Smith Chart Essentials

Why Smith Chart?

$$Z = R + jX \xrightarrow{\text{Normalized with } Z_0} \bar{Z} = r + jx$$

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\bar{Z} - 1}{\bar{Z} + 1}$$

$$= \frac{r + jx - 1}{r + jx + 1} = \frac{(r-1) + jx}{(r+1) + jx}$$

$$\bar{Z} = r + jx$$

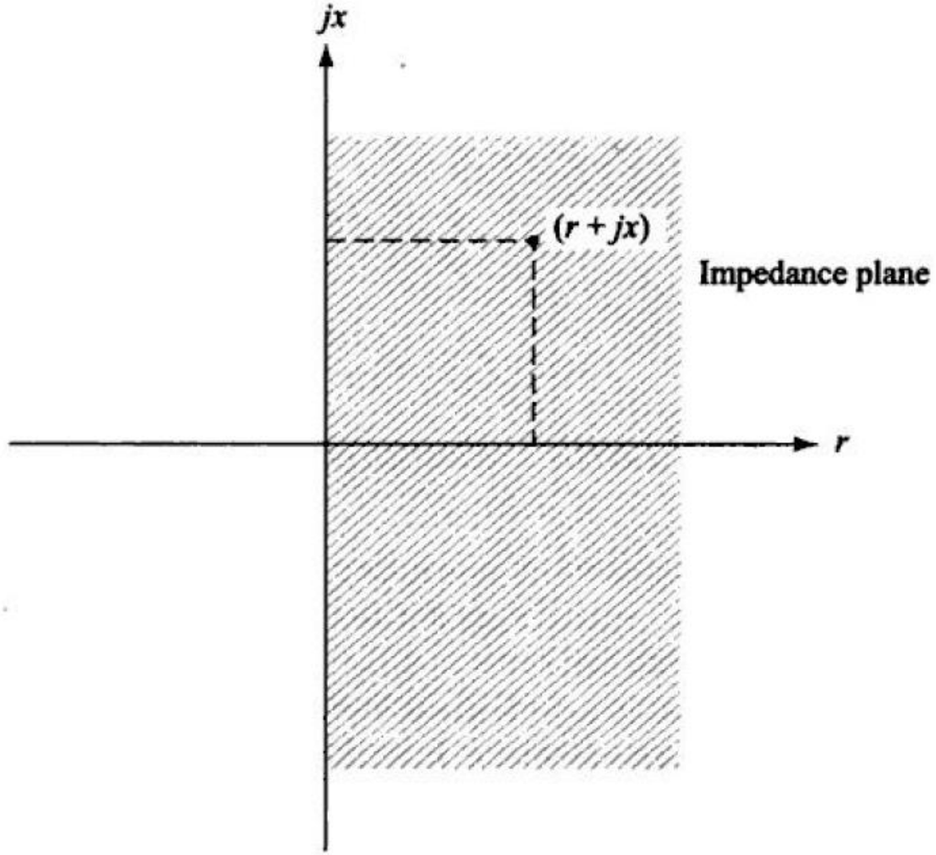


Fig. 2.12 *Complex impedance plane.*

$$\Gamma \equiv u + jv \equiv Re^{j\theta}$$

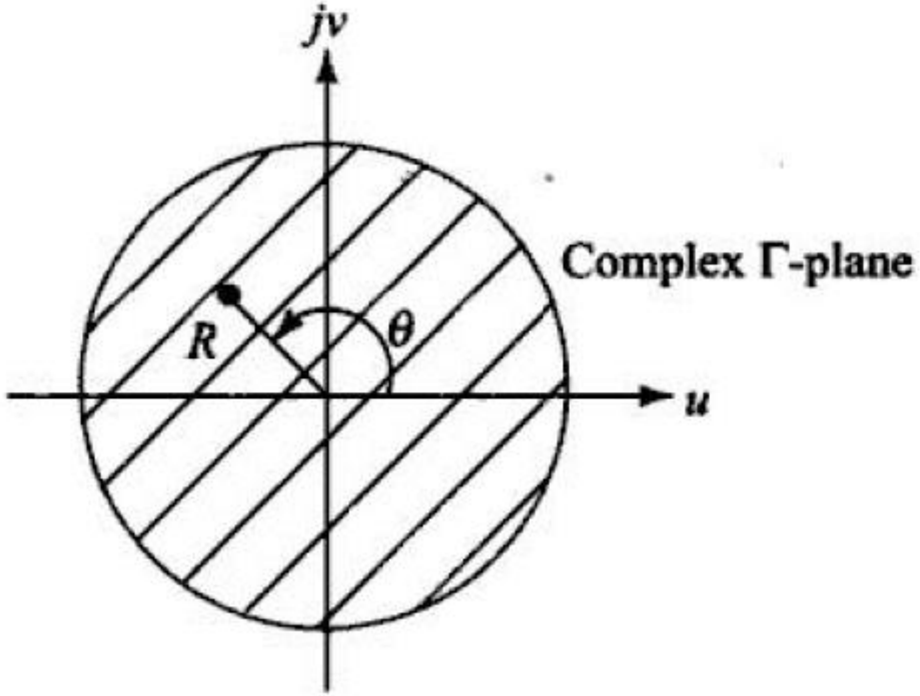


