

# Fingerprint Quality Validation

Lưu Nam Đạt

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# Part 1: Variance of Gray Levels

$$mean(I) = \mu = \frac{1}{N} \times \sum_{i=1}^N E_i$$

$$var(I) = \sigma^2 = \frac{\sum_{i=1}^N (E_i - \mu)^2}{N}$$

$$std(I) = \sigma = \sqrt{var(I)}$$

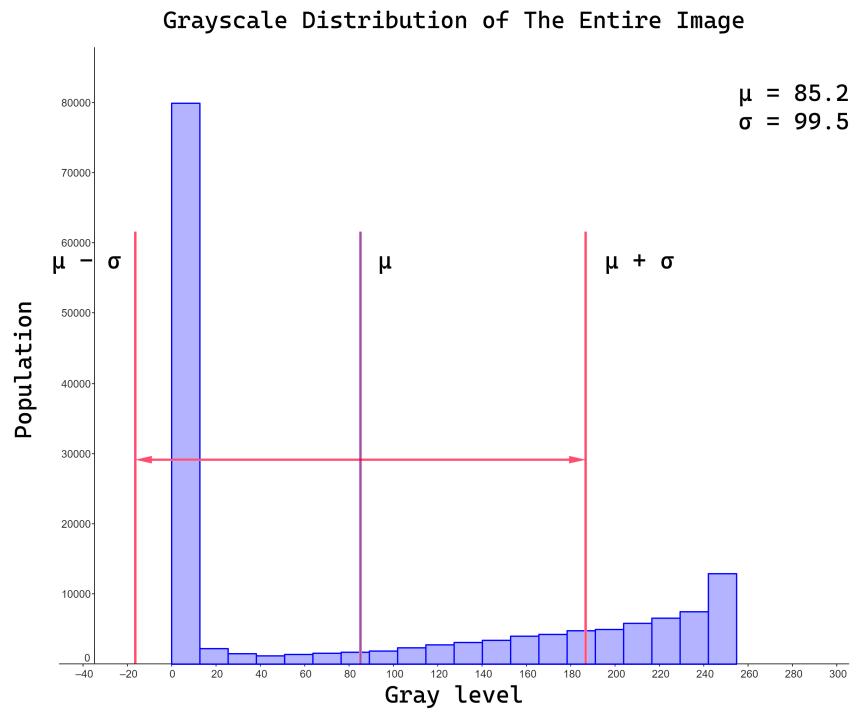


Figure 1: Grayscale Distribution of a Fingerprint Image

Let  $\sigma_{base}$  be the standard deviation of the image

The contrast quality ( $cq$ ) of a block  $\beta$  is determined by:

$$cq_\beta = \frac{\sigma_\beta}{\sigma_{base}}$$

High  $cq_\beta$  value means that the block  $\beta$  contains both clear ridges and clear valleys, which promises useful data.

If  $cq_\beta$  is too low,  $\beta$  can either be a background block, or a block without any helpful information at all (bad block).

