Fingerprint Quality Validation

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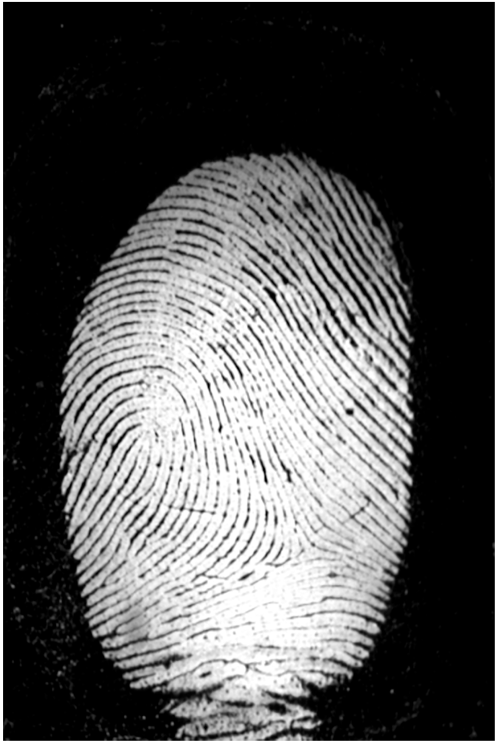
Part 1:
Variance of Gray Levels

$$mean(I) = \mu = \frac{1}{N} \times \sum_{i=1}^{N} E_i$$

$$var(I) = \sigma^2 = \frac{\sum_{i=1}^{N} (E_i - \mu)^2}{N}$$

$$std(I) = \sigma = \sqrt{var(I)}$$



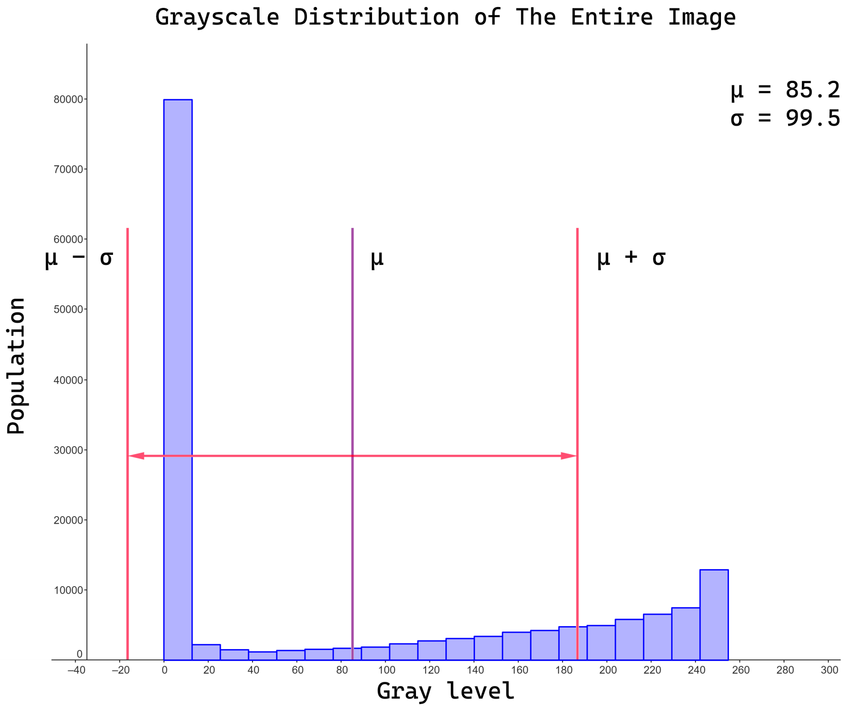


Figure 1: Grayscale Distribution of a Fingerprint Image

Let σ_{base} be the standard deviation of the image

The contrast quality (cq) of a block β is determined by:

$$cq_\beta = \frac{\sigma_\beta}{\sigma_{base}}$$

High cq_β value means that the block β contains both clear ridges and clear valleys, which promises useful data.

If cq_β is too low, β can either be a background block, or a block without any helpful information at all (bad block).

