

# CM 2062 - Statistical Computing with R

## Lab Sheet 2

### List

Lists are the R objects which contain elements of different types like - numbers, strings, vectors and another list inside it. List is created using **list()** function.

```
> list_data <- list("Red", c(21,32,11), TRUE, 51.23)
> list_data
[[1]]
[1] "Red"

[[2]]
[1] 21 32 11

[[3]]
[1] TRUE

[[4]]
[1] 51.23
```

### Give names to the elements in the list

```
> names(list_data) <- c("Colour", "Vector", "Logical", "Number")
> list_data
$Colour
[1] "Red"

$Vector
[1] 21 32 11

$Logical
[1] TRUE

$Number
[1] 51.23
```

## Accessing the elements in the list

```
> list_data[1]
$Colour
[1] "Red"
```

```
> list_data[2]
$Vector
[1] 21 32 11
```

#The \$ operator returns a named element of a list.

```
> list_data$Vector
[1] 21 32 11
```

Add element at the end of the list.

```
> list_data[5] <- "New element"
> list_data
```

Remove the last element.

```
> list_data[4] <- NULL
> list_data
```

## Merging Lists

```
> merged.list <- c(list1, list2)
> merged.list
```

## Convert the lists to vectors

```
> list1 <- list(1:5)
> list2 <- list(10:14)
> v1 <- unlist(list1)
> v2 <- unlist(list2)
> result <- v1+v2
> result
[1] 11 13 15 17 19
```

## Arrays

An array can be considered as a multiply subscripted collection of data entries, for example numeric. R allows simple facilities for creating and handling arrays. You can use `array()` function to create an array.

```
> a1 <- array(1:5,dim = c(1,5))
> a1
      [,1] [,2] [,3] [,4] [,5]
[1,]      1      2      3      4      5
```

```
> a2 <- array(1:3,dim = c(3,1))
> a2
      [,1]
[1,]      1
[2,]      2
[3,]      3
```

```
> m1 <- array(1:15,dim = c(3,5))
> m1
      [,1] [,2] [,3] [,4] [,5]
[1,]      1      4      7     10     13
[2,]      2      5      8     11     14
[3,]      3      6      9     12     15
```

# Multidimensional Array

Example 1

```
> arr <- array(2:13, dim = c(2, 3, 2))
> arr
, , 1
```

```
      [,1] [,2] [,3]
[1,]      2      4      6
[2,]      3      5      7
```

```
, , 2
```

```
      [,1] [,2] [,3]
[1,]      8     10     12
[2,]      9     11     13
```

Example 2

```
array1 <- c(5, 10, 15, 20)
array2 <- c(25, 30, 35, 40, 45, 50, 55, 60)
```

```
final <- array(c(array1, array2),dim =c(4,4,3))
```

```
> final <- array(c(array1, array2),dim =c(4,4,3))
> final
```

```
, , 1
```

	[ ,1]	[ ,2]	[ ,3]	[ ,4]
[1 ,]	5	25	45	5
[2 ,]	10	30	50	10
[3 ,]	15	35	55	15
[4 ,]	20	40	60	20

```
, , 2
```

	[ ,1]	[ ,2]	[ ,3]	[ ,4]
[1 ,]	25	45	5	25
[2 ,]	30	50	10	30
[3 ,]	35	55	15	35
[4 ,]	40	60	20	40

```
, , 3
```

	[ ,1]	[ ,2]	[ ,3]	[ ,4]
[1 ,]	45	5	25	45
[2 ,]	50	10	30	50
[3 ,]	55	15	35	55
[4 ,]	60	20	40	60

```
> final <- array (c (array1 , array2),dim =c(4,3,3))
```

```
> final
```

```
, , 1
```

	[ ,1]	[ ,2]	[ ,3]
[1 ,]	5	25	45
[2 ,]	10	30	50
[3 ,]	15	35	55
[4 ,]	20	40	60

```
, , 2
```

	[ ,1]	[ ,2]	[ ,3]
[1 ,]	5	25	45
[2 ,]	10	30	50
[3 ,]	15	35	55
[4 ,]	20	40	60

```
, , 3
```

```
      [,1] [,2] [,3]
[1,]    5   25   45
[2,]   10   30   50
[3,]   15   35   55
[4,]   20   40   60
```

```
> final <- array (c (array1 , array2),dim =c(3,3,3))
```

```
> final
```

```
, , 1
```

```
      [,1] [,2] [,3]
[1,]    5   20   35
[2,]   10   25   40
[3,]   15   30   45
```

```
, , 2
```

```
      [,1] [,2] [,3]
[1,]   50    5   20
[2,]   55   10   25
[3,]   60   15   30
```

```
, , 3
```

```
      [,1] [,2] [,3]
[1,]   35   50    5
[2,]   40   55   10
[3,]   45   60   15
```