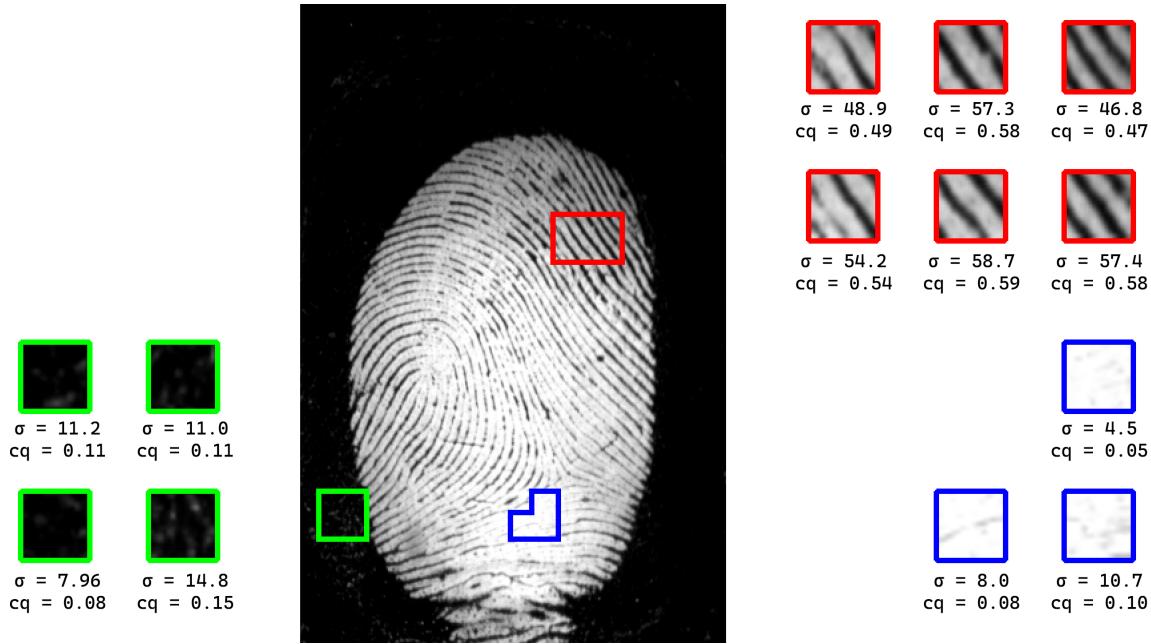


Figure 2: Standard Deviation and Contrast Quality of some blocks

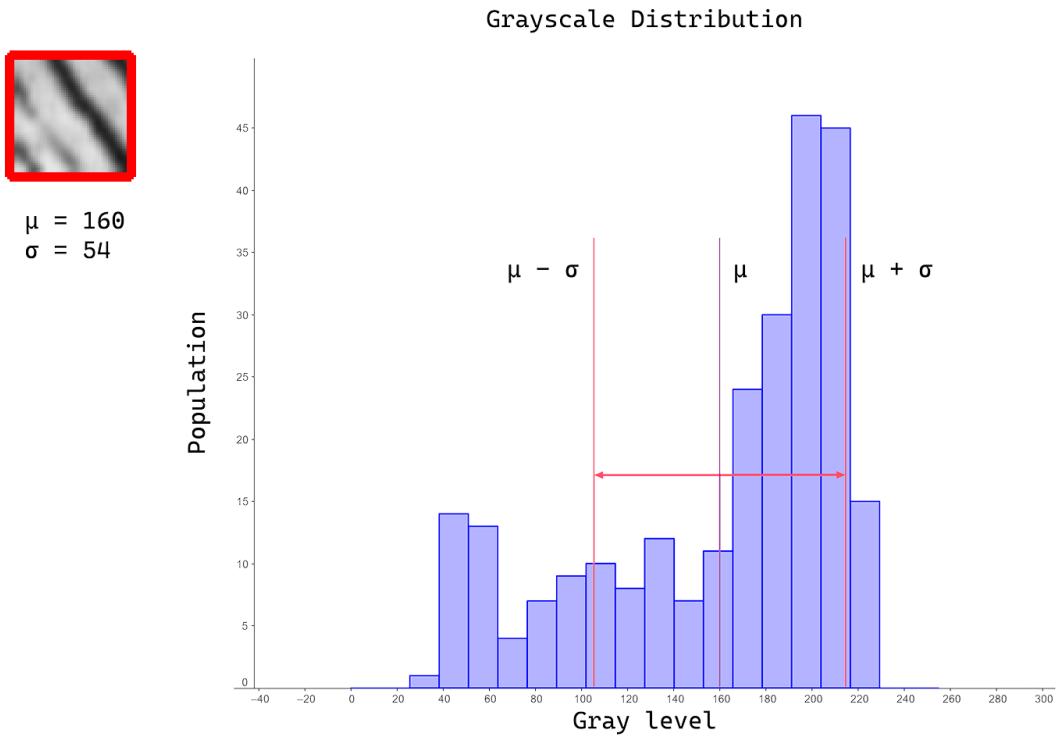


Figure 3: Grayscale Distribution of a good block

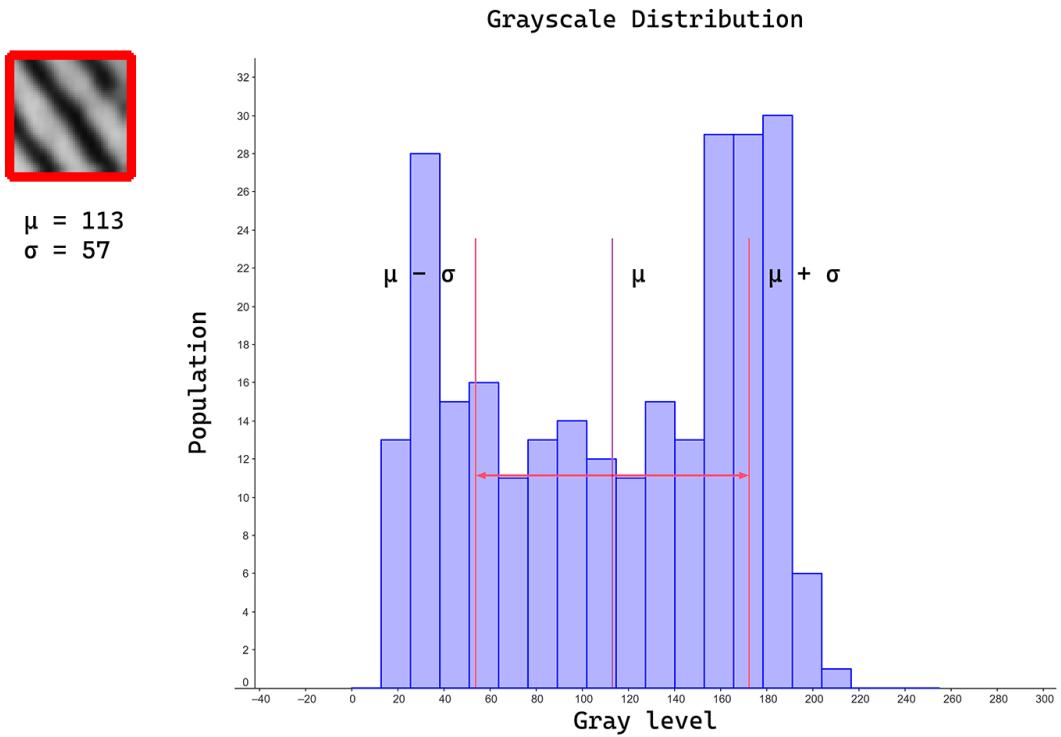


Figure 4: Grayscale Distribution of another good block

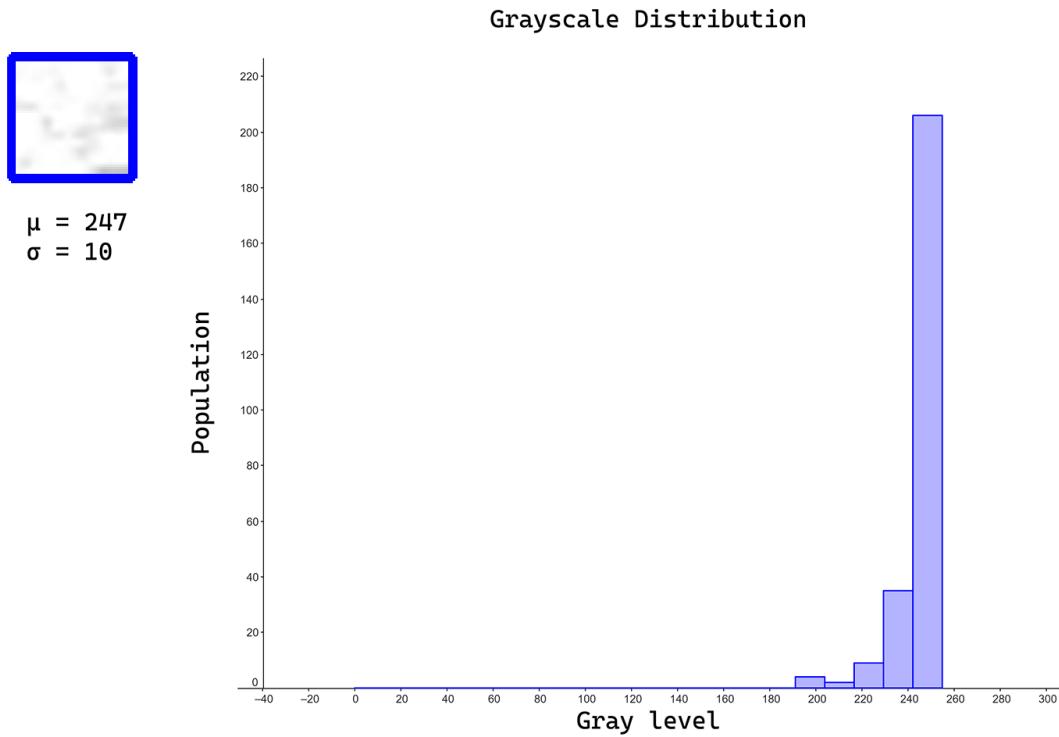


