The number of information N and the orbital parameters in our Solar system

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***Abstract*—In this paper, the relationship between the number of information N and the orbital parameters in the solar system is investigated. This is achieved by considering non-Newtonian Yukawa-type correction to the classical gravitational potential as a function of the orbital mean motion for circular and elliptical orbits, where an expression for the mass of the Graviton is calculated using the Compton wavelength expression derived from the Yukawa-type potential. Finally, relating the the derived mass of the Graviton in terms of the orbital parameters, and what Wesson [3] obtained for the limit on gravitational mass by combing the cosmological constant, Planck's constant, the speed of light and the gravitational constant G, and taking into account that the number of information bits N is related to the entropy S; an expression for the gravitational mass in terms of the N is found. The aim is to relate both expressions for the gravitational mass to find an expression for the orbital parameters in the solar system in terms the number of information N.**

I. Yukawa potential orbital energy and the mass of the graviton mediating the interaction in Celestial bodies [2]

A. Non-Newtonian Yukawa-type correction

Even though Newtonian gravity still holds in most cases, still some anomalies arise to challenge our understanding of gravity. Modified gravity models were necessary to find alternatives or correct Newtonian gravity. In weak field limit, some modified gravity models predict the existence of massive gravitons that may carry the gravitational interaction over a certain scale depending by the mass of these particles, and one of the current modified theories of gravity is the Yukawa potential which is a proposed correction to the Newtonian gravity.

Following Haranas et al. (2018), in this section, we summarizing the paper's results of finding an expression for the graviton mass m_{gr}, by solving for the range of the graviton λ in the Yukawa-type correction for i.e., $V(r) \propto r^-1 e^{-r/\lambda_{gr}}$.

Yukawa-type potential correction can be described [2]:

$$V = -\frac{GMm}{r}\left(1 + \alpha\, e^{-\frac{r}{\lambda}}\right) \tag{1}$$

Where,

r is the distance between the two bodies

G is the Newtonian gravitational constant

$\alpha = \frac{kK}{GMm}$, k and K are the coupling constants of the new force to the bodies relative to the gravitational one

The total Energy, where the semi-major axis a is the average value of the radial distance r along the orbit:

$$\mathscr{E} = \frac{-GMm}{2r}(\alpha e^{\frac{-r}{\lambda}}) \tag{2}$$

Such that, the total time rate of the energy of the Yukawa correction is:

$$\frac{d\mathscr{E}}{dt} = \frac{GMm}{2r^2}[\alpha(1+\frac{r}{\lambda})e^{\frac{-a}{\lambda}}]\frac{dr}{dt} \tag{3}$$

B. Circular and Elliptical orbits

For circular orbits, we let eccentricity=0, and r = a:

$$\frac{d\mathscr{E}}{dt} = \frac{GMm}{2a^2}[\alpha(1+\frac{a}{\lambda})e^{\frac{-a}{\lambda}}]\frac{da}{dt} \tag{4}$$

The third Keplerian law:

$$GM = n^2 a^3 \tag{5}$$

Such that, the rate of change of the mean motion can be expressed at:

$$\frac{d\mathscr{E}}{dn} = -\frac{V_N(a)}{3n}[\alpha(1+\frac{a}{\lambda})e^{\frac{-a}{\lambda}}] \tag{6}$$

Where $V_N(a) = -\frac{GMm}{a}$

Then the orbital energy dependence on the mean motion is obtained, and following Haranas et al. (2018), The range potential λ is found to be:

$$\lambda = -\frac{a}{1 + W[3(\mathscr{E}-\mathscr{E}_0)/e\alpha V(a) ln(\frac{n}{n_0})]} \tag{7}$$

Similarly, for elliptical orbits, the range of potential λ:

$$\lambda = -\frac{a(1-e_0 cos(E))}{1 + W[-3(\mathscr{E}-\mathscr{E}_0)(1-e_0 cos(E)/\alpha e V_N(a) ln(n/n_0)]} \tag{8}$$

C. The mass of the Graviton field mediating interaction

In theories that are concerned with understanding long-range forces, a hypothetical quantum of gravity arise: graviton. In these theories, the graviton is the elementary particle that mediates the force of gravity. Assumptions are placed on the graviton, where it assumes that it's is essentially a photon with some special attributes and it's range is the Compton's wavelength λ. Such that:

$$\lambda = \frac{\hbar}{mc} \tag{9}$$

Equating equation 9 and 8, the mass of the graviton is obtained:

$$m_{gr} = -\frac{\hbar}{ca(1-ecos(E))}[1+W(-\frac{3(1-e_0 cos(E))}{\alpha e V_N(a) ln(n/n_0)})(\mathscr{E}-\mathscr{E}_0)] \tag{10}$$

