Fingerprint Quality Validation

Lưu Nam Đạt

Mar-22-2023

Part 1:
Variance of Gray Levels

$$mean(I) = \mu = \frac{1}{N} \times \sum_{i=1}^{N} E_i$$

$$var(I) = \sigma^2 = \frac{\sum_{i=1}^{N} (E_i - \mu)^2}{N}$$

$$std(I) = \sigma = \sqrt{var(I)}$$

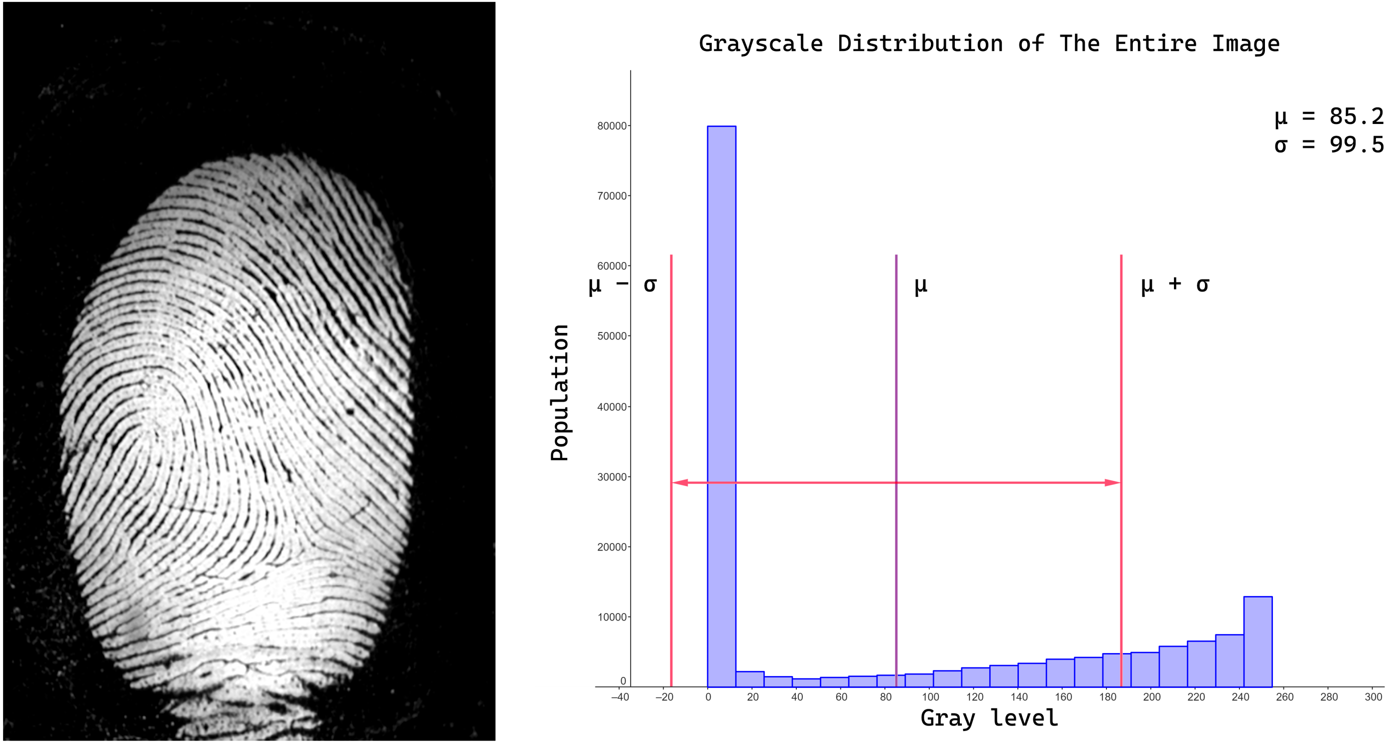

Figure 1: Grayscale Distribution of a Fingerprint Image

Let σ_{base} be the standard deviation of the image

The contrast quality (cq) of a block β is determined by:

$$cq_\beta = \frac{\sigma_\beta}{\sigma_{base}}$$

High cq_β value means that the block β
contains both clear ridges and clear valleys, which promises useful data.

If cq_β is too low, β can either be a background block,
or a block without any helpful information at all (bad block).