Figure 5: Grayscale Distribution of a bad block

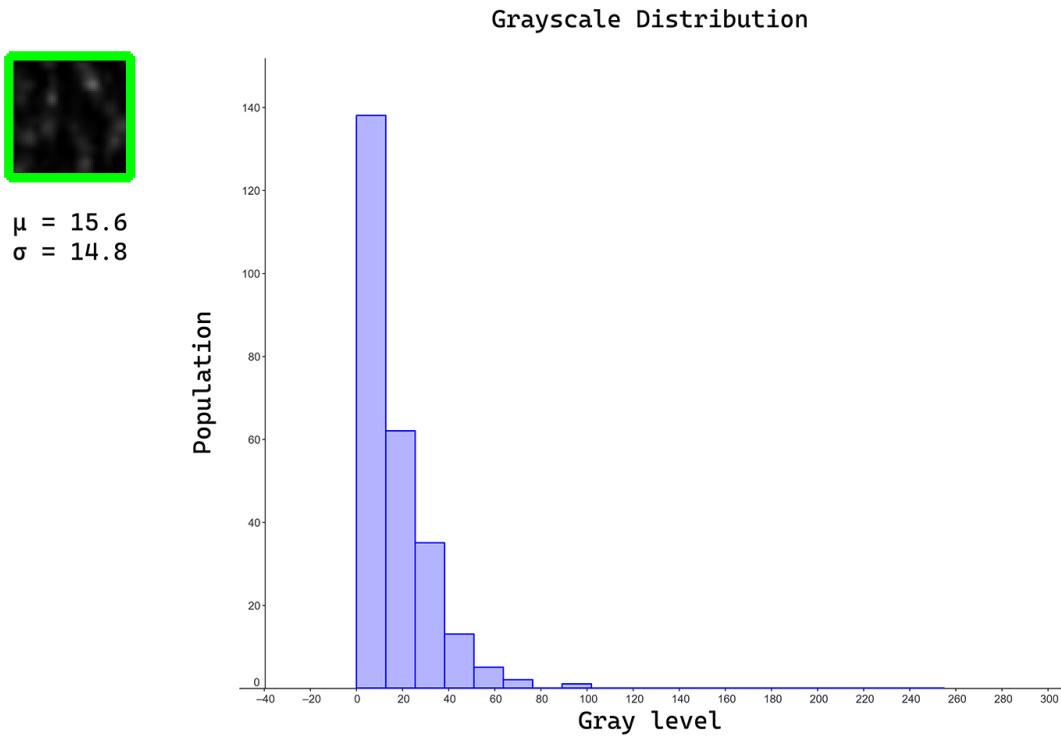


Figure 6: Grayscale Distribution of a background block



Figure 7: Masking blocks with  $\text{std} < 0.2$

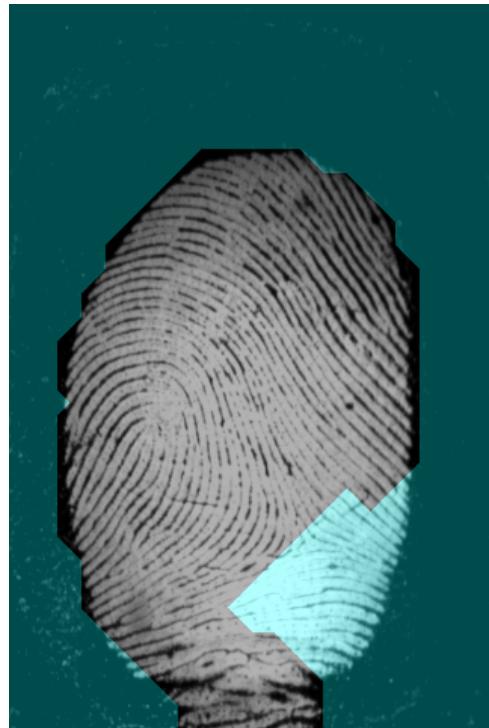


Figure 8: The smoothed mask

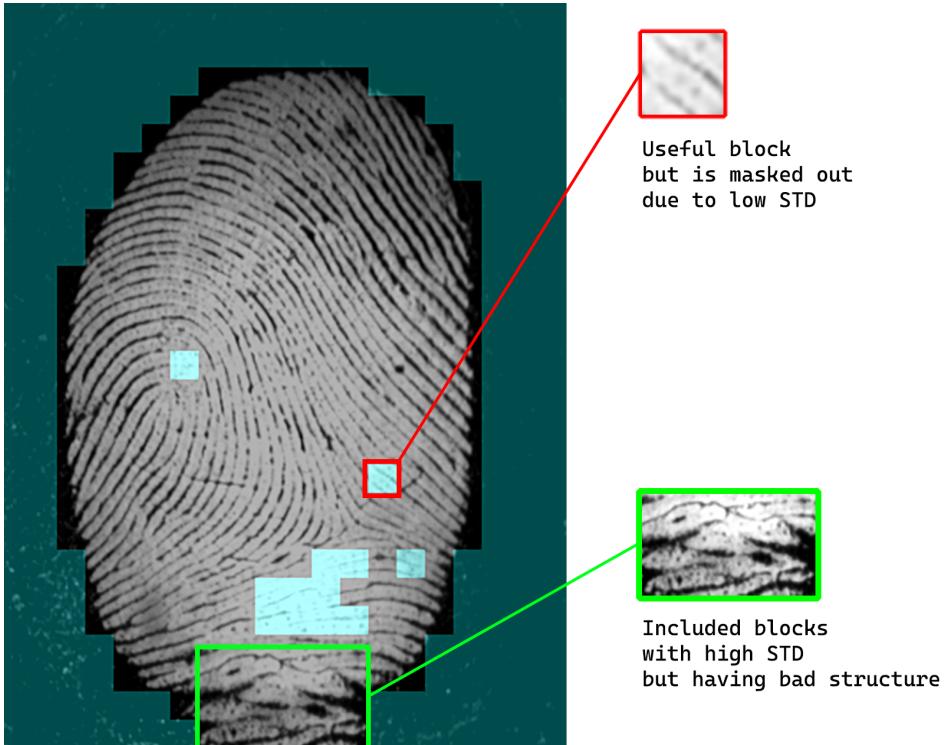
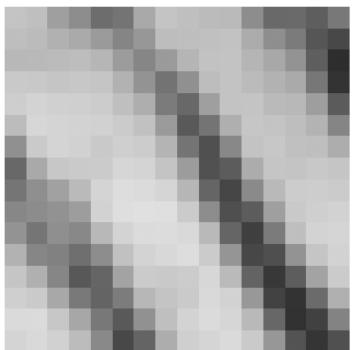