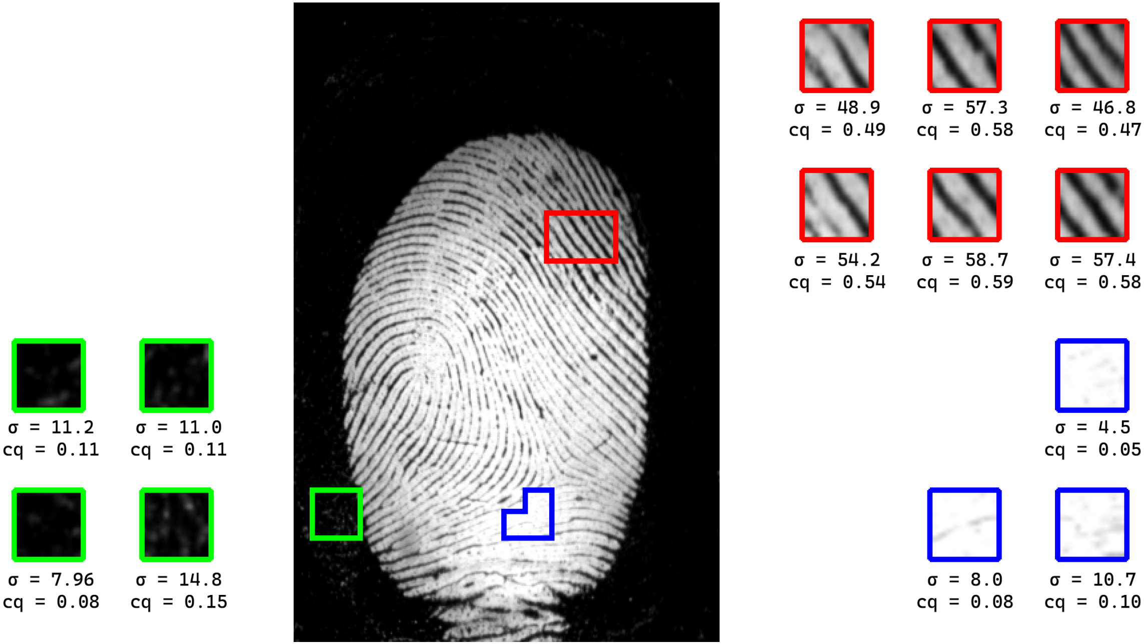


Figure 2: Standard Deviation and Contrast Quality of some blocks

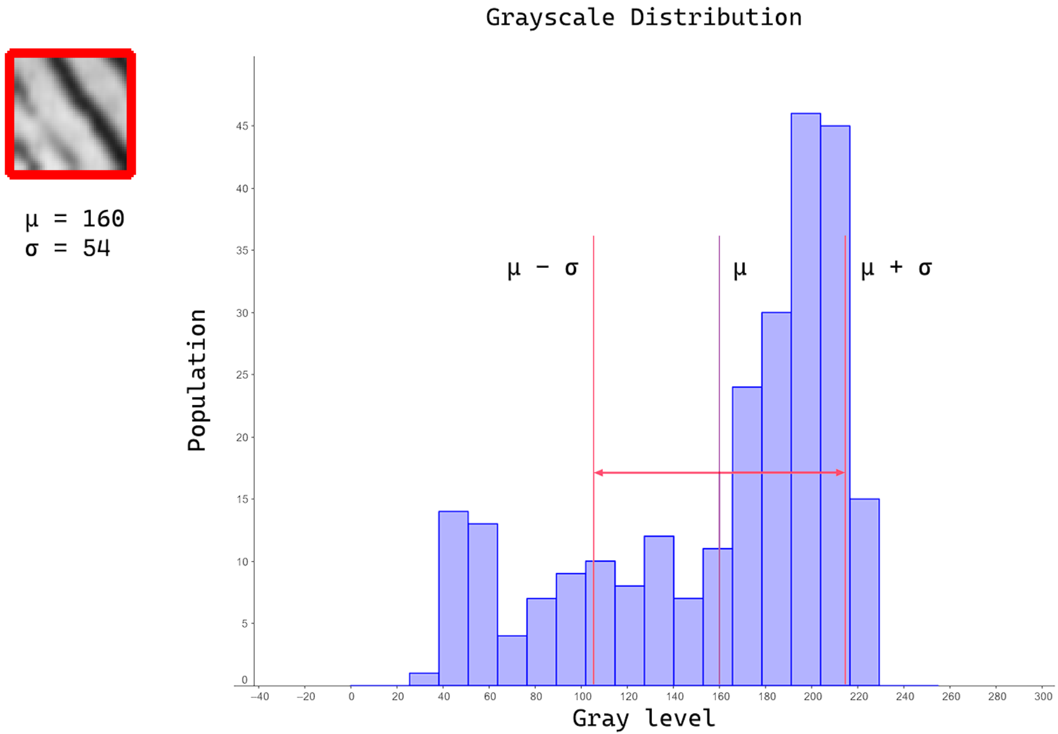


Figure 3: Grayscale Distribution of a good block

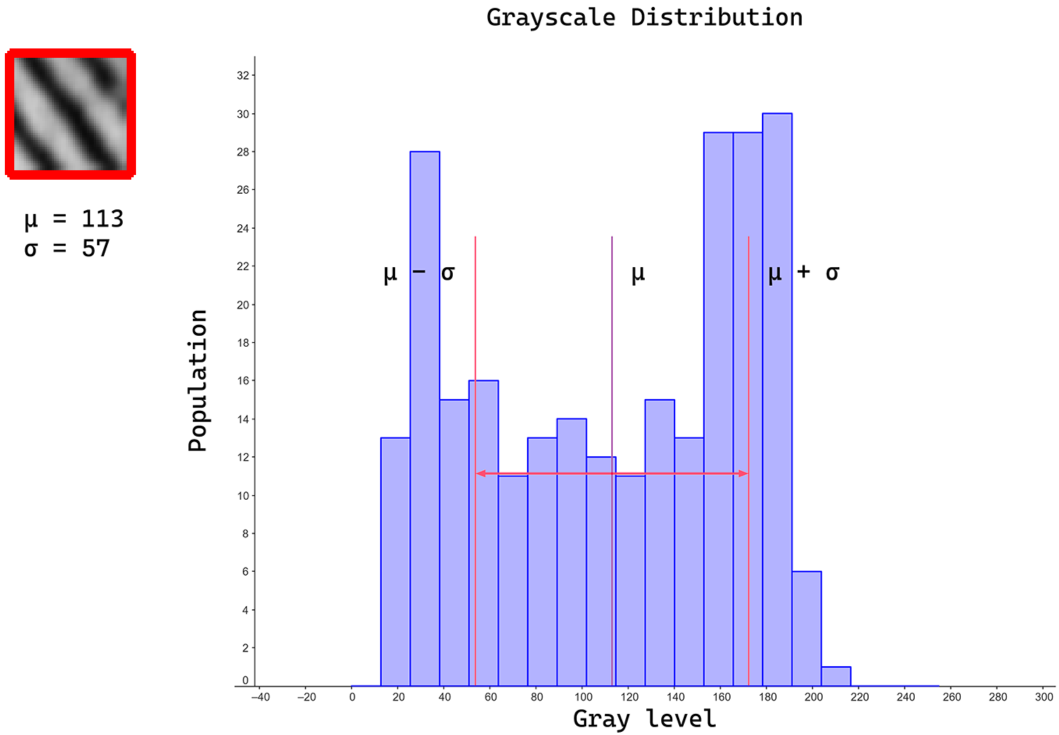


Figure 4: Grayscale Distribution of another good block

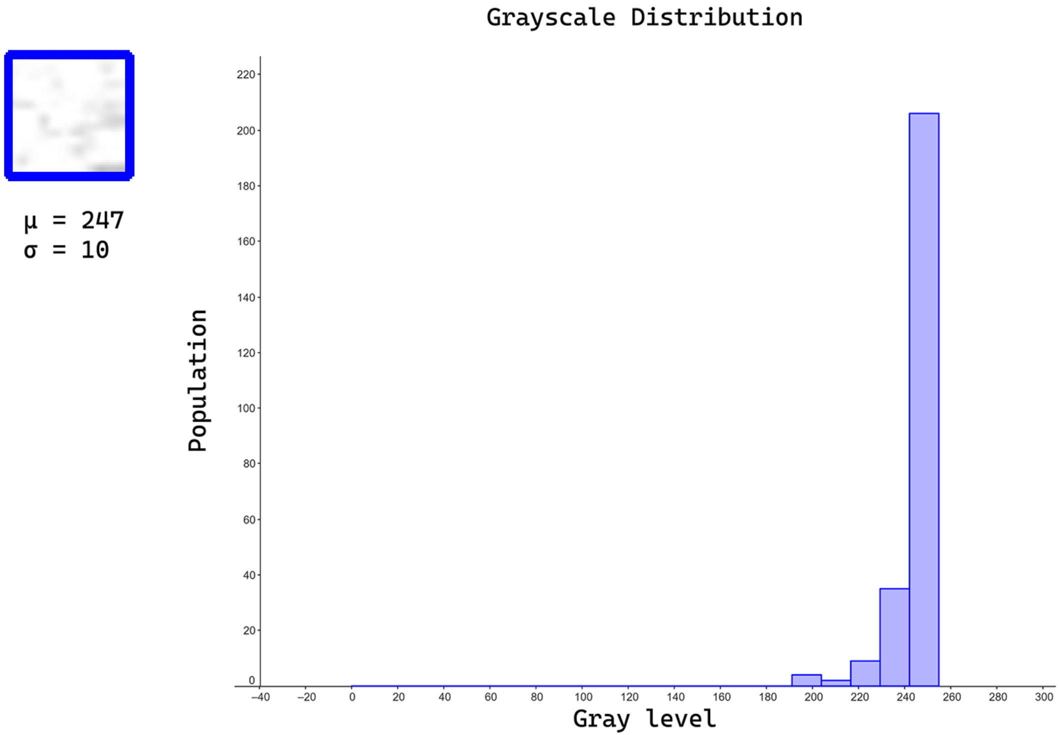


Figure 5: Grayscale Distribution of a bad block

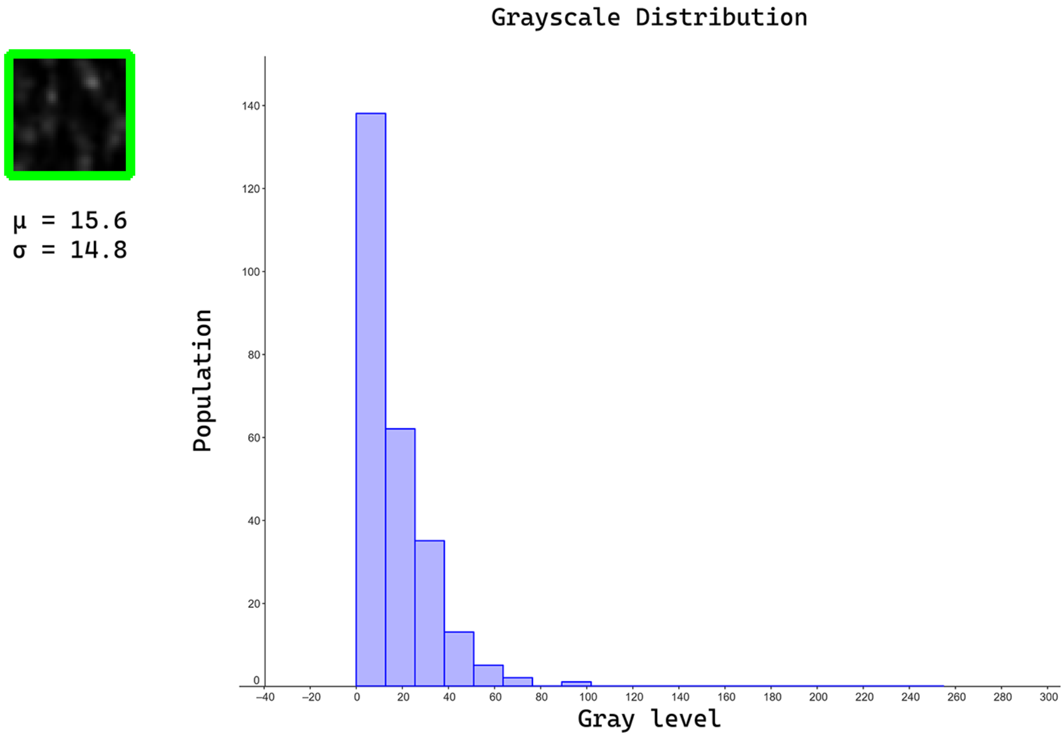


Figure 6: Grayscale Distribution of a background block

Figure 7: Masking blocks with std < 0.2

Figure 8: The smoothed mask