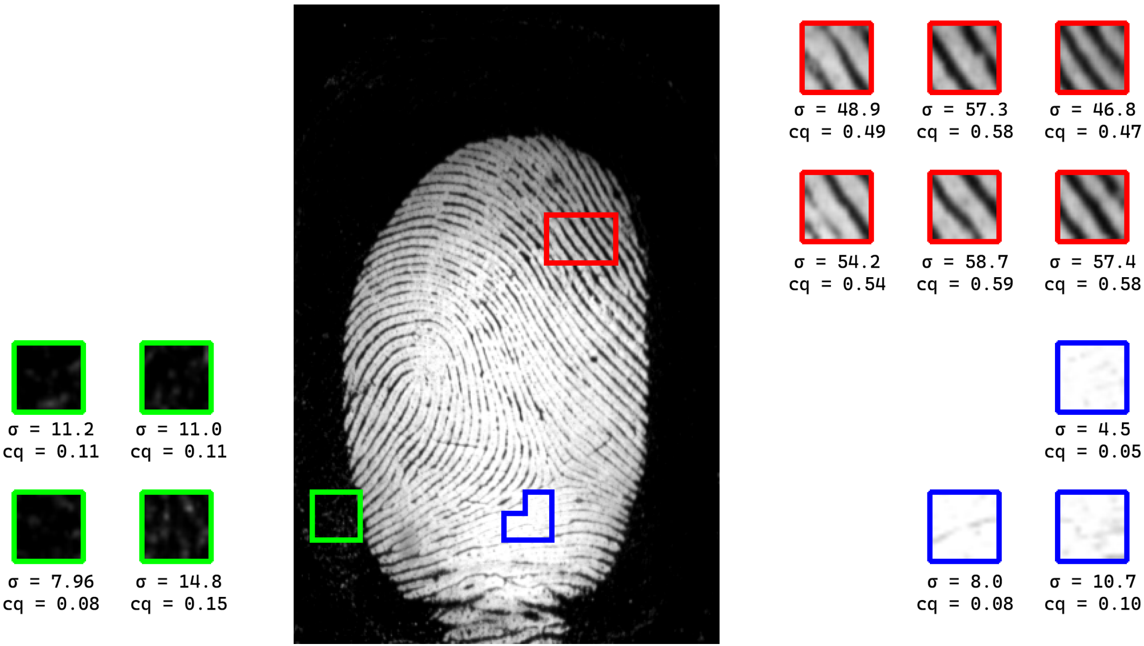

Figure 2: Standard Deviation and Contrast Quality of some blocks

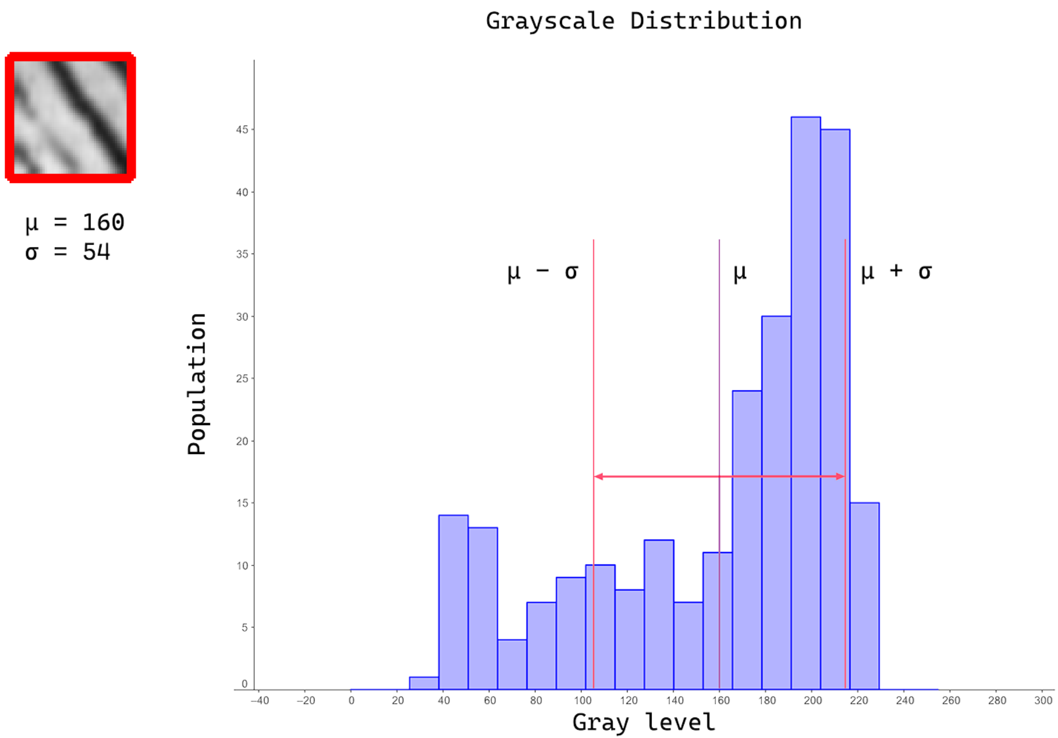

Figure 3: Grayscale Distribution of a good block