Figure 9: STD masking fails

## Part 2: Orientation Certainty

$$gx = I * \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$gy = I * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$



size: 16\*16

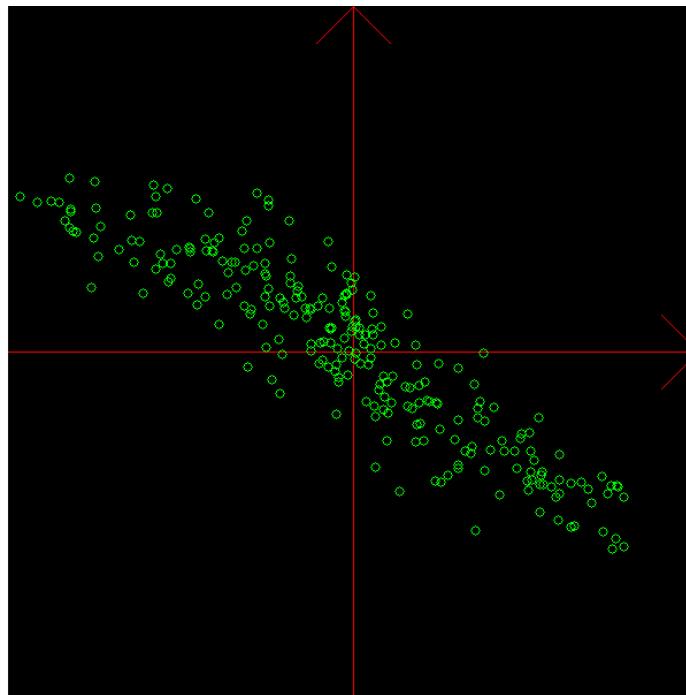
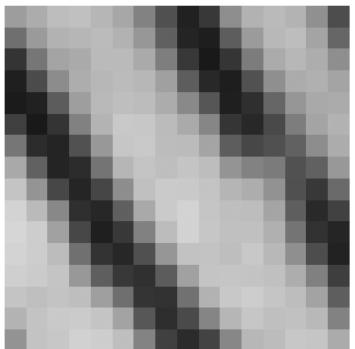


Figure 10: Relation between  $gx$  (horizontal axis) and  $gy$  (vertical axis) on a good block (1)



size: 16\*16

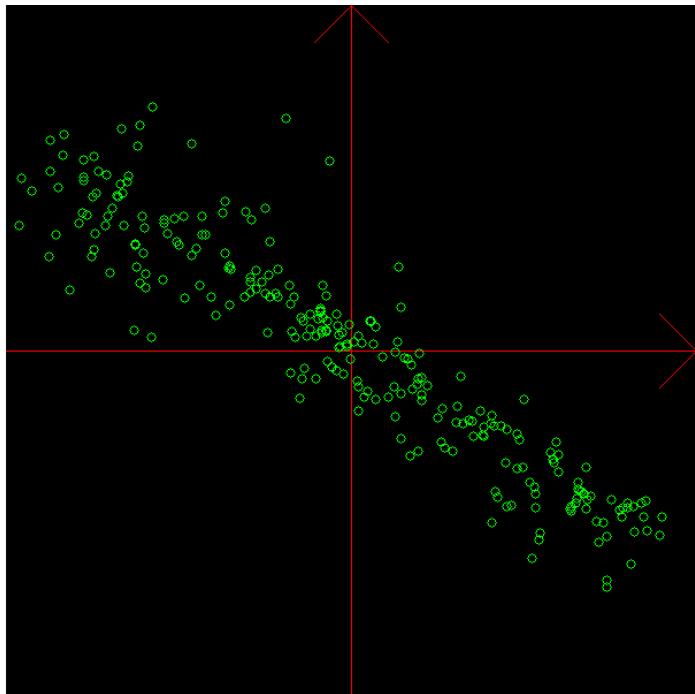


Figure 11: Relation between  $gx$  (horizontal axis) and  $gy$  (vertical axis) on a good block (2)



size: 16\*16

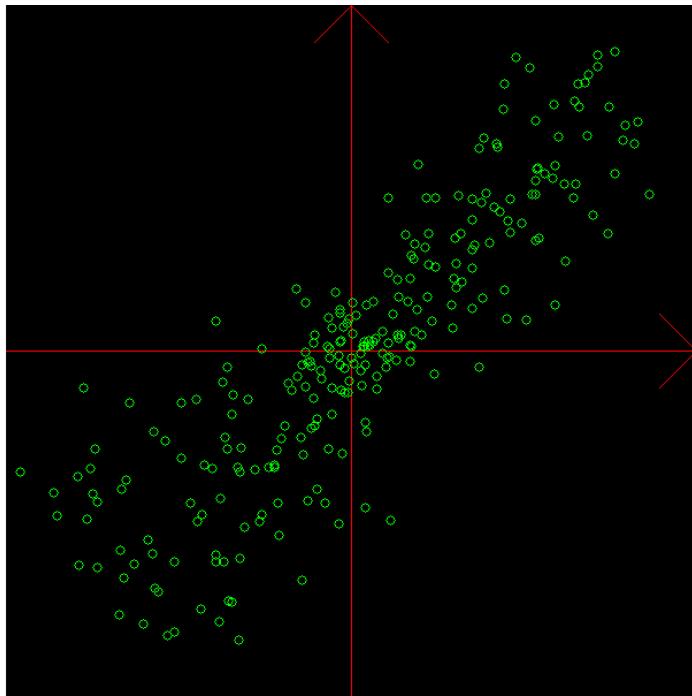


Figure 12: Relation between  $gx$  (horizontal axis) and  $gy$  (vertical axis) on a good block (3)



size: 16\*16

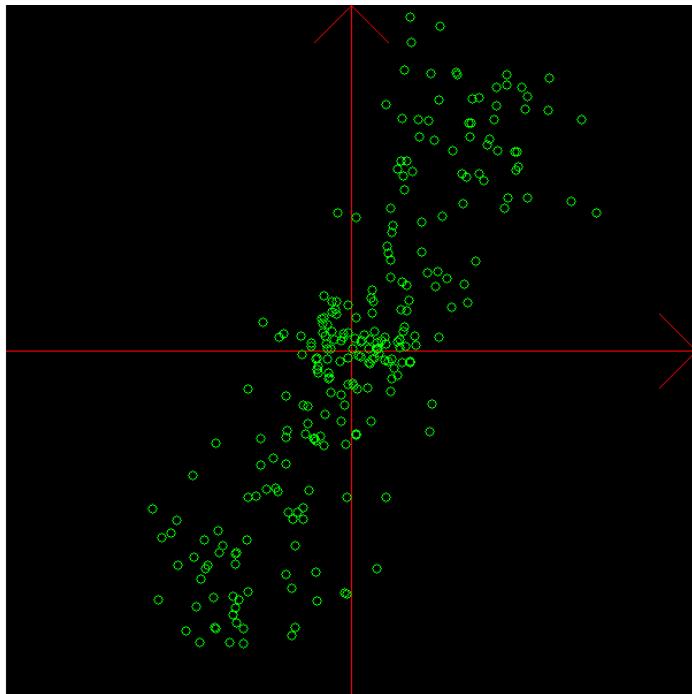
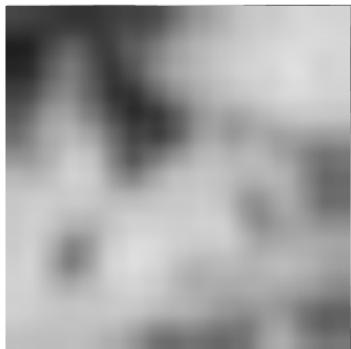


