Measuring Speed of Light

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

Measuring Speed of Light

Abstract—**In this experiment, we measured the speed of light using two methods. For the first method, we used Michelson-Foucault method where we used a rotating mirror and a travelling microscope to find the relation between the position and the mirror speed over a set of frequency range and so the speed of light can be determined. The speed of light was found to be $3.02 \pm 0.01 (10^8 m/s)$ with an error difference of 0.7% from the accepted value of the speed of light.**

In the second method, we used Pulsed Laser method where we sent a pulsed laser beam along a certain path and then the taken time for it to reflect back is shown on a digital oscilloscope, so we can measure the speed of light, given the path length and time taken. The speed of light was found to be $3.02 \pm 0.01 (10^8 m/s)$ with an error difference of 4% from the accepted value of the speed of light.

I. INTRODUCTION [1]

Throughout history, many scientists tried measuring the speed of light as it was such a major concern among scientists. This value, c, is one of the universal physical constants that are employed in many important fields. Using our current best technology, this constant is best estimated to be about three hundred thousand kilometers per second, which when you think about it, is quite huge! So the question is, how those scientists were able to measure the speed of light?

The first to measure the speed of light experimentally was the Italian scientist, Galileo, as he tried measuring the speed of light with the help of his two assistants. He asked each one of them to stand on a top of a hill which are separated by 1km. Each one was holding a lamp while covering it with his hand, then one of them should uncover the lamp for the other one to see the light and wave his hands at the same time, and at the moment the other assistant saw the light, he should wave with his hands too. Galileo was observing them both and measuring the time both of them took to uncover the lamp and wave to each other. Galileo used this measured time and the distance to calculate the speed of light. He found that the time taken is zero and he conclude that the speed of light is infinite. Of course it's far from right, as Galileo didn't account for the human reaction time and the very short travelled distance, given that the speed of light is so huge.

The second attempt to measure the speed of light was by the Danish scientist, Olaf Roemer, in 1970, when he observed that one of Jupiter's moons, Io, takes more time than the predicted time it should take for the eclipse to happen. For that, he concluded that the light might be travelling longer distance, and if the speed of light is finite and not infinite then that means it takes longer time to travel this longer distance. He measured the speed of light to be 200 km per second as he was off in his data of the eclipse's taken time.

Then in in 1728, the British astronomer James Bradley, calculated the speed of light to be 301,000 km/s, which is quite amazing to be this close to the known value now. He used this a phenomenon called Light aberration. Astronomers before believed that the stars are fixed until the French astronomer Picard noted that the fixed stars are not completely stationary. However, It would have been logical for Bradley to think that this movement was not because of a starry motion but because of the earth's rotation around the sun, however, he thought if the speed of light was infinite, then no one should noticed this delinquency. Bradley used a tilted telescope guiding it towards the star till the light from it is completely inside the telescope. Since the earth changes its course every day, he had to change the position of the telescope every day so that we get the best vision of the stars, so the stars look like they are doing a complete cycle. Bradley was able to calculate the speed of light by measuring the amount of the telescope tilt.

In 1849, French scientist Armand Fizeau, was able to measure the speed of light, as shown in figure 1. Fizeau calculated the speed of light to be 315,000 km/s. Then in 1862, the French scientist, Jean Léon Foucault, was able to develop the Fizeau's method and reduce its size so that it could be placed in the laboratory. Then in 1887, Michelson and Morley invented a very precise device, interferometer, to measure the speed of light. This device measured the value we now know. [2]

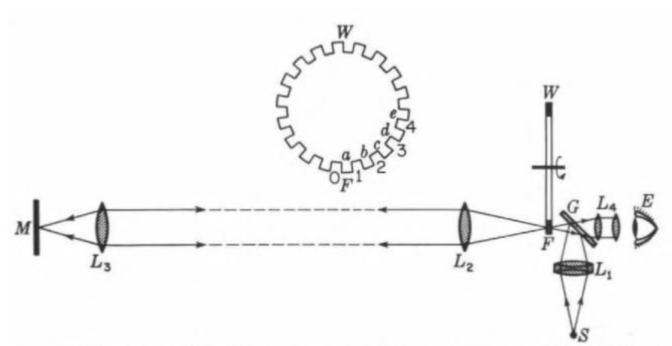

Fig. 1. Fizeau's experimental arrangement diagram. [1]

II. THEORETICAL BACKGROUND

1) Michelson-Foucault method derivations: During time t, light travels between the rotating mirror M_1 and the fixed mirrors M_2 and M_3 in the following pathway: $M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow M_4 \rightarrow M_5$. The distance travelled in this pathway is labelled as 2D. Thus, the speed of light c in air (assumed n_{air} = 1) can be expressed as:

$$t = \frac{2D}{c} \tag{1}$$

Meanwhile, in time t, the rotating mirror M_1 also undergoes constant rotation speed of $\omega = 2\pi f$. During time t, M_1 would have rotated by

$$\Delta\theta = \omega t \tag{2}$$

where r is the distance between M1 and the beam splitter M_4.