Let's rename our array to "Arr1" and "Arr2" by using "matrix.names". Also, the rows name changed to ("row1", "row2") and column names will be changed to ("column1", "column2", "column3") respectively. The dimension of the matrix is 2 rows and 3 columns.

```
> array1 <- c (9 , 18 )
> array2 <- c (27, 36)
> r.names = c ("row1", "row2")
> c.names = c ("column1", "column2", "column3")
> m.names = c ("Arr1", "Arr2")
> final <- array (c (array1, array2), dim=c (2,3,2),
+               dimnames=list (r.names, c.names, m.names))
> final
```

```
, , Arr1
```

	column1	column2	column3
row1	9	27	9
row2	18	36	18

```
, , Arr2
```

	column1	column2	column3
row1	27	9	27
row2	36	18	36

### Exercise

Create an array with the name "arr" and dim=c (3,3,1), with the below two vectors.

```
a1= c (1,2,3,4)
```

```
a2= c (5,6,7,8,9)
```

Rename the column names as (c1,c2,c3) and row names as (r1,r2,r3) and the matrix name as "first".

### Indexing in an array

In the previous Exercise array is,

```
, , first
```

	c1	c2	c3
r1	1	4	7
r2	2	5	8
r3	3	6	9

Let's see how the elements in the array can be extracted with the following example

1. Let's extract number '7' from the above array 'arr'.

```
> arr[1,3,1]  
[1] 7
```

2. To access multiple values at once, you need to specify the range you want.

```
> arr[1:2,1:2,1]  
      c1 c2  
r1     1  4  
r2     2  5
```

3. You can access the entire array 'arr' with the following syntax where 'arr[ , ,1]' specifies to include all rows and columns each separated by commas, which are indicated by space. The 1 specifies the array 'arr' to be extracted.

```
> arr[, ,1]
      c1 c2 c3
r1    1  4  7
r2    2  5  8
r3    3  6  9
```

4. You can get the entire second row by following code where `arr[2, ,1]` gets the second row with space, and 1 is the 'arr' to be extracted.

```
> arr[2, ,1]
c1 c2 c3
 2  5  8
```

5. You can get the entire second column by following code where `arr[,2,1]` space with 2 is the second column, and 1 is the 'arr' to be extracted.

```
> arr[,2,1]
r1 r2 r3
 4  5  6
```

5. You can change the value of an element in the array.

```
> arr[1,3,1] <- 5
> arr
, , first
```

```
      c1 c2 c3
r1    1  4  5
r2    2  5  8
r3    3  6  9
```

## Matrices

A matrix is a two dimensional array. A matrix can be generated by the `array(...)` function with the `dim` argument set to be a vector of length 2. Alternatively, we can use the `matrix(...)` command or provide a `dim` attribute for a vector.

```
> m1 <- array(1:15,dim = c(3,5))
> m1
      [,1] [,2] [,3] [,4] [,5]
[1,]     1     4     7    10    13
[2,]     2     5     8    11    14
[3,]     3     6     9    12    15
```

```
> m2 <- matrix(1:15,3,5)
> m2
```

```
> m2 <- matrix(1:15,nrow=3,ncol=3)
> m2
```

```
> m3 <- 1:15
> dim(m3) <- c(3,5)
> m3
```

By default the matrix function reorders a vector into columns, but we can also tell R to use rows instead.

```
m3 <- matrix(1:15, nrow=3, ncol=3, byrow = TRUE)
m3
```

Define the column and row names

```
> rownames = c("row1", "row2", "row3")
> colnames = c("col1", "col2", "col3")
> m3 <- matrix(1:15, nrow=3, ncol=3, dimnames = list(rownames, colnames))
> m3
```

	col1	col2	col3
row1	1	4	7
row2	2	5	8
row3	3	6	9

If their sizes match, vectors can be combined to form matrices, and matrices can be combined with vectors or matrices to form other matrices. The functions that do this are `cbind` and `rbind`.