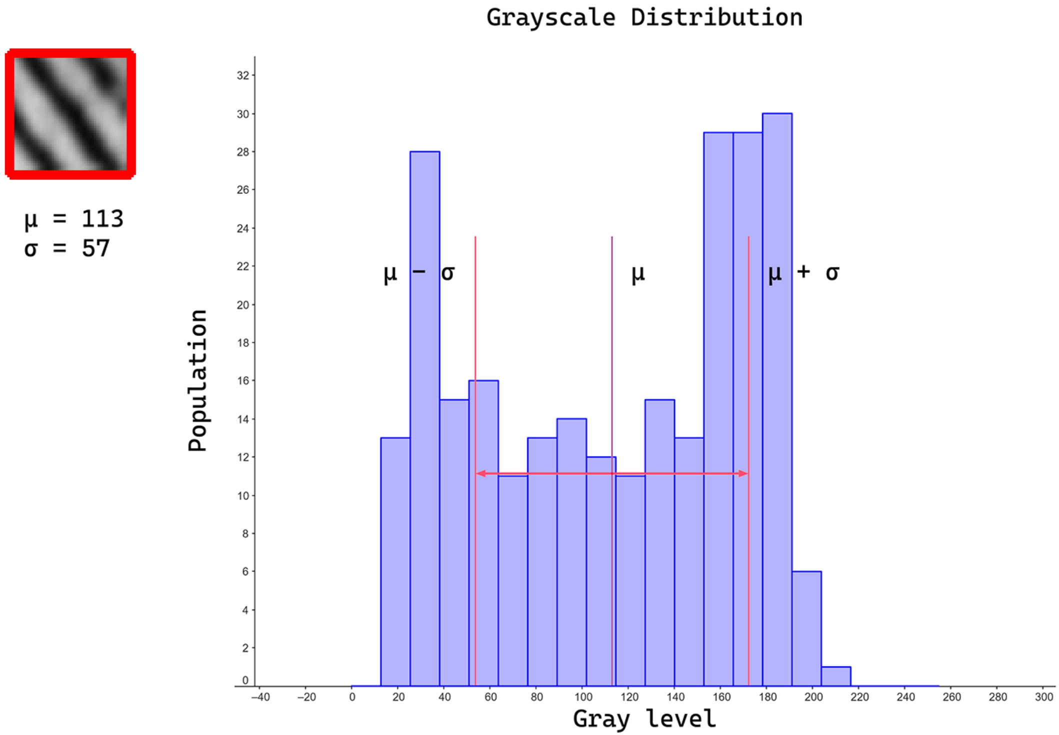

Figure 4: Grayscale Distribution of another good block

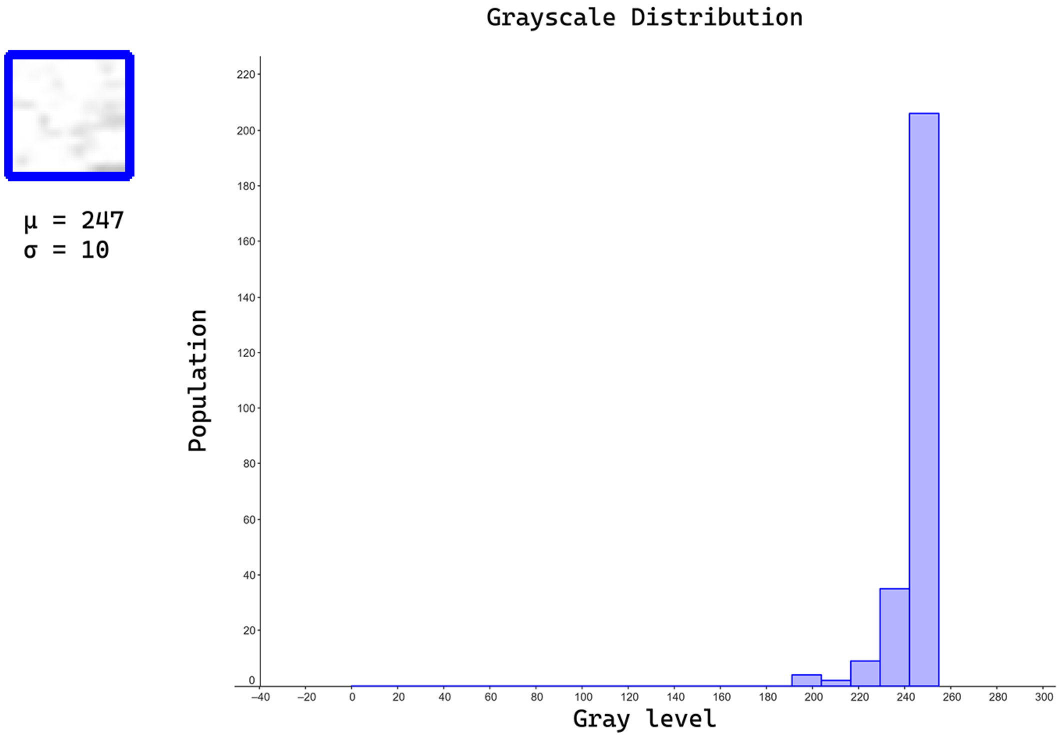

Figure 5: Grayscale Distribution of a bad block

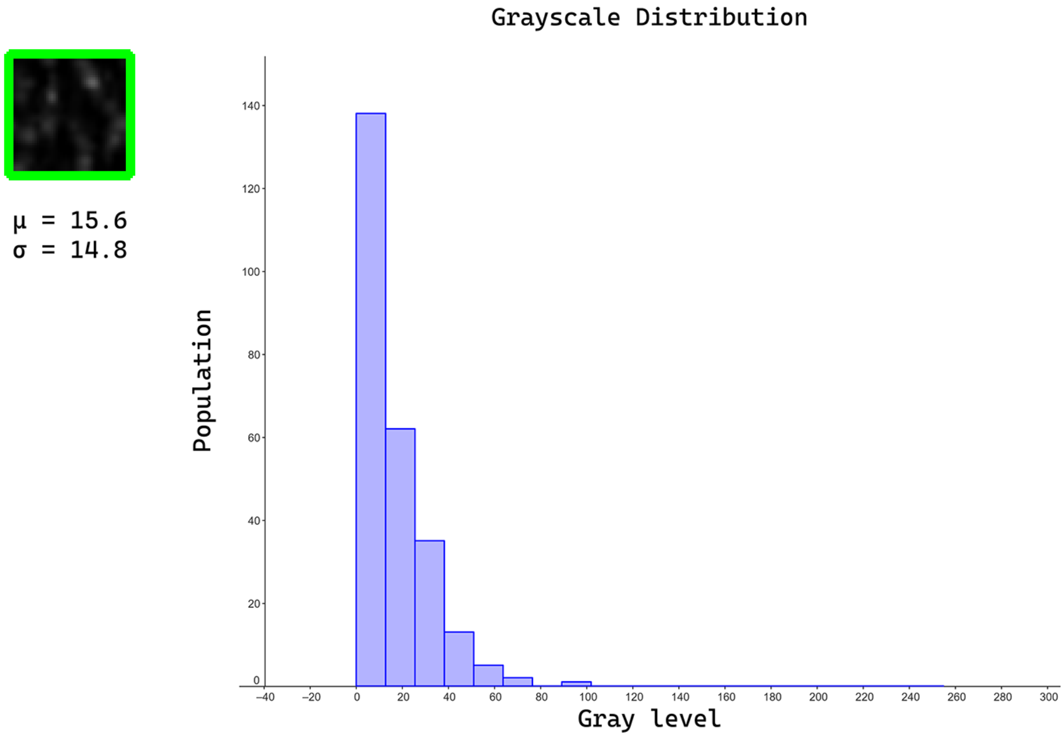

Figure 6: Grayscale Distribution of a background block

Part 2:
Orientation Certainty

$$gx = I * \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$gy = I * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