Figure 13: Relation between  $gx$  (horizontal axis) and  $gy$  (vertical axis) on a good block (4)



size: 16\*16

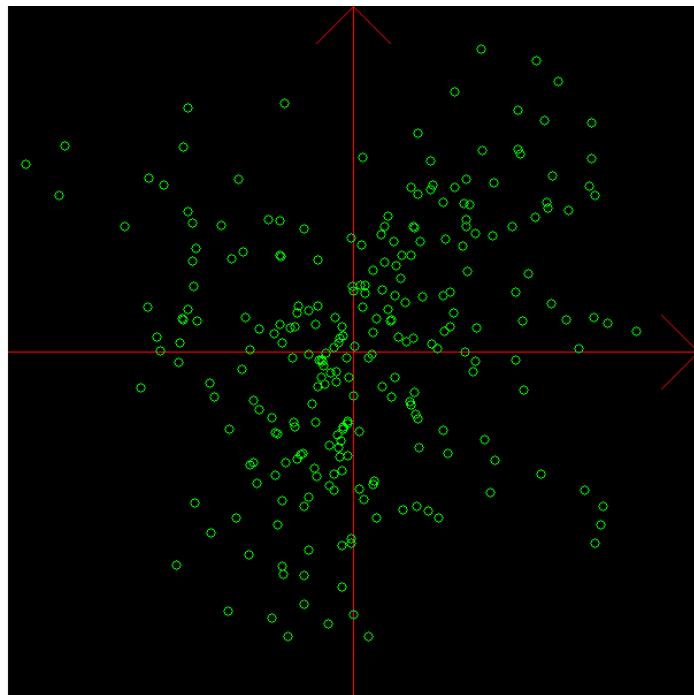


Figure 14: Relation between  $gx$  (horizontal axis) and  $gy$  (vertical axis) on a bad block (1)

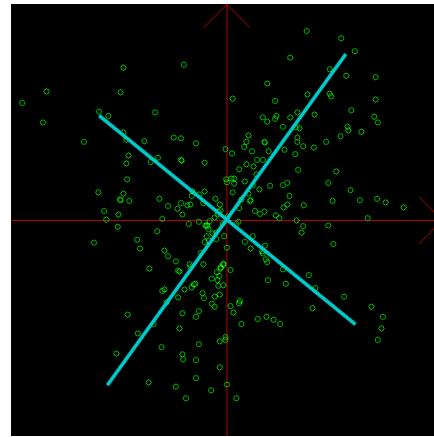
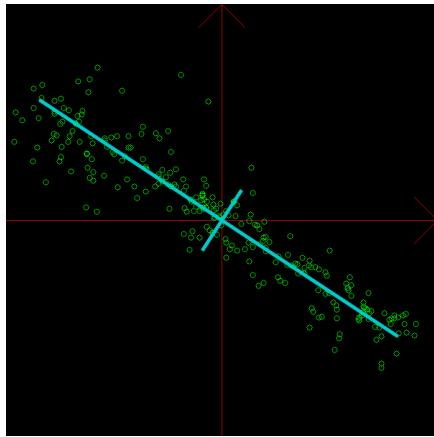


Figure 15: The direction along which the data set has minimum/maximum variance  
(the length of each line indicates the variance along respective direction)

Find unit vectors that minimize/maximize variance

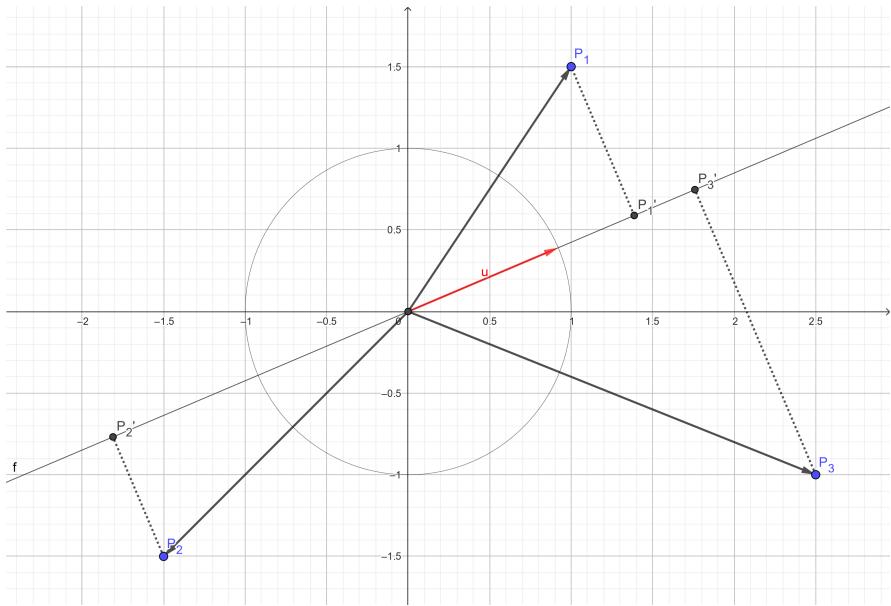


Figure 16: Projections of some points onto a unit vector

If  $\vec{v}$  is a unit vector:

$$dist(O, P'_i) = \vec{P}_i \cdot \vec{v} = \sum_{d=1}^D P_{id} v_d$$

Variance of projections:

$$V = \frac{1}{N} \times \sum_{i=1}^N \sum_{d=1}^D (p_{id} v_d - \mu)^2$$

By performing a geometric transformation such that

$$\mu_x = \mu_y = 0$$

Variable  $\mu$  of the equation above then has the value of 0

Thus, the formula to calculate the variance of projections is simplified to:

$$V = \frac{1}{N} \times \sum_{i=1}^N \sum_{d=1}^D (p_{id} v_d)^2$$

The goal is to find a vector  $\vec{v}$  with the length of 1 unit  
such that  $V$  is maximized.

And thus, I add a Lagrange multiplier  $\lambda$  to the equation:

$$V = \frac{1}{N} \times \sum_{i=1}^N \sum_{d=1}^D (p_{id} v_d)^2 - \lambda \left( \sum_{d=1}^D v_d^2 - 1 \right)$$

To find local min/max of  $V$ , I derive the equation into:

$$\frac{\delta V}{\delta v_a} = \frac{2}{N} \times \sum_{i=1}^N \left( p_{ia} \sum_{d=1}^D (p_{id} v_d) \right) - 2\lambda v_a$$

$$\text{At } \frac{\delta V}{\delta v_a} = 0:$$

$$\begin{aligned} & \frac{2}{N} \sum_{i=1}^N \left( p_{ia} \sum_{d=1}^D (p_{id} v_d) \right) = 2\lambda v_a \\ \Leftrightarrow & \sum_{d=1}^D v_d \left( \frac{2}{N} \sum_{i=1}^N p_{ia} p_{id} \right) = 2\lambda v_a \\ \Leftrightarrow & \sum_{d=1}^D v_d (2\text{cov}(p_a, p_d)) = 2\lambda v_a \end{aligned}$$