`cbind` binds together columns of two objects. One thing it can do is put vectors together to form a matrix:

```
> m4 <- cbind(1:3, 4:6, 5:7)
> m4
```

	[,1]	[,2]	[,3]
[1,]	1	4	5
[2,]	2	5	6
[3,]	3	6	7

```
> m5 <- rbind(1:3, 4:6)
> m5
```

	[,1]	[,2]	[,3]
[1,]	1	2	3
[2,]	4	5	6

The **`diag()`** function can be used create diagonal Matrix.

```
> diag(3)
```

	[,1]	[,2]	[,3]
[1,]	1	0	0
[2,]	0	1	0
[3,]	0	0	1



```
> diag(1:3)
      [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    2    0
[3,]    0    0    3
```

## Index and Sub-set in Matrices

```
> m3[1,2]
[1] 2
```

Here we accessed the element in the first row and the second column. We could also subset an entire row or column.

```
> m3[1,]
[1] 1 2 3
```

```
> m3[,2]
[1] 2 5 8
```

We can also use vectors to subset more than one row or column at a time. Here we subset to the first and third column of the second row.

```
> m3[2, c(1, 3)]
[1] 4 6
```

We can also subset the diagonal elements.

```
> diag(m3)
[1] 1 5 9
```

## Arithmetic Operations in Matrices

You can perform arithmetic operations element by element in matrices using arithmetic operators.

```
> rowSums(m1)
```

```
[1] 35 40 45
```

```
> colSums(m1)
```

```
[1] 6 15 24 33 42
```

```
> m1+m1
```

```
      [,1] [,2] [,3] [,4] [,5]
[1,]    2    8   14   20   26
[2,]    4   10   16   22   28
[3,]    6   12   18   24   30
```

```
> m1-m2
      [,1] [,2] [,3] [,4] [,5]
[1,]    0    0    0    0    0
[2,]    0    0    0    0    0
[3,]    0    0    0    0    0
```

```
> 3*m1
      [,1] [,2] [,3] [,4] [,5]
[1,]     3    12    21    30    39
[2,]     6    15    24    33    42
[3,]     9    18    27    36    45
```

```
> m1*m2
      [,1] [,2] [,3] [,4] [,5]
[1,]     1    16    49   100   169
[2,]     4    25    64   121   196
[3,]     9    36    81   144   225
```

# Notice that m1\*m2 does not result the standard matrix multiplication. It is element-wise multiplication.

```
> t(m1)
      [,1] [,2] [,3]
[1,]     1     2     3
[2,]     4     5     6
[3,]     7     8     9
[4,]    10    11    12
[5,]    13    14    15
```

# Transpose of m1

If two matrices are conformal (that is, matrices for which the number of columns of the first is equal to the number of rows of the other), then can be multiplied using the operator `%*%` to result the matrix multiplication.

```
> m3 <- matrix(1:6,3,2)
> m3
```

```
      [,1] [,2]
[1,]     1     4
[2,]     2     5
[3,]     3     6
```

```
> m4 <- matrix(1:6,2,3)
> m4
```

```
      [,1] [,2] [,3]
[1,]     1     3     5
[2,]     2     4     6
```

```
> m3%*%m4
      [,1] [,2] [,3]
[1,]    9   19   29
[2,]   12   26   40
[3,]   15   33   51
```

Matrix inversion and (equivalent) solution of linear systems of equations can be performed using the function `solve(...)`. For example, to solve the system  $ax = b$ , we would use `solve(a, b)`. If a single matrix argument is passed to `solve(...)`, it will return the inverse of the matrix.

### Example

Consider the following system of equations.

$$\begin{aligned}x_1 + 2x_2 &= 1 \\ 3x_1 + x_2 &= 0\end{aligned}$$

In matrix form, this becomes  $ax = b$  where

$$a = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

```
> m6 <- array(c(1,3,2,1),c(2,2))
> m6
      [,1] [,2]
[1,]    1    2
[2,]    3    1

> v1 <- array(c(1,0), c(2,1))
> v1
      [,1]
[1,]    1
[2,]    0

> solve(m6,v1)
      [,1]
[1,] -0.2
[2,]  0.6

> solve(m6)
      [,1] [,2]
[1,] -0.2  0.4
```

```
[2,] 0.6 -0.2
```

```
> solve(m6) %*% v1 # does the same as solve(m6,v1)
```

```
[,1]
```

```
[1,] -0.2
```

```
[2,] 0.6
```