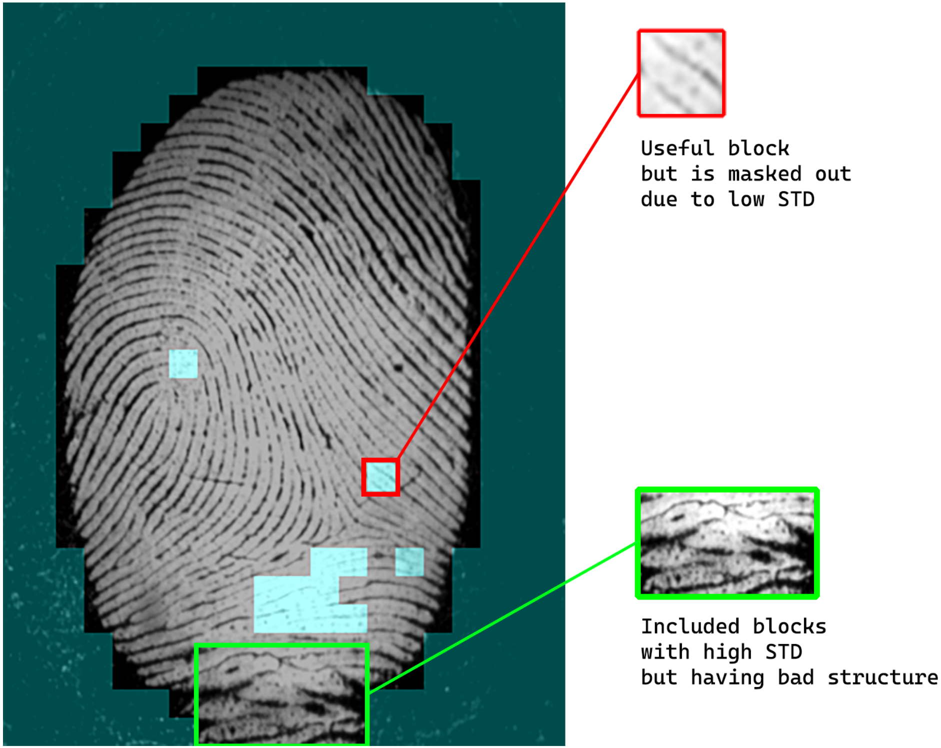


Figure 9: STD masking fails

Part 2: Orientation Certainty

$$gx = I * \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$gy = I * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

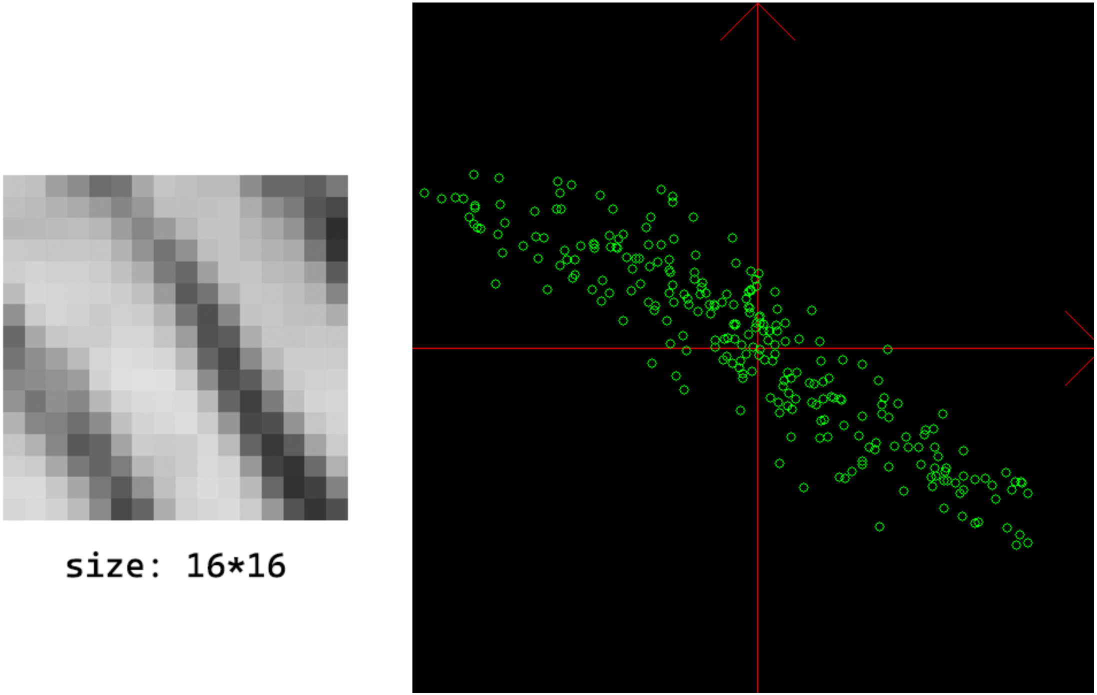


Figure 10: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (1)

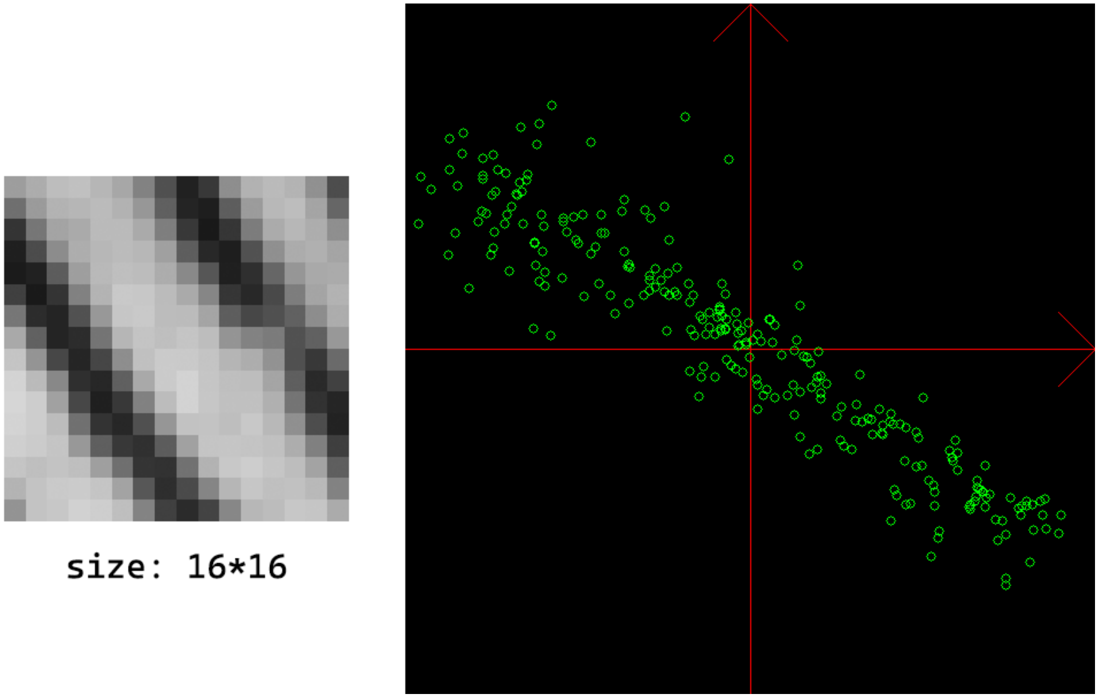


Figure 11: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (2)

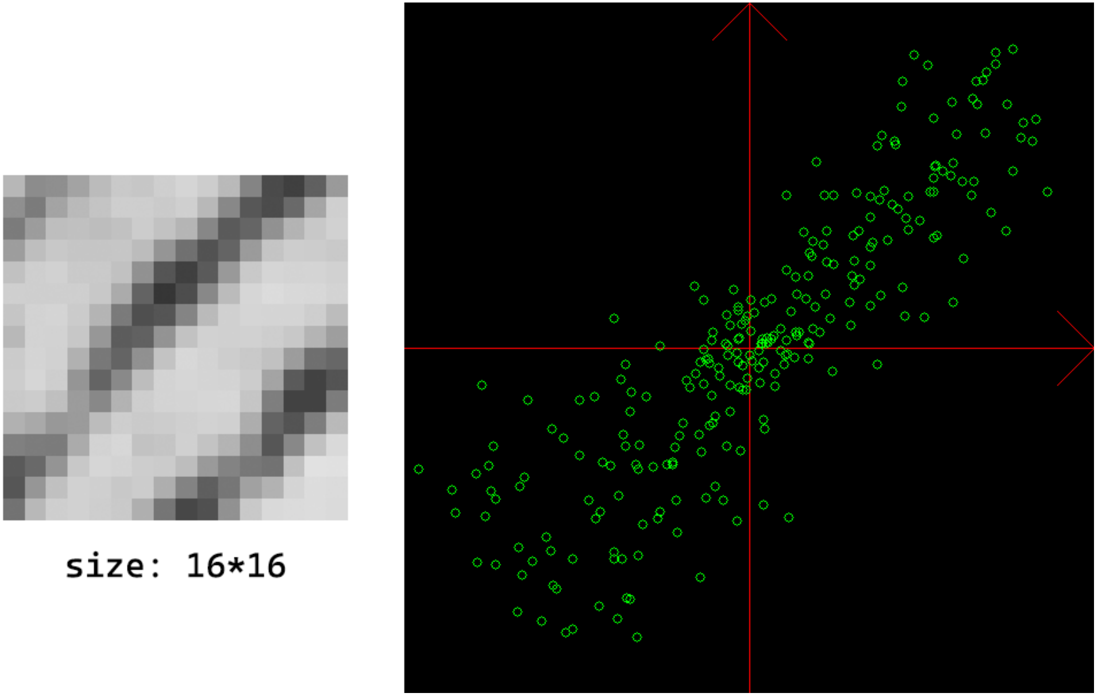


Figure 12: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (3)