II. THE NUMBER OF INFORMATION N AND THE GRAVITATIONAL MASS IN A VACUUM DOMINATED UNIVERSE [1]

To establish a relation between the graviton mass and the function of the information bits number N, firstly, we derive an expression for the cosmological constant as a function of information bits. This path is taken as the number of information N is related to entropy, and entropy is a factor in calculating the cosmological constant as well. Using Wesson (2004), the gravitational mass scale is found to be:

$$m_g r = \frac{c^2}{G}\sqrt{3/\Lambda} \tag{11}$$

As we mentioned above, we can express the cosmological constant in terms of entropy according to Bousso et al. (2002):

$$\Lambda = 3\pi\left(\frac{k_M}{S_{UH} l_p^2}\right) \tag{12}$$

Where the entropy S can be expressed in the number of information bits N:

$$S = k_B N ln2 \tag{13}$$

Substituting equation 13 in equation 12, we get a relation between the cosmological constant and the number of information bits:

$$\Lambda = \frac{3\pi}{N ln2 l_p^2} \tag{14}$$

Now, to get a relationship between the graviton mass and the number of information bits N, we substitute equation 14 in equation 11:

$$m_{gr} = \frac{c^2}{G}\sqrt{\frac{3Nl_p^2 ln2}{3\pi}} \tag{15}$$

III. THE NUMBER OF INFORMATION N AND THE ORBITAL PARAMETERS IN THE SOLAR SYSTEM

The final goal of this paper is to find a relation between the number of Information N and the orbital parameters in our solar system. We have two equations to calculate the graviton mass, equation (10) and equation (15). Where equation (10) describes the mass of the graviton in terms of the orbital parameters, and equation (15) describes the mass of the graviton in terms the number of the information. Equating both equations and solving for the semi-major axis a:

$$a = \frac{G\hbar}{c^3\sqrt{\frac{Nl_p^2 ln(2)}{\pi}}(e_0 cos(E) - 1)}\left[1 - \frac{W(3 - 3e_0 cos(E))(\mathscr{E} - \mathscr{E}_0)}{\alpha e V_N(a) ln(\frac{n}{n_0})}\right] \tag{16}$$

When we tried to solve for the other parameters, Maple couldn't isolate for them. So we simplified it to the circular orbit case, where the eccentricity is zero and $(\mathscr{E} - \mathscr{E}_0) = V(a)$, we solved for the semi-major axis a, and the coupling constant α and graviton's range λ:

$$a = \frac{G\hbar}{c^3\sqrt{\frac{Nl_p^2 ln(2)}{\pi}}}\left[1 + W\left(\frac{3}{e\alpha ln(\frac{n}{n_0})}\right)\right] \tag{17}$$

$$\alpha = -\frac{3G\hbar e\pi}{ln(\frac{n}{n_0}) e^{\frac{ac^3\sqrt{\pi N ln(2)} l_p}{G\hbar\pi}} e(-ac^3\sqrt{\pi N ln(2)} l_p + G\hbar\pi)} \tag{18}$$

$$\lambda = -\frac{G\hbar\sqrt{\pi}}{c^3\sqrt{ln(2) N l_p^2}} \tag{19}$$

IV. DISCUSSION AND NUMERICAL RESULTS

To obtain numerical results, we are considering the orbit of mercury where the semi-major axis $a = 5.79 \times 10^7 km$ and Yukawa coupling constant $\alpha = 3.57 \times 10^{-10}$ and the range $\lambda = 4.937 \times 10^{15} m$.

From equations (17), (18) and (19), we can substitute the known constants and the orbital parameters (a, α and λ) in the equations to get the number of the Information bits N. However, one of the variables in the equations is the relation $ln(\frac{n}{n_0})$, we can find it by rearranging equation (7) to solve for $ln(\frac{n}{n_0})$, where $(\mathscr{E} - \mathscr{E}_0) = V(a)$, such that:

$$ln\left(\frac{n}{n_0}\right) = -8.403 \times 10^9 \tag{20}$$

For semi-major axis a, equation (17), we can see it's $\frac{1}{\sqrt{N}}$ dependence. where the coupling constant α, equation (18), is $\sqrt{N}e^{\sqrt{N}}$. Finally, λ (equation 19), is $\frac{1}{\sqrt{N}}$ dependence.

V. CONCLUSION

In this paper, we have used the Yukawa-type correction to find an expression for the mass of the graviton in terms of the orbital parameters (i.e. semi-major axis, Coupling constant α, and graviton range λ), and the results for the graviton mass predicted by Wesson in terms of the number of information bits number N, in order to derive a relation between these orbital parameters and N number.

From Yukawa-correction, the mass of the gravtiton m_{gr} is equation (10), and it's equation (15) according to Wesson. Finally, three relations were found for semi-major axis a, the coupling constant α and graviton's range λ, according to equation (17), (18) and (19) respectively.

The motivation behind this work is to open the possibilities of studying and understating the unclear concept of Information number N and how the orbit of a celestial object can change with the change of the N number.

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