Fingerprint Quality Validation

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Part 1:
Variance of Gray Levels

$$mean(I) = \mu = \frac{1}{N} \times \sum_{i=1}^{N} E_i$$

$$var(I) = \sigma^2 = \frac{\sum_{i=1}^{N} (E_i - \mu)^2}{N}$$

$$std(I) = \sigma = \sqrt{var(I)}$$

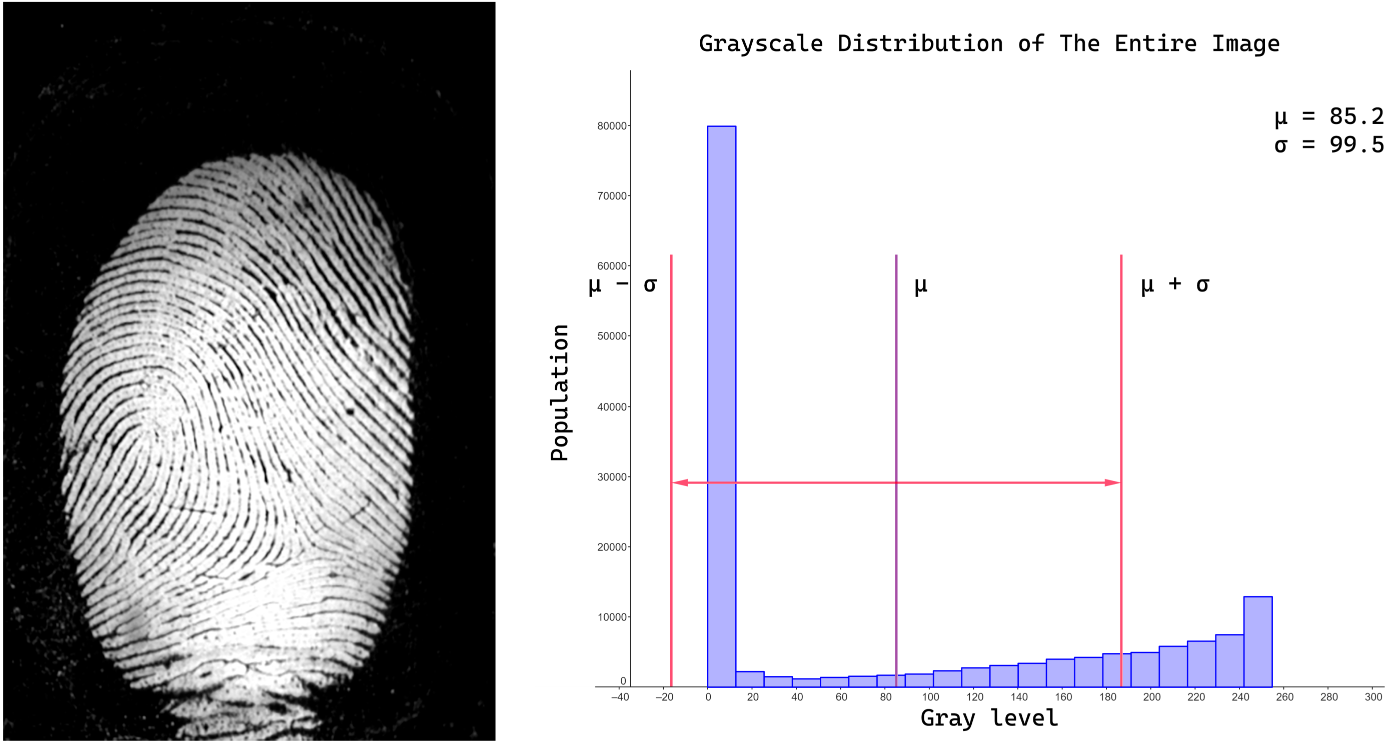

Figure 1: Grayscale Distribution of a Fingerprint Image

Let σ_{base} be the standard deviation of the image

The contrast quality (cq) of a block β is determined by:

$$cq_\beta = \frac{\sigma_\beta}{\sigma_{base}}$$

High cq_β value means that the block β
contains both clear ridges and clear valleys, which promises useful data.

If cq_β is too low, β can either be a background block,
or a block without any helpful information at all (bad block).

