Armature windings

EE6303 Electric Machines II

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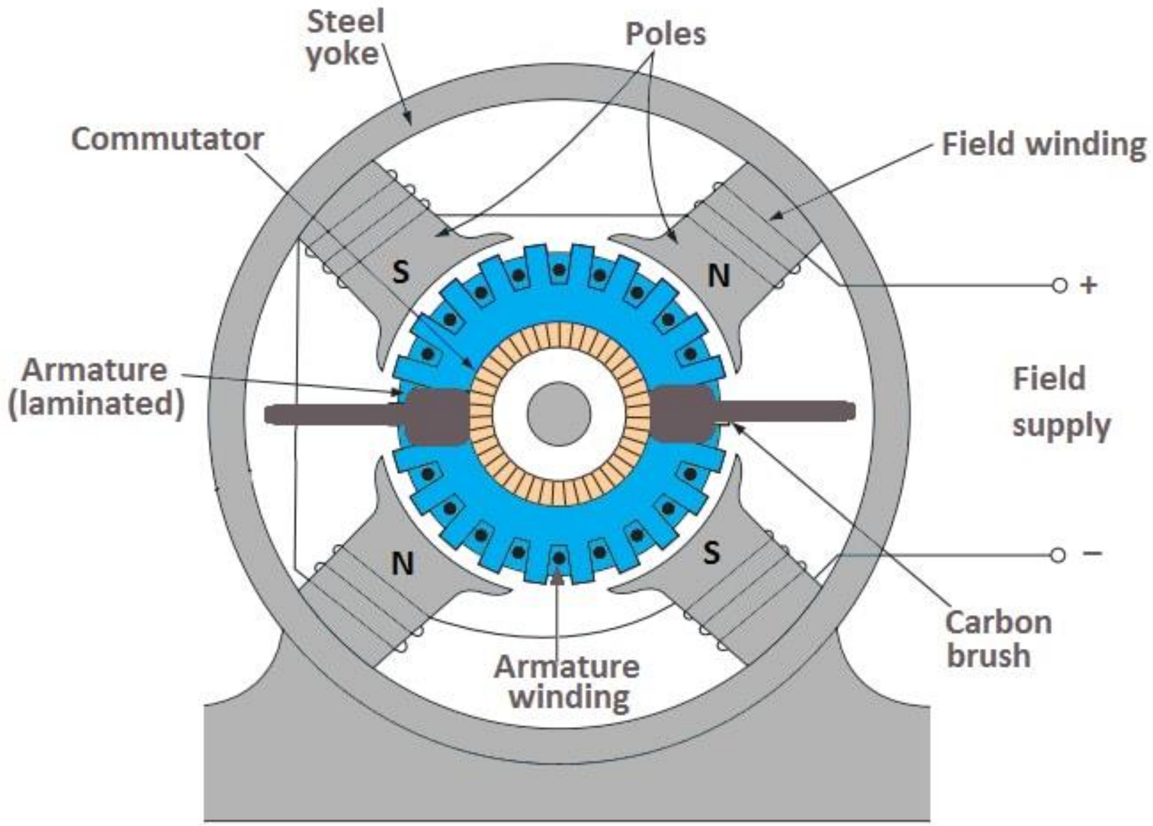
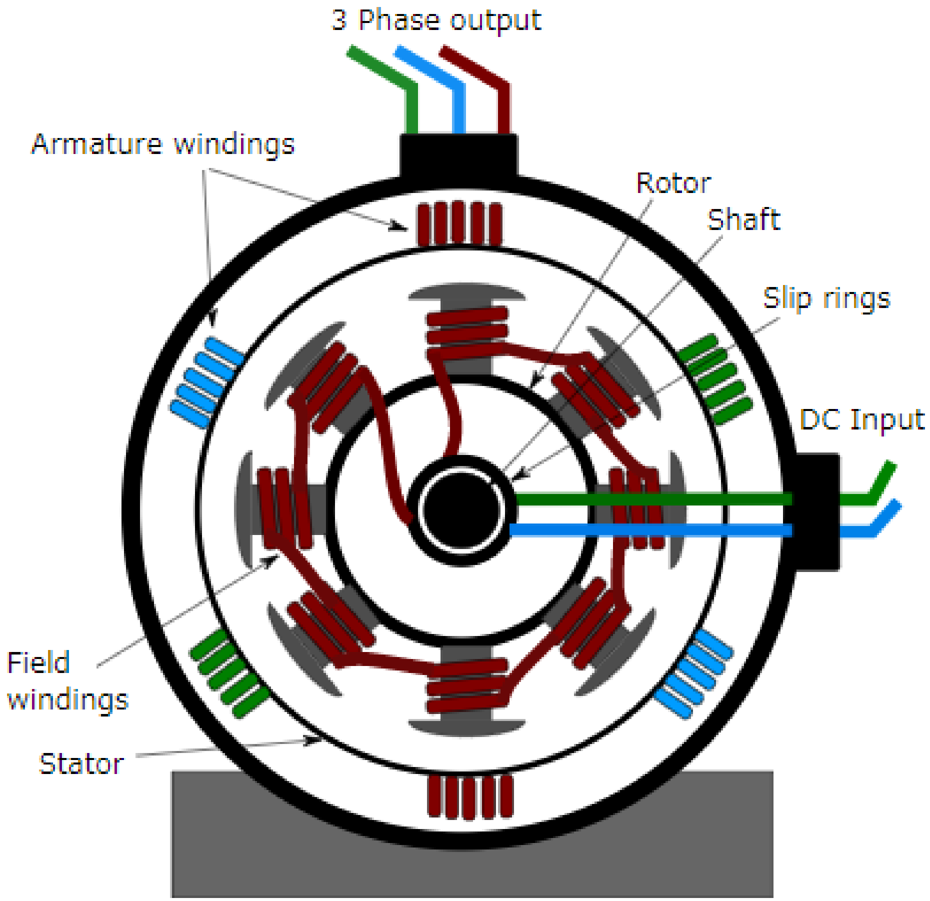
Electrical machine windings

- Field and armature windings are the essential features of electric machines (both DC and AC).



Construction of a DC machine.



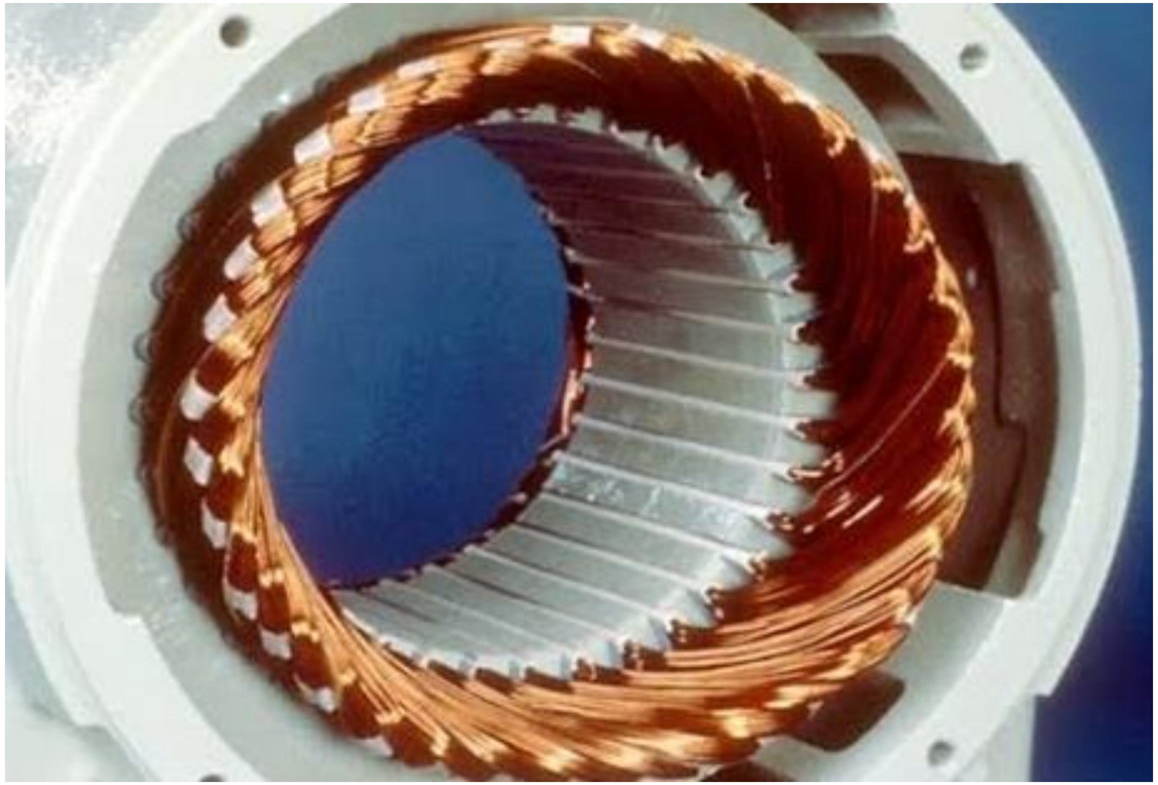
Construction of a salient pole type AC generator.

Electrical machine windings

- The field windings are simple arrangements with concentrated coils.
- Concentrated coils: Coils in which all the turns have the same magnetic axis

Electrical machine windings

- Armature windings comprise a set of coils embedded in the slots, uniformly spaced round the armature periphery.



Classification of windings

The windings (field and armature windings) used in rotating electrical machines (DC and AC) can be classified as:

- Concentrated windings
- Distributed Windings

Armature windings (both in DC and AC machines) in general can be classified as

- Closed Windings
- Open Windings

Concentrated and Distributed windings

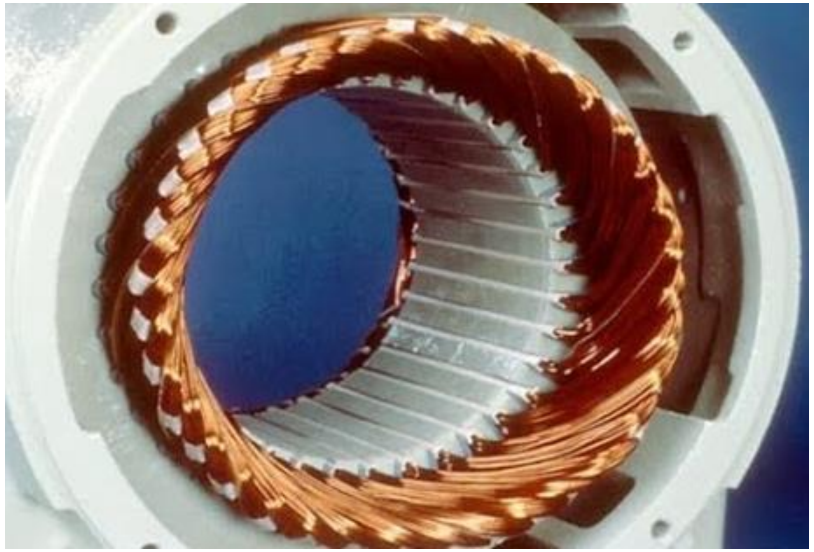
<u>Concentrated windings</u>

- In concentrated windings, all the winding turns are wound together in series to form one multi-turn coil and all the turns have the same magnetic axis.
- Examples of concentrated winding are:
  - Field windings of salient-pole synchronous machines.
  - Field windings of DC machines

Concentrated and Distributed windings

Distributed Windings

- All the winding turns are arranged in several full-pitch or fractional-pitch coils.
- These coils are then housed in the slots spread around the armature periphery.
- Examples of distributed winding are
  - Stator and rotor of induction machines
  - The armatures of both synchronous and DC machines



Closed and Open windings

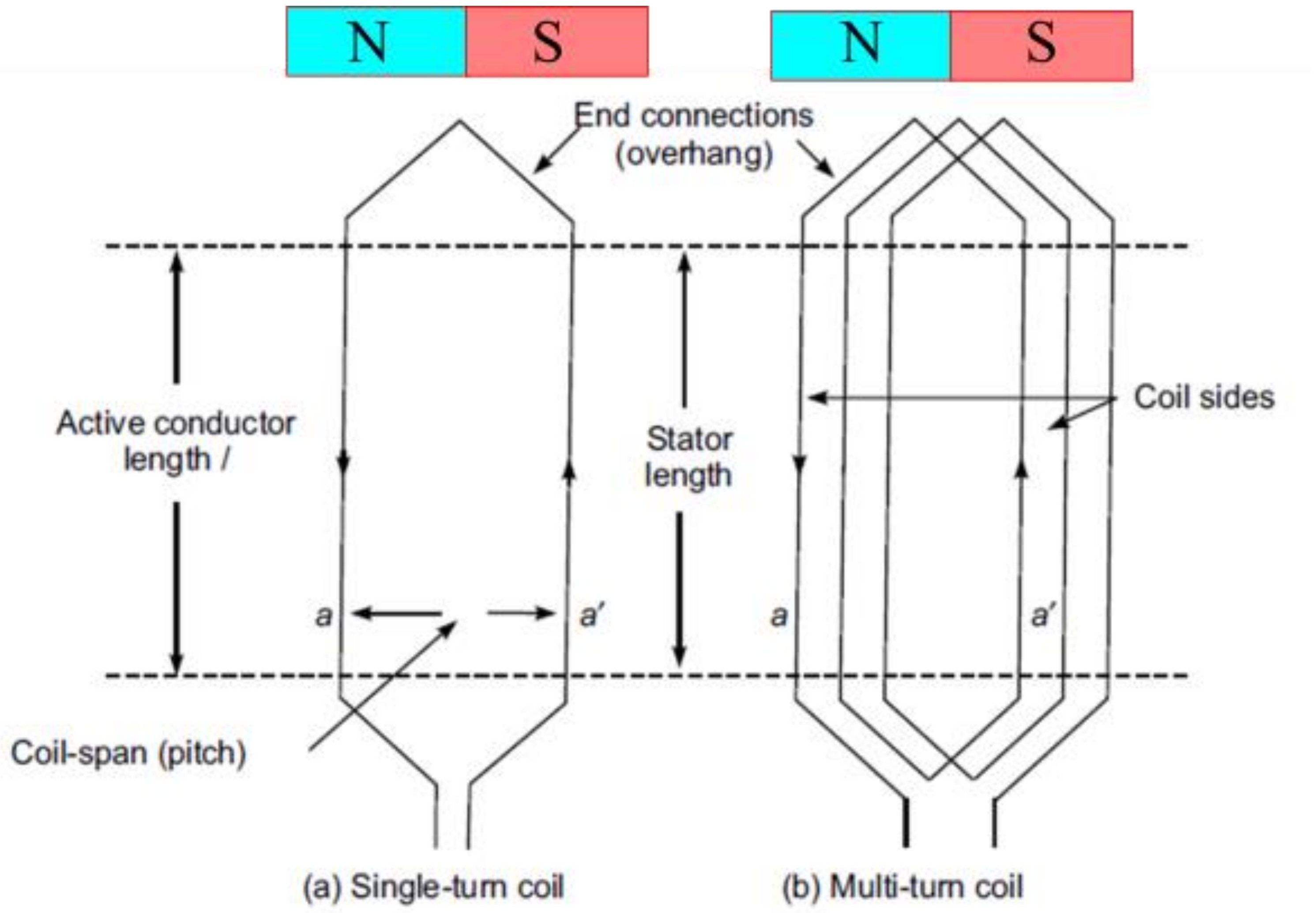
Closed Windings

- There is a closed path in the sense that if one starts from any point on the winding and traverses it, one again reaches the starting point from where one had started.
- Used only in DC machines and AC commutator machines.

Open Windings

- There is no closed path in the windings.
- Used only for AC machines.

Common terminologies associated with armature windings



Coils.

Common terminologies associated with armature windings

- Conductor:

  An individual piece of wire (active length) placed in the slots in the machine in the magnetic field.

- Turn:

  A turn consists of two conductors separated from each other by a coil pitch and connected in series so that the emf induced will be additive.

- Coil:

  A coil is formed by connecting number of turns together. A coil may consist of a single turn or may consist of many turns. Each turn is insulated from the other turns and from the rotor slot.

- Coil side:

  A coil consists of two coil sides, which are placed in two different slots. The group of conductors on one side of the coil form one coil side while the conductors on the other side of the coil forms the second coil side.

Common terminologies associated with armature windings

- Coil throw:

  The location in an armature core of the sides of a coil starting in slot number one.

  Example 1:The coil throw 1 to 9 meaning that one side of the coil is inserted in slot number 1, and the other side in slot number 9.

- Slots per pole:

  The number of slots in the armature core divided by the number of poles constituting the field.

  Example 2: In a 4 pole three phase generator having 48 slots on the armature, the number of slots per pole is 48/4 = 12 slots.

Common terminologies associated with armature windings

- Pole pitch:

  The distance between the central line of one pole to the central line of the next pole. The coil pitch could be expressed in terms of its angular span or in terms of slots.

  Pole pitch is always 180 electrical degrees regardless of the number of poles on the machine.

$$\theta_e = \frac{P}{2}\theta_m$$

  where θ_e is the electrical angle, θ_m is the mechanical angle, and P is the number of poles.

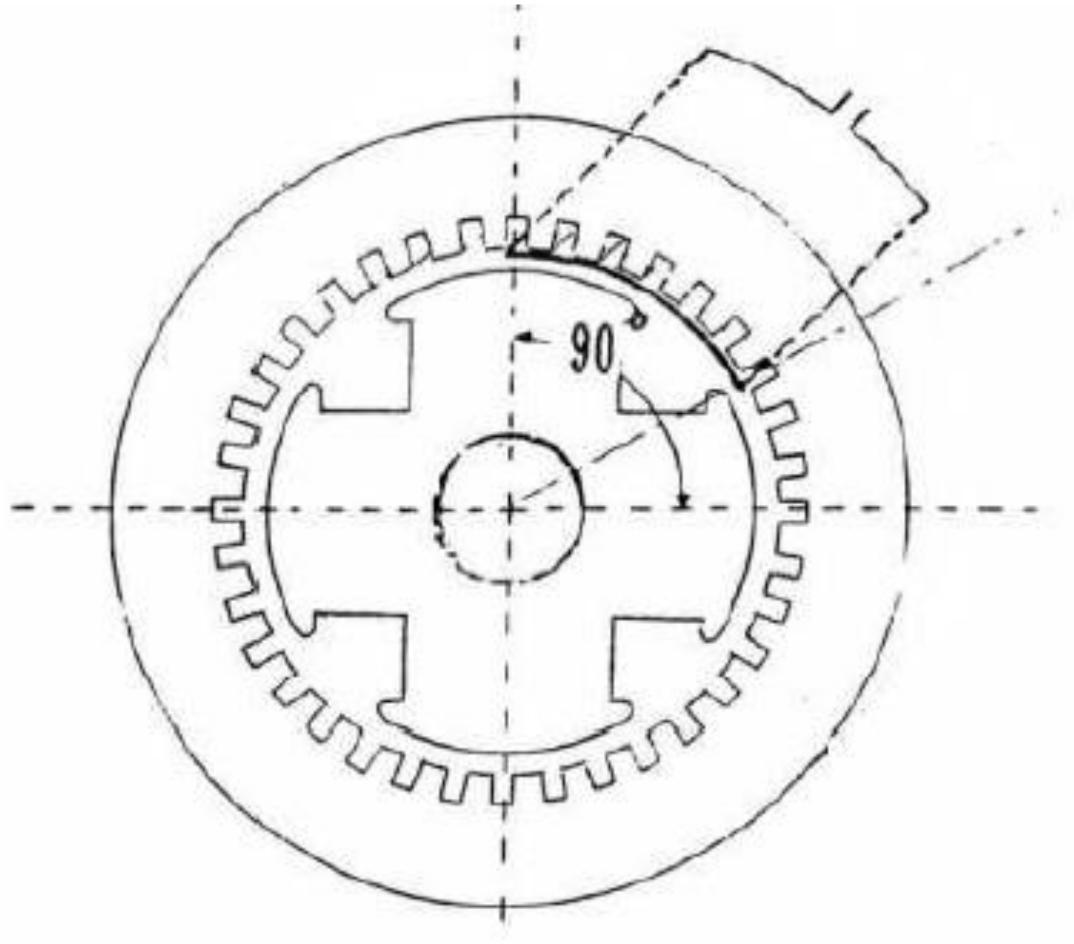
- Coil pitch or coil span:

  The distance between the two coil sides of a coil is called coil span or coil pitch. The coil pitch could be expressed in terms of its angular span or in terms of slots.

  Example 2: In example 1 mentioned in slide 11, the coil pitch is 8 slots (9 - 1).

Questions

Example: Find the slots per pole, coil pitch, and the pole pitch of the following machine.



Number of slots = 36
Number of poles = 4
Slots per pole = 9 slots
Coil Pitch = 6 slots,
Pole pitch = 9 slots.

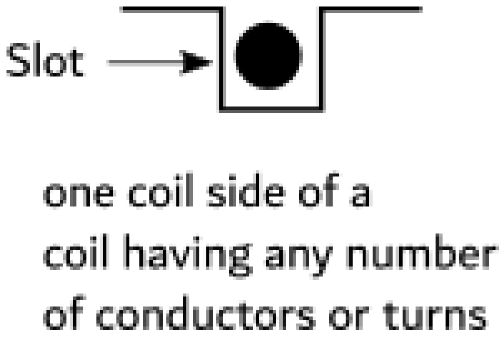
Common terminologies associated with armature windings

- <u>Single-layer winding and Double-layer windings</u>

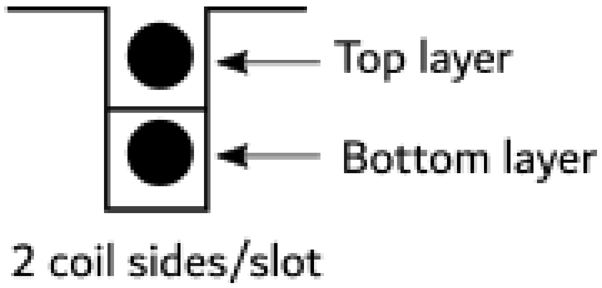
  In a single-layer winding, only one coil side is placed in one slot.

  In a double-layer winding, two coil sides are placed in a single slot and arranged in two layers.

  In a double-layer winding, all the coils are identical in shape and size with two coil-sides lying in two different planes. Each slot has one coil-side entering its bottom half from one side and the other coil-side leaving its top half on the other side.



Single-layer windings.



Double-layer windings.

Common terminologies associated with armature windings

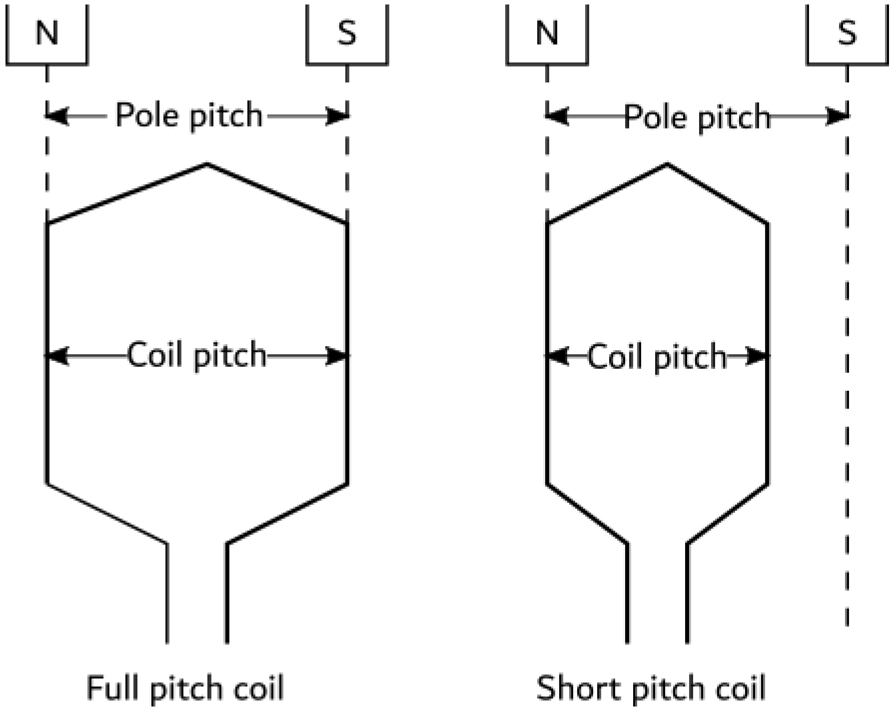
Full pitch coil and short pitch (chorded) coil

- Full pitch coil

  When the angular distance between the sides of a coil is exactly equal to the angular distance between the centers of adjacent field poles (coil pitch is equal to the pole pitch), the coil is termed to be a full pitch coil.

- Short pitch (chorded) coil

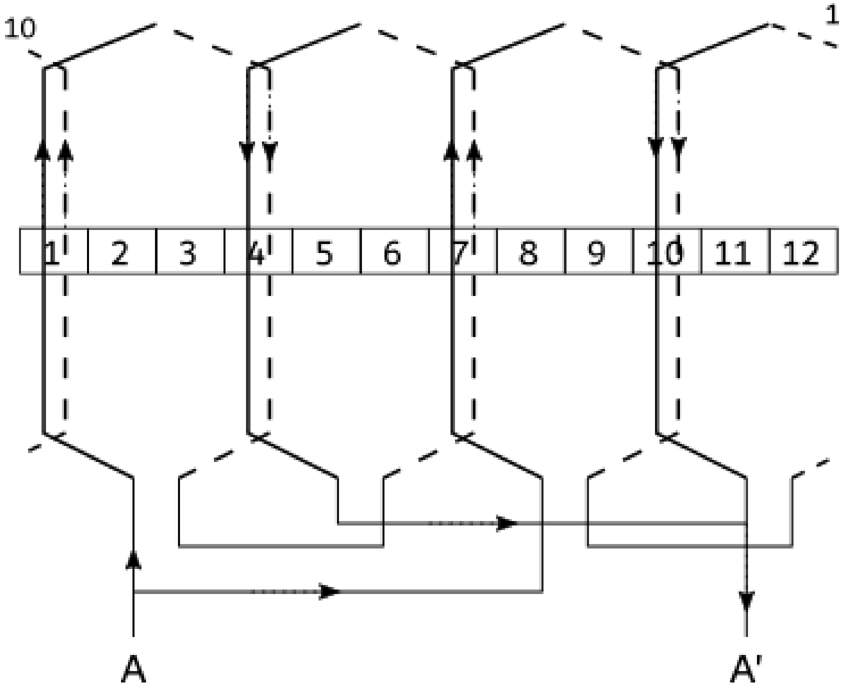
  When the angular distance between the sides of a coil is less than the angular distance between the centers of adjacent field poles (coil pitch is less than pole pitch), the coil is termed to be a short pitch coil.



Full pitch and short pitch coils.

Developed winding diagram

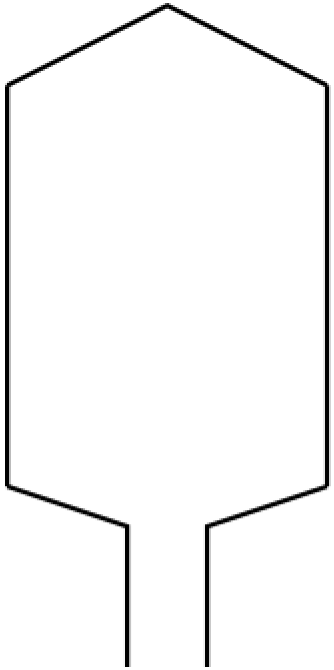
- The arrangement of coils round the armature periphery and their interconnections is best illustrated in form of a *developed winding diagram.*
- For the purpose of drawing a winding diagram, it is convenient to imagine the armature to be laid out flat in a developed form with slots parallel to each other.



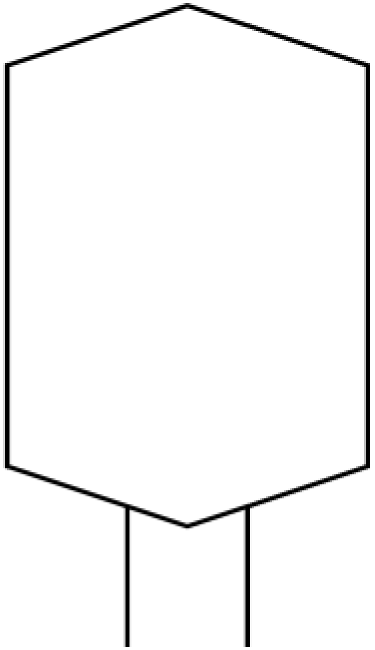
Developed winding diagram.

Representation of a single turn coil and a multi turn coil

- In a developed winding diagram, single turn coils and multi-turn coils are represented as follows.



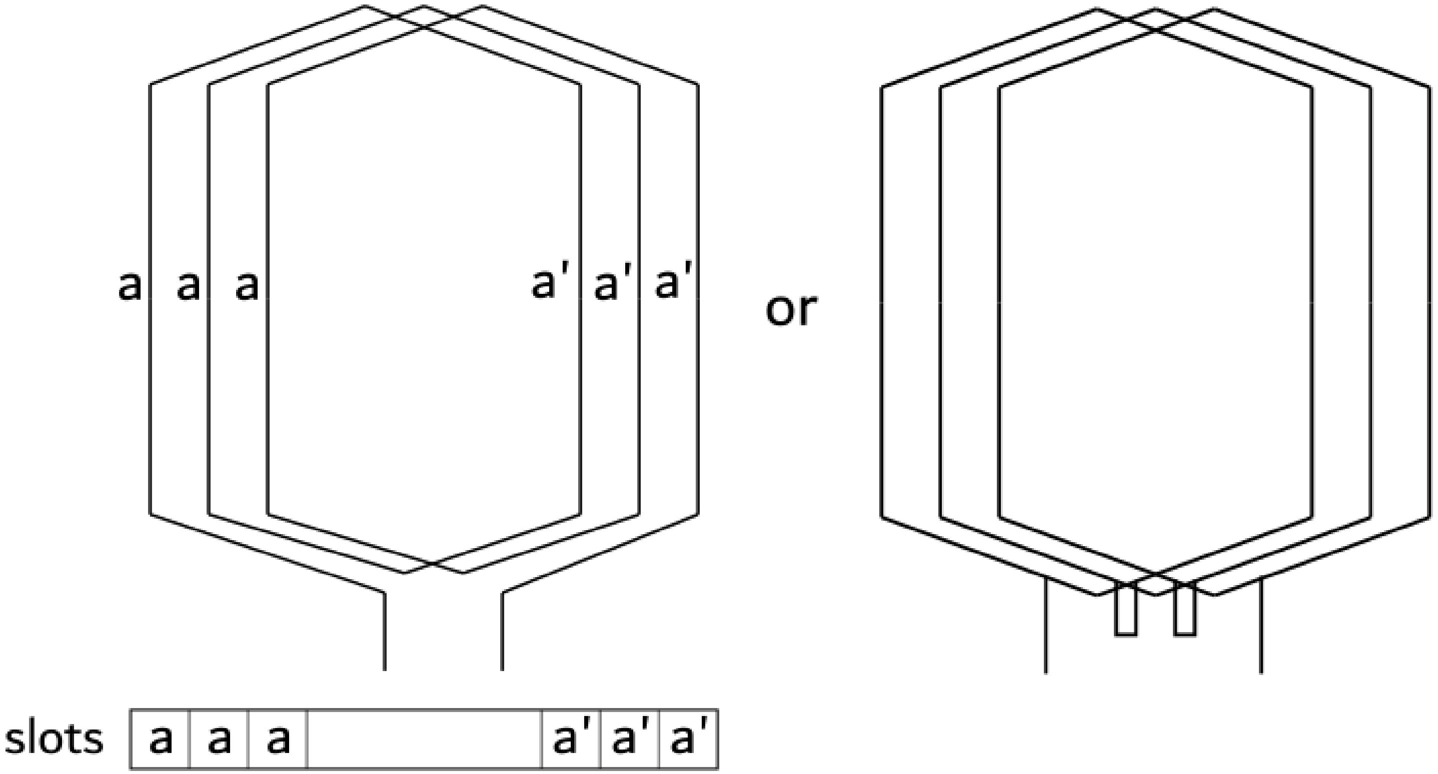
Single turn coil



Multi turn coil

Representation of Single-layer winding

Example: Single-layer representation of a winding with three series connected coils distributed in three consecutive slots.

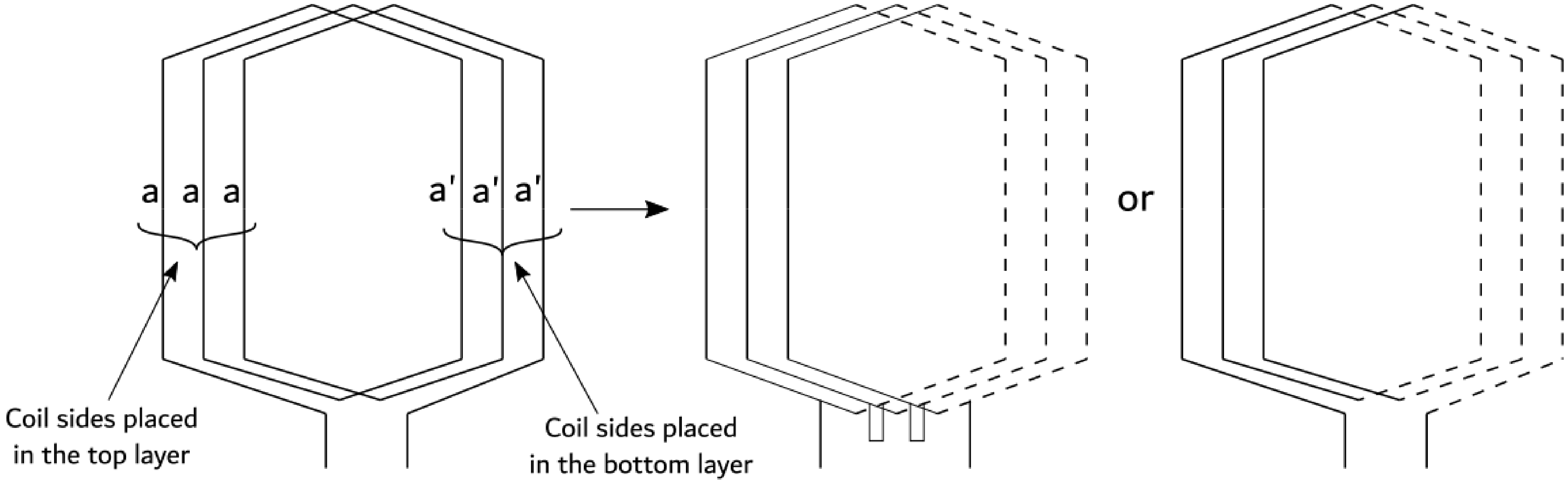


Single-layer winding representation in a winding diagram.

Representation of Double-layer winding

Example: Double-layer representation of a winding with three coils distributed in three consecutive slots.

The top layer coil sides are indicated by continuous lines and the bottom layer coil sides are indicated by dotted lines.



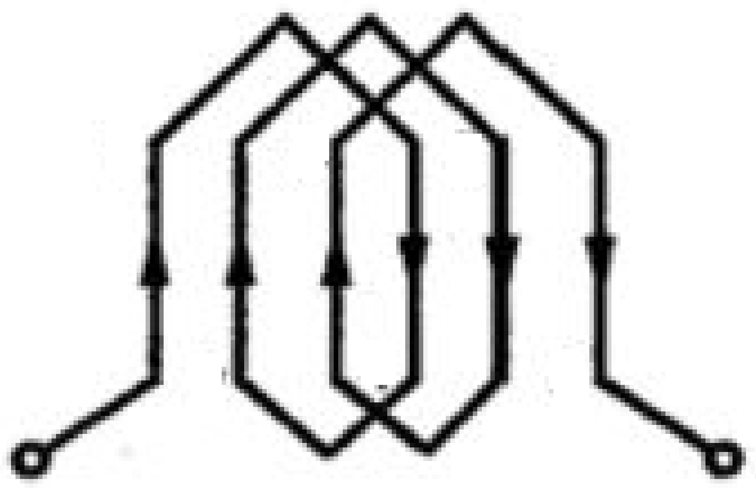
Top layer	a	a	a				
Bottom layer					a'	a'	a'

Double-layer winding representation in a winding diagram.

DC machine armature windings

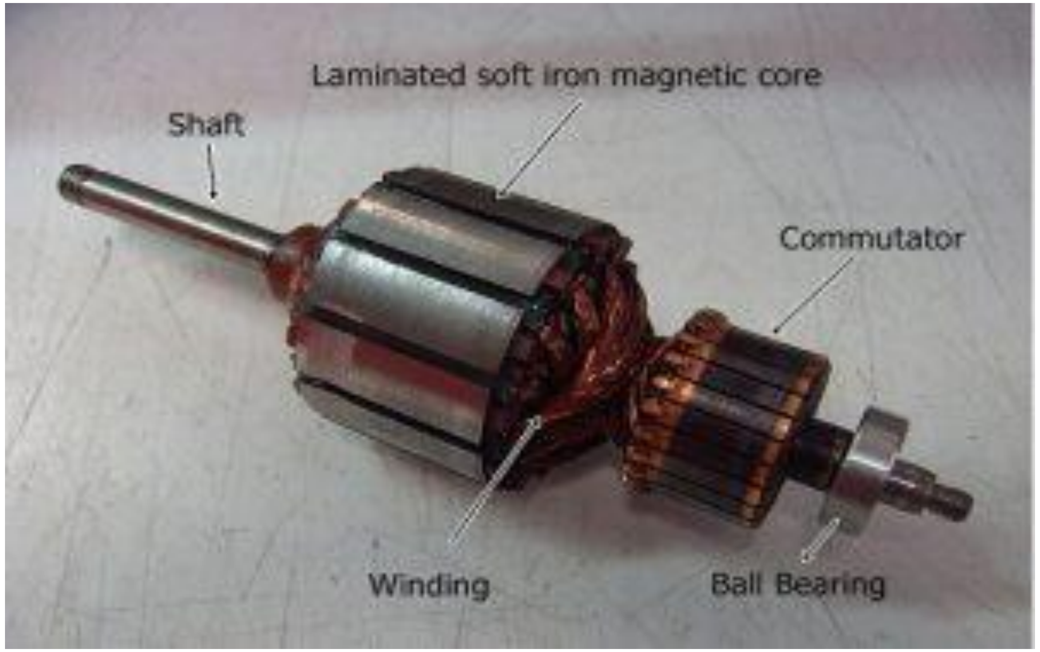
DC machine armature windings

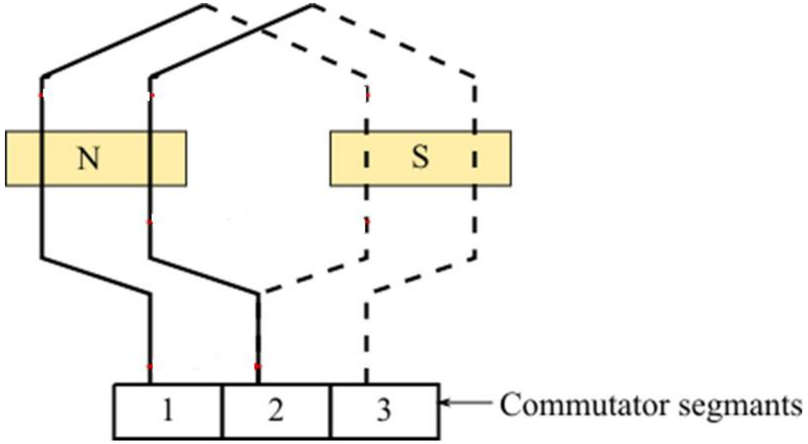
- Armature winding of a DC machine is always *closed and of double layer type*.
- Closed winding essentially means that all the coils are connected in series forming a closed circuit.



DC machine armature windings

- The junctions of the consecutive coils are terminated on copper bars called commutator segments.
- Each commutator segment is insulated from the adjacent segments by mica insulation.
- On a commutator segment, two coil sides (belonging to two different coils) terminate.
- Number of commutator segments must be equal to the number of slots.





Common terminologies

- *Commutator pitch (y_c)*

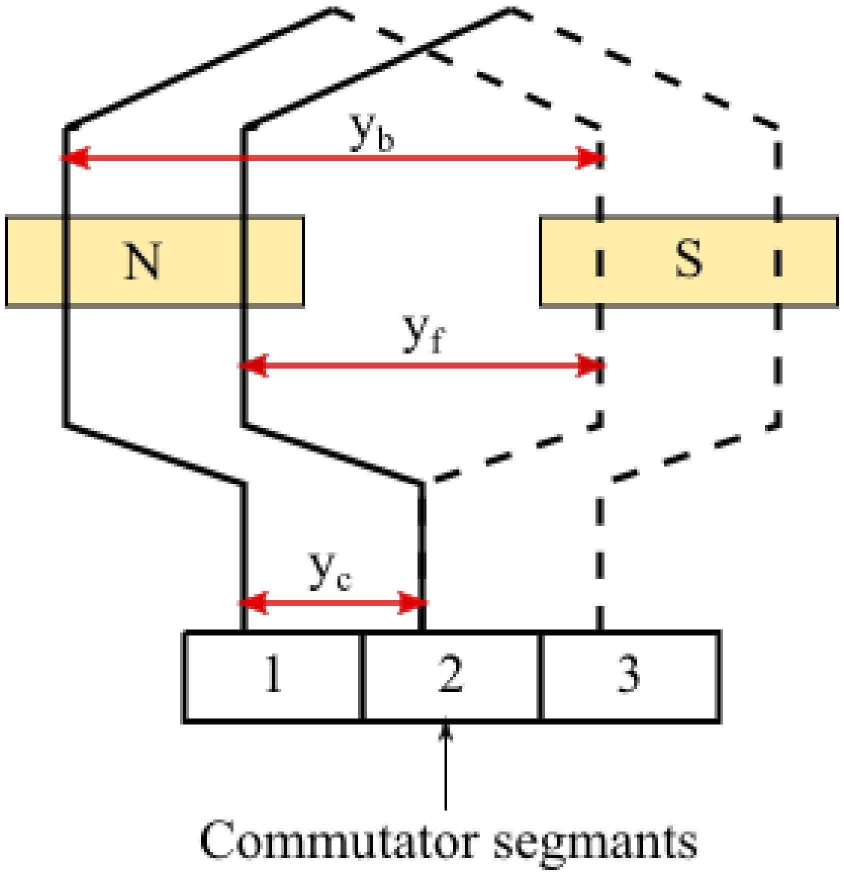
  The separation of coil sides of a coil in terms of number of commutator segments.

- *Back pitch (y_b)*

  The back pitch is similar to that of coil pitch i.e., the distance between the two coil sides of a coil. It is the distance between the armature conductors, which a coil advance on the back of the armature.

- *Front pitch (y_f)*

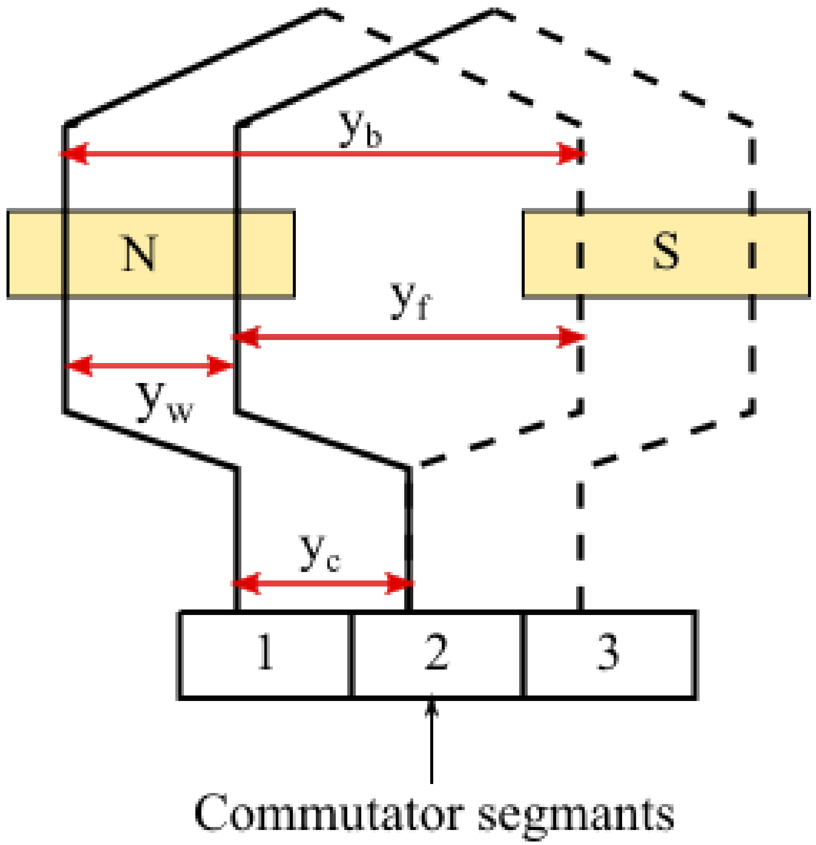
  The distance between the two coil sides connected to the same commutator segment.



Common terminologies

- *Winding pitch* (y_w)

  The distance between the two consecutive and similar coil sides as the winding progresses.



DC machine armature windings types

- The different armature coils in a DC machine armature winding must be connected in series with each other by means of end connections (back connections and front connections) in a manner so that the generated voltages of the respective coils will aid other in the production of the terminal emf of the winding.
- Two basic methods of making these end connections are;
  - Simplex Lap winding
  - Simplex Wave winding
- In each type, there are two modes:
  - Progressive
  - Retrogressive

Design procedure of DC machine armature windings

- Type of winding (lap or wave), total number of slots (S), and total number of poles (P) will be given.
- Calculate the front (y_f) and back pitches (y_b).
- Calculate the commutator pitch (y_c).
- Draw the developed view of the winding diagram showing
  - the positions of the coil sides in slots,
  - interconnection of the coils through commutator segments
- Place the stationary brushes on the correct commutator segment.
  - The function of brushes in the DC machine is to collet current from commutator segments.
  - The number of brushes in the DC machine equal to the number of parallel paths.

Simplex Lap winding

- In lap winding, the end of one coil is connected to the beginning of the next coil with the two ends of each coil coming out at adjacent commutator segments.

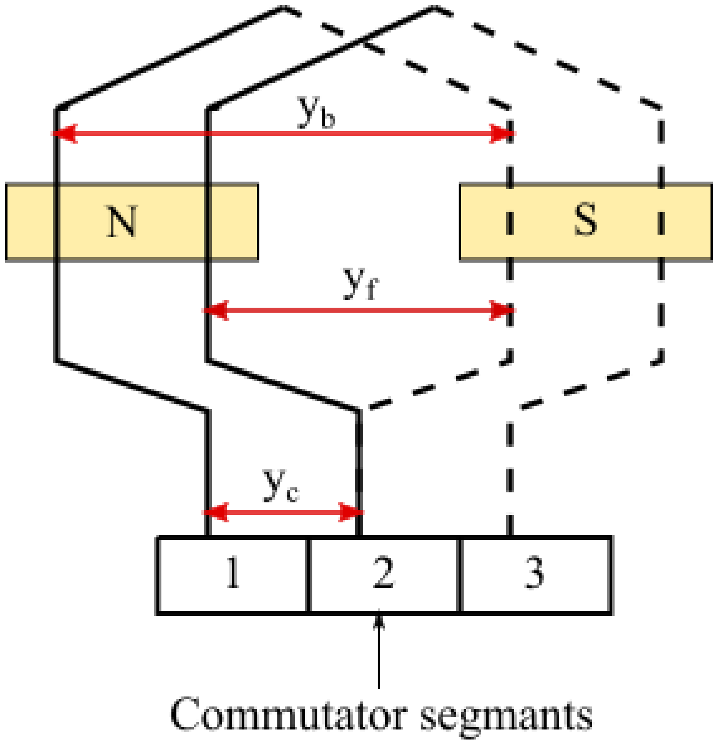
- Commutator pitch (y_c) for a lap winding is ± 1.
  - Progressive lap winding: $y_c = +1$,
  - Retrogressive lap winding: $y_c = -1$

- In simplex lap winding;

$$y_b \approx \frac{S}{P}$$

$$y_f = y_b - y_c$$

$$y_w = y_b - y_f$$



- For Lap winding, the numbers of parallel path are equal to the total number of poles.

  Therefore, the number of brushes is equal to the total number of poles.

Simplex Lap winding

- This winding is used in large current applications, because it has more parallel paths.
- It is suitable for low voltage and high current applications.
- This winding required more number of conductors for generating certain emf, hence the winding cost is high.

Example 1

Draw the developed armature winding diagram of a DC machine with 8 slots, four-pole, two-layer, progressive simplex lap winding. The number of turns per coil is one.

Slots per pole $= \frac{S}{P} = 2$ slots

Back pitch: $y_b \approx \frac{S}{P} = 2$ slots

Front pitch: $y_f = y_b - y_c = 2 - 1 = 1$ slots

The commutator pitch, $y_c = +1$

Example 1: Solution

- Winding table

Slot numbers where the coils sides are placed $[x, (x + y_b)']$	Commutator segments where the coil ends terminate $[x, x + y_c]$
1,1+2 = 1,3’	1,1+1=1,2
2,4’	2,3
3,5’	3,4
4,6’	4,5
5,7’	5,6
6,8’	6,7
7,1’	7,8
8,2’	8,1

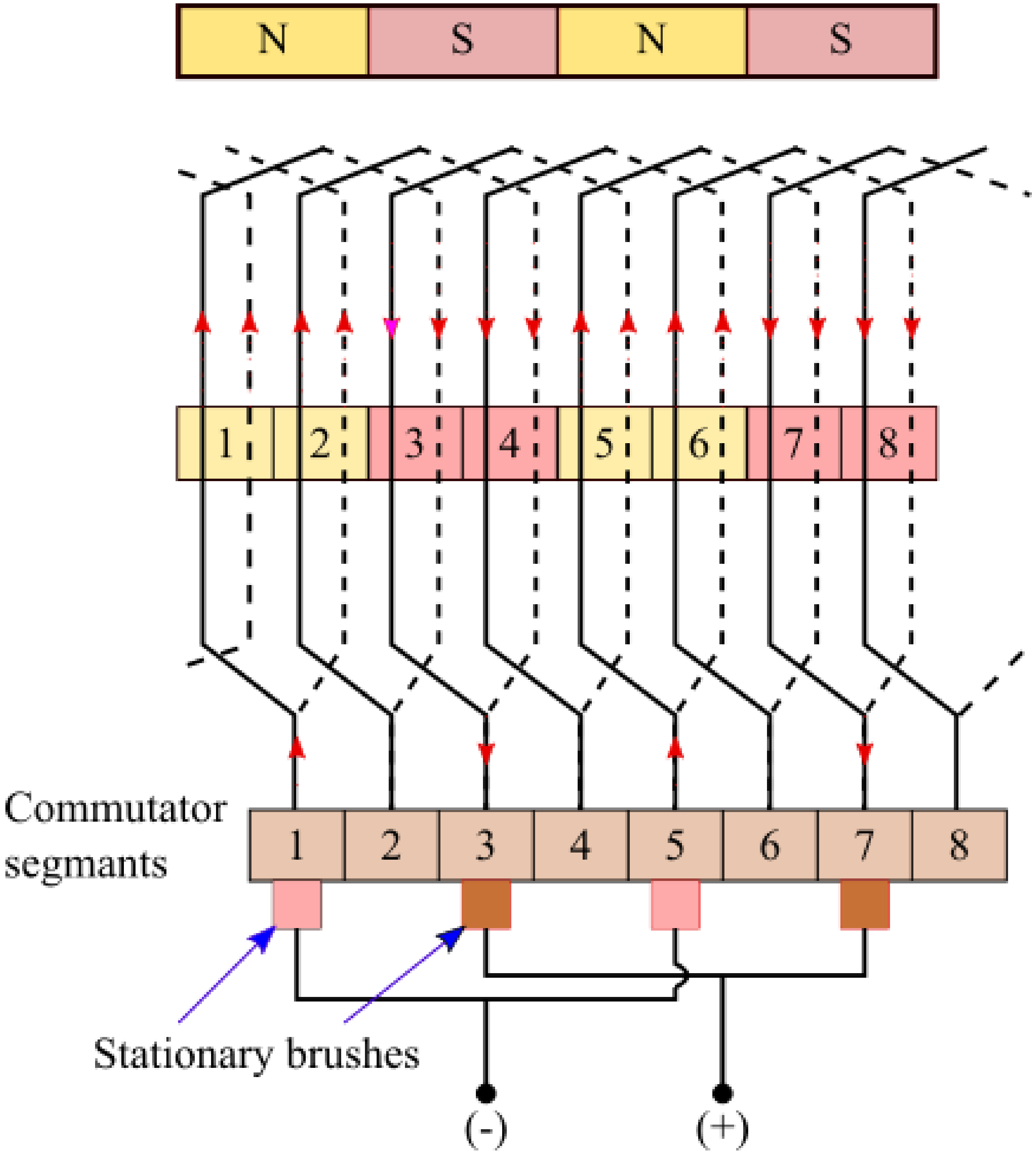
1,3’

Slot # 1, top layer

Slot # 3, bottom layer

Number of brushes = 4

Figure shows the winding connection where the two ends of a coil are connected to adjacent commutator segments. The coils in each path are connected in series and the winding close upon itself.



Example 2

Draw the developed armature winding diagram of a DC Machine with 4 poles, 16 slots, progressive, double layer simplex lap winding. Show the position of brushes and direction of induced emf. The number of turns per coil is one.

The commutator pitch, $y_c = +1$

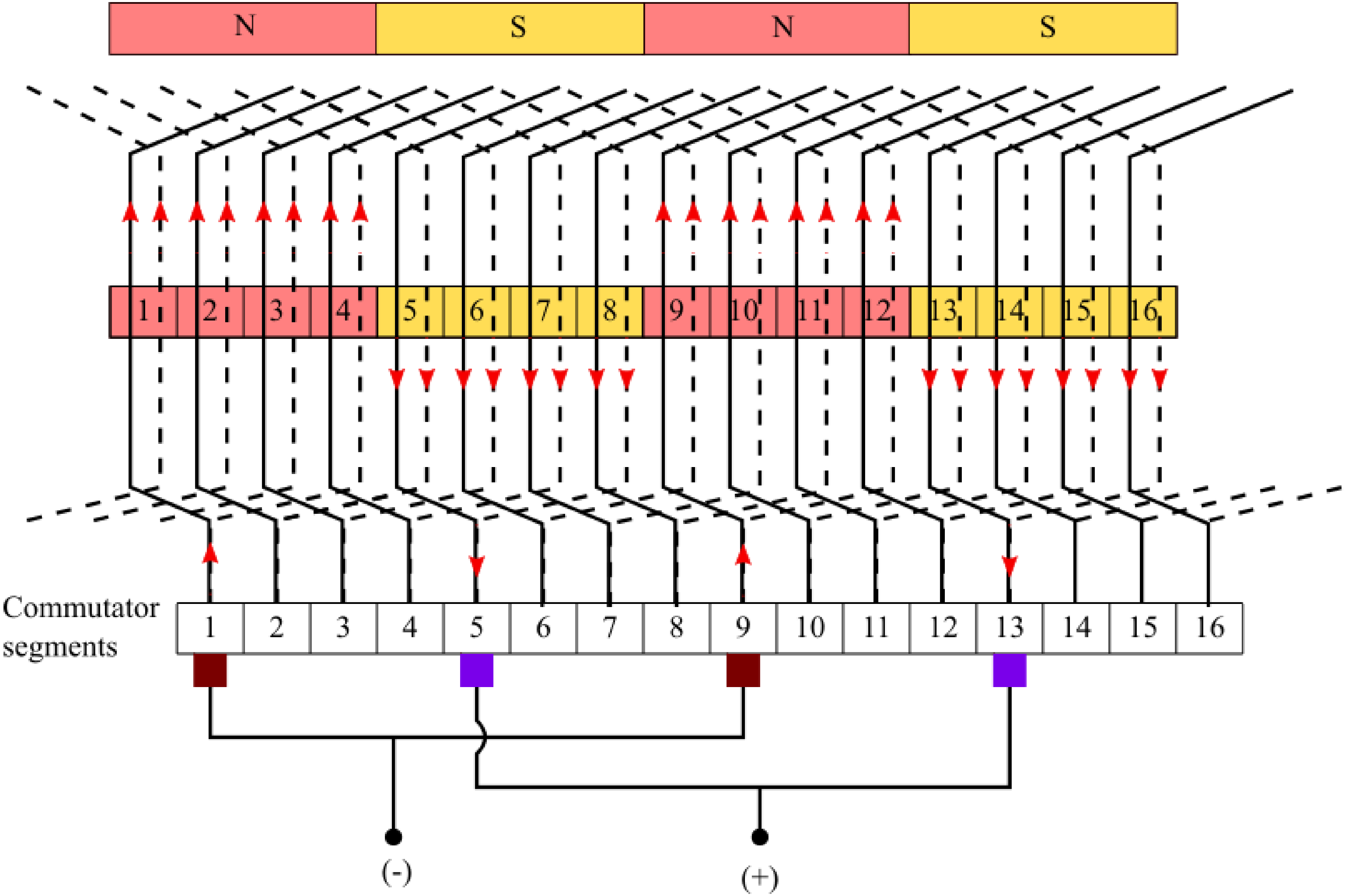
Back pitch: $y_b \approx \frac{S}{P} = 4$

Front pitch: $y_f = y_b - y_c = 4 - 1 = 3$

Number of brushes = 4

Winding table

Slot numbers where the coils sides are placed $[x, (x+y_b)']$	Commutator segments where the coil ends terminated $[x, x+y_c]$
1,1+4=1,5’	1,1+1=1,2
2,6’	2,3
3,7’	3,4
4,8’	4,5
5,9’	5,6
6,10’	6,7
7,11’	7,8
8,12’	8,9
9,13’	9,10
10,14’	10,11
11,15’	11,12
12,16’	12,13
13,1’	13,14
14,2’	14,15
15,3’	15,16
16,4’	16,1


N
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11
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16
Commutator segments
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(+)

Simplex wave winding

- In wave winding, commutator pitch is selected to be closely equal to two pole pitch in terms of commutator segments (approximately 360 electrical degrees apart).

- Commutator pitch (y_c) for a wave winding is

$$y_c = \frac{2(S \pm 1)}{P}$$

  – Progressive wave winding: $y_c = \frac{2(S+1)}{P}$,

  – Retrogressive wave winding, $y_c = \frac{2(S-1)}{P}$

- For wave windings, number of parallel paths are always equal to two. Therefore, the number of brushes is always equal to two.

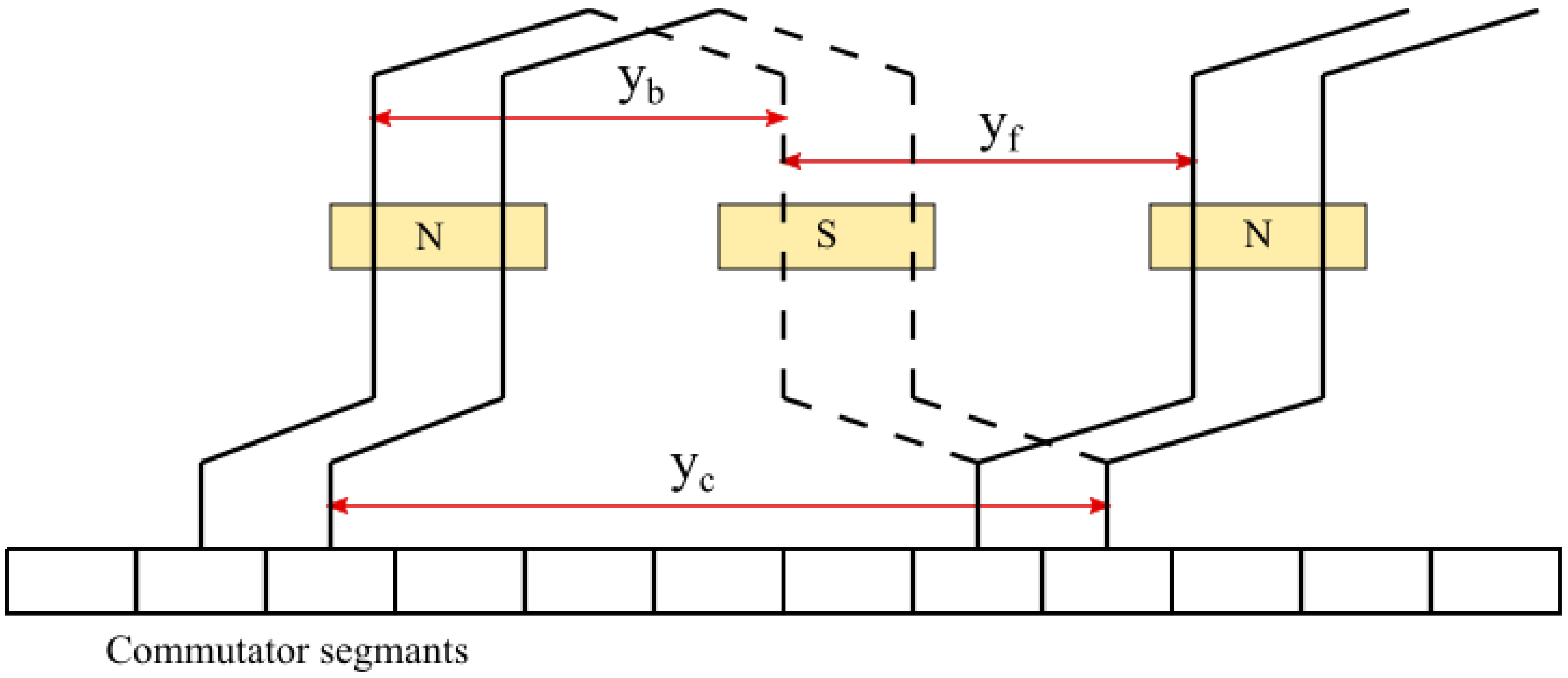
Simplex wave winding

- In simplex wave winding;

$$y_b \approx \frac{S}{P}$$

$$y_f = y_c - y_b$$

- The following diagram shows a part of simplex wave winding.



Example

Draw the developed armature winding diagram of a DC generator with 4 poles, 17 slots, progressive, double layer simplex wave winding.

Number of poles, $P = 4$

Number of slots, $S = 17$

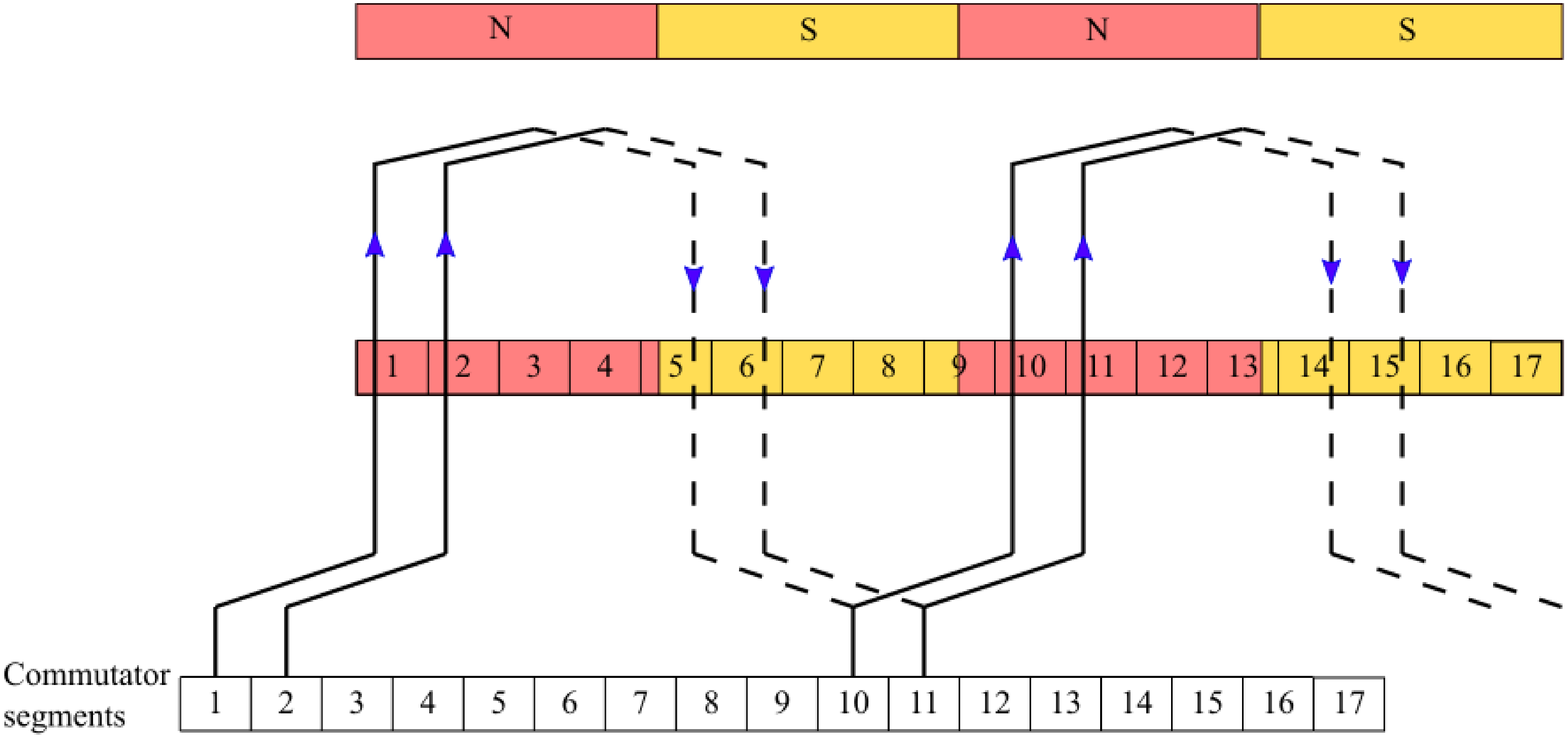
Commutator pitch, $y_c = \frac{2(S+1)}{P} = \frac{2(17+1)}{4} = 9$

Back pitch: $y_b \approx \frac{S}{P} = 4$

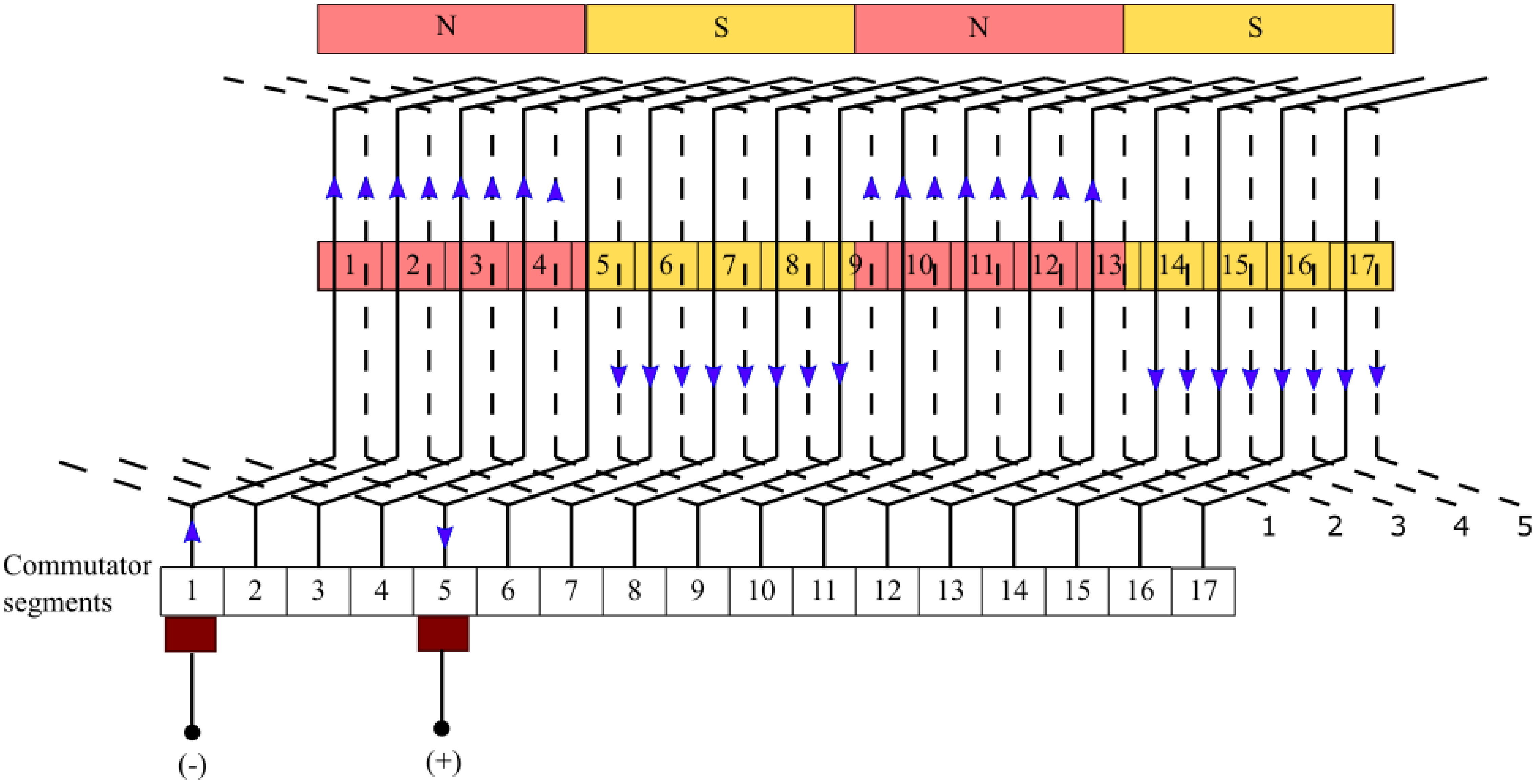
Front pitch: $y_f = y_c - y_b = 9 - 1 = 8$

Winding table

Slot numbers where the coils sides are placed $[x, (x+y_b)']$	Commutator segments where the coil ends terminated $[x, x+y_c]$
1,1+4= 1,5’	1, 1+9 = 1,10
10, 14’	10,2
2,6’	2,11
11,15’	11,3
3,7’	3,12
12,16’	12,4
4,8’	4,13
13,17’	13,5
5,9’	5,14
14,1’	14,6
6,10’	6,15
15,2’	15,7
7,11’	7,16
16,3’	16,8
8,12’	8,17
17,4’	17,9
9,13’	9,1



Starting a simplex progressive wave winding.



Complete simplex progressive wave winding.

AC machine armature windings

AC machine armature windings

- AC armature winding can be either single-phase or 3-phase.
- AC windings are generally of a 3-phase kind because of the inherent advantages of a 3-phase machine.
- In 3-phase windings it is essential that:
  - The generated emfs of all the phases are of equal magnitude.
  - The waveforms of the phase emfs are identical.
  - The frequency of the phase emfs are equal.
  - The phase emfs have mutual time-phase displacement of 120 electrical degrees.

Terminologies associated with AC machine armature windings

The following terminologies are associated only with AC armature windings.

- Slots per pole per phase (SPP): The number of slots in the armature core divided by the number of poles constituting the field times the number of phases.

  Example: In a 4 pole three phase generator having 48 slots on the armature, the number of slots per pole per phase is 48/(4x3) = 4.

- Slot angle pitch (γ): The phase difference contributed by one slot in electrical degrees is called slot angle pitch.

$$\gamma = \frac{180^{o}}{\text{Slots per pole}}$$

Phase belt and phase spread

Phase belt

A group of adjacent slots belonging to one phase under one pole pair is known as *phase belt.*

Phase spread

The angle (electrical) subtended by a phase belt is known as *phase spread*.

Phase spread = Phase belt x Slot angle pitch

3-phase armature winding

- In a three phase machine, armature consists of three windings, i.e. phase A winding, phase B winding, and phase C winding.
- The phase emfs of a 3-phase winding should have mutual time-phase displacement of 120° electrical radians.
- In order to achieve 120° phase displacement, the placement of coil sides of windings A, B, and C should be as follows.

Phase A: slot #1

Phase B: slot # $\left(1 + \frac{120^o}{\gamma}\right)$

Phase C: slot # $\left(1 + \frac{240^o}{\gamma}\right)$

where $\gamma = \frac{180^o}{\text{Slots per pole}}$

Examples: Phase belt and phase spread

1) Field winding on the rotor produce 2 poles and the stator carries 12 conductors housed in 12 slots as shown in the Figure.

$$\gamma = \frac{180^{\circ}}{\text{Slots per pole}} = \frac{180^{\circ}}{6} = 30^{\circ}$$

Phase A: slot #1

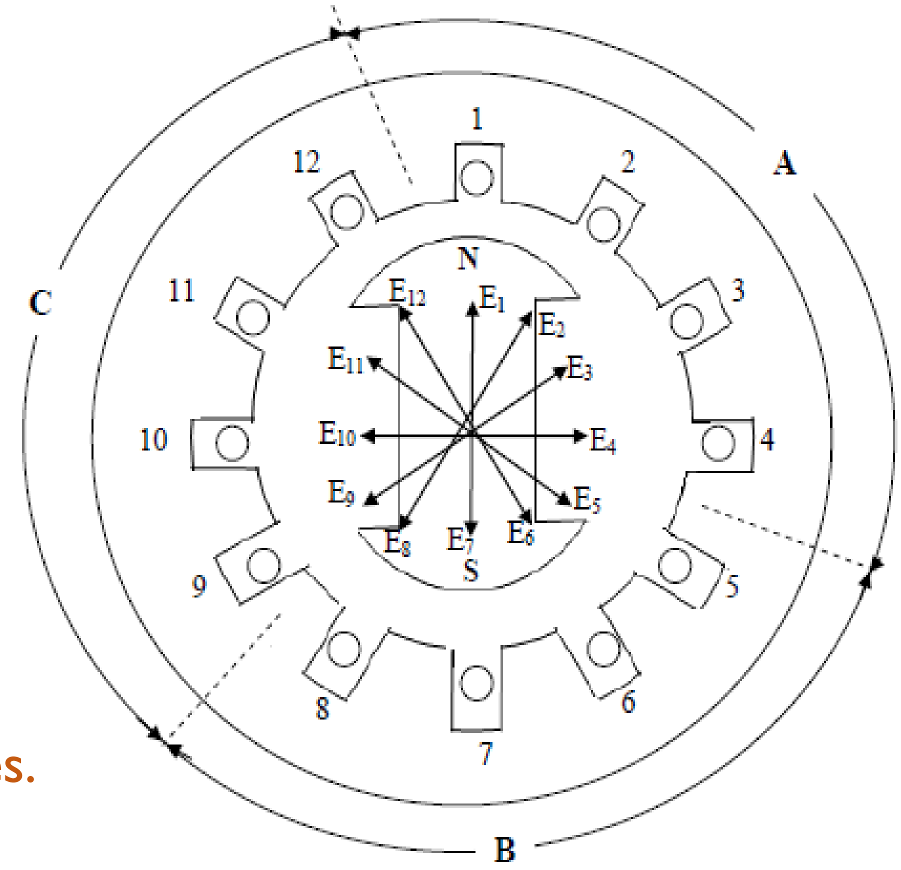
Phase B: slot # $\left(1 + \frac{120^{o}}{\gamma}\right) = \left(1 + \frac{120^{o}}{30^{o}}\right)$ = slot # 5

Phase C: slot # $\left(1 + \frac{240^{o}}{\gamma}\right) = \left(1 + \frac{240^{o}}{30^{o}}\right)$ = slot # 9

Phase belt = 4

Phase spread = $4 \times 30^{o} = 120^{\circ}$

Note: Phase spread of 120° is rarely adopted in AC machines.



Three phase winding with phase spread 120°.

Examples: Phase belt and phase spread

2) Field winding on the rotor produce 2 poles and the stator carries 12 conductors housed in 12 slots as shown in the Figure.

$$\gamma = \frac{180^{o}}{\text{Slots per pole}} = \frac{180^{o}}{6} = 30^{o}$$

Phase A: slot #1

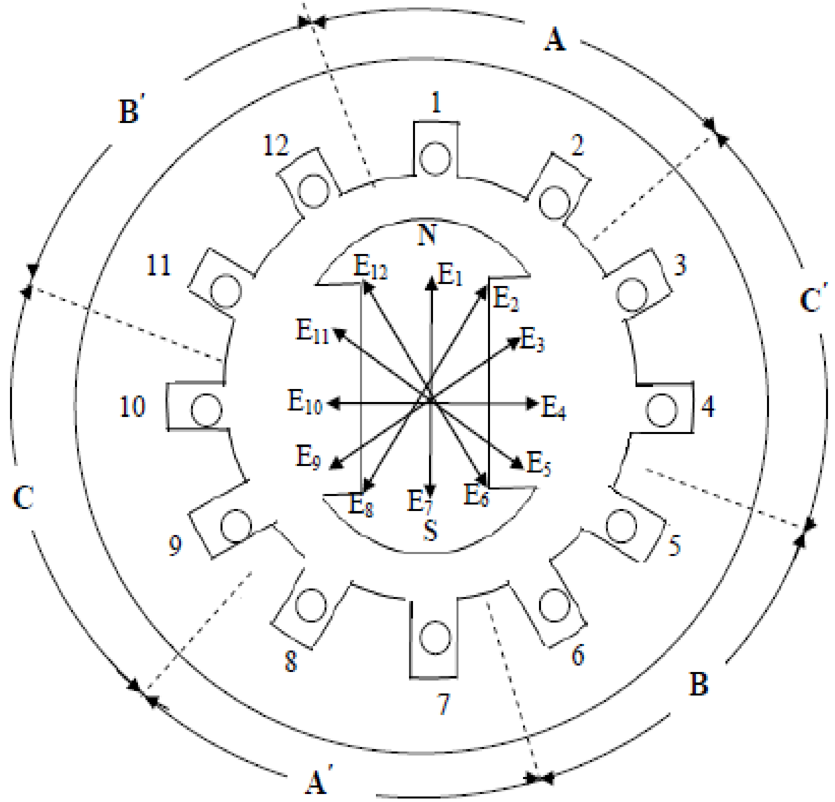
Phase B: slot # $\left(1+\frac{120^{o}}{\gamma}\right) = \left(1+\frac{120^{o}}{30^{o}}\right)$ = slot # 5

Phase C: slot # $\left(1+\frac{240^{o}}{\gamma}\right) = \left(1+\frac{240^{o}}{30^{o}}\right)$ = slot # 9

Phase belt = 2

Phase spread = 2×30^{o} = 60°

Note: Phase spread of 60° is commonly used in AC machines.



Three phase winding with phase spread 60°.

Types of AC armature windings

AC armature windings can be classified based on the distribution of coil sides in the armature slots as follows.

- Double layer windings
  - Integral slot winding
  - Integral slot chorded winding
  - Fractional slot winding
- Single layer windings
  - Concentric winding
  - Mush winding

End connections of coils

- Coil ends could be connected in series or various series-parallel connections could be employed.
- The most common connection methods are:
  - Lap
  - Wave
  - Concentric

Double layer winding

- Double-layer windings are the most widely used class of windings.
- Though both lap and wave types are possible for connecting coil ends, because of inherent problems of wave windings, lap type is commonly used.
- Double layer winding types:
  - Integral slot winding
  - Integral slot chorded winding
  - Fractional slot winding

Integral slot winding

- When the number of slots per pole per phase (SPP) is an integer, then the winding is called integral-slot winding.
- Example:

  Armature winding of a 3-phase machine with total number of slots = 24 and number of poles = 4.

  Slots per pole per phase = 24/(4x3) = 2 $\Rightarrow$ An integer, therefore winding type is integral slot winding.

Example 1: Integral slot winding

Draw the armature winding diagram of a 3-phase machine with total number of slots = 12; number of poles = 4, phase spread = 60°, double-layer, full-pitch winding.

SPP = 12/(4x3) = 1 slot.

Pole pitch = $\frac{12}{4} = 3$ slots.

Full-pitch winding: coil pitch = pole pitch = 3 slots.

Slot angle pitch $\gamma = \dfrac{180^\circ}{\text{Slots per pole}} = \dfrac{180^\circ}{12/4} = 60^\circ$

Phase spread = 60°

Phase belt = Phase spread/Slot angle pitch = 60°/60° = 1 slot.

Example 1: Integral slot winding

Winding placement:

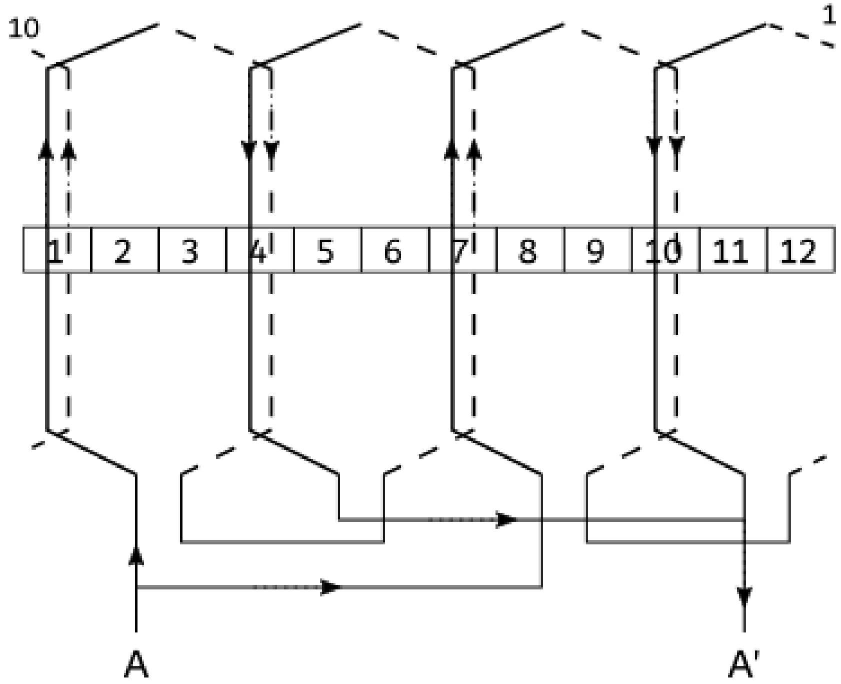
Phase A	One side of the first coil is placed at slot #1 in the top layer; other coil side should be placed at 1+coil pitch = 1+3 = slot #4 in the bottom layer.
Phase B	Starts at slot # $\left(1+\frac{120^o}{\gamma}\right)$ = slot $\left(1+\frac{120^o}{60^o}\right)$ = slot #3 in the top layer; other side of the coil should be placed at slot #6 in the bottom layer.
Phase C	Starts at slot # $\left(1+\frac{240^o}{\gamma}\right)$ = slot $\left(1+\frac{240^o}{60^o}\right)$= slot #5 in the top layer; other side of the coil should be placed at slot #8 in the bottom layer.

Example 1: Integral slot winding

- The four coils of one phase are connected in parallel in two series groups of two coils each.

Slot #	1	2	3	4	5	6	7	8	9	10	11	12
Top layer	a	c'	b	a'	c	b'	a	c'	b	a'	c	b'
Bottom layer	a	c'	b	a'	c	b'	a	c'	b	a'	c	b'

Coil placement in slots.

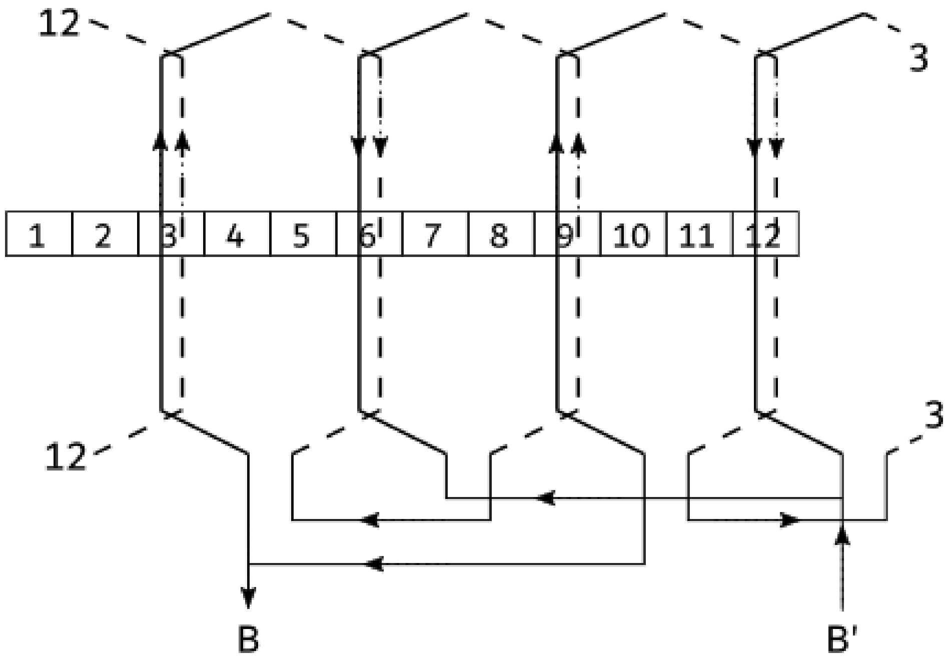


Winding diagram of phase A.

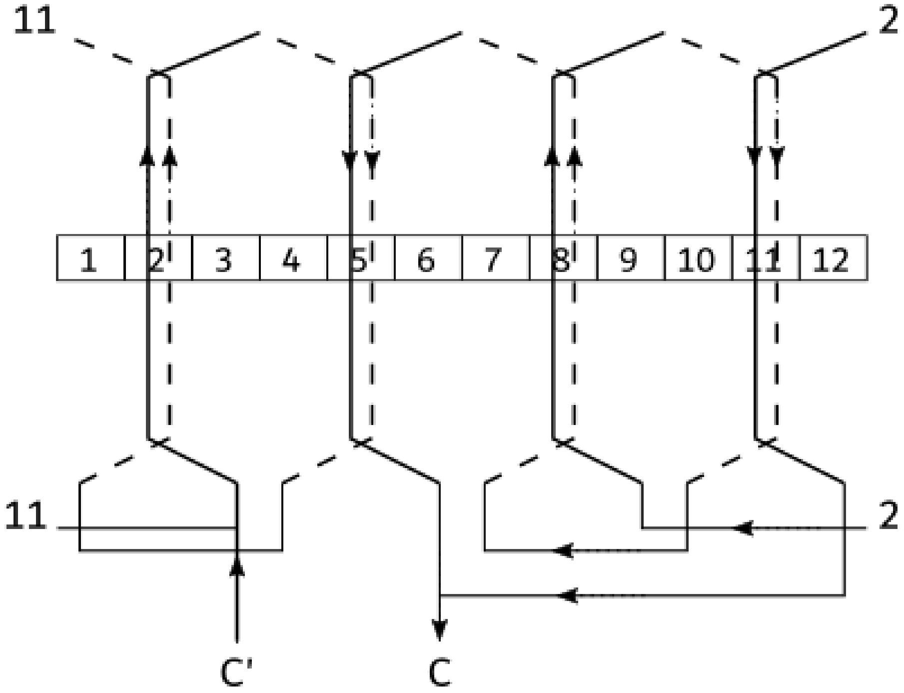
Example 1: Integral slot winding

Slot #	1	2	3	4	5	6	7	8	9	10	11	12
Top layer	a	c'	b	a'	c	b'	a	c'	b	a'	c	b'
Bottom layer	a	c'	b	a'	c	b'	a	c'	b	a'	c	b'

Coil placement in slots.



Winding diagram of phase B.



Winding diagram of phase C.

Example 2: Integral slot winding

Draw the armature winding diagram of a 3-phase machine with total number of slots = 24; number of poles = 4, phase spread = 60°, double-layer, full-pitch lap winding.

SPP = 24/(4x3) = 2 slots.

Pole pitch = $\frac{24}{4} = 6$ slots.

Full-pitch winding: coil pitch = pole pitch = 6 slots.

Slot angle pitch $\gamma = \frac{180^{o}}{\text{Slots per pole}} = \frac{180^{o}}{{}^{24}/_{4}} = 30^{o}$

Phase spread = 60°

Phase belt = Phase spread/Slot angle pitch = 60°/30° = 2 slots.

Example 2: Integral slot winding

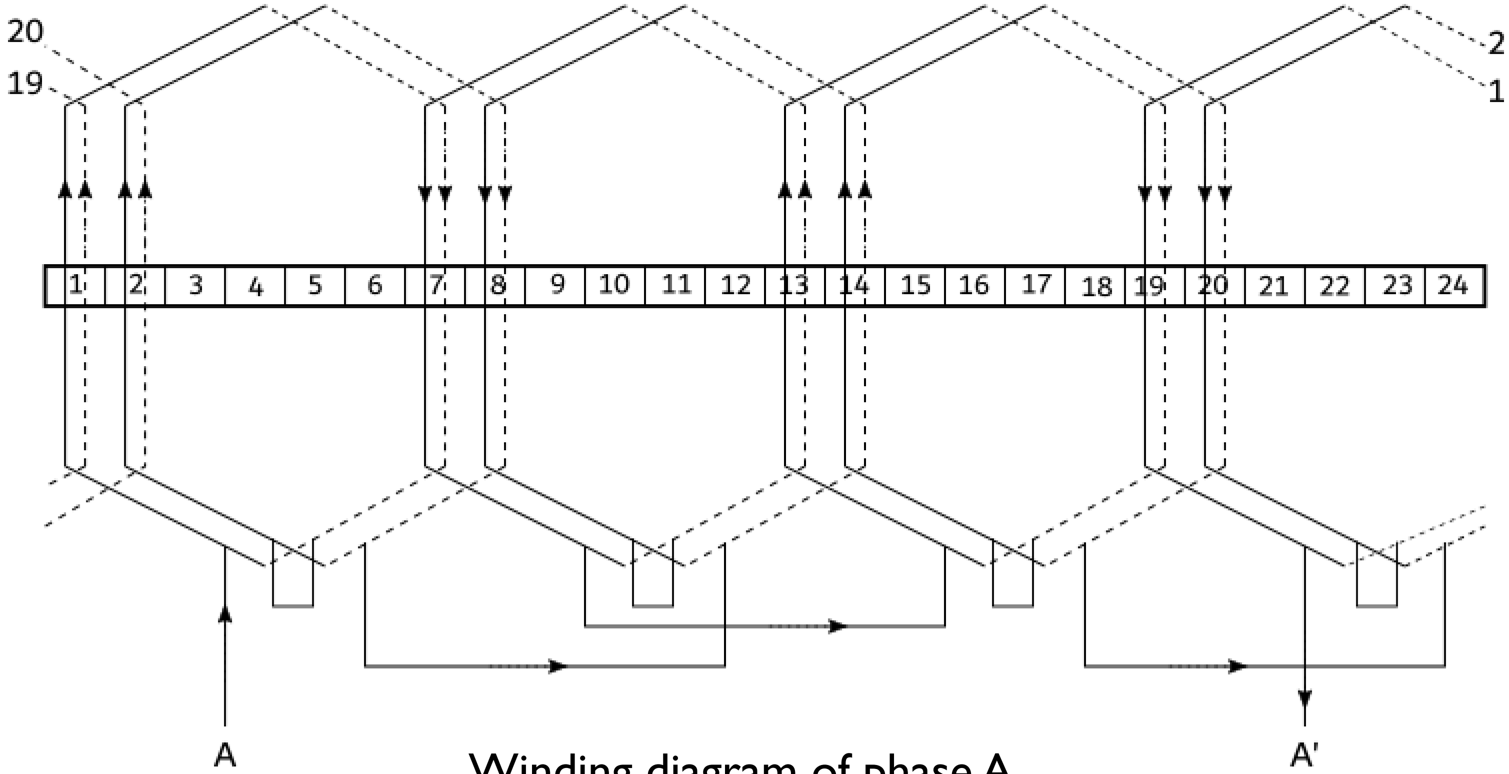
Winding placement:

Phase A	One side of the coil is placed at slot #1 in the top layer; other coil side should be placed at 1+coil pitch = 1+6 = slot #7 in the bottom layer.
Phase B	Starts at slot # $\left(1+\frac{120^o}{\gamma}\right)$ = slot $\left(1+\frac{120^o}{30^o}\right)$ = slot #5 in the top layer; other side of the coil should be placed at slot #11 in the bottom layer.
Phase C	Starts at slot # $\left(1+\frac{240^o}{\gamma}\right)$ = slot $\left(1+\frac{240^o}{30^o}\right)$= slot #9 in the top layer; other side of the coil should be placed at slot #15 in the bottom layer.

Example 2: Integral slot winding

Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Top layer	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'
Bottom layer	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'

Coil placement in slots.



Winding diagram of phase A.

- The four coils of one phase are connected in series.

Integral slot chorded winding

- When the number of slots per pole per phase (SPP) is an integer, and the coil pitch is less than the pole pitch, then the winding is called integral-slot chorded winding.
- The coil pitch generally varies from 2/3 pole pitch to full pole pitch.
- The advantages of using chorded coils are:
  - To reduce the amount of copper required for the end-connections (or over hang).
  - To reduce the magnitude of certain harmonics in the waveform of phase emfs and mmfs.

Example: Integral slot chorded winding

Draw the armature winding diagram of a three phase machine: double-layer lap winding with total number of slots = 36, Number of poles = 4, chorded coils with coil pitch = 7, and pole pitch = 9.

SPP = 36/(4x3) = 3 slots.

Pole pitch = 9 slots.

Coil pitch = 7 slots.

Coil pitch < Pole pitch and SPP is an integer, hence winding is integral slot chorded type.

Pole pitch – Coil pitch = 9 – 7 = 2 slots, hence the coils are chorded by 2 slots.

Phase spread = 60°

Slot angle pitch $\gamma = \dfrac{180^\circ}{\text{Slots per pole}} = \dfrac{180^\circ}{36/4} = 20^\circ$

Phase belt = Phase spread/Slot angle pitch = 60°/20° = 3 slots.

Example: Integral slot chorded winding

Winding placement:

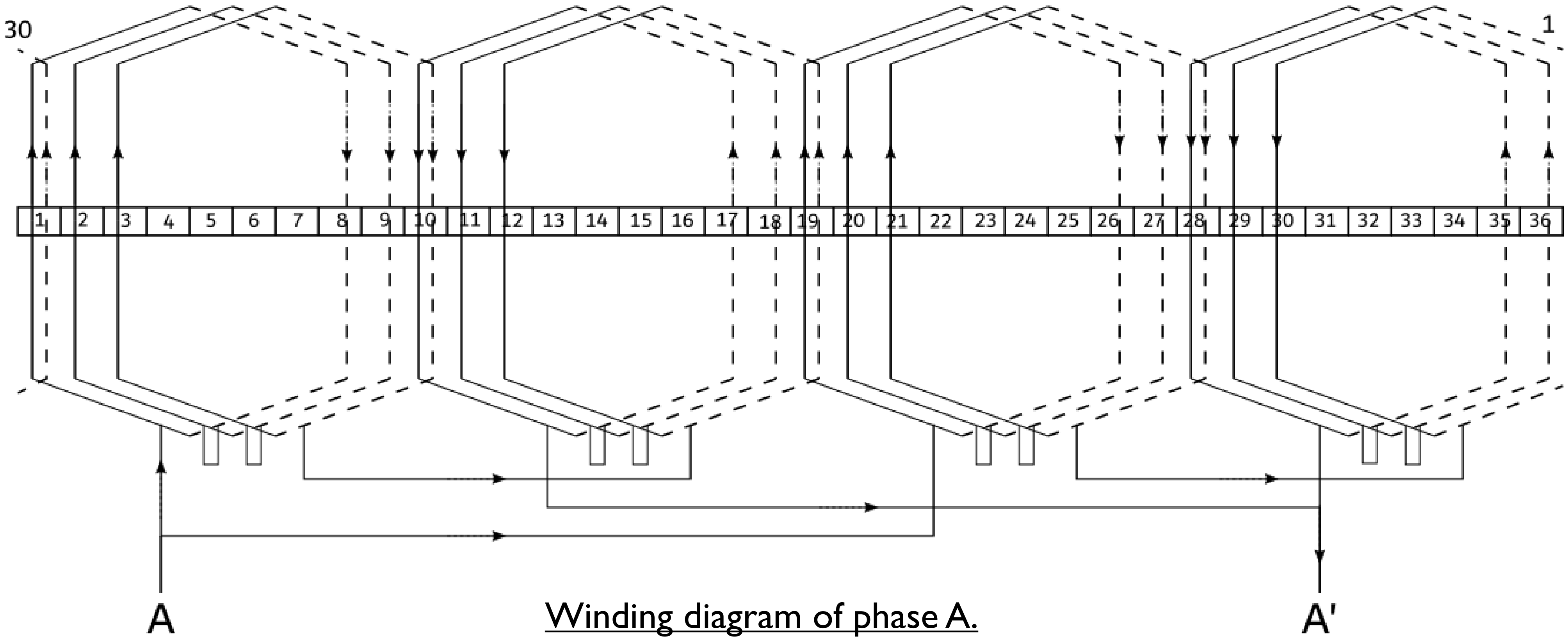
Phase A	One side of the coil is placed at slot #1 in the top layer; other coil side should be placed at 1+coil pitch = 1+7 = slot #8 in the bottom layer.
Phase B	Starts at slot # $\left(1+\frac{120^o}{\gamma}\right)$ = slot $\left(1+\frac{120^o}{20^o}\right)$ = slot #7 in the top layer; other side of the coil should be placed at slot #14 in the bottom layer.
Phase C	Starts at slot # $\left(1+\frac{240^o}{\gamma}\right)$ = slot $\left(1+\frac{240^o}{20^o}\right)$ = slot #13 in the top layer; other side of the coil should be placed at slot #20 in the bottom layer.

Example: Integral slot chorded winding

Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Top layer	a	a	a	c'	c'	c'	b	b	b	a'	a'	a'	c	c	c	b'	b'	b'	a	a	a	c'	c'	c'	b	b	b	a'	a'	a'	c	c	c	b'	b'	b'
Bottom layer	a	c'	c'	c'	b	b	b	a'	a'	a'	c	c	c	b'	b'	b'	a	a	a	c'	c'	c'	b	b	b	a'	a'	a'	c	c	c	b'	b'	b'	a	a

chorded by 2 slots

Coil placement in slots.



Winding diagram of phase A.

- The four coils of one phase are connected in parallel in two series groups of two coils each.

Fractional slot winding

- When the number of slots per pole per phase (SPP) of a winding is a fraction, then the winding is called a *fractional slot winding*.
- Advantages of fractional slot windings when compared with integral slot windings are:
  - a great freedom of choice with respect to the number of slots.
  - this winding allows more freedom in the choice of coil pitch.
  - the amount of copper used in the overhang is reduced and hence a saving on copper.
  - this winding reduces the high-frequency harmonics in the emf and mmf waveforms.

Fractional slot winding

- In fractional slot winding, SPP is a fraction.
- For symmetry, the total number of armature slots should be divisible by the number of phases, i.e. 3.
- Each phase should occupy the same number of slots otherwise the winding would be unbalanced.
- The slots per pole per phase is expressed as a whole number plus a fraction or as an improper fraction.

Example: A three phase ac machine armature with 36 slots constituting 8 field poles.

$$\text{SPP} = \frac{36}{8 \times 3} = 1\frac{1}{2} \text{ or } \frac{3}{2}$$

Arranging fractional slot windings with the aid of tables

- The coil groups in a fractional-slot winding are easily arranged with the aid of a table.
- Taking a sheet paper, the table is drawn with as many horizontal lines as there are poles, and each line is divided into $3C$ boxes, where C is the numerator of the improper fraction representing the slots per pole per phase and 3 is no. of poles.
- The table is next divided by vertical lines forming three equal columns for the three phases with C boxes per phase.
- Following this, in ordinal succession, the boxes are filled in with the numbers of the slots at intervals of d boxes, where d is the denominator of the fraction expressing the number of slots per pole per phase.

Arranging fractional slot windings with the aid of tables

Example: Produce the winding table of a three-phase ac machine with armature slots of 27 and number of poles 6.

SPP = $\frac{27}{6\times 3} = \frac{27}{18} = \frac{3}{2} = 1\frac{1}{2}$

Winding table:

Draw a table where no. rows = no. of poles= 6, and each column of three phases with C no. of sub columns, where, C is the numerator of the improper fraction, i.e. 3.

Fill the boxes starting from the extreme left top box with cross (representing adjacent slots). Proceed to the right marking crosses separated from each other by denominator of the improper fraction of no. of slots per phase per pole, i.e. 2.

Arranging fractional slot windings with the aid of tables

Winding table

No. Of Poles	Phase A			Phase C			Phase B		
N	X		X		X		X		X
S		X		X		X		X	
N	X		X		X		X		X
S		X		X		X		X	
N	X		X		X		X		X
S		X		X		X		X	

Arranging fractional slot windings with the aid of tables

Winding table Interpretation

- Reading the table horizontally line by line, write down the letter of the respective phase each time a cross appears in its column.
- This reveals the following sequence of the coils of each phase under consecutive poles.

aacbb, accb, aacbb,accb, aacbb, accb

- Each letter indicates the coils of each phase, and like letters succeeding one another indicate how many coils of the same phase the group will contain.

Coil groups: 2,1,2; 1,2,1; 2,1,2; 1,2,1; 2,1,2; 1,2,1.

- Thus, in our example, the sequence shows that it is necessary to prepare nine groups of two coils each and nine single coils.
- They will occupy (9x2)+9 = 27 slots.

Example

Design and draw the developed winding diagram of an AC motor with following details: No of poles= 6 no. of phases = 3, No. of slots = 27, coil pitch = 4, double layer lap winding.

SPP = $\frac{27}{6\times3} = \frac{27}{18} = \frac{3}{2} = 1\,\frac{1}{2}$

SPP is a fraction, therefore fractional slot winding is used.

Pole pitch = $\frac{27}{6} = 4.5$ slots

Coil pitch = 4.

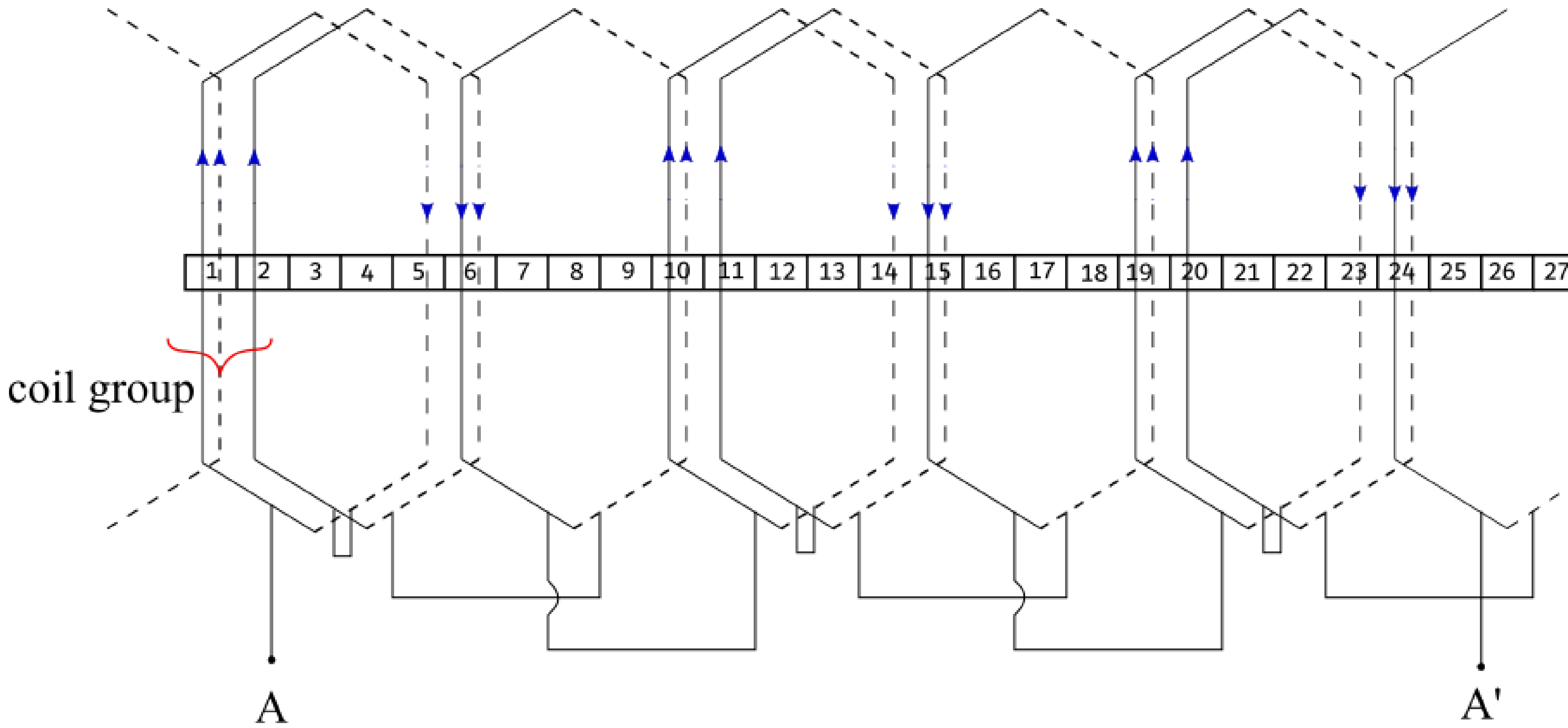
From the previous example, the sequence of coils of each phase under consecutive poles for the machine is

aacbb, accb, aacbb,accb, aacbb, accb.

Example: Solution

Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Top layer	a	a	c'	b	b	a'	c	c	b'	a	a	c'	b	b	a'	c	c	b'	a	a	c'	b	b	a'	c	c	b'
Bottom layer	a	c'	c'	b	a'	a'	c	b'	b'	a	c'	c'	b	a'	a'	c	b'	b'	a	c'	c'	b	a'	a'	c	b'	b'

Coil placement in slots.

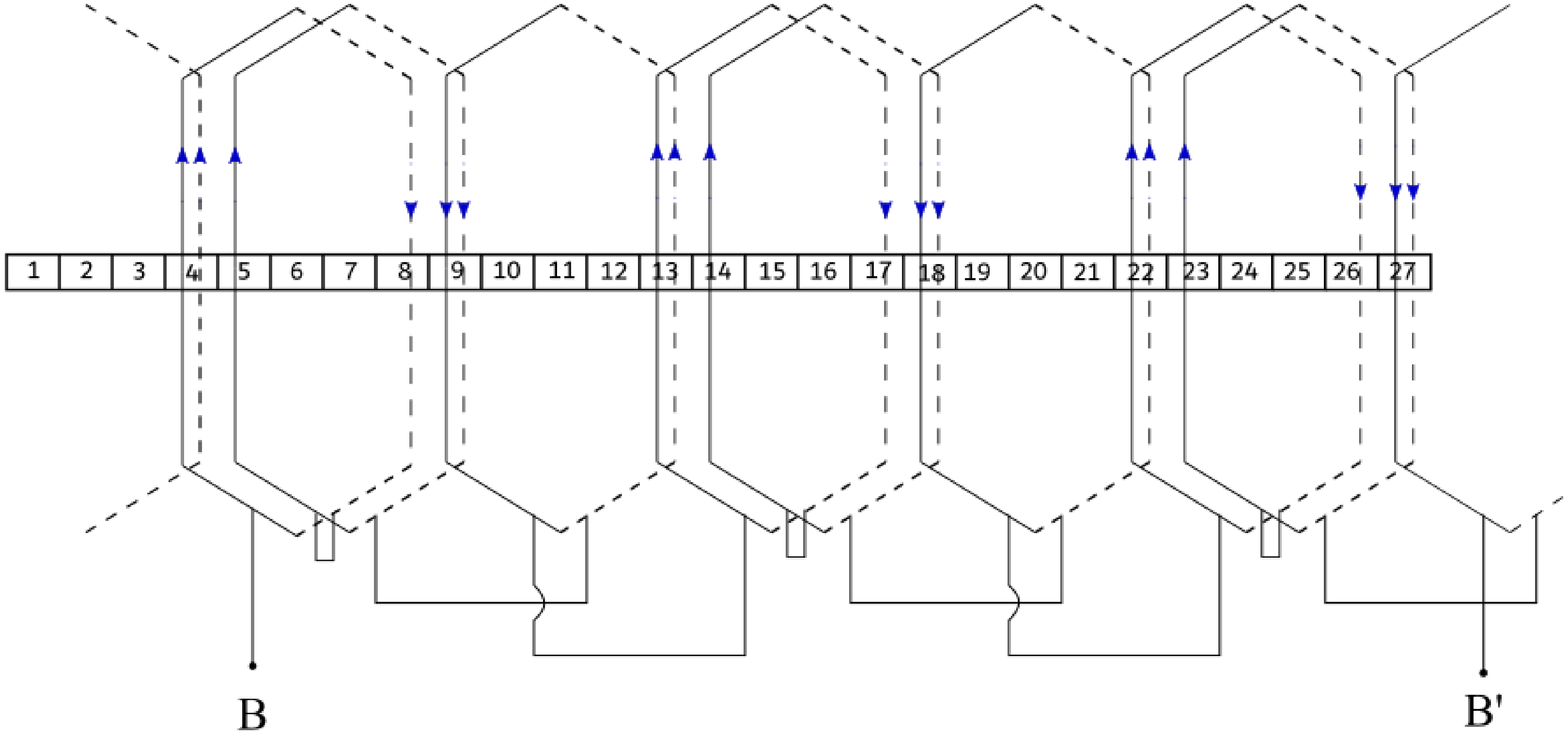


Winding diagram of phase A.

Example: Solution

Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Top layer	a	a	c'	b	b	a'	c	c	b'	a	a	c'	b	b	a'	c	c	b'	a	a	c'	b	b	a'	c	c	b'
Bottom layer	a	c'	c'	b	a'	a'	c	b'	b'	a	c'	c'	b	a'	a'	c	b'	b'	a	c'	c'	b	a'	a'	c	b'	b'

Coil placement in slots.

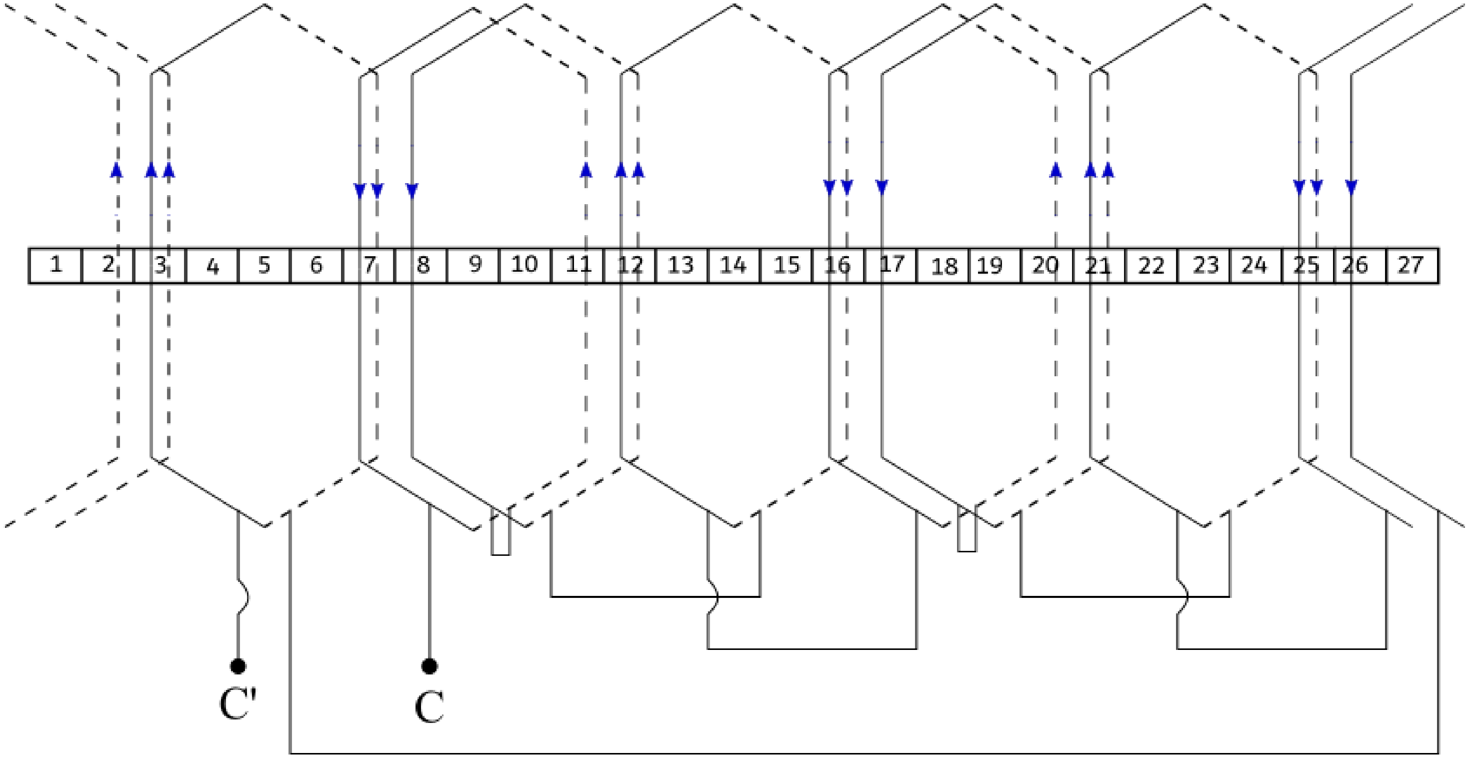


Winding diagram of phase B.

Example: Solution

Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Top layer	a	a	c'	b	b	a'	c	c	b'	a	a	c'	b	b	a'	c	c	b'	a	a	c'	b	b	a'	c	c	b'
Bottom layer	a	c'	c'	b	a'	a'	c	b'	b'	a	c'	c'	b	a'	a'	c	b'	b'	a	c'	c'	b	a'	a'	c	b'	b'

Coil placement in slots.



Winding diagram of phase C.

Single layer winding

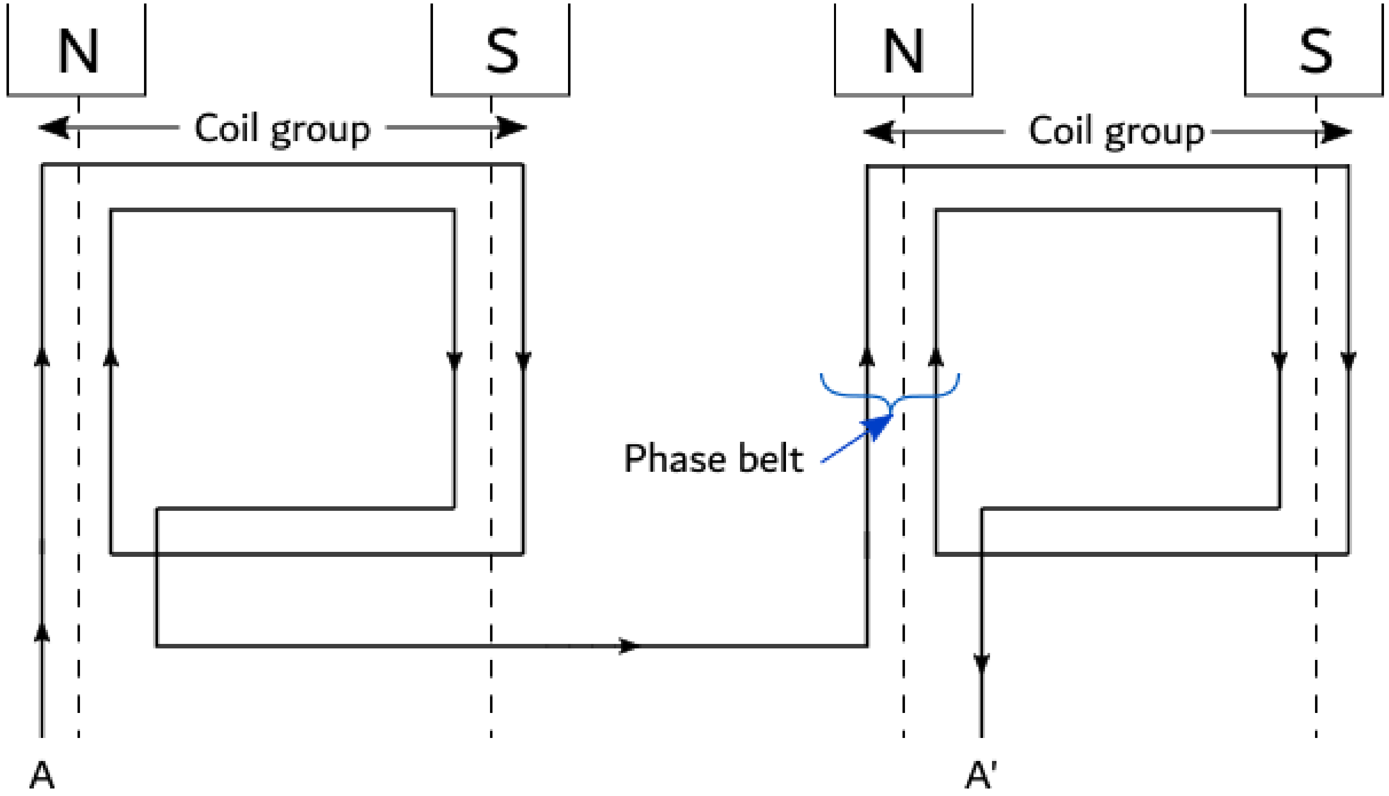
- Single-layer windings are not commonly used in practice except for machines of a few kW.
- One coil side occupies one slot completely, therefore number of coils is equal to half the number of slots.
- The 3-phase single-layer windings types:
  - Concentric winding
  - Mush winding
- In case of three phase machine, single layer winding must be designed to allow the end-connections to be accommodated in separate tiers or planes.

Concentric Windings

- The coils under one pole pair are wound such that the coils have one center.
- Coil Group: The group of coils having the same center is defined as *coil group*.
- The concentric winding can further be sub-divided into
  - half coil winding or unbifurcated winding
  - whole coil winding or bifurcated winding

Half coil winding or unbifurcated winding

- The number of coils in each coil group = the number of coil sides in each phase belt.
- The direction of current in all coil groups are the same.



Half coil winding of phase A.

Example: Half coil winding

Draw the half coil single layer concentric windings for a 3-phase machine with 24-slots, 4-poles for phase spread of 60°.

Total number of slots = 24

Pole pitch = 24/4 = 6 slots

SPP = 24/(3x4) = 2 slots

$$\gamma = \frac{180^{o}}{\text{Slots per pole}} = \frac{180^{o}}{24/4} = 30^{o}$$

Phase spread = 60°

Phase belt = Phase spread/Slot angle pitch = 60°/30° = 2 slots.

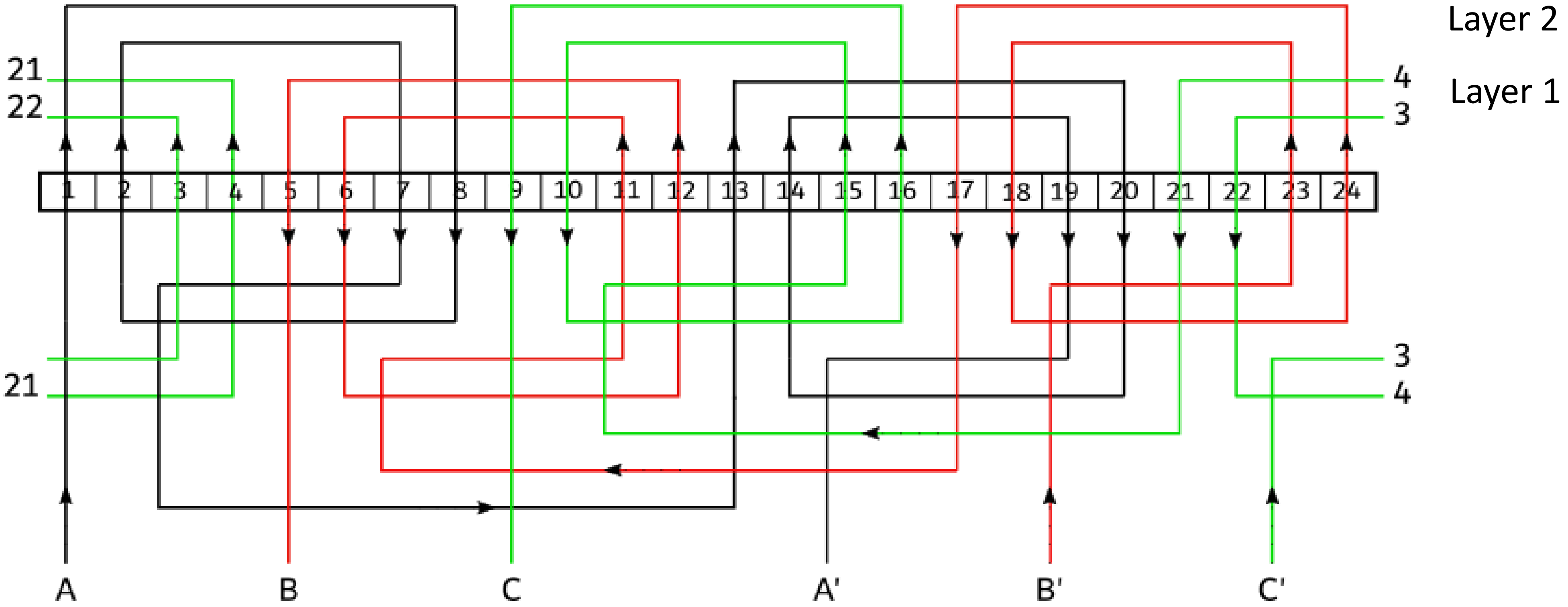
<u>Winding placement:</u>

Phase A	One side of the coil is placed at slot #1
Phase B	Starts at slot # $\left(1+\frac{120^{o}}{\gamma}\right)$ = slot $\left(1+\frac{120^{o}}{30^{o}}\right)$ = slot #5
Phase C	Starts at slot # $\left(1+\frac{240^{o}}{\gamma}\right)$ = slot $\left(1+\frac{240^{o}}{30^{o}}\right)$ = slot #9

Example: Half coil winding

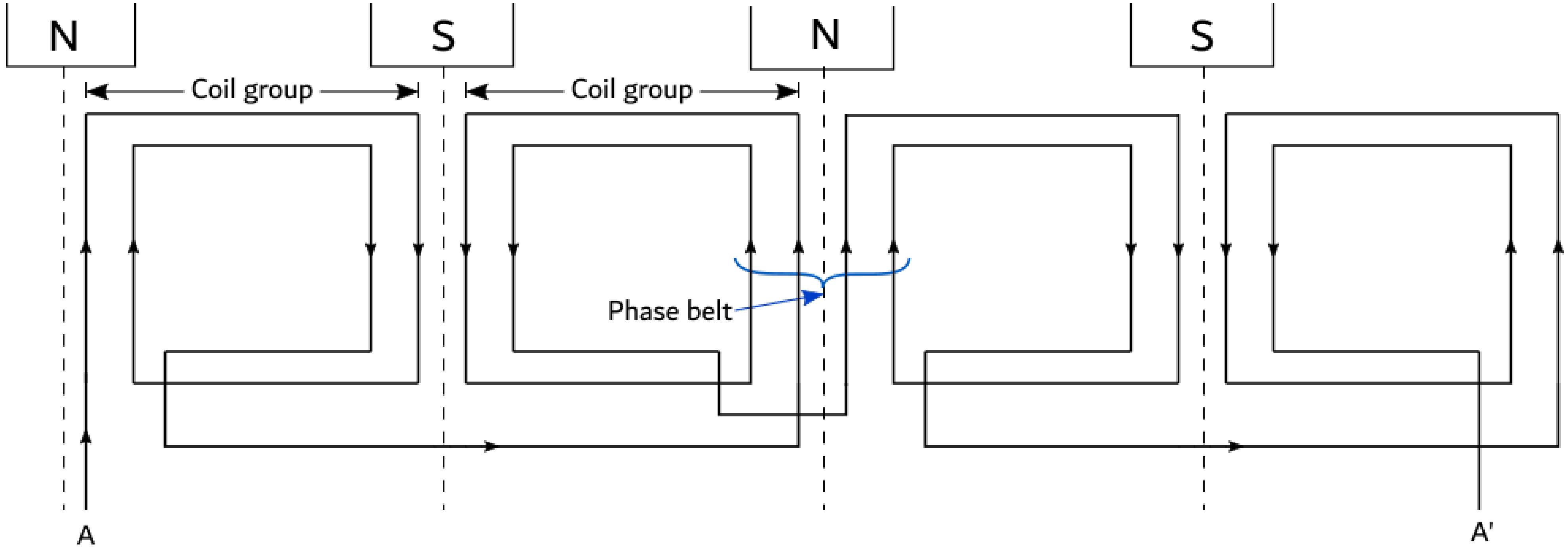
Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'

Coil placement in slots.



Whole coil winding or bifurcated winding

- In whole coil winding, the number of coil sides in each phase belt are double the number of coils in each coil group.
- The number of coil groups equals the number of poles.
- The adjacent coil groups carry currents in opposite directions.



Example: Whole coil winding

Draw the whole coil single layer concentric windings for a 3-phase machine with 24-slots, 4-poles for phase spread of 60°.

SPP = 24/(3x4) = 2 slots

Pole pitch = 24/4 = 6 slots

$\gamma = 30^{\mathrm{o}}$, Phase spread = 60°

The number of coils per phase belt = 2

The number of coils in each coil group = 1

The coil pitch of 6 slot pitches does not result in proper arrangement of the winding.

Therefore a coil pitch of 5 is chosen.

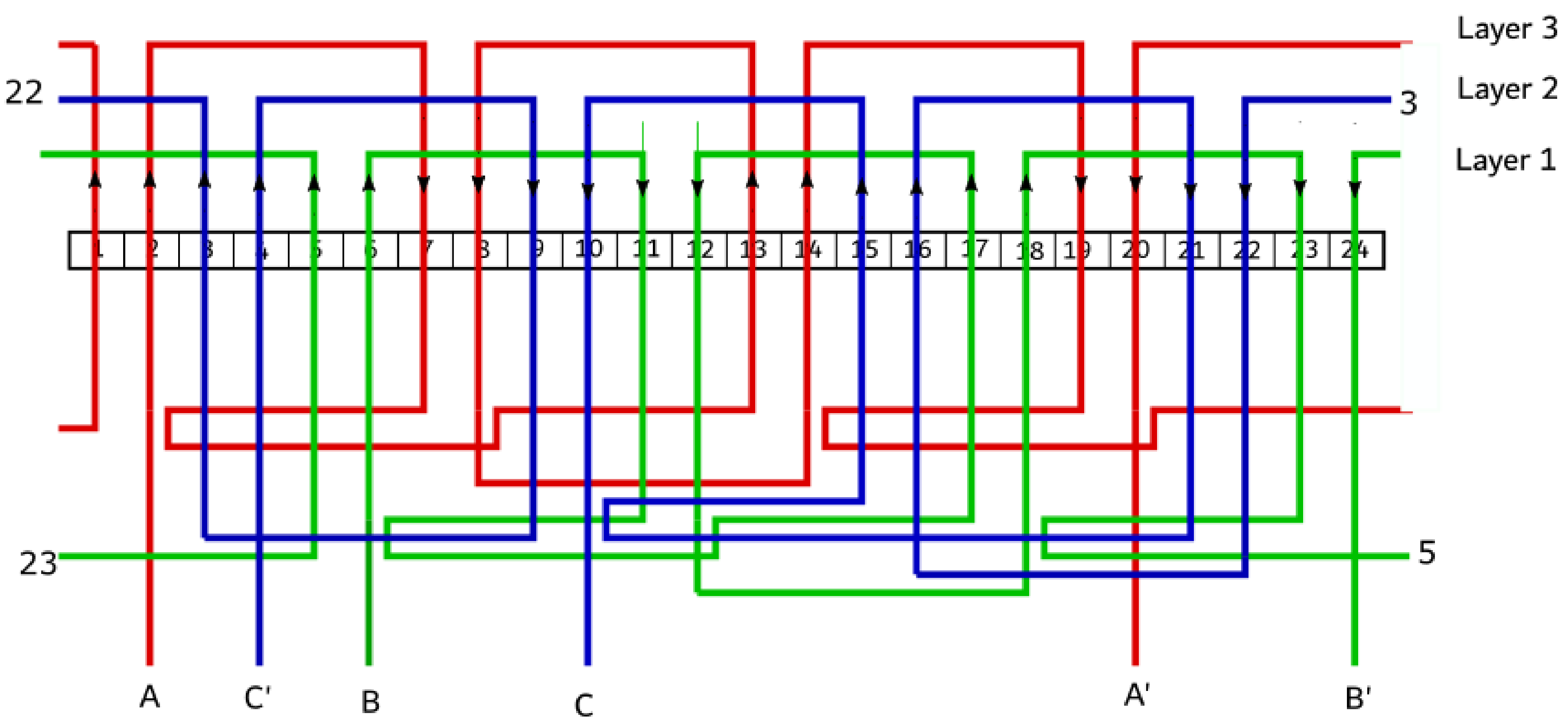
<u>Winding placement:</u>

Phase A	One side of the coil is placed at slot #1
Phase B	Starts at slot # $\left(1+\frac{120^{o}}{\gamma}\right)$ = slot $\left(1+\frac{120^{o}}{30^{o}}\right)$ = slot #5
Phase C	Starts at slot # $\left(1+\frac{240^{o}}{\gamma}\right)$ = slot $\left(1+\frac{240^{o}}{30^{o}}\right)$ = slot #9

Example: Whole coil winding

Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'

Coil placement in slots.



Mush winding

- The coil pitch is the same for all the coils.
- Each coil is wound on a trapezoidal shaped.
- The number of slots per pole per phase must be a whole number.
- The coil pitch is always odd.

Example: Mush winding

Winding diagram of 24 slots, 4 poles, single-layer mush winding for a 3-phase machine.

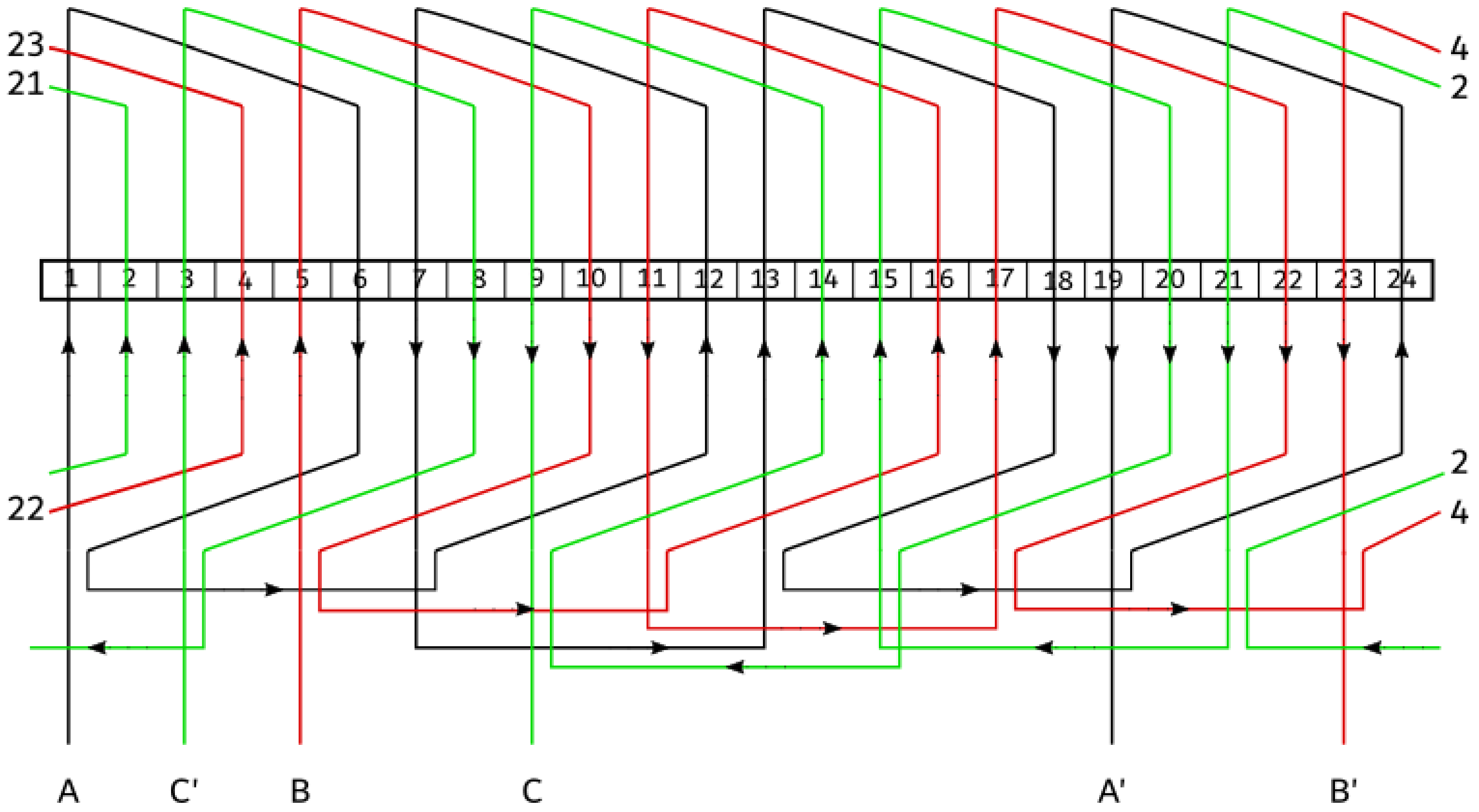
SPP = 2 slots

Since the coil pitch must be odd, it can be taken as 5 or 7. Choosing here a coil pitch of 5 slot pitches.

Example: Mush winding

Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	a	c'	c'	b	b	a'	a'	c	c	b'	b'	a	a	c'	c'	b	b	a'	a'	c	c	b'	b'	a

Coil placement in slots.



Choice of winding

- *Type of coil:* lap, wave, concentric
- *Overhang:* diamond, multiplane, mush
- *Layers:* single, double
- *Phase spread:* 60°, 120°
- *Slotting:* integral, fractional
- *Coil-span:* full-pitched, chorded
- *Circuits:* series, parallel
- *Coils:* single-turn, multiturn

AC Windings

1 Winding factor

Winding factor for a specific winding expresses the ratio of flux linked by that winding compared to flux that would have been linked by a single-layer full-pitch integer-slot winding with the same number of turns and one single slot per pole per phase. The torque of an electric motor is proportional to the fundamental winding factor.

The winding factor K_w can generally be expressed as the product of two factors, the pitch factor K_p (sometimes also called coil-span or chording factor) and the breath coefficient or distribution factor K_d.

$$K_w = K_p.\, K_d$$

According to the definition of winding factor, the winding factor of a single-layer full-pitch non-skewed integer-slot winding with one single slot per pole per phase must be 1.0.

Examples of winding layouts that have a winding factor of 1.0:

Single-layer 2-pole 6-slot integer-slot winding.
Single-layer 4-pole 12-slot integer-slot winding.
Single-layer 6-pole 18-slot integer-slot winding.
Single-layer 8-pole 24-slot integer-slot winding.

Pitch factor

In full pitched coil, the phase angle between the induced emfs of two coil sides is exactly 180° (electrical). Hence, the resultant emf of a full pitched coil is just the arithmetic sum of the emfs induced on both sides of the loop.

However, in short pitched coil, the phase angle between the induced emfs of two opposite coil sides is less than 180° (electrical). The resultant emf of the short pitched coil is found by adding two phasor values. The phasor sum of two quantities is always less than their arithmetic sum.

The pitch factor is the measure of resultant emf of a short pitched coil in comparison with resultant emf of a full pitched coil.

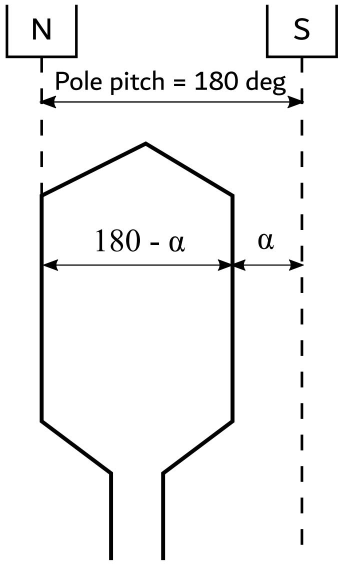


Figure 1

$$K_p = \frac{\text{emf generated in short pitched coil}}{\text{emf generated in full pitched coil}}$$
$$K_p = \frac{\text{Phasor sum of the induced emf}}{\text{Arithmatic sum of the induced emf}}$$

Lets consider a coil, which is short pitched by an angle α (electrical degree) as shown in Fig 1. Emf induced per coil side is E. The induced voltage across the coil terminals, if the coil would have been full pitched is 2E.

Fig. 2 shows the phasor diagram of the resultant emf of the short pitched coil. From the figure

$$E_R = 2E\cos\frac{\alpha}{2}$$

The pitch factor

$$K_p = \frac{2E\cos\frac{\alpha}{2}}{2E}$$
$$K_p = \cos\frac{\alpha}{2}$$

Example:

Calculate the pitch factors of the following windings.

1. 2-pole 6-slot winding with coil span of 3 slot pitches (i.e. full pitch).
   $\alpha = 0$ elec.deg
   $K_p = \cos 0 = 1$

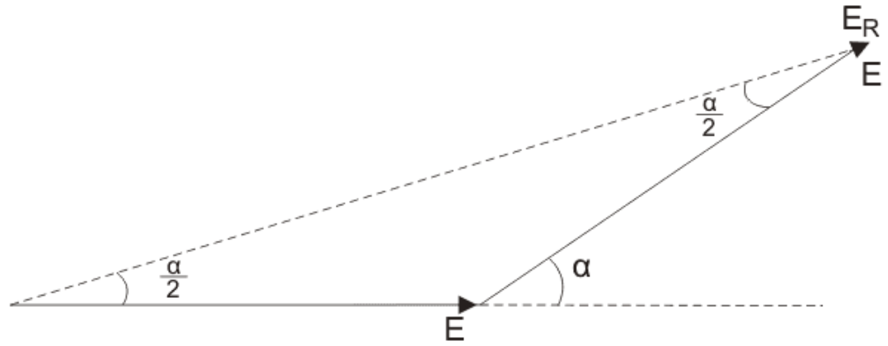


Figure 2: Induced voltage in a short pitched coil - phasor diagram.

2. 2-pole 6-slot winding with coil span of 2 slot pitches.
   $\alpha = 60$ elec.deg
   $K_p = \cos 30 = 0.866$

3. 2-pole 6-slot winding with coil span of 1 slot pitch.
   $\alpha = 120$ elec.deg
   $K_p = \cos 60 = 0.5$

Distribution factor

The distribution factor K_d reflects the fact that the winding coils of each phase are distributed in a number of slots. Since the emf induced in different slots are not in phase, their phasor sum is less than their numerical sum.

If all the coil sides of any one phase under one pole are bunched in one slot, the winding obtained is known as concentrated winding and the total emf induced is equal to the arithmetic sum of the emfs induced in all the coils of one phase under one pole.

But in practical cases, for obtaining smooth sinusoidal voltage waveform, armature winding is not concentrated but distributed among the different slots.

The distribution factor is a measure of resultant emf of a distributed winding in compared to a concentrated winding.

$$K_d = \frac{\text{emf induced in distributed winding}}{\text{emf induced if the winding would have been concentrated}}$$
$$K_d = \frac{\text{Phasor sum of the induced emf}}{\text{Arithmatic sum of the induced emf}}$$

Let consider an armature with n number of slots per pole and m number of slots per pole per phase. The induced emf per coil side is E_c.

Angular displacement between two consecutive slots also called slot angle pitch (γ)

$$\gamma \text{ (elec degree)} = \frac{P}{2}\frac{360}{nP} = \frac{180}{n}$$

The emfs induced (E_c) in each coils of one phase under one pole will be equal in magnitude but they differ from each other by an angle γ.

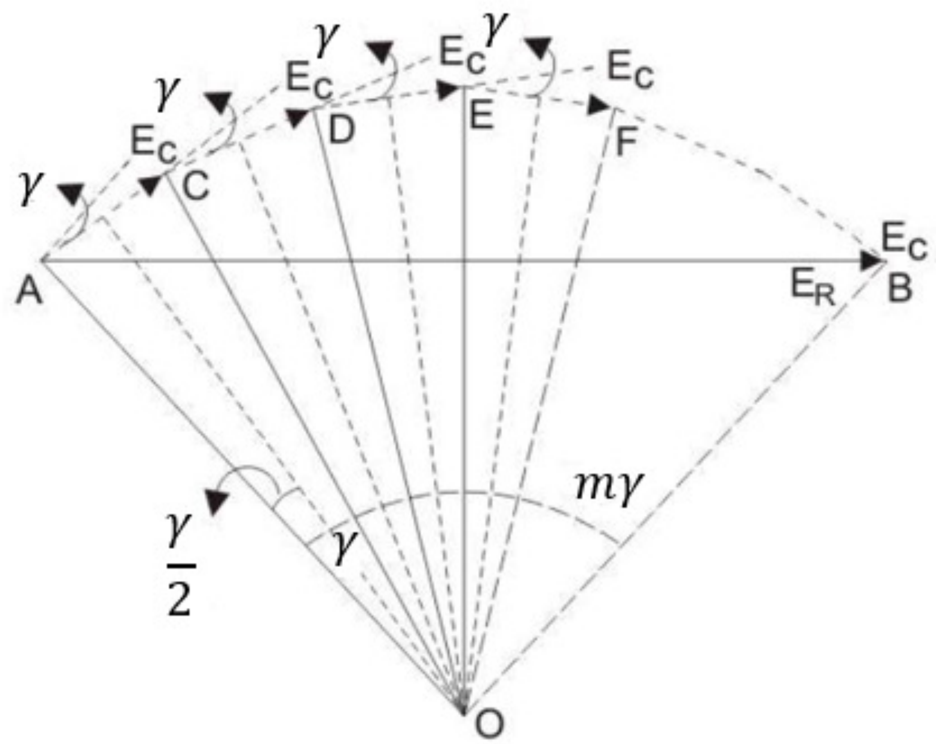


Figure 3: Induced voltage in distributed winding - phasor diagram.

From the phasor diagram shown in Fig. 3, emf induced in each coil side is given by

$$E_c = \text{AC} = 2 \times \text{OA} \times \sin\frac{\gamma}{2}$$

From the phasor diagram, the resultant emf

$$E_R = \text{AB} = 2 \times \text{OA} \times \sin\frac{m\gamma}{2}$$

As the slot per pole per phase is m, the total arithmetic sum of all induced emfs per coil sides per pole per phase is

$$2 \times m \times \text{OA} \times \sin\frac{\gamma}{2}$$

Therefore the distribution factor is

$$K_d = \frac{2 \times \text{OA} \times \sin\frac{m\gamma}{2}}{2 \times m \times \text{OA} \times \sin\frac{\gamma}{2}}$$

$$K_d = \frac{\sin \frac{m\gamma}{2}}{m \sin \frac{\gamma}{2}}$$

Example:

Calculate the distribution factors of the following windings.

1. 2-pole 6-slot 3-phase winding.
   $m = 1$
   $\gamma = 180/3 = 60$ elec.deg,
   $K_d = 1.0$

2. 2-pole 12-slot 3-phase winding.
   $m = 2$
   $\gamma = 180/6 = 30$ elec.deg,
   $K_d = \frac{\sin 30}{2 \sin 15} = 0.966$

3. 2-pole 18-slot 3-phase winding.
   $m = 3$
   $\gamma = 180/9 = 20$ elec.deg,
   $K_d = \frac{\sin 30}{3 \sin 10} = 0.96$

4. 2-pole 24-slot 3-phase winding.
   $m = 4$
   $\gamma = 180/12 = 15$ elec.deg,
   $K_d = \frac{\sin 30}{4 \sin 7.5} = 0.958$

2 Generated Voltage of AC Windings

In ac machines, field windings of each phase ideally should produce a sinusoidally distributed, radial field (B-wave) in the air gap. Figure 4 illustrates the sinusoidal B-wave of a synchronous machine and cross sectional form of a single full-pitched coil (coil-side space separation π elect. rad).

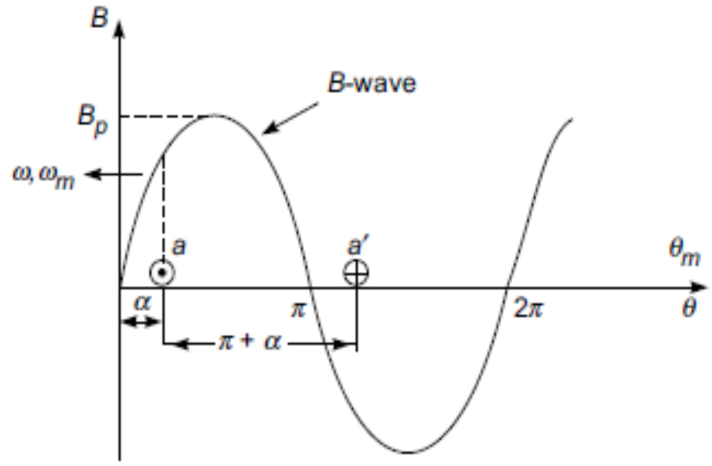


Figure 4: Relative location of the B-wave and armature coil at any time t.

The B-wave moves towards left with a speed of ω elec. rad/s or ω_m mech. rad/s. At the origin of time the coil-sides are located in the interpolar region where the pole flux links the coil. At any time t, the coil has relatively moved by α elec.rad to the right of the B-wave.

$$\alpha = \omega t \text{ elec.rad}$$

The B-wave can be expressed as

$$\begin{aligned} B &= B_p \ \sin \theta \\ &= B_p \ \sin \left(\frac{P}{2} \theta_m \right) \end{aligned}$$

where B_p is the peak flux density.

Since the flux is physically spread over the mechanical angle, the flux Φ linking the coil can be computed by integrating over the mechanical angle.

$$\phi = \int_{2\alpha/P}^{2(\pi+\alpha)/P} B_p \ \sin \left(\frac{P}{2} \theta_m \right) lr \ d\Theta_m$$

where
l = active coil-side length
r = mean radius of the stator at the air-gap

Since

$$\theta_m = \frac{2}{P}\theta$$

$$\begin{aligned}
\phi &= \frac{2}{P}\int_{\alpha}^{\pi+\alpha} B_p\ lr\ \sin\theta\ d\Theta \\
&= \frac{2}{P}\ 2B_p\ lr\ \cos\alpha \\
&= \frac{2}{P}\ 2B_p\ lr\ \cos\ \omega t \\
&= \Phi\ \cos\ \omega t
\end{aligned}$$

According to the above equation, the flux linking the coil varies sinusoidally. The maximum value of the flux is

$$\Phi = \frac{4}{P}\ B_p\ lr\ \ \text{(flux/pole)} \tag{1}$$

The flux linkage of the coil at any time t is

$$\lambda = N\phi = N\Phi\ \cos\ \omega t$$

where N is the number of turns of the coil.

The induces emf in the coil is

$$e = -\frac{d\lambda}{dt} = \omega N\Phi\ \sin\ \omega t$$

The rms value of emf induced in the coil

$$E = \sqrt{2}\,\pi f N\Phi\ \ \text{Volts} \tag{2}$$

The spatial flux density wave upon rotation causes time-varying flux linkages with the coil and hence the production of emf.

The sinusoidally varying flux linking the coil leads the sinusoidally varying emf by 90^o. This difference is caused by the negative sign in the induced emf.

2.1 Distributed windings

It may be seen from eq.(1) that the flux/pole is limited by the machine dimensions and the peak flux density which cannot exceed a specified value dictated by saturation characteristic of iron.

Therefore, for inducing an emf of an appropriate value in a practical machine, a large number of coil turns are needed and it is not possible to accommodate all these in a single slot-pair.

Furthermore, it may be also noticed that with one coil/pole pair/phase, i.e. one slot/pole/phase, the periphery of the stator is far from being fully utilized. Therefore, it is natural to create more slots/pole/phase on the stator periphery.

In a practical machine with S slot distributed uniformly around the stator periphery,

$$\text{SPP} = m = \frac{S}{qP}$$

where q is the number of phases, which is generally 3.

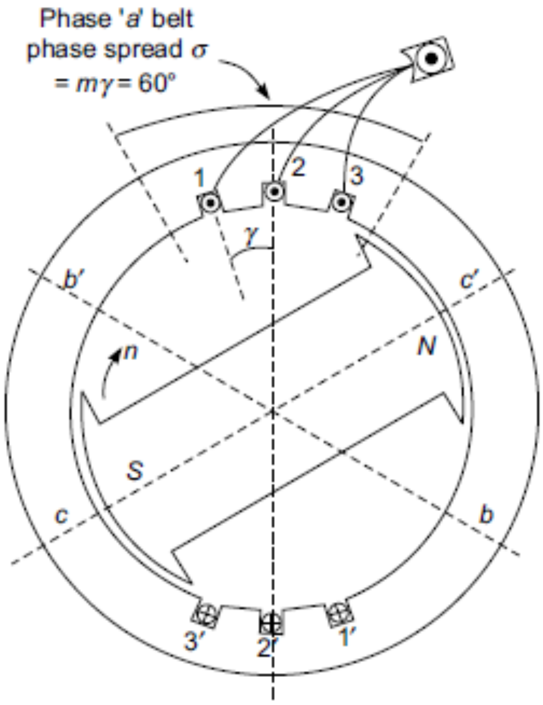


Figure 5: Synchronous generator with distributed windings

Figure 5 illustrates a 2-pole, 3-phase machine with $m = 3$.

The angle between two adjacent slots is

$$\gamma = \frac{\pi P}{S} \text{ elec. rad}$$

The winding of phase a in the machine has three coils ($11'$, $22'$ and $33'$) which are placed in three slot-pairs distributed in space with an angular separation of γ elec.rad.

The total angle $\sigma = m\gamma$ occupied by the phase winding along the armature periphery is called the phase spread. Such a winding is referred to as the distributed winding.

Since the machine is always wound with identical coils, the sinusoidal emfs induced in coils $11'$, $22'$, and $33'$ have the same rms value (E) but have a progressive time phase difference of γ because coil are uniformly distributed in space.

These coils are series connected to yield the phase voltage, which is the phasor sum of the coil emf's. Because of distribution, the rms phase voltage is less than the algebraic sum of the rms coil voltages. This reduction ratio called the distribution factor K_d.

The induced phase emf for a distributed winding is obtained by multiplying Eq. (2) by K_d.

$$E = \sqrt{2}\pi\ K_d f N_{ph} \Phi \ \text{ Volts} \tag{3}$$

where N_{ph} is the total turns per phase.

2.2 Short-pitch (Chorded) Windings

Lets consider a coil, which is short pitched by an angle α (electrical degree) as shown in Fig 1.

The reduction of the coil emf due to short-pitching is governed by the pitch factor. Therefore, the induced phase emf of a short pitched concentrated winding is given by

$$\sqrt{2}\pi\ K_p f N_{ph} \Phi \ \text{ Volts}$$

where K_p is the pitch factor.

The induced phase emf of a short pitched distributed winding is given by

$$E = \sqrt{2}\pi\ K_p K_d f N_{ph} \Phi \ \text{ Volts}$$

where $K_w = K_p K_d =$ winding factor.

$$E = \sqrt{2}\pi\ K_w f N_{ph} \Phi \ \text{ Volts} \tag{4}$$

Example

Calculate the flux/pole required for a three-phase, 4-pole synchronous machine to generate 50 Hz ac voltage of 6.6 kV (phase voltage). The armature of the synchronous machine consists of 36 slots. Each slot has 4 conductors. The coil pitch is 7 slots.

Pole pitch $= \dfrac{36}{4} = 9$ slots

slot angle pitch, $\gamma = \dfrac{180^o}{9} = 20^o$

$$\alpha = \gamma(9-7) = 40^o$$
$$K_p = \cos\frac{\alpha}{2} = \cos\frac{40^o}{2} = 0.94$$
$$m = \frac{36}{3\times 4} = 3$$
$$K_d = \frac{\sin\frac{m\gamma}{2}}{m\sin\frac{\gamma}{2}} = \frac{\sin\frac{3\times 20^o}{2}}{3\sin\frac{20^0}{2}} = 0.96$$
$$E = \sqrt{2}\pi K_p K_d f N_{ph}\Phi$$
$$N_{ph} = \frac{36\times 4}{3\times 2} = 24$$
$$\Phi = \frac{E}{\sqrt{2}\pi K_p K_d f N_{ph}} = \frac{6600}{\sqrt{2}\pi\times 0.94\times 0.96\times 50\times 24} = 2.43 \text{ Wb}$$