Since the image is two-dimensional,  $\vec{v}$  has 2 components  $v_x$  and  $v_y$   
The equation above is simplified into:

$$\begin{aligned} & \Leftrightarrow \begin{cases} v_x \text{cov}(p_x, p_x) + v_y \text{cov}(p_x, p_y) = \lambda v_x \\ v_x \text{cov}(p_y, p_x) + v_y \text{cov}(p_y, p_y) = \lambda v_y \end{cases} \\ & \Leftrightarrow \begin{bmatrix} \text{var}(p_x) & \text{cov}(p_x, p_y) \\ \text{cov}(p_y, p_x) & \text{var}(p_y) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \lambda \begin{bmatrix} v_x \\ v_y \end{bmatrix} \Leftrightarrow A\vec{v} = \lambda\vec{v} \end{aligned}$$

The fact that  $A$  is a matrix and  $\lambda$  is a scalar implies that  
 $\vec{v}$  must be an eigenvector

Consequently, the eigenvectors are the directions along which  
the data set has minimum/maximum variance

$$\left( \frac{\delta V}{\delta v_a} = 0 \right)$$

And the eigenvalues  $\lambda$  indicate the variance  
along those directions pointed by their respective eigenvectors

Why eigenvalues are equal to the data's variance along eigenvectors ?

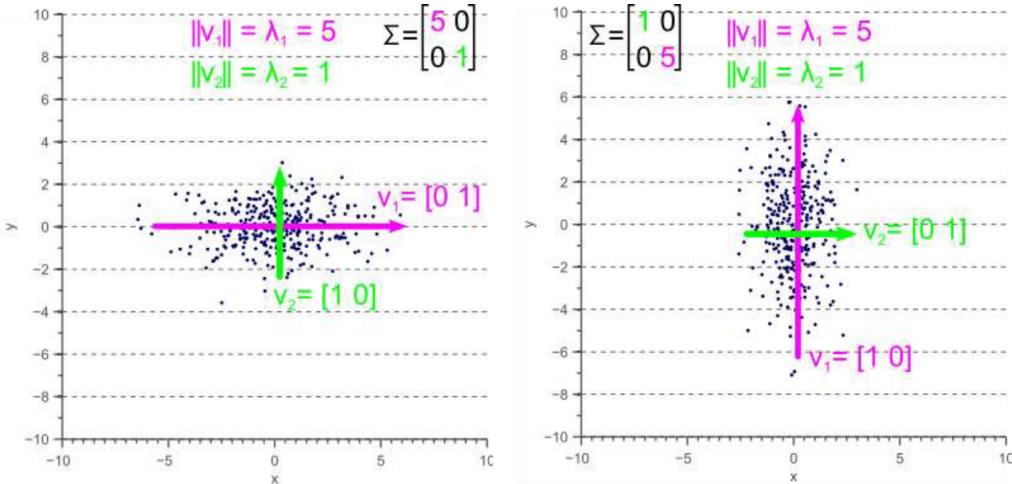


Figure 17: If  $\text{cov}(y, x) = 0$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \vec{v} = \lambda \vec{v} \Leftrightarrow \begin{bmatrix} a - \lambda & 0 \\ 0 & b - \lambda \end{bmatrix} \vec{v} = \vec{0} \Leftrightarrow (a - \lambda)(b - \lambda) = 0 \Leftrightarrow \begin{cases} \lambda_1 = a \\ \lambda_2 = b \end{cases}$$

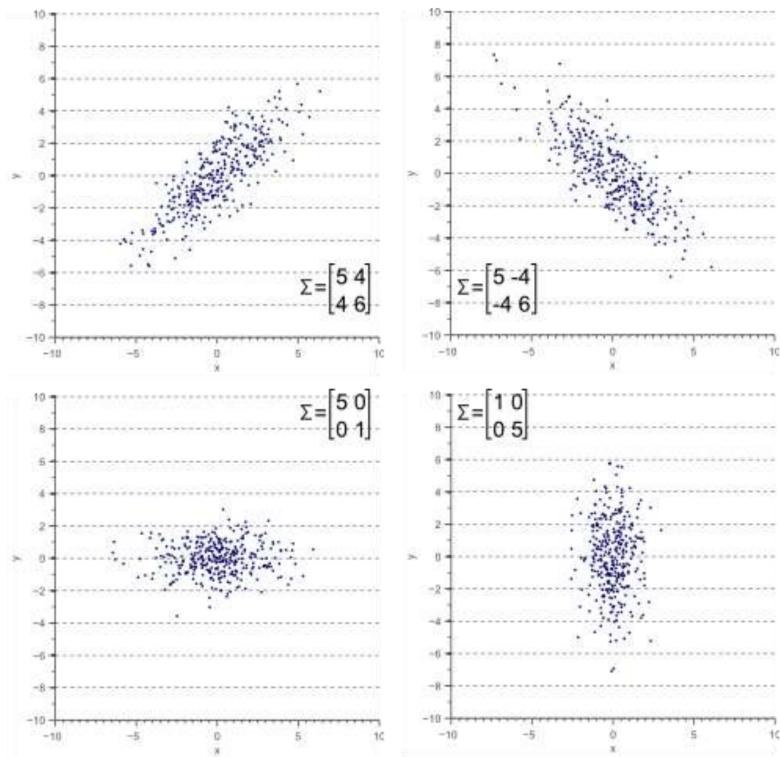


Figure 18: All distributions...

