高等数学A23秋期中统考

一. 求极限

(1) $\lim\limits_{n\to+\infty} \dfrac{3^n}{n!}$

(2) $\lim\limits_{n\to+\infty}\left[\dfrac{1}{(n+1)^2} + \dfrac{1}{(n+2)^2} + \cdots + \dfrac{1}{(2n)^2}\right]$

(3) $\lim\limits_{x\to+\infty} \sin\left[\pi\left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)\right]$

(4) $\lim\limits_{n\to+\infty}\left[\dfrac{1}{n^2}\sum\limits_{k=1}^{n} k\ln(n+k) - \dfrac{n+1}{2n}\ln n\right]$

二.

(1) $y = x\sqrt{1+x^2} + \ln(x+\sqrt{1+x^2})$ 求 $\dfrac{dy}{dx}$

(2) $y = \begin{cases} x^4 \sin\dfrac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$ 求 $\dfrac{d^2y}{dx^2}$

(3) $y = \displaystyle\int_{\cot x}^{\tan x} \sqrt{1+t^2}\,dt$ 求 $\dfrac{dy}{dx}$

(4) $F(x) = f(x) - f^{(2)}(x) + f^{(4)}(x) + \cdots + (-1)^n f^{(2n)}(x)$

$f(x) = x^n(1-x)^n$

求 $\dfrac{d}{dx}\left[F'(x)\sin x - F(x)\cos x\right]$

三. 求积分

(1) $\displaystyle\int \sqrt{1+x^2}\,dx$

(2) $\displaystyle\int \dfrac{\arctan e^x}{e^x + e^{-x}}\,dx$

(3) 由 $y^2(x-y) = x^2$ 确定 $y = y(x)$ 求 $\displaystyle\int \dfrac{1}{y^2}dx$

四. $f(x) = \lim\limits_{n\to+\infty} \dfrac{x^{2n-1} + ax^2 + bx}{x^{2n} + 1}$ 求 a, b 使 $f(x)$ 在 R 连续

五. 求积分

(1) $\displaystyle\int_0^1 \dfrac{\sqrt{x}}{1+\sqrt{x}}\,dx$

(2) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \dfrac{\sin^2 x}{1+e^x}\,dx$

(3) $\displaystyle\int_0^\pi \left(\int_0^x \dfrac{\sin t}{\pi - t}\,dt\right)dx$

六. 求 $\lim\limits_{n\to\infty} \dfrac{1}{n}\sum\limits_{k=1}^{n}(-1)^{k-1} f\left(\dfrac{k}{n}\right)$

七. $f(x) \in C[0, +\infty)$, $f(0) = 0$ $x>0$ 时 $0 < f(x) < x$

$a_1 = f(1)$ $a_2 = f(a_1)$ $\cdots$ $a_n = f(a_{n-1})$ 证明 $\lim\limits_{n\to\infty} a_n = 0$