Fig. 2.13 *Complex reflection coefficient plane.*

Mapping a point in **Z** plane to Γ plane

$$\overline{Z} = r + jx$$

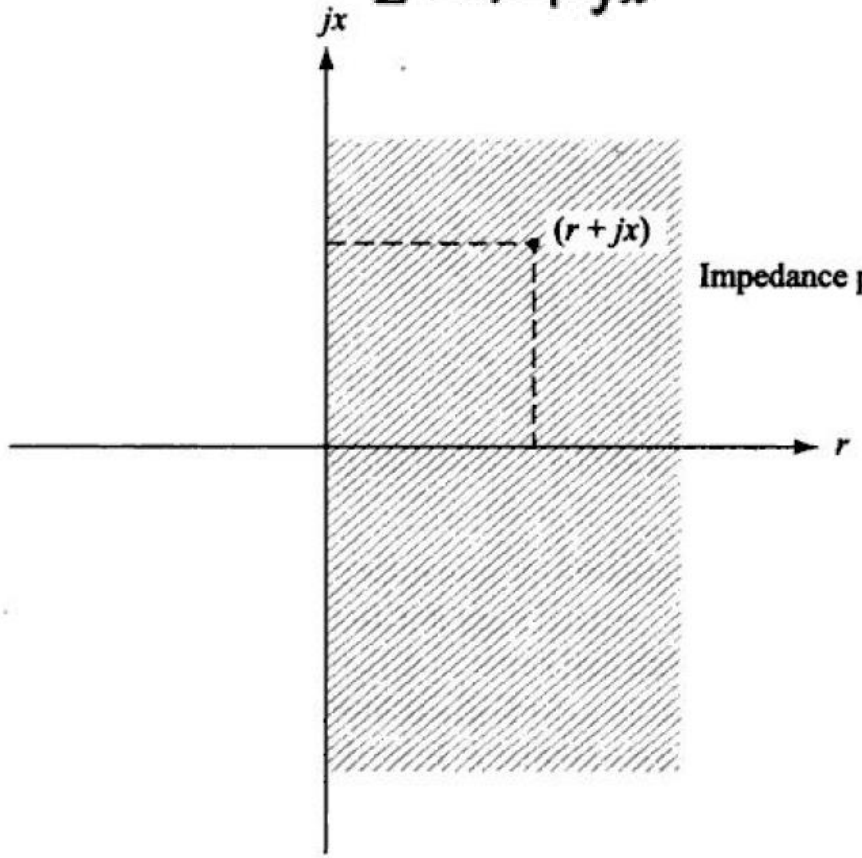


Fig. 2.12 *Complex impedance plane.*

$$\Gamma \equiv u + jv \equiv Re^{j\theta}$$

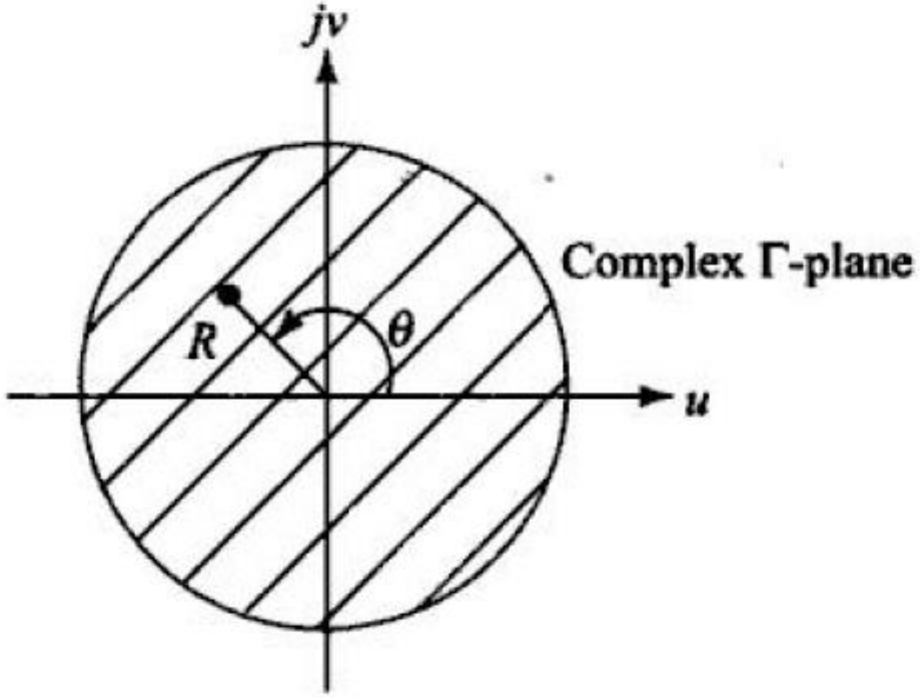


Fig. 2.13 *Complex reflection coefficient plane.*

Inverting the equation and changing the subject,

$$\bar{Z} = \frac{1+\Gamma}{1-\Gamma}$$

$$r + jx = \frac{1+(u+jv)}{1-(u+jv)}$$

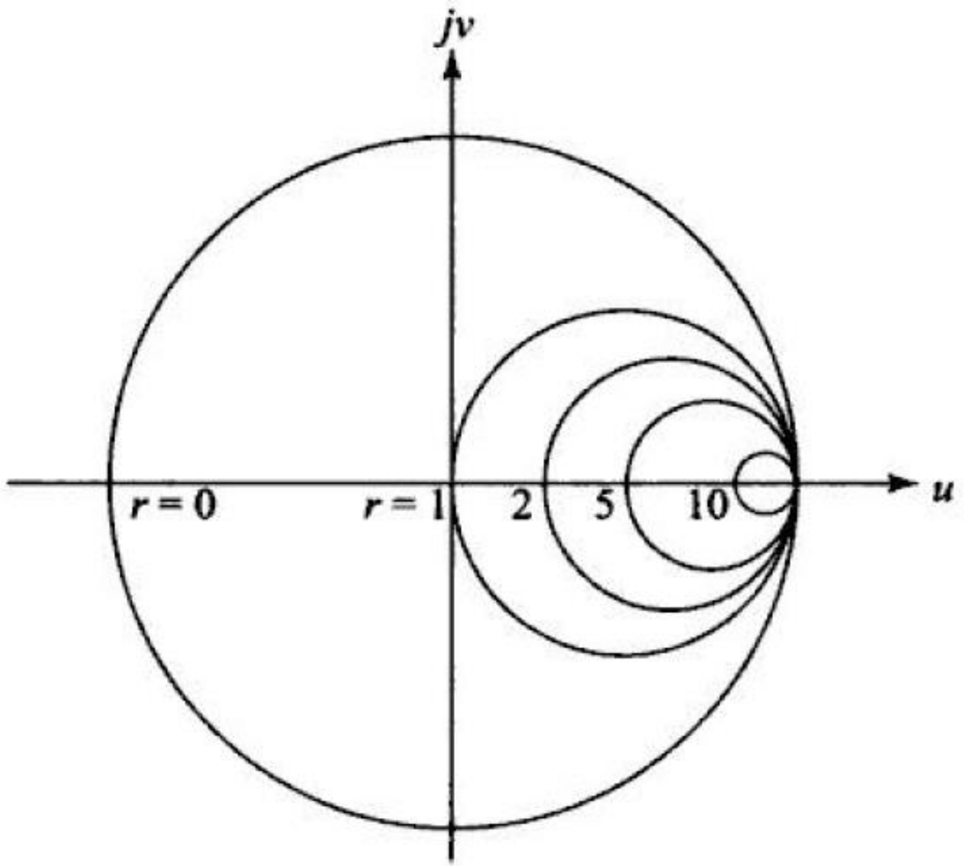
Separate in to real and imaginary parts.

$$u^2 - 2\left(\frac{r}{r+1}\right)u + v^2 + \left(\frac{r-1}{r+1}\right) = 0$$

$$u^2 + v^2 - 2u - \left(\frac{2}{x}\right)v + 1 = 0$$

Two Equations representing circles in the Complex Reflection coefficient plane

Constant Resistance Circles



ig. 2.14 *Constant resistance circles in the complex Γ-plane.*

$$u^2 - 2\left(\frac{r}{r+1}\right)u + v^2 + \left(\frac{r-1}{r+1}\right) = 0$$

What is the center and radius for constant r?