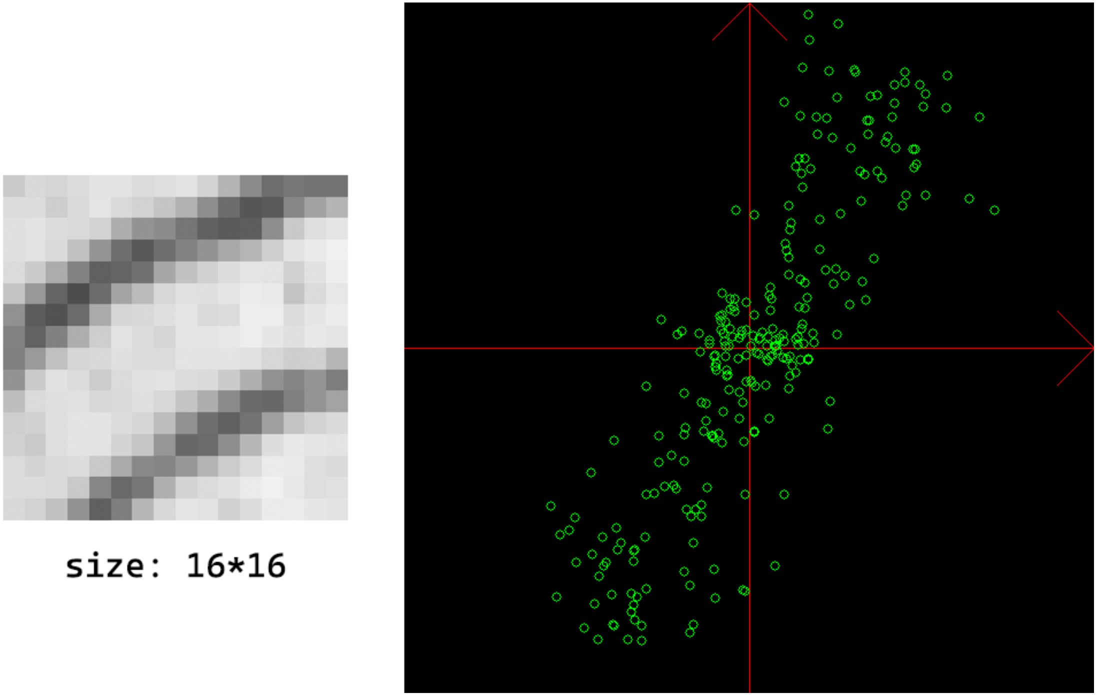


Figure 13: Relation between gx (horizontal axis) and gy (vertical axis) on a good block (4)

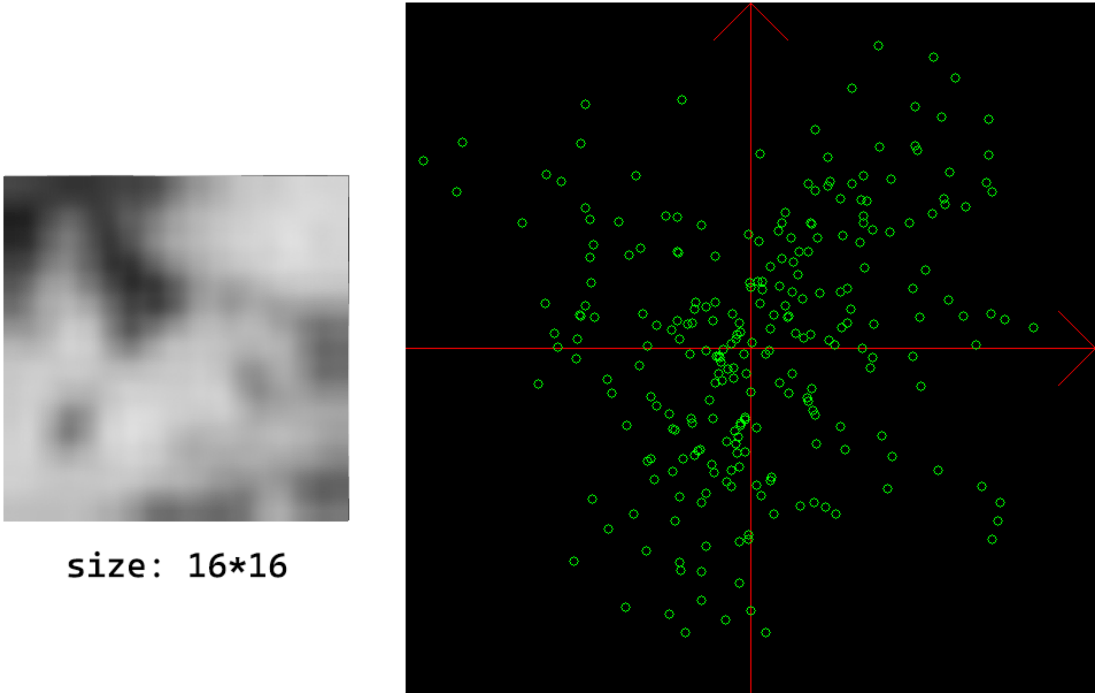


Figure 14: Relation between gx (horizontal axis) and gy (vertical axis) on a bad block (1)

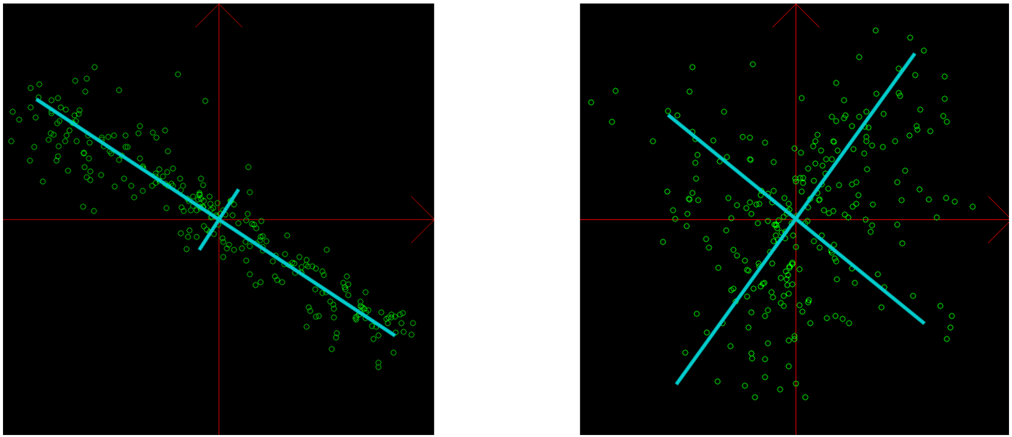


Figure 15: The direction along which the data set has minimum/maximum variance (the length of each line indicates the variance along respective direction)