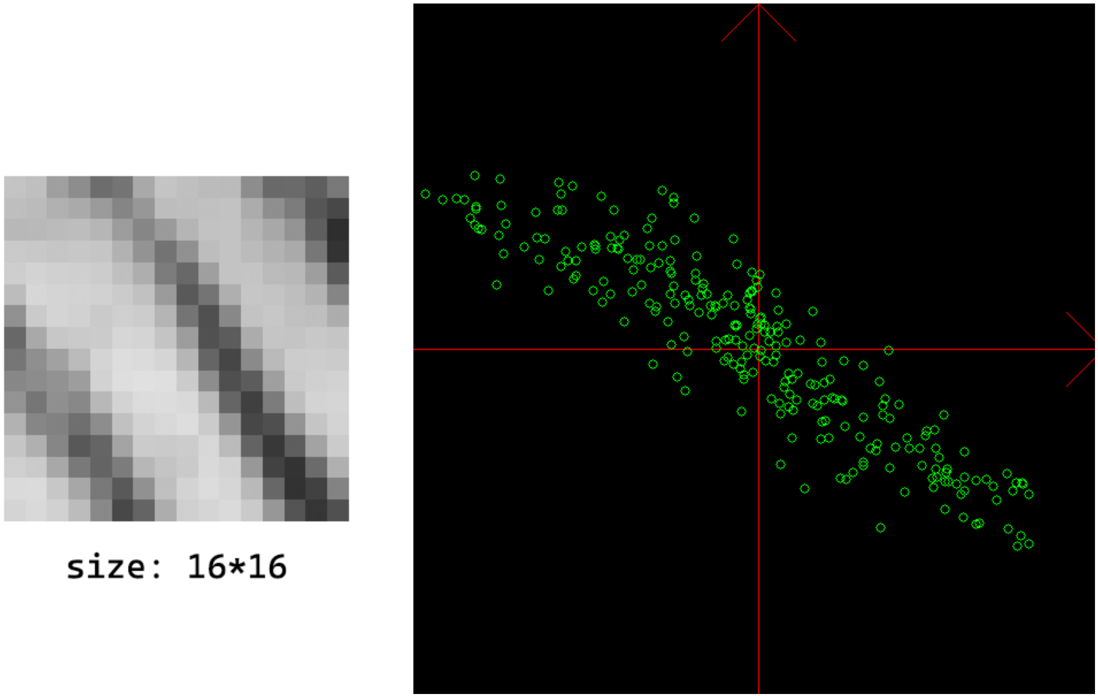

Figure 7: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (1)

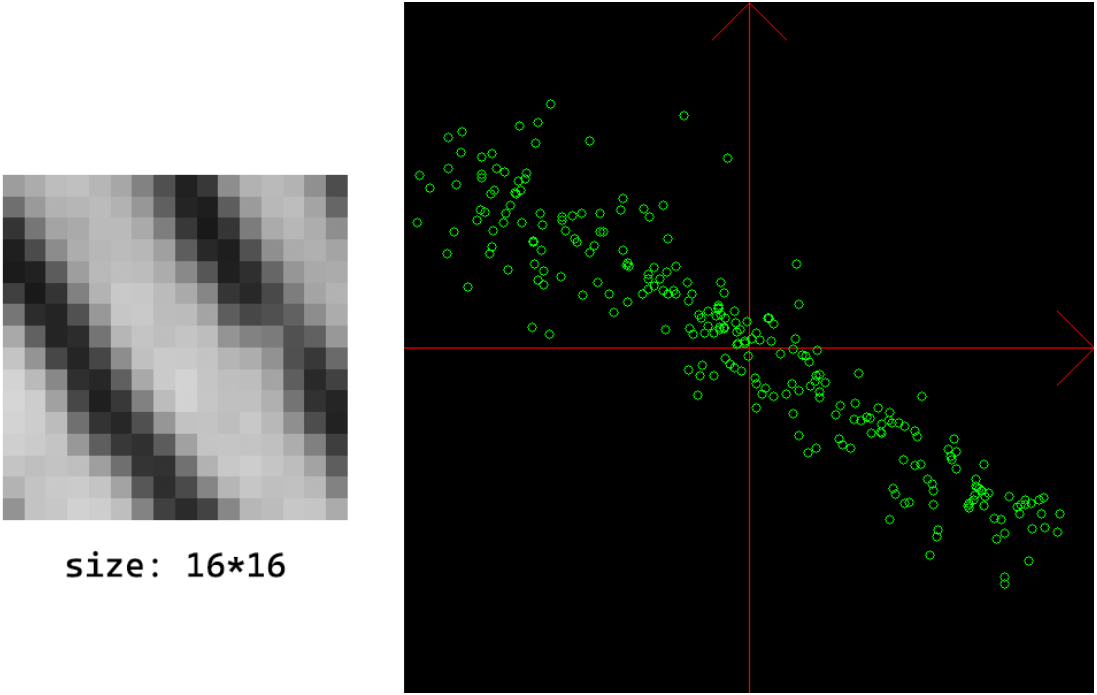

Figure 8: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (2)