We can note following things about the constant resistance circles.

(a) The circles always have centres on the real Γ-axis (u-axis).

(b) All circles pass through the point(1, 0) in the complex Γ plane.

(c) For $r = 0$ the center lies at the origin of the Γ plane and it shifts to the right as r increases.

(d) As r increases the radius of the circle goes on reducing and for $r \to \infty$ the radius approaches zero, i.e. the circle reduces to a point.

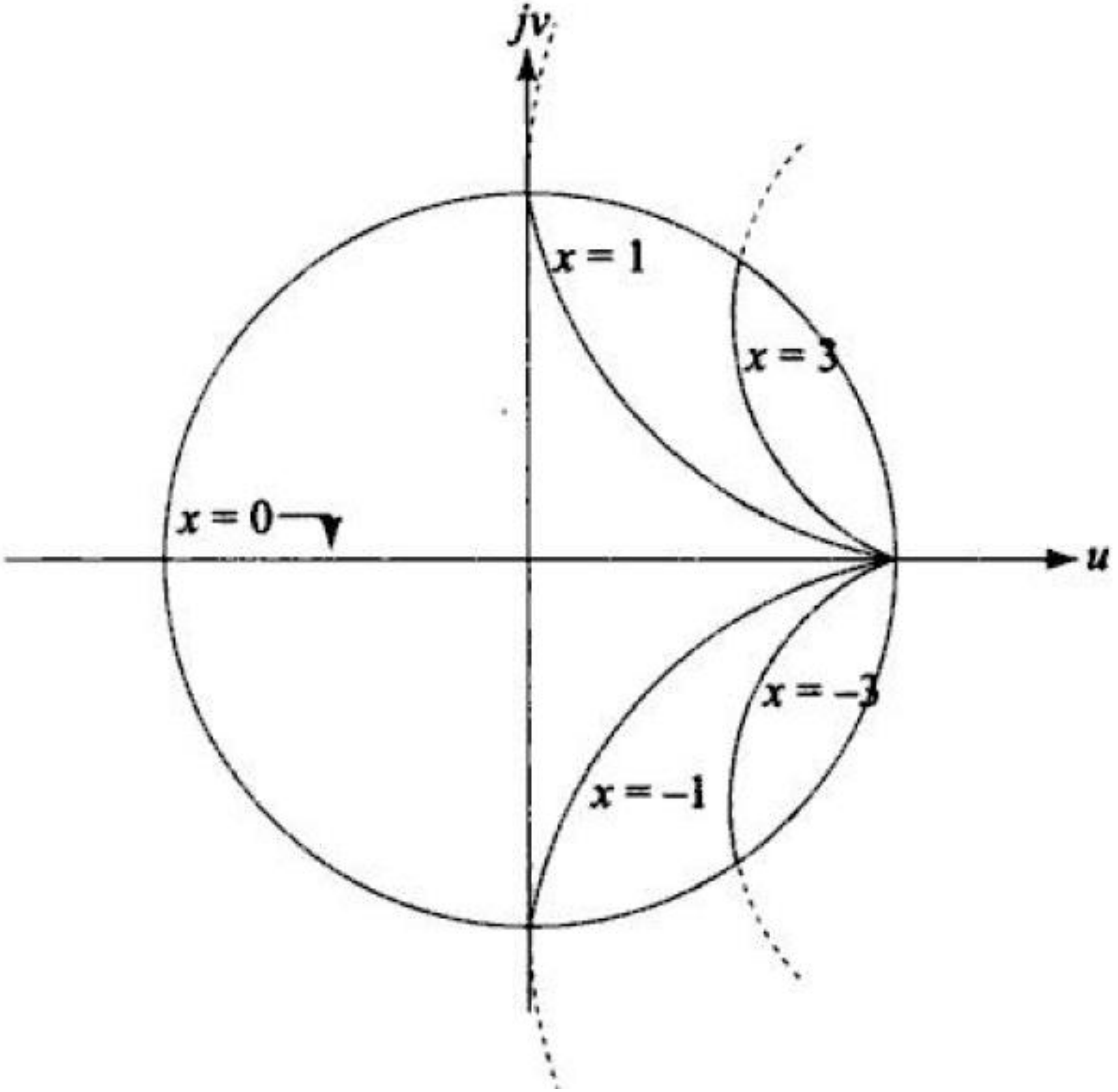
(e) The outermost circle with center (0, 0) and radius unity, corresponds to $r = 0$ or in other words represents reactive loads only.

(f) The right most point on the unit circle $(1, 0)$ represents $r = 0$ as well as $r = \infty$.