Find unit vectors that minimize/maximize variance

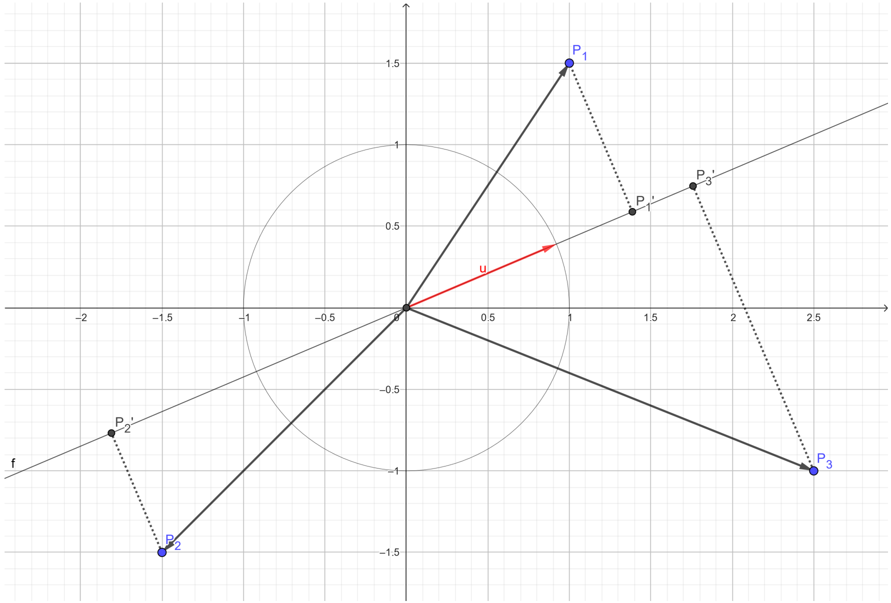


Figure 16: Projections of some points onto a unit vector

If $\vec{v}$ is a unit vector:

$$dist(O, P_i') = \vec{P}_i \cdot \vec{v} = \sum_{d=1}^{D} P_{id} v_d$$

Variance of projections:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d - \mu)^2$$

By performing a geometric transformation such that

$$\mu_x = \mu_y = 0$$

Variable μ of the equation above then has the value of 0

Thus, the formula to calculate the variance of projections is simplified to:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d)^2$$

The goal is to find a vector $\vec{v}$ with the length of 1 unit
such that V is maximized.
And thus, I add a Lagrange multiplier λ to the equation:

$$V = \frac{1}{N} \times \sum_{i=1}^{N} \sum_{d=1}^{D} (p_{id} v_d)^2 - \lambda(\sum_{d=1}^{D} v_d^2 - 1)$$

To find local min/max of V, I derive the equation into:

$$\frac{\delta V}{\delta v_a} = \frac{2}{N} \times \sum_{i=1}^{N} \left(p_{ia} \sum_{d=1}^{D} (p_{id} v_d) \right) - 2\lambda v_a$$

$$\text{At } \frac{\delta V}{\delta v_a} = 0:$$

$$\frac{2}{N}\sum_{i=1}^{N}\left(p_{ia}\sum_{d=1}^{D}(p_{id}v_d)\right) = 2\lambda v_a$$

$$\Leftrightarrow \sum_{d=1}^{D} v_d \left(\frac{2}{N}\sum_{i=1}^{N} p_{ia}p_{id}\right) = 2\lambda v_a$$

$$\Leftrightarrow \sum_{d=1}^{D} v_d \left(2cov(p_a, p_d)\right) = 2\lambda v_a$$

Since the image is two-dimensional, $\vec{v}$ has 2 components v_x and v_y
The equation above is simplified into:

$$\Leftrightarrow \begin{cases} v_x cov(p_x, p_x) + v_y cov(p_x, p_y) = \lambda v_x \\ v_x cov(p_y, p_x) + v_y cov(p_y, p_y) = \lambda v_y \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} var(p_x) & cov(p_x, p_y) \\ cov(p_y, p_x) & var(p_y) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \lambda \begin{bmatrix} v_x \\ v_y \end{bmatrix} \Leftrightarrow A\vec{v} = \lambda\vec{v}$$

The fact that A is a matrix and λ is a scalar implies that
$\vec{v}$ must be an eigenvector

Consequently, the eigenvectors are the directions along which
the data set has minimum/maximum variance

$$\left(\frac{\delta V}{\delta v_a} = 0\right)$$

And the eigenvalues λ indicate the variance
along those directions pointed by their respective eigenvectors

Why eigenvalues are equal to the data's variance along eigenvectors ?

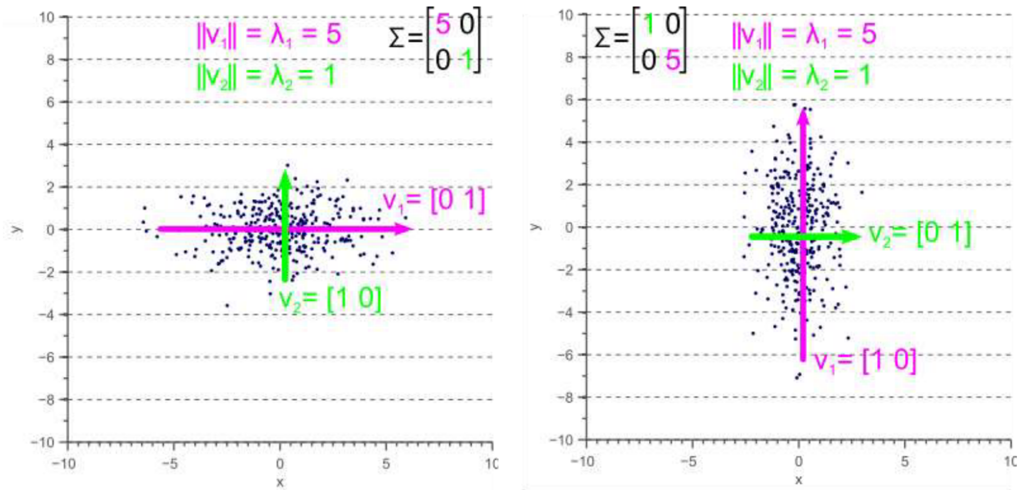


Figure 17: If cov(y, x) = 0

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \vec{v} = \lambda \vec{v} \Leftrightarrow \begin{bmatrix} a-\lambda & 0 \\ 0 & b-\lambda \end{bmatrix} \vec{v} = \vec{0} \Leftrightarrow (a-\lambda)(b-\lambda) = 0 \Leftrightarrow \begin{cases} \lambda_1 = a \\ \lambda_2 = b \end{cases}$$

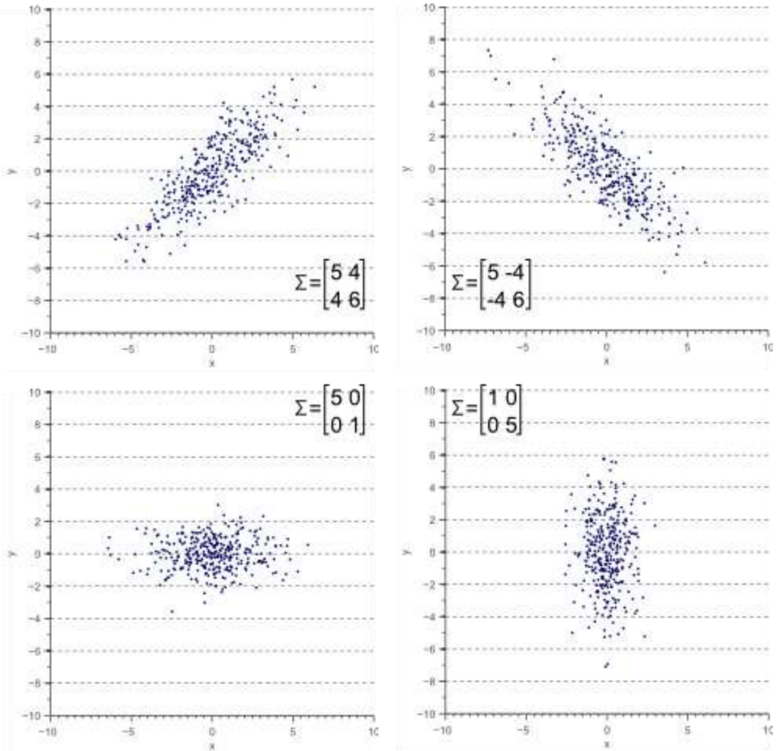


Figure 18: All distributions...