Subsequently, the return of the light beam from M_1 to the beam splitter experiences a divergence due to this angular change of plane of incidence at M_1. More specifically, the incidence angle changes from θ_i to $\theta_i + \Delta\theta$. Since the reflectance angle θ_r is always equal to the incidence angle, θ_r also changes to $\theta_i + \Delta\theta$. In overall, the returning beam diverges by $2\Delta\theta$ at the beam splitter, as shown in Figure 9.

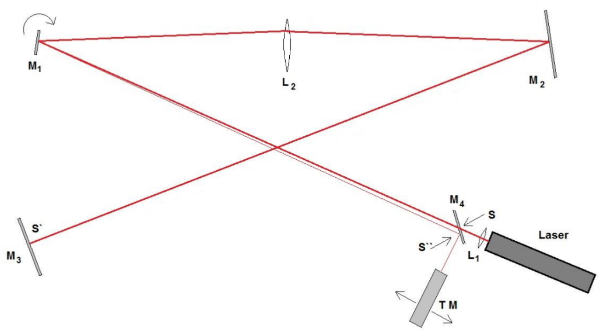

Fig. 2. Schematics of the experimental set-up, with clear illustration of the beam divergence.

Based on small angle approximation, the lateral displacement Δx of the beam image shown on the travelling microscope can be calculated from the equation, where r is the distance between M_1 and M_4:

$$\Delta x = 2r\Delta\theta \tag{3}$$

Combining all equations 1, 2 and3,

$$c = \frac{2D}{t} = \frac{2D\omega}{\Delta\theta} = \frac{4rD\omega}{\Delta x} = \frac{8\pi rDf}{\Delta x} \tag{4}$$

It must be noted that the rotating frequency measured using a photocell detector is twice the value of that of M1 as both sides of M_1 reflect light beam onto the photodetector, so the value of frequency read off the detector $f_recorded$ must be divided by two. Thus, equation 4 can be expressed as:

$$c = \frac{4\pi rDf_{recorded}}{\Delta x} \tag{5}$$

2) Pulsed Laser method derivations: For the Pulsed Laser method, we just need the time difference Δt that the pulsed laser takes to go out from the source and then reflects back to the detector, travelling the distance D. If we have these two information Δt and D, we can apply the equation:

$$c = \frac{D}{\Delta t} \tag{6}$$

III. EXPERIMENTAL DESIGN AND PROCEDURE [3]

A. Description of the apparatus

1) Michelson-Foucault method apparatus: For the first method, the used apparatus consists of :

A) Laser source
B) Travelling microscope
C) Lens with 17 cm focal length
D) Variable autotransformer
E) Frequency counter
F) half-silvered mirror
G) Lens with 5.0m focal length
H) Rotating mirror
I) Polarizer
J) Photodiode sensor
K) Oscilloscope
L) Two flat fixed mirrors (not shown)

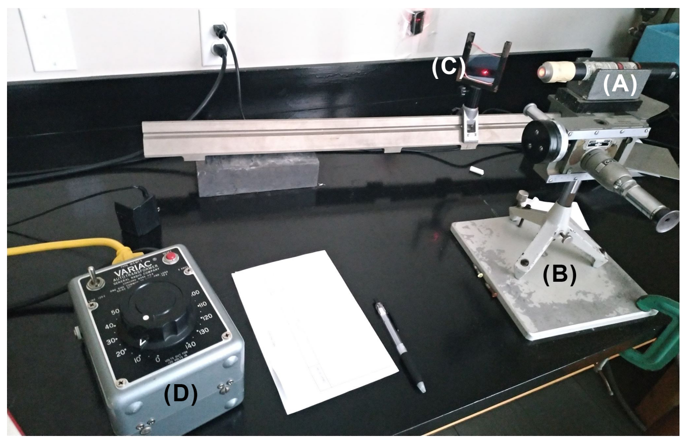

Fig. 3. Michelson-Foucault method instruments (1)