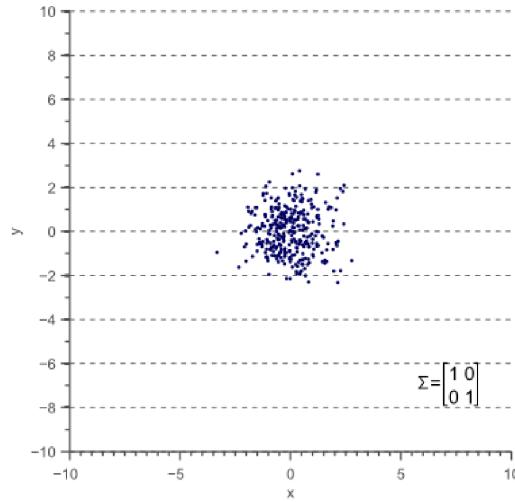


Figure 19: Are just a linearly transformed instance of this

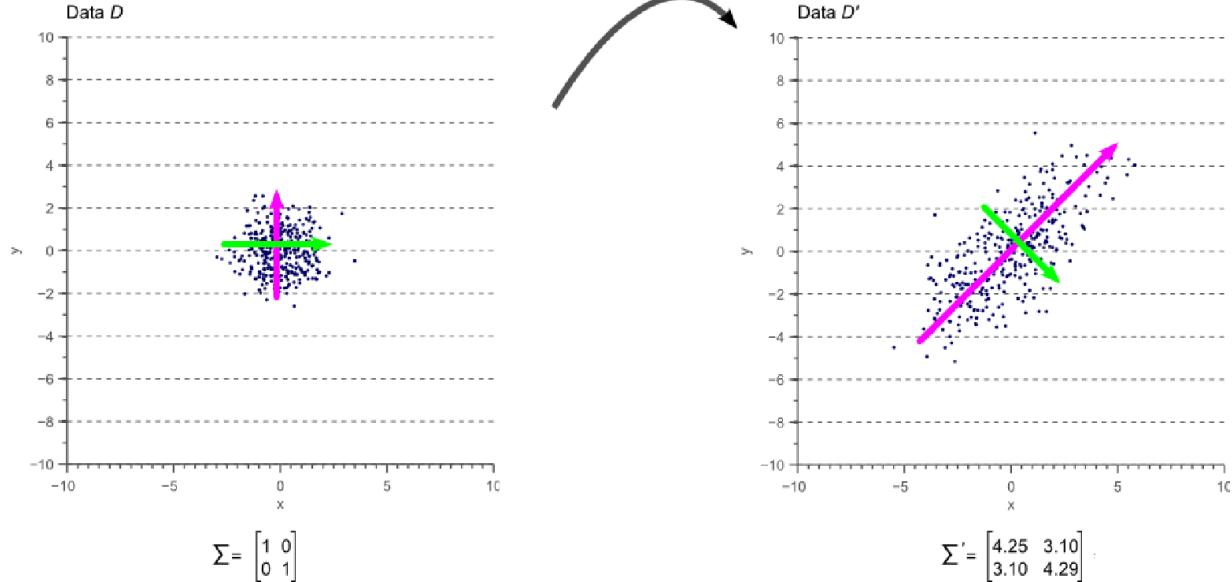


Figure 20: Therefore, the eigenvectors and eigenvalues  
are just simply linearly transformed along

The covariance matrix of 2 gradient images  $gx$  and  $gy$  is calculated as:

$$A = \begin{bmatrix} var(dx) & cov(dx, dy) \\ cov(dy, dx) & var(dy) \end{bmatrix} = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

We then find the eigenvalues  $\lambda_1$  and  $\lambda_2$  of  $A$ , which indicate the maximum and minimum variance of the data set while being projected onto 1 line

$$\begin{aligned} A\vec{v} &= \lambda\vec{v} \\ \Leftrightarrow A\vec{v} &= \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\vec{v} \\ \Leftrightarrow A\vec{v} - \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\vec{v} &= \vec{0} \\ \Leftrightarrow \left(A - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\vec{v} &= \vec{0} \\ \Leftrightarrow \begin{bmatrix} a - \lambda & c \\ c & b - \lambda \end{bmatrix}\vec{v} &= \vec{0} \end{aligned}$$

Because  $\vec{v}$  is non-zero vector:

$$\begin{aligned} \det \begin{pmatrix} a - \lambda & c \\ c & b - \lambda \end{pmatrix} &= 0 \\ \Leftrightarrow (a - \lambda)(b - \lambda) - c^2 &= 0 \\ \Leftrightarrow \begin{cases} \lambda_1 = \frac{1}{2} \left( (a + b) + \sqrt{(a - b)^2 + 4c^2} \right) \\ \lambda_2 = \frac{1}{2} \left( (a + b) - \sqrt{(a - b)^2 + 4c^2} \right) \end{cases} \end{aligned}$$

Thus,  $\lambda_1$  is the maximum variance of the data set along one direction  
while  $\lambda_2$  is the minimum.

The orientation certainty level (*ocl*) of a block  
can be then calculated as follow:

$$ocl = \frac{\lambda_2}{\lambda_1} \quad (0 \leq ocl \leq 1)$$

With low (high) ocl values, the local structure and orientation  
of ridges and valleys are very regular (irregular), and therefore  
the block has good (bad) quality.