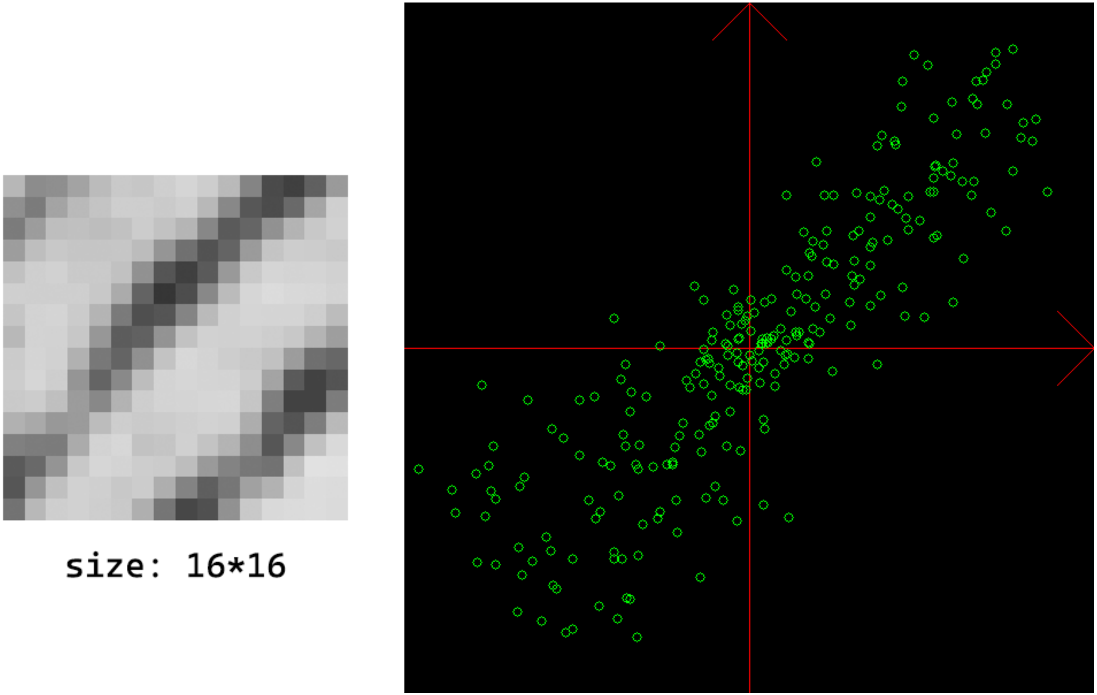

Figure 9: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (3)

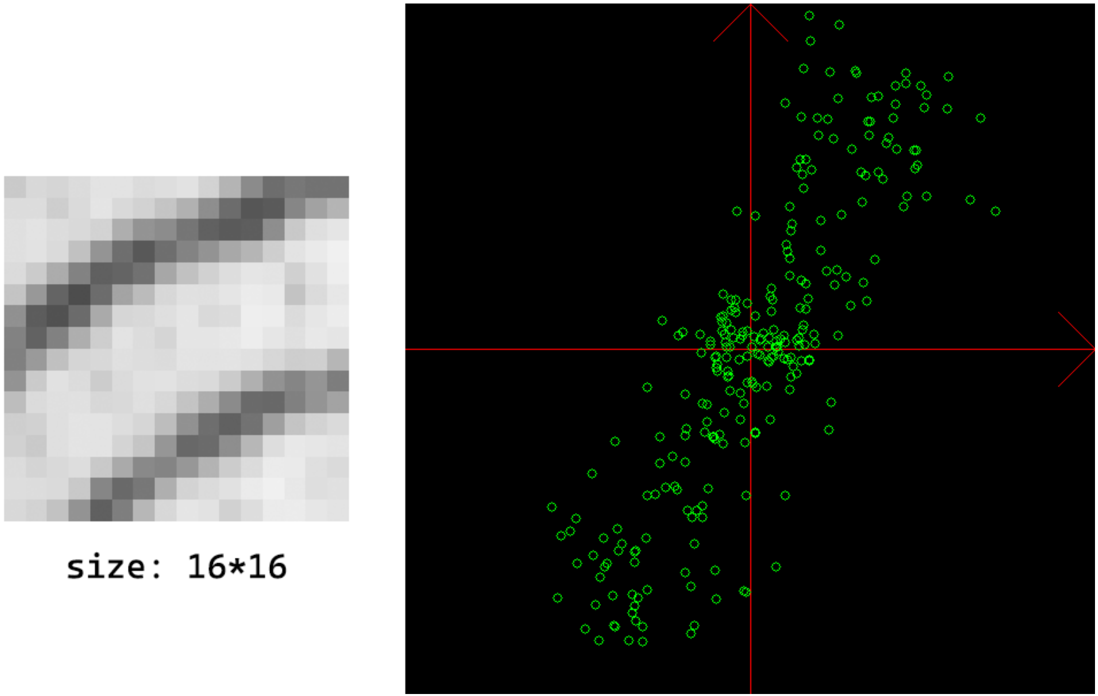

Figure 10: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (4)