Constant Reactance Circles

$$u^2 + v^2 - 2u - \left(\frac{2}{x}\right)v + 1 = 0$$

What is the center and radius for constant x?



(a) These circles have their centers on a vertical line passing through point $(1, 0)$.

(b) For positive x the center lies above the real Γ-axis and for negative x, the center lies below the real Γ-axis.

(c) For $x = 0$, the center is at $(1, \pm\infty)$ and radius is ∞. This circle therefore represents a straight line.

(d) As the magnitude of the reactance increases the center moves towards the real Γ-axis and it lies on the real Γ-axis at (1,0) for $x = \pm\infty$.

(e) As the magnitude of the reactance increases, the radius of the circle $(\frac{1}{x})$ decreases and it approaches zero as $x \to \pm\infty$.

(f) All circles pass through the point $(1, 0)$.

(g) The real Γ-axis (u-axis) corresponds to $x = 0$ and therefore represents real load impedances, i.e. purely resistive impedances.

(h) The right most point on the unit circle, $(1, 0)$, corresponds to $x = 0$ as well as $x = \pm\infty$.

The Smith Chart

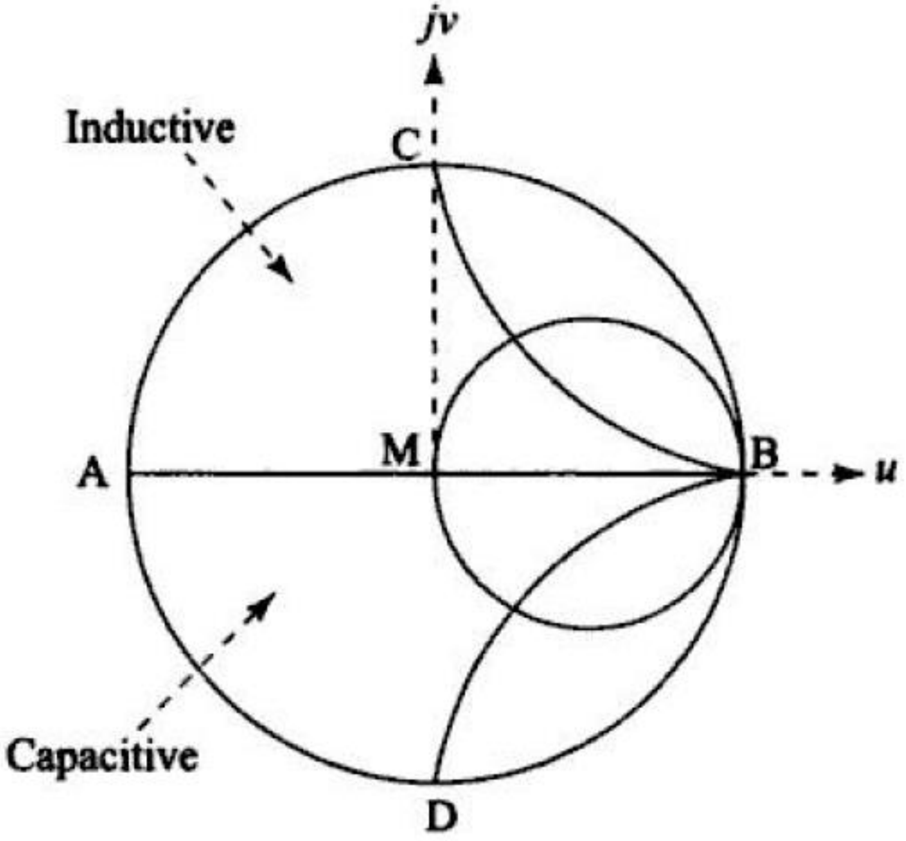


Fig. 2.16 *Smith chart: Superposition of constant resistance and constant reactance circles in the complex Γ-plane.*

(a) The left most point A on the Smith chart corresponds to $r = 0, x = 0$ and therefore represents ideal short-circuit load.

(b) The right most point B on the Smith chart corresponds to $r = \infty, x = \infty$ and therefore represents ideal open circuit load.

(c) The center of the Smith chart M , corresponds to $r = 1, x = 0$ and hence represents the matched load.

(d) Line AB represents pure resistive loads and the outermost circle passing through A and B represents pure reactive loads.

(e) The upper most point C represents a pure inductive load of unity reactance and the lower most point D represents a pure capacitive load of unity reactance.

(f) In general the upper half of the Impedance Smith chart represents the complex inductive loads and the lower half represents the complex capacitive loads.