Figure 21: Masking blocks using STD

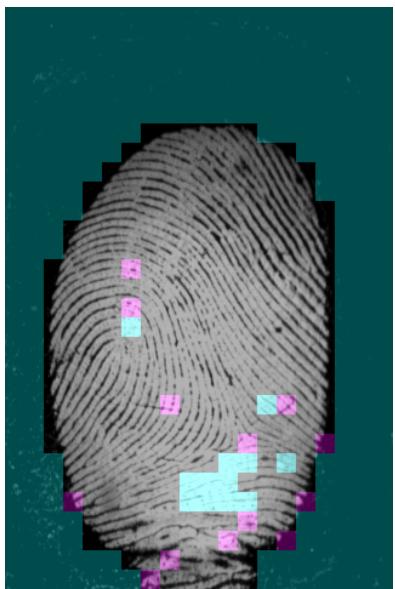


Figure 22: Combine the STD mask with OCL mask



Figure 23: Smoothing the combined mask



Figure 24: Original Image



Figure 25: Segmented Image

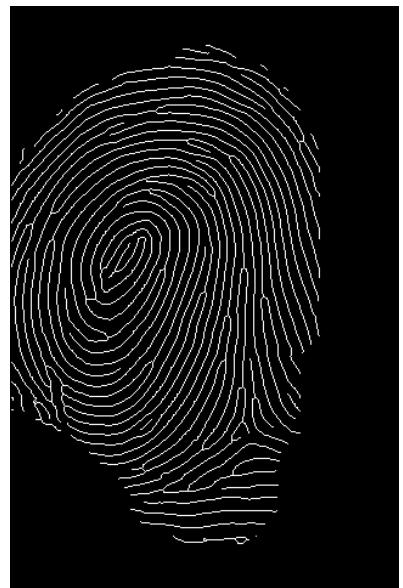


Figure 26: Skeleton



Figure 27: Original Image



Figure 28: Segmented Image

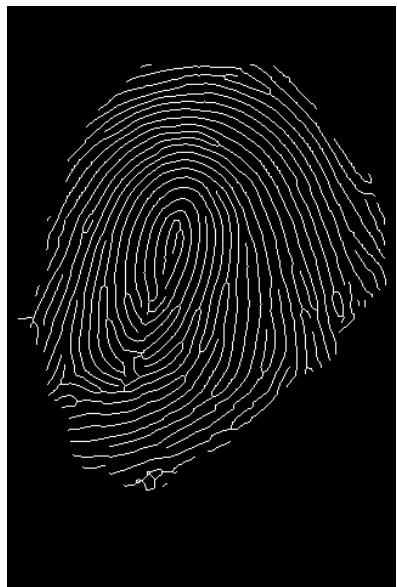


Figure 29: Skeleton



Figure 30: Original Image



Figure 31: Segmented Image

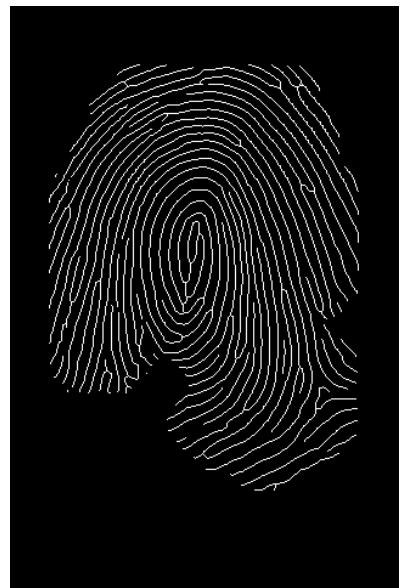


Figure 32: Skeleton



Figure 33: Original Image



Figure 34: Segmented Image

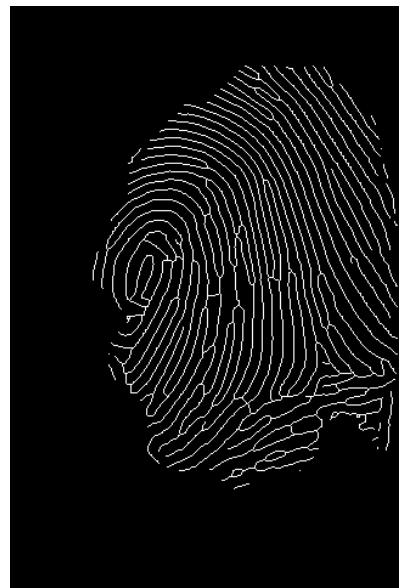


Figure 35: Skeleton



Figure 36: Original Image

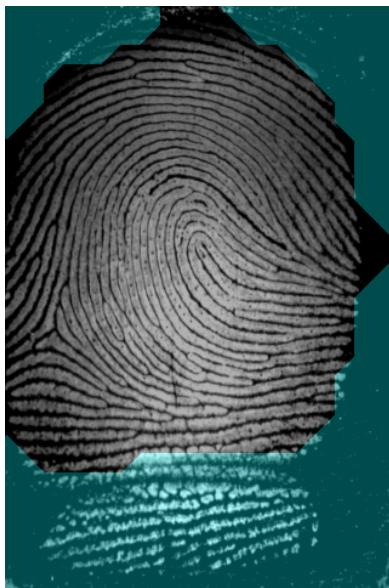


Figure 37: Segmented Image

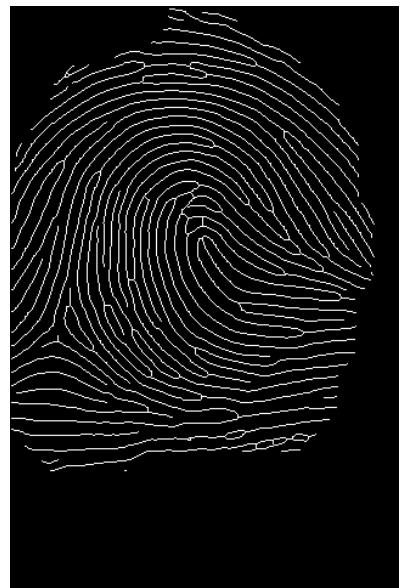


Figure 38: Skeleton



Figure 39: Original Image

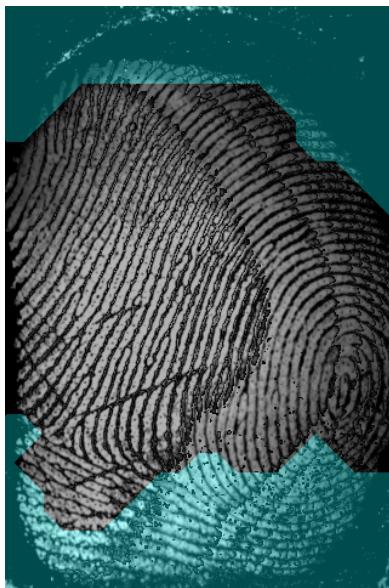


Figure 40: Segmented Image



Figure 41: Skeleton



Figure 42: Original Image

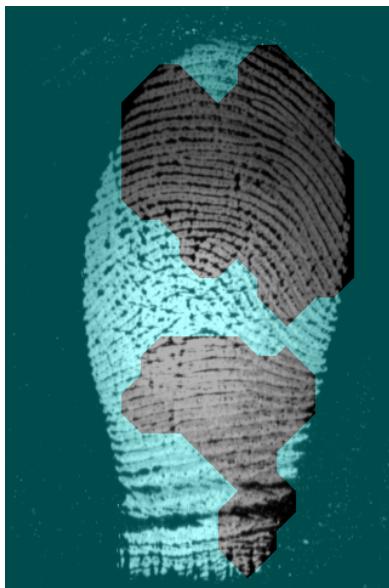


Figure 43: Segmented Image

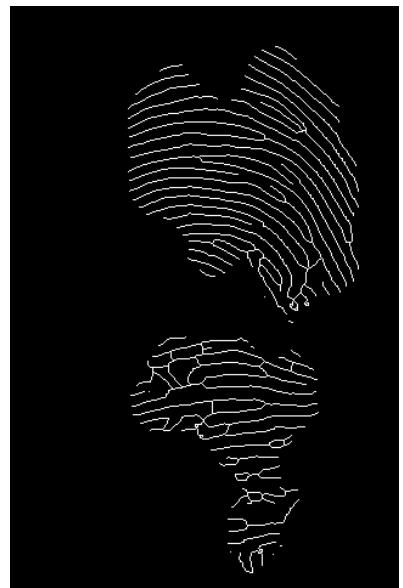


Figure 44: Skeleton



Figure 45: Original Image

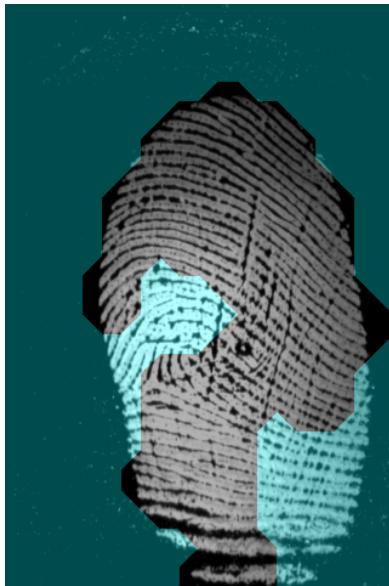


Figure 46: Segmented Image

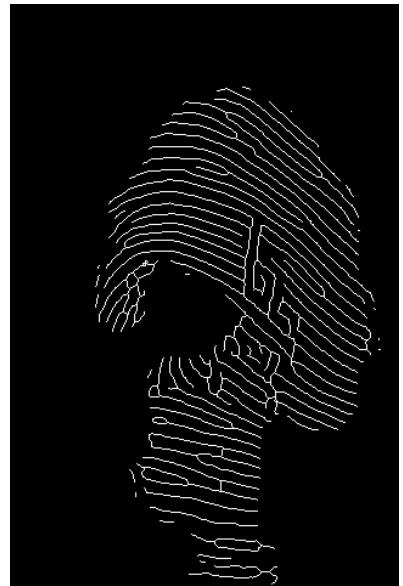


Figure 47: Skeleton