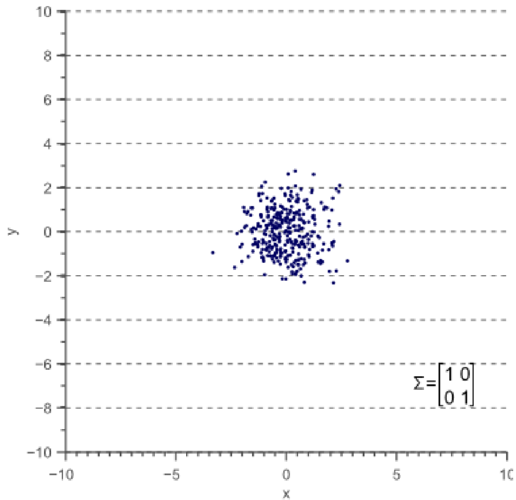


Figure 19: Are just a linearly transformed instance of this

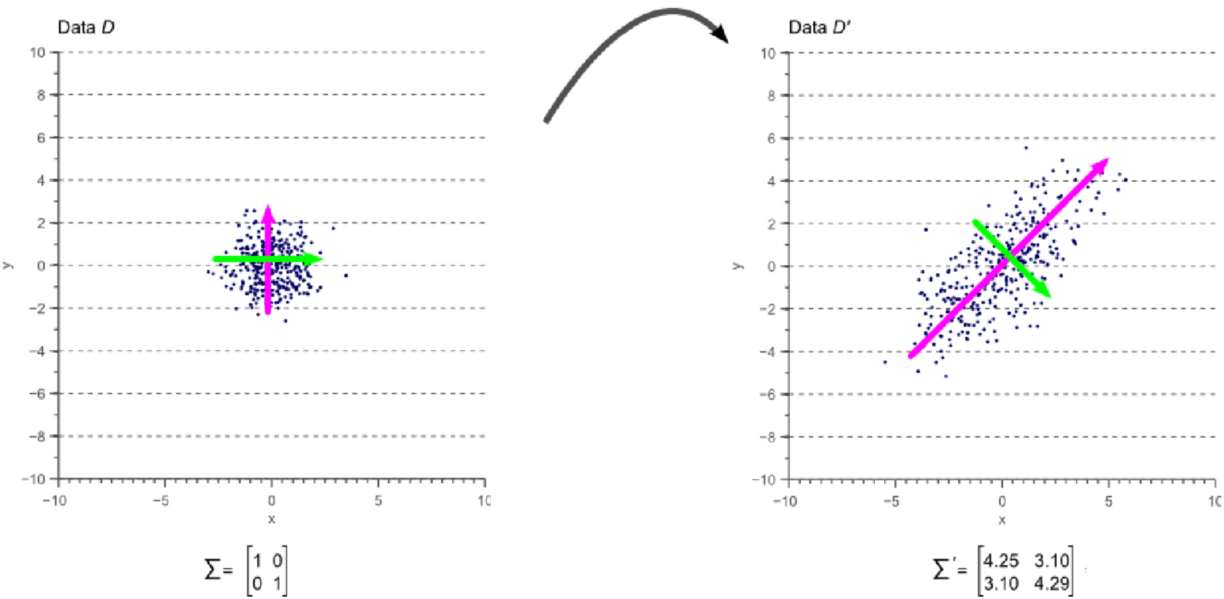


Figure 20: Therefore, the eigenvectors and eigenvalues are just simply linearly transformed along

The covariance matrix of 2 gradient images gx and gy is calculated as:

$$A = \begin{bmatrix} var(dx) & cov(dx, dy) \\ cov(dy, dx) & var(dy) \end{bmatrix} = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

We then find the eigenvalues λ_1 and λ_2 of A, which indicate
the maximum and minimum variance of the data set while being projected onto 1 line

$$A\vec{v} = \lambda\vec{v}$$

$$\Leftrightarrow A\vec{v} = \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \vec{v}$$

$$\Leftrightarrow A\vec{v} - \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \vec{v} = \vec{0}$$

$$\Leftrightarrow \left(A - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \vec{v} = \vec{0}$$

$$\Leftrightarrow \begin{bmatrix} a - \lambda & c \\ c & b - \lambda \end{bmatrix} \vec{v} = \vec{0}$$

Because $\vec{v}$ is non-zero vector:

$$det\left(\begin{bmatrix} a-\lambda & c \\ c & b-\lambda \end{bmatrix}\right) = 0$$

$$\Leftrightarrow (a-\lambda)(b-\lambda) - c^2 = 0$$

$$\Leftrightarrow \begin{cases} \lambda_1 = \frac{1}{2}\left((a+b) + \sqrt{(a-b)^2 + 4c^2}\right) \\ \lambda_2 = \frac{1}{2}\left((a+b) - \sqrt{(a-b)^2 + 4c^2}\right) \end{cases}$$

Thus, λ_1 is the maximum variance of the data set along one direction while λ_2 is the minimum.

The orientation certainty level (ocl) of a block can be then calculated as follow:

$$ocl = \frac{\lambda_2}{\lambda_1} \qquad (0 \leq ocl \leq 1)$$

With low (high) ocl values, the local structure and orientation of ridges and valleys are very regular (irregular), and therefore the block has good (bad) quality.

Figure 21: Masking blocks using STD

Figure 22: Combining the STD mask with OCL mask

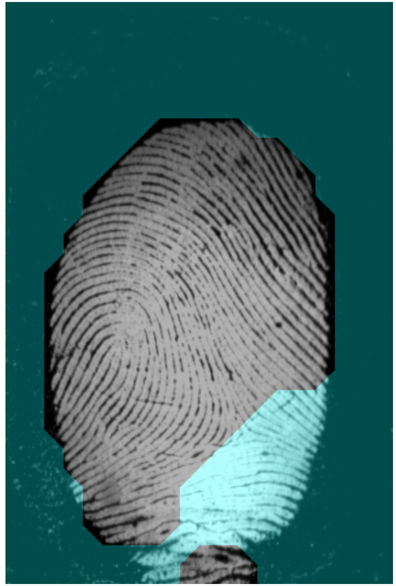
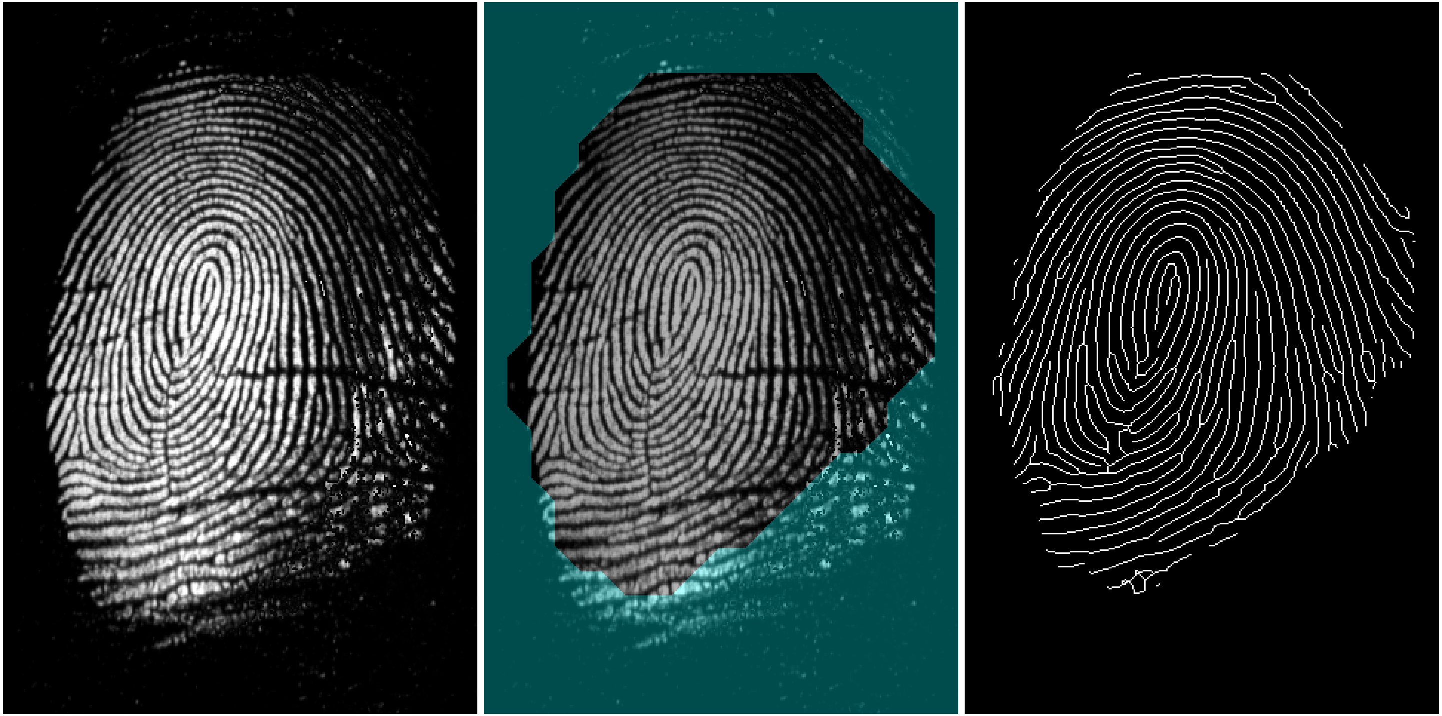
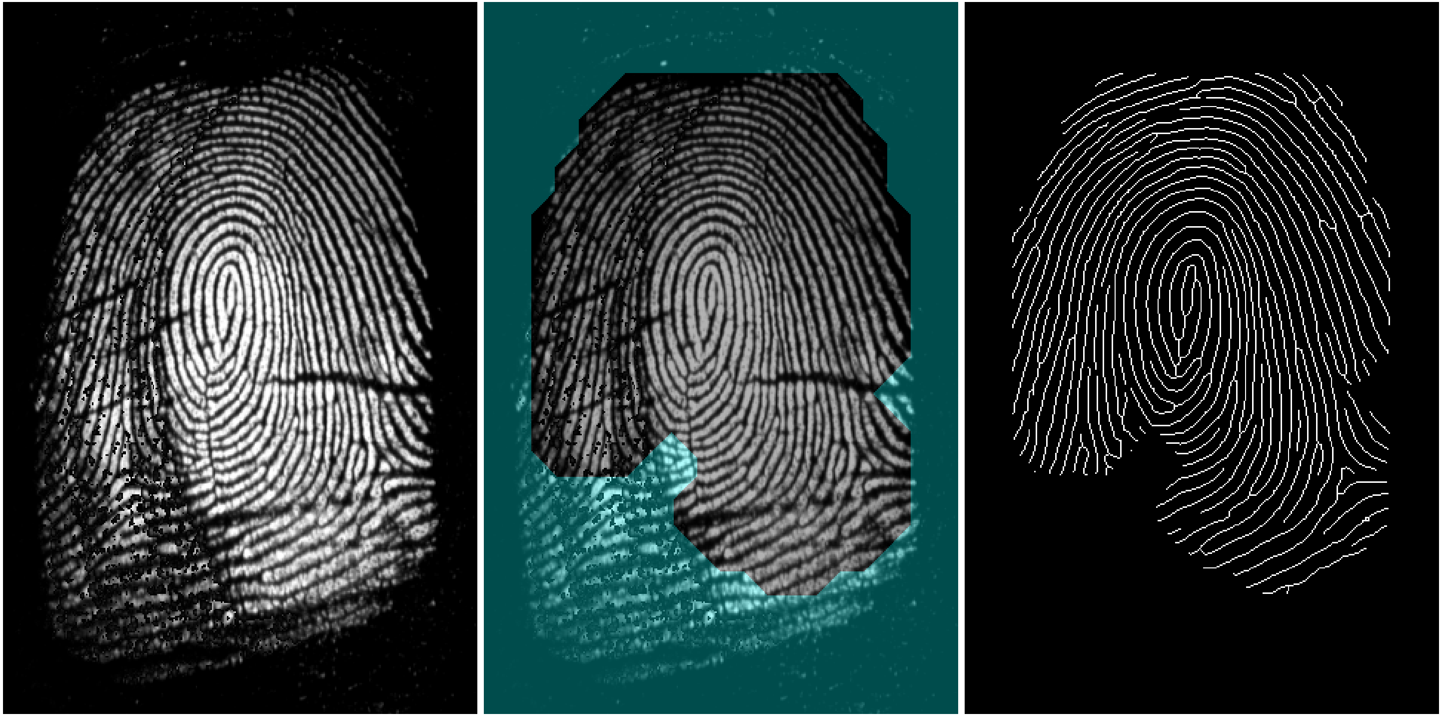