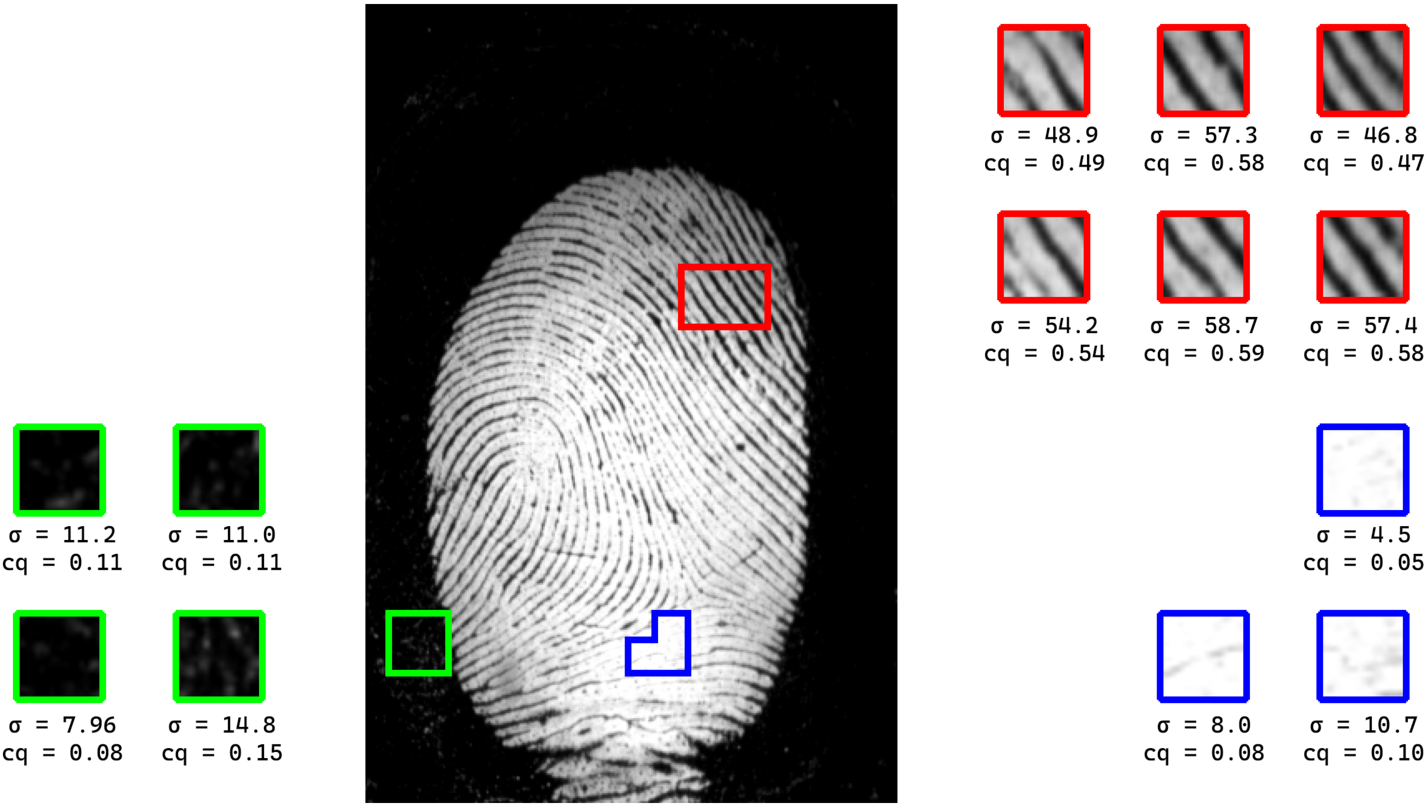

Figure 2: Standard Deviation and Contrast Quality of some blocks

Part 2:
Orientation Certainty

$$gx = I * \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$gy = I * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

The covariance matrix of 2 gradient images gx and gy are calculated as:

$$A = \frac{1}{N} \times \sum \begin{bmatrix} dx \\ dy \end{bmatrix} \begin{bmatrix} dx & dy \end{bmatrix} = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

Why does the Eigenvector maximizes variance?

Variance of projections:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d - \mu)^2$$

Thus, the formula to calculate the variance of projections is simplified to:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d)^2$$

The goal is to find a vector $\vec{v}$ with the length of 1 unit
such that V is maximized.
And thus, I add a Lagrange multiplier λ to the equation:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d)^2 - \lambda(\sum_{d=1}^{D} v_d^2 - 1)$$

To find local min/max of V, I derive the equation into:

$$\frac{\delta V}{\delta v_a} = \frac{2}{N} \times \sum_{i=1}^{N} \left(p_{ia} \sum_{d=1}^{D} (p_{id} v_d) \right) - 2\lambda v_a$$

$$\text{At } \frac{\delta V}{\delta v_a} = 0:$$

$$\frac{2}{N}\sum_{i=1}^{N}\left(p_{ia}\sum_{d=1}^{D}(p_{id}v_d)\right) = 2\lambda v_a$$

$$\Leftrightarrow \sum_{d=1}^{D} v_d\left(\frac{2}{N}\sum_{i=1}^{N} p_{ia}p_{id}\right) = 2\lambda v_a$$

$$\Leftrightarrow \sum_{d=1}^{D} v_d\left(2cov(p_a, p_d)\right) = 2\lambda v_a$$

Since the image is two-dimensional, $\vec{v}$ has 2 components v_x and v_y

The equation above is simplified into:

$$\Leftrightarrow \begin{cases} v_x cov(p_x, p_x) + v_y cov(p_x, p_y) = \lambda v_x \\ v_x cov(p_y, p_x) + v_y cov(p_y, p_y) = \lambda v_y \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} var(p_x) & cov(p_x, p_y) \\ cov(p_y, p_x) & var(p_y) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \lambda \begin{bmatrix} v_x \\ v_y \end{bmatrix} \Leftrightarrow A\vec{v} = \lambda\vec{v}$$

The fact that A is a matrix and λ is a scalar implies that $\vec{v}$ must be an eigenvector (3blues1brown video for proof)

Consequently, the eigenvectors are the directions along which the data set has minimum/maximum variance

$$\left(\frac{\delta V}{\delta v_a} = 0\right)$$