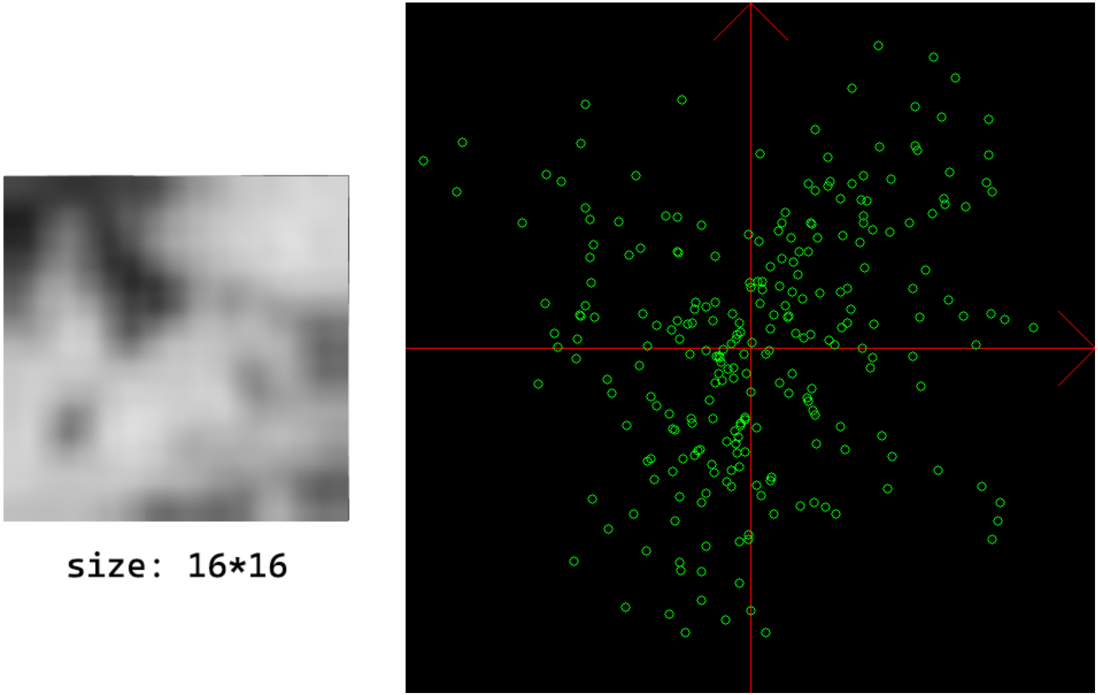

Figure 11: Relation between gx (horizontal axis) and gy (vertical axis) on a bad block (1)

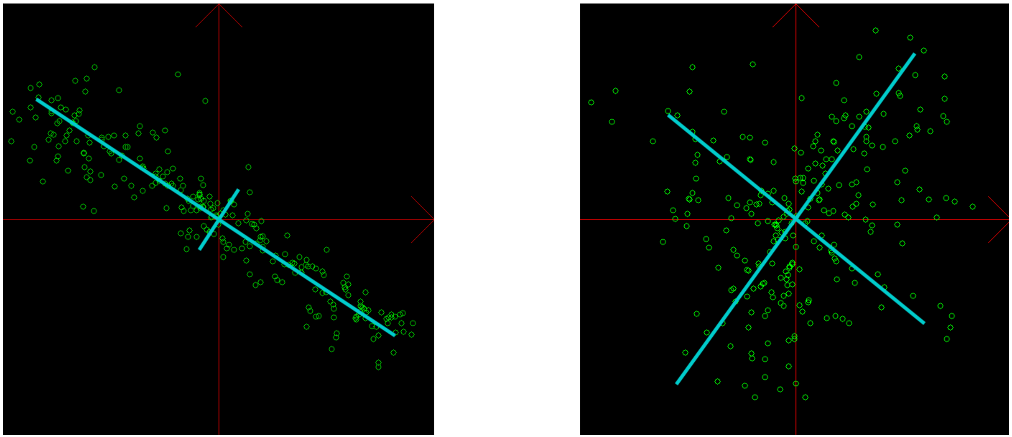

Figure 12: The direction along which the data set has minimum/maximum variance (the length of each line indicates the variance along respective direction)

Find unit vectors that minimize/maximize variance

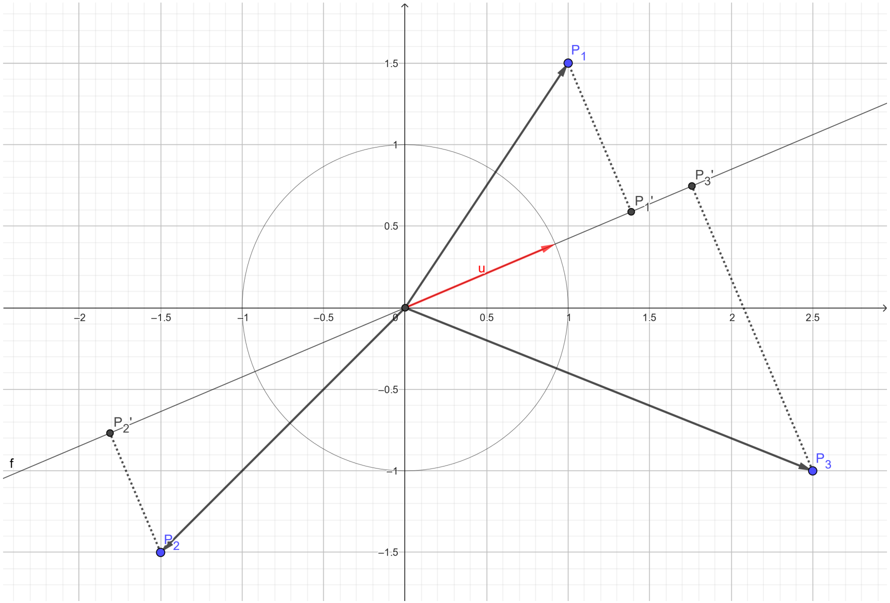

Figure 13: Projections of some points onto a unit vector

If $\vec{v}$ is a unit vector:

$$dist(O, P_i') = \vec{P}_i \cdot \vec{v} = \sum_{d=1}^{D} P_{id} v_d$$

Variance of projections:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d - \mu)^2$$