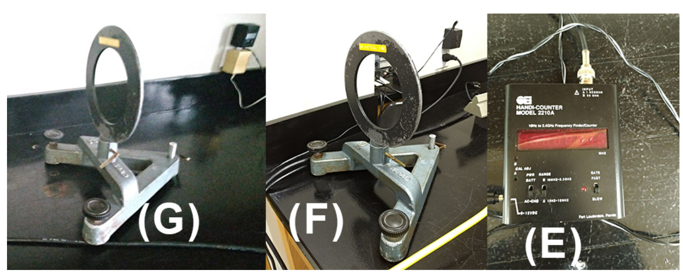

Fig. 4. Michelson-Foucault method instruments (2)

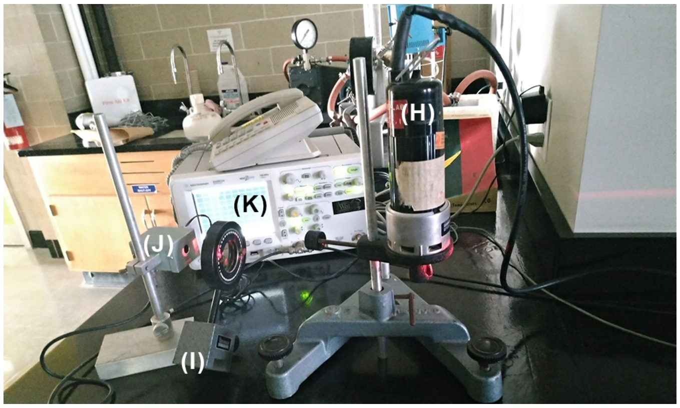

Fig. 5. Michelson-Foucault method instruments (3)

2) Pulsed Laser method apparatus: For the second method, the used apparatus consists of :

A) Laser source
B) Pulse generator
C) Laser power switch
D) Oscilloscope
E) Flat mirror
F) Beam splitter
G) Detector
H) Lens with 5.0m focal length
I) Flat fixed mirror
J) Flat fixed mirror
K) Retro-reflector (corner cube)

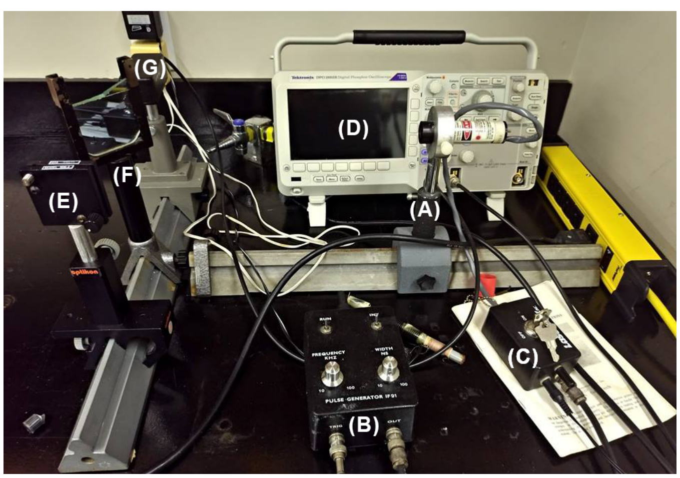

Fig. 6. Pulsed Laser method instruments (1)

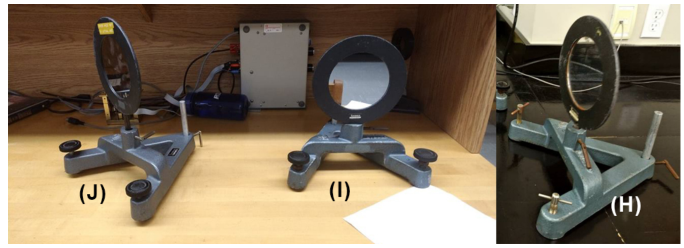

Fig. 7. Pulsed Laser method instruments (2)

Fig. 8. Pulsed Laser method instruments (3)

B. Description of the experimental procedure

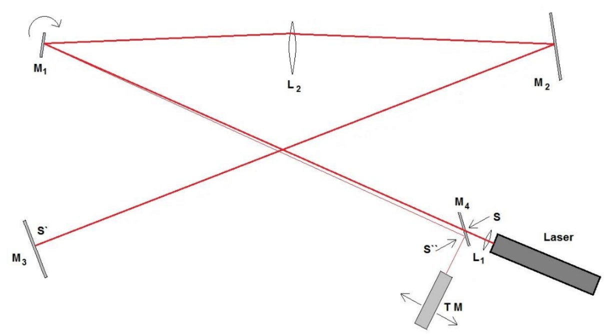

Fig. 9. Michelson-Foucault method setup diagram

1) Michelson-Foucault method procedure: We arranged the system such that the beam is set to go through an arrangement of mirrors as shown in figure 9 and then reflect back returning to M_1.