Figure 23: Smoothing the combined mask

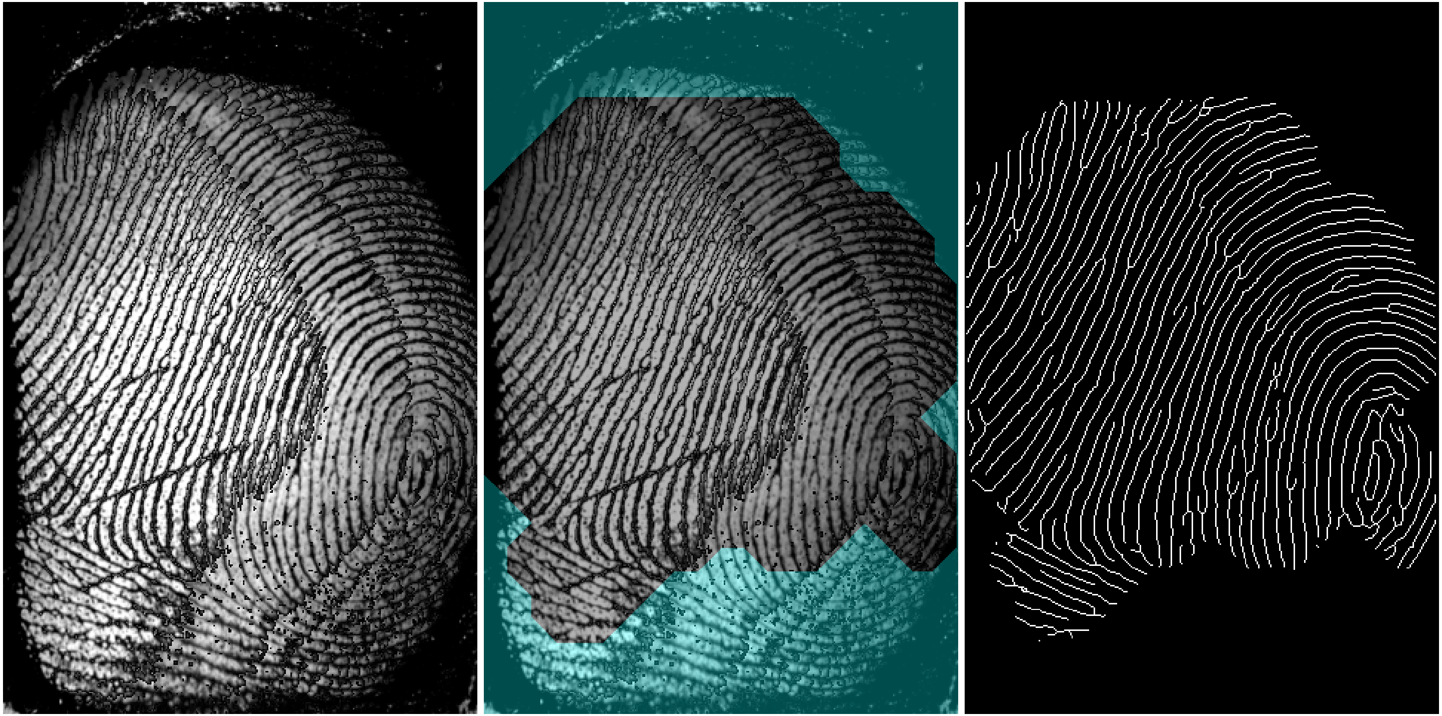
Using STD and OCL to mask bad blocks out



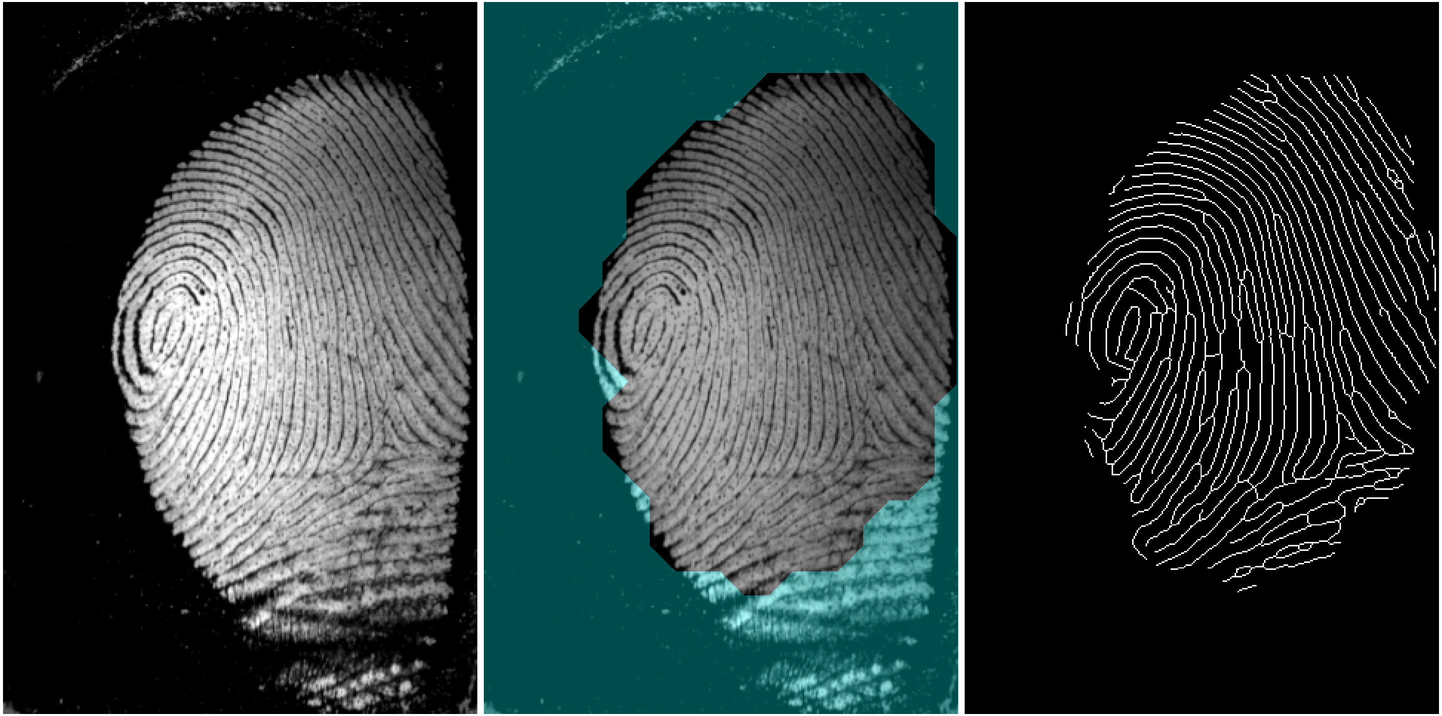
Using STD and OCL to mask bad blocks out



