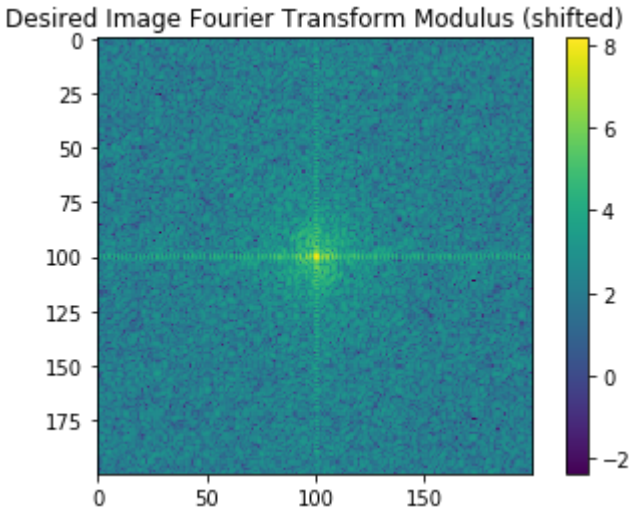
25
50
75
100
125
150
175
0
50
100
150
8
6
4
2
0


Averaged Final Image Fourier Transform Modulus (shifted)
0
25
50
75
100
125
150
175
0
50
100
150
8
6
4
2
0


Desired Image Fourier Transform Modulus (shifted)
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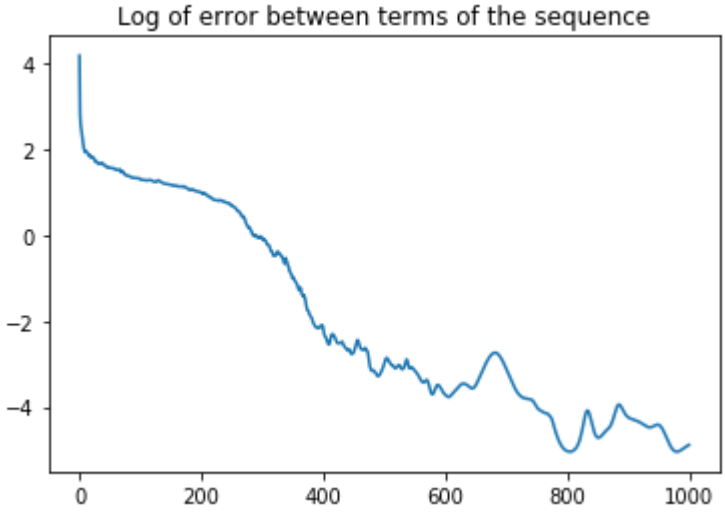
25
50
75
100
125
150
175
0
50
100
150
8
6
4
2
0
−2


Log of error between terms of the sequence
4
2
0
−2
−4
0
200
400
600
800
1000

In [ ]: