Question 1 (20 marks)

Consider the following two linear programs:

(1) Model A

$$max\ 5x_1 + 7x_2$$

$$s.t.\ \ 2x_1 - x_2 \le -1$$

$$-x_1 + 2x_2 \le -1$$

$$x_1, x_2 \ge 0$$

(2) Model B

$$max\ x_1 - x_2$$

$$s.t.\ -x_1 + 3x_2 \le 30$$

$$-3x_1 + x_2 \le 30$$

$$x_1, x_2 \ge 0$$

(a) (10 marks) Sketch the feasible region of the LP model A and B. Are the feasible regions unbounded?

(b) (10 marks) Solve the LP models A and B using the graphical method. Explain your approach.

Question 2:

The data below gives an indication of cashier's productivity at a Tesco store. x: the average items scan rate per hour and y: the average checkout sales per hour. The data is obtained for 10 full time checkout staff in a London store working on a Monday.

| Staff Name	Sam	Jay	Katie	Sophie	Dan	Tim	Lee	Ali	Paul	Amanda
x: Scan rate	100	115	45	145	70	55	55	100	85	85
y: Average Sales	450	525	315	495	495	390	330	465	495	540

a) a) By applying a scatter plot diagram and the mean formula to the above table, is there a connection (association) between the sales per hour achieved by the 10 member of staff and the item scan rate per hour? Justify (describe) your answer in the context of the variables mean and the plotted scatter graph.

b) Calculate the covariance cov(x,y) to then describe the tendency of the pattern based on the obtained calculation.

c) Calculate the Pearson's product moment correlation coefficient r and describe the association between both Item Scan Rate and the average sales in relation to r.

Question 3 (25 marks)

A gas company owns a pipeline network, sections of which are used to pump natural gas from its main field to its distribution centre. The network is shown below, where the direction of the arrows indicates the only direction in which the gas can be pumped. The pipeline links of the system are numbered one through six, and the intermediate nodes are large pumping stations. At the present time, the company needs 1200 mcf (million cubic feet) of gas per month from its main field and must transport that entire amount to the distribution centre. The following are the maximum usage rates and costs associated with each link:

	1	2	3	4	5	6
Maximum usage: mcf/month	500	900	700	400	600	1000
Tariff: $/mcf	20	25	10	15	20	40

The gas company wants to find those usage rates that minimise total cost of transportation.
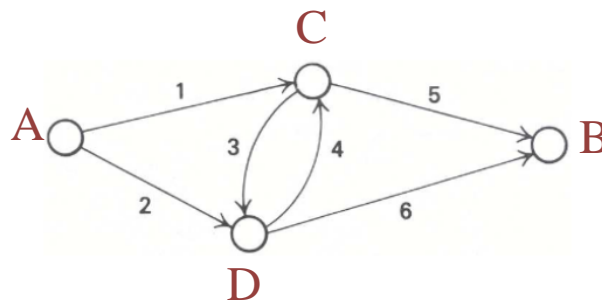
(a) **(5 marks)** What are the decision variables?

(b) **(20 marks in total)** Formulate the problem as a linear program.