Firstly, we aligned S with L_2 such that the distance from S to L_2 is set to be $2f_2$ which is equal to 10 m, which is also the same distance from L_2 to M_3 where the final image is formed. For the rotating Mirror M_1, we used a pair of tweezers to orient the mirror such that we want the beam to pass through the centre of lens L_2 and then reflect to the centre of Mirror

M_2. Then we used the knobs on Mirror M_2 to direct the final image on mirror M_3. Again, we did the same with mirror M_3 such that we wanted it to reflect the beam taking the same exact path to Mirror M_2. After getting the right alignment, we were able to see the image S"at M_4. Now, we got to set up the travelling microscope to focus on the beam's image, however, we saw two spots in travelling microscope which was because of the reflections from the beam splitter.

Finally, we started the Autotransformer to change the frequency of the rotating mirror to take the measurements of the beam's spot displacement while we vary the mirror's frequency.

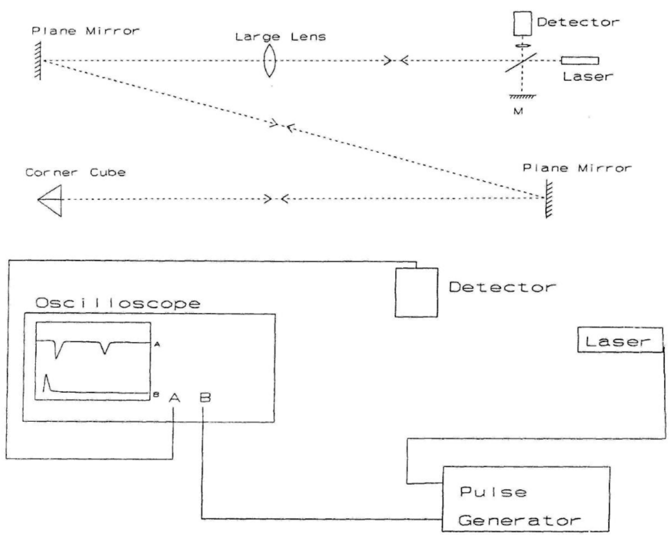

Fig. 10. Using Pulsed Laser method diagram

2) Pulsed Laser method procedure: we adjusted the system such that the pulsed laser go through the arrangement of mirrors as shown in figure 10 then it hits the retroflector then reflects back to the detector. We used the oscilloscope to measure the time the pulsed takes for the detector to detect, relative to the laser's trigger signal. We measured the distance traveled by the beam giving our arrangement of the system. Then we repeated the same procedure for other two arrangements where we added another mirror in the system and moved the position of the reflector.

IV. Analysis and Discussion

A. *Part A: Michelson-Foucault Method Analysis*

As derived in previous section, a linear model of $4\pi rDf$ versus Δx can be plotted to obtain the value of speed of light from the measured gradient, where f refers to the recorded value of frequency from photodetector. The corresponding values of Δx was recorded for each varying f. A set of about 12 data points were plotted in a scatter graph, followed by a linear best-fitting to estimate the gradient value, as shown in Figure 11. The same set of procedures was repeated for two more times, producing results shown in Figure 12 and 13.

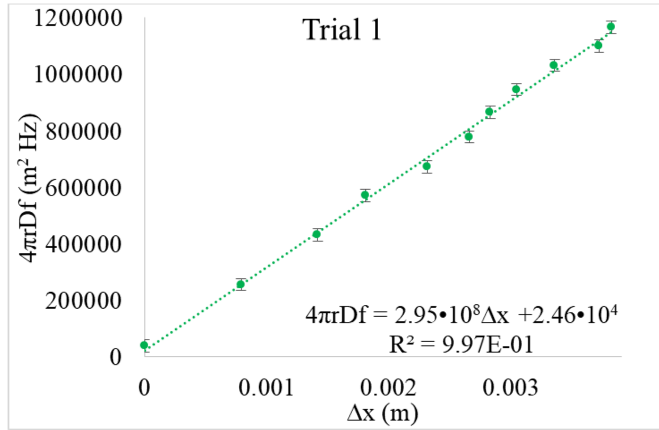

Fig. 11. Linear plot of $4\pi rDf$ versus Δx from the first trial of experiment.

