Assignment : Multiplying 2 square matrices

Let A & B be 2 square matrices $(n \times n)$

$$A = a_{ij} \text{ for } i,j \in [1,n]$$
$$B = b_{ij} \text{ for } i,j \in [1,n]$$

Product of A & B is $C = AB$
which is a $n \times n$ matrix where
$C = c_{ij}$ for $i,j \in [1,n]$

Where

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj} \text{ for } i,j\ [1,n]$$

Visualised

$$\begin{bmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & & \\ a_{i1} & & a_{ij} \end{bmatrix} \qquad \begin{bmatrix} b_{11} & \cdots & b_{1j} \\ \vdots & & b_{ij} \\ b_{i1} & & \end{bmatrix}$$

$$\begin{bmatrix} c_{11} & \cdots & c_{1j} \\ \vdots & & \\ c_{i1} & & c_{ij} \end{bmatrix}$$

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix}_{3\times 3}$$

$$c_{ij} = \sum_{k=1}^{3} a_{ik} \cdot b_{kj}$$

$$c_{11} = \sum_{k=1}^{3} a_{1k} \cdot b_{k1}$$

$$c_{11} = (a_{11} \cdot b_{11}) + (a_{12} \cdot b_{21}) + (a_{13} \cdot b_{31})$$
$$= (1 \times 9) + (2 \times 6) + (3 \times 7)$$
$$= 30$$

Similarly

$$c_{12} = \sum_{k=1}^{3} a_{1k} \cdot b_{k2}$$
$$= (a_{11} \cdot b_{12}) (a_{12} \cdot b_{22}) + (a_{13} \cdot b_{32})$$
$$= (1 \times 8) + (2 \times 5) + (3 \times 2)$$
$$= 24$$

$$c_{13} = \sum_{k=1}^{3} a_{1k}\, b_{k3}$$

$$c_{13} = (a_{11} \cdot b_{13}) + (a_{12} \cdot b_{23}) + (a_{13} \cdot b_{33})$$
$$= (1 \times 7) + (2 \times 4) + (3 \times 1)$$
$$= 18$$

Similarly, the rest can be calculated.

$$C = AB = \begin{bmatrix} 30 & 24 & 18 \\ 84 & 69 & 54 \\ 138 & 114 & 90 \end{bmatrix}$$

Strassen's algorithm for matrix multip

Example using Strassen's algo.

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \\ 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 3 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 \end{bmatrix}$$

Partitioning the matrices:

$$A_{11} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A_{21} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A_{22} = \begin{bmatrix} \; \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad B_{12} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \quad B_{21} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \quad B_{22} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

Using Strassen's definitions for M_k

$$M_1 = (A_{11} + A_{22})(B_{11} + B_{22})$$
$$M_2 = (A_{21} + A_{22})(B_{11})$$
$$M_3 = A_{11}(B_{12} - B_{11})$$
$$M_4 = A_{22}(B_{21} - B_{11})$$
$$M_5 = (A_{11} + A_{12}) B_{22}$$
$$M_6 = (A_{21} - A_{11})(B_{11} + B_{12})$$
$$M_7 = (A_{12} - A_{22})(B_{21} + B_{22})$$

$M_1 \Rightarrow$

$$\begin{bmatrix} 1+1 & 2+2 \\ 3+3 & 4+4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1+4 & 1+4 \\ 1+4 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \times \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 30 & 30 \\ 70 & 70 \end{bmatrix}$$

$M_2 \Rightarrow$

$$\begin{bmatrix} 1+1 & 2+2 \\ 3+3 & 4+4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 14 & 14 \end{bmatrix}$$

$M_3 \Rightarrow$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} -6 & -6 \\ -14 & -4 \end{bmatrix}$$

$M_4 \Rightarrow$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 14 & 14 \end{bmatrix}$$

$$M_5 \Rightarrow \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 24 & 24 \\ 56 & 56 \end{bmatrix}$$

$$M_6 \Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_7 \Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 7 & 7 \\ 7 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_{11} = M_1 + M_4 - M_5 + M_7$$
$$C_{12} = M_3 + M_5$$
$$C_{21} = M_2 + M_4$$
$$C_{22} = M_1 - M_2 + M_3 + M_6$$

$$C_{11} = \begin{bmatrix} 30 & 30 \\ 70 & 70 \end{bmatrix} + \begin{bmatrix} 6 & 6 \\ 14 & 14 \end{bmatrix} - \begin{bmatrix} 24 & 24 \\ 56 & 56 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ 28 & 28 \end{bmatrix}$$

$$C_{12} = \begin{bmatrix} -6 & -6 \\ -14 & -14 \end{bmatrix} + \begin{bmatrix} 24 & 24 \\ 56 & 56 \end{bmatrix} = \begin{bmatrix} 18 & 18 \\ 42 & 42 \end{bmatrix}$$

$$C_{21} = \begin{bmatrix} 6 & 6 \\ 14 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 6 \\ 14 & 14 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ 28 & 28 \end{bmatrix}$$

$$C_{22} = \begin{bmatrix} 30 & 30 \\ 70 & 70 \end{bmatrix} - \begin{bmatrix} 6 & 6 \\ 14 & 14 \end{bmatrix} + \begin{bmatrix} -6 & -6 \\ -14 & -14 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 18 & 18 \\ 42 & 42 \end{bmatrix}$$

$$C_{final} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 12 & 18 & 18 \\ 28 & 28 & 42 & 42 \\ 12 & 12 & 18 & 18 \\ 28 & 28 & 42 & 42 \end{bmatrix}$$