Using STD and OCL to mask bad blocks out

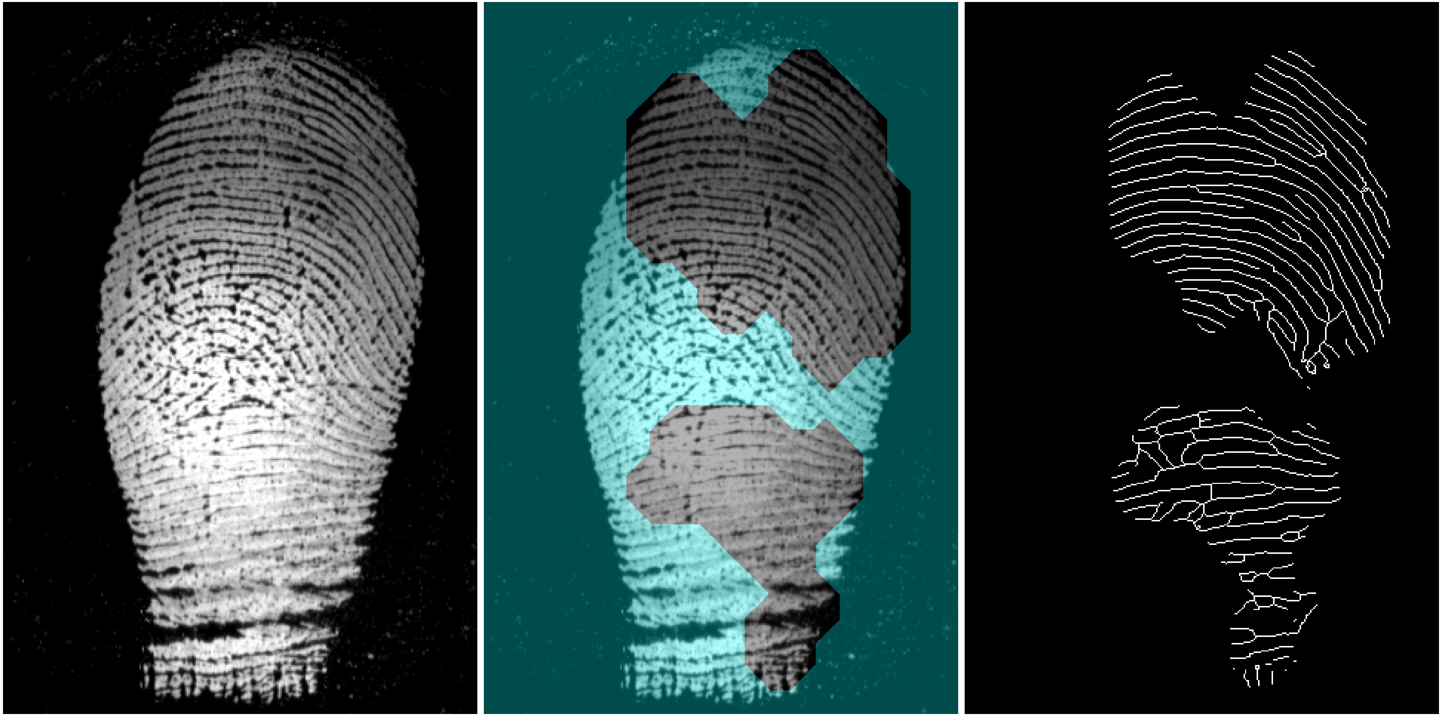


Using STD and OCL to mask bad blocks out

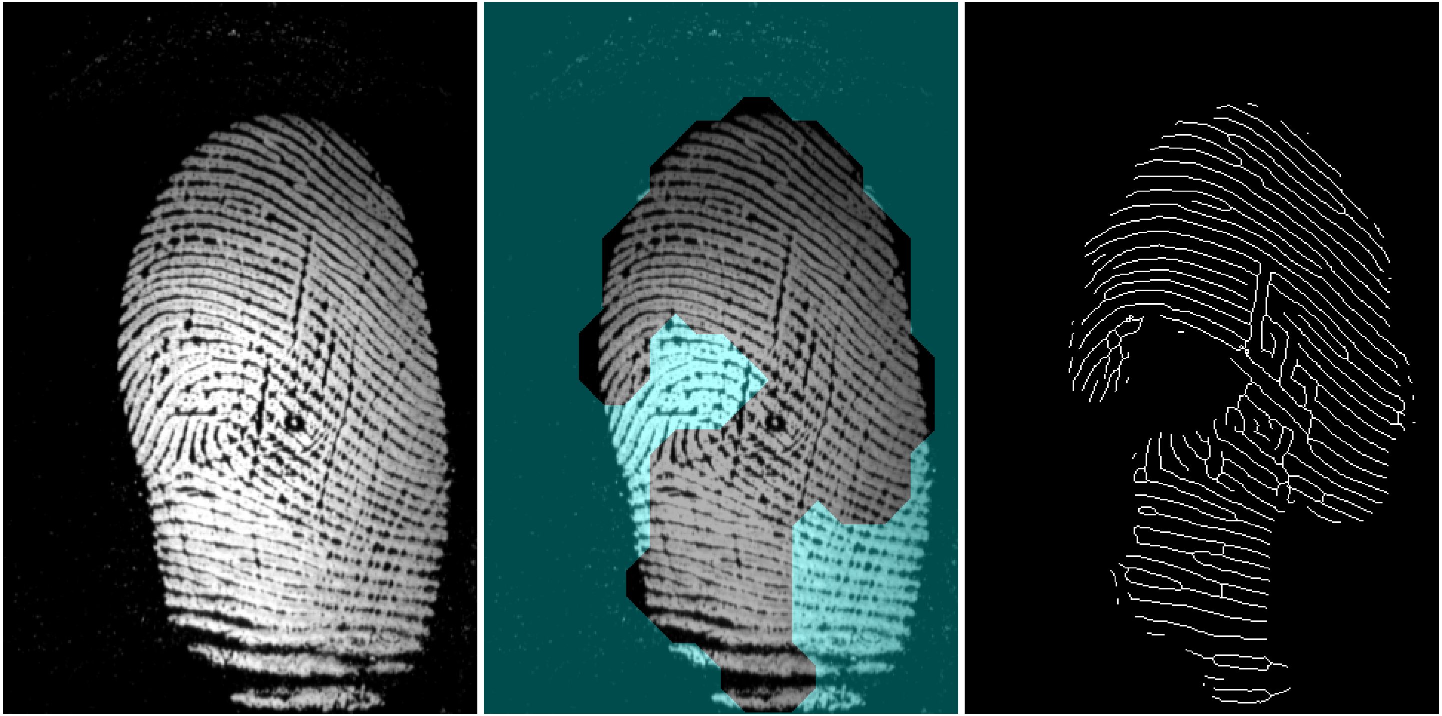


Using STD and OCL to mask redundant parts out

Using STD and OCL to mask redundant parts out



May be combined with the upcoming modules to discard a bad image



May be combined with the upcoming modules to discard a bad image