By performing a geometric transformation such that

$\mu_x = \mu_y = 0$

Variable μ of the equation above then has the value of 0

Thus, the formula to calculate the variance of projections is simplified to:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d)^2$$

The goal is to find a vector $\vec{v}$ with the length of 1 unit
such that V is maximized.
And thus, I add a Lagrange multiplier λ to the equation:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d)^2 - \lambda (\sum_{d=1}^{D} v_d^2 - 1)$$

To find local min/max of V, I derive the equation into:

$$\frac{\delta V}{\delta v_a} = \frac{2}{N} \times \sum_{i=1}^{N} \left(p_{ia} \sum_{d=1}^{D} (p_{id} v_d) \right) - 2\lambda v_a$$

$$\text{At } \frac{\delta V}{\delta v_a} = 0:$$

$$\frac{2}{N}\sum_{i=1}^{N}\left(p_{ia}\sum_{d=1}^{D}(p_{id}v_d)\right) = 2\lambda v_a$$

$$\Leftrightarrow \sum_{d=1}^{D} v_d\left(\frac{2}{N}\sum_{i=1}^{N} p_{ia}p_{id}\right) = 2\lambda v_a$$

$$\Leftrightarrow \sum_{d=1}^{D} v_d\left(2cov(p_a, p_d)\right) = 2\lambda v_a$$

Since the image is two-dimensional, $\vec{v}$ has 2 components v_x and v_y
The equation above is simplified into:

$$\Leftrightarrow \begin{cases} v_x cov(p_x, p_x) + v_y cov(p_x, p_y) = \lambda v_x \\ v_x cov(p_y, p_x) + v_y cov(p_y, p_y) = \lambda v_y \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} var(p_x) & cov(p_x, p_y) \\ cov(p_y, p_x) & var(p_y) \end{bmatrix}\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \lambda \begin{bmatrix} v_x \\ v_y \end{bmatrix} \Leftrightarrow A\vec{v} = \lambda\vec{v}$$

The fact that A is a matrix and λ is a scalar implies that
$\vec{v}$ must be an eigenvector

Consequently, the eigenvectors are the directions along which
the data set has minimum/maximum variance

$$\left(\frac{\delta V}{\delta v_a} = 0 \right)$$

And the eigenvalues λ indicate the variance
along those directions pointed by their respective eigenvectors

The covariance matrix of 2 gradient images gx and gy is calculated as:

$$A = \begin{bmatrix} var(dx) & cov(dx,dy) \\ cov(dy,dx) & var(dy) \end{bmatrix} = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

We then find the eigenvalues λ_1 and λ_2 of A, which indicate
the maximum and minimum variance of the data set while being projected onto 1 line

$$A\vec{v} = \lambda\vec{v}$$

$$\Leftrightarrow A\vec{v} = \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \vec{v}$$

$$\Leftrightarrow A\vec{v} - \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \vec{v} = \vec{0}$$

$$\Leftrightarrow \left(A - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \vec{v} = \vec{0}$$

$$\Leftrightarrow \begin{bmatrix} a-\lambda & c \\ c & b-\lambda \end{bmatrix} \vec{v} = \vec{0}$$

Because $\vec{v}$ is non-zero vector:

$$det\left(\begin{bmatrix} a-\lambda & c \\ c & b-\lambda \end{bmatrix}\right) = 0$$

$$\Leftrightarrow (a-\lambda)(b-\lambda) - c^2 = 0$$

$$\Leftrightarrow \begin{cases} \lambda_1 = \frac{1}{2}\left((a+b) + \sqrt{(a-b)^2 + 4c^2}\right) \\ \lambda_2 = \frac{1}{2}\left((a+b) - \sqrt{(a-b)^2 + 4c^2}\right) \end{cases}$$

Thus, λ_1 is the maximum variance of the data set along one direction while λ_2 is the minimum.

The orientation certainty level (ocl) of a block can be then calculated as follow:

$$ocl = \frac{\lambda_2}{\lambda_1} \qquad (0 \leq ocl \leq 1)$$

With low (high) ocl values, the local structure and orientation of ridges and valleys are very regular (irregular), and therefore the block has good (bad) quality.