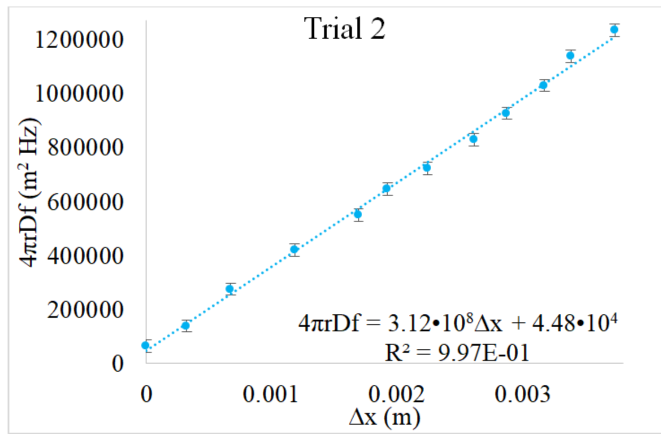

Fig. 12. Linear plot of $4\pi rDf$ versus Δx from the second trial of experiment.

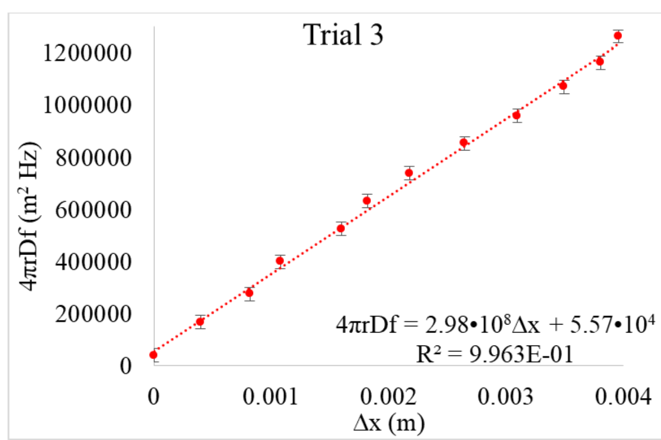

Fig. 13. Linear plot of $4\pi rDf$ versus Δx from the third trial of experiment.

Using the equation, where e is the corresponding absolute uncertainty:

$$c_1 = \frac{D_1 \pm e_D}{t_1 \pm e_t} = \frac{l_1}{t_1} \pm \sqrt{(\%e_D)^2 + (\%e_t)^2 = c_1 \pm e_1} \quad (7)$$

the corresponding uncertainty propagation can be calculated for each trial of speed determination.

The main contributor of uncertainty in length determination e_D comes from the measurement of distance between mirrors using a tape ruler. As the distance extends beyond 1 m, the weight of the tape tends to cause sagging, even though two ends of the tape were pulled as tightly as possible. Sagging of tape leads to overestimation of D by approximately 2 to 3 cm. In addition, finite thickness and curvature of mirror surface present uncertainty of roughly 1 cm as the tape can only be used for measurement from the side of the mirrors. Another contributor is the inherent precision of a tape ruler which is ± 0.05 cm. Lastly, it is hard to ensure that the separation distance measured coincides exactly with the path length of the reflected light beam. Though arguably, the deviation between the measured separation and path length is minimal. This is because the measurement is performed from the edge of mirror (due to design limitation of the tape ruler) while the beam spot typically lies near the middle. By estimation, the radius of mirror is 10 cm while the distance between two mirrors is 7 m. The difference between the hypotenuse (of $\sqrt{0.1^2 + 7^2} \approx 7.0007$) and the measured length of 7 m is minimal at 0.07 cm. Thus,

$$e_D = \approx \sqrt{3^2 + 0.05^2 + 1^2} = 3.16 \approx 3cm \quad (8)$$

It is assumed that $\%e_D$ remains the same for all three trials since the apparatus set-up is not changed between trials:

$$\%e_D = \frac{e_D}{D} = \frac{3cm}{1303.2cm} = 0.2\% \quad (9)$$

The uncertainty of time determination is a collective result of few variable measurements:

$$t = \frac{\Delta x}{2r\omega} \rightarrow \frac{e_t}{t} \times 100\% = \quad (10)$$

$$\sqrt{(e_{\frac{\Delta x}{\Delta x}})^2 + (\frac{e_r}{r})^2 + (\frac{e_f}{f})^2} \times 100\% =$$

$$\sqrt{(\%e_{\Delta x})^2 + (\%e_r)^2 + (\%e_f)^2} = \%e_t$$

e_r is similar to e_D, with the same sources of error as aforementioned. Hence,

$$\%e_r = \frac{e_r}{r} = \frac{3cm}{696.8cm} = 0.4\% \quad (11)$$

is the same for all three trials, without adjustments of the apparatus set-up between trials.

$e_{\Delta x}$ is attributed to two factors, namely the uncertainty in determining the displacement in the light beam image and the inherent limitation in the precision level of a travelling microscope. In the first trial, the displacement of beam spot was estimated using the center of the spot. However, the determination of the center position is not consistent with naked eye observation, especially since the beam spot is not a perfect circle. Thus, subsequent trials were performed using the edge of the beam spot, by aligning the crosshair of the microscope eyepiece with the spot edge. Admittedly, the uncertainty is improved but not eliminated entirely, owing to the blurry or diffusive edge. The absolutely uncertainty is estimated to be ± 0.001 cm. On the other hand, the travelling microscope has a precision of up to ± 0.0005 cm, the half of the least scale division. Thus,

$$e_{\Delta x} \approx \sqrt{0.0005^2 + 0.001^2} = 0.001 \quad (12)$$

It can be seen from the calculation that the error from displacement determination due to human eye limitation overrides the instrument precision. Average recorded value of Δx_{ave} is used to estimate $e_{\Delta x}$, for example in Trial 1:

$$\%e_{\Delta x} = \frac{e_{\Delta x}}{\Delta x_{ave}} \times 100\% = \frac{0.001}{0.234} \times 100\% = 0.4\% \quad (13)$$

Similar to $e_{\Delta x}$, both instrumental limitation and human random error contribute to e_f. It was observed that the reading of rotation frequency from the photocell detector fluctuated and decreased with time. To ensure accurate reading, one of the two experimenters recorded the frequency reading while the other experimenter adjusted the travelling microscope displacement simultaneously. Synchronised recordings eliminate the time lag between the two readings since rotation frequency changes with time invariably. Thus, e_f is primarily the precision of photocell detector i.e. ± Hz. Average recorded value of f is used to estimate $\%e_f$:

$$\%e_f = \frac{e_f}{f_{ave}} \times 100\% \quad (14)$$

The value of c and its corresponding uncertainty from each trial is calculated and tabulated as follow:

Trial	1	2	3
c (10^8 m/s) from plot	2.95	3.12	2.98
% e_D (%)	0.2	0.2	0.2
% e_r (%)	0.4	0.4	0.4
Δx_{ave} (cm)	0.234	0.207	0.198
% $e_{\Delta x} = \frac{e_{\Delta x}}{\Delta x_{ave}} \times 100\%$	0.4	0.5	0.5
f_{ave} (Hz)	626	590	582
% $e_f = \frac{e_f}{f_{ave}} \times 100\%$	0.2	0.2	0.2
% $e_t = \sqrt{(\%e_{\Delta x})^2 + (\%e_r)^2 + (\%e_f)^2}$	0.6	0.7	0.7
% $e_c = \sqrt{(\%e_D)^2 + (\%e_t)^2}$	0.6	0.7	0.7
e_C = c × % e_c (10^8 m/s)	0.02	0.02	0.02

Fig. 14.

It is noteworthy that $e_{\Delta x}$ contributes mainly to the uncertainty. Thus, the human error in the determination of spot displacement is the most significant source of error. Furthermore, all three linear plots in Figure 11, 12 and 13 have a non-zero y-intercept which is rather large in magnitude. This is partly attributed to the experimental mistake in taking the first reading with the lowest frequency i.e. 34 Hz as the zero marking of Δx, rather than that at f = 0. Nevertheless, the plot gradient does not change with the zero value so the determination of c is not affected by this mistake.

All in all, average $c \pm e_{mean}$ can be determined as follow:

$$\bar{c} = \frac{\sum \frac{c_n}{e_n^2}}{\sum \frac{1}{e_n^2}} = \frac{\frac{2.95}{0.02^2} + \frac{3.12}{0.02^2} + \frac{2.98}{0.02^2}}{\frac{1}{0.02^2} + \frac{1}{0.02^2} + \frac{1}{0.02^2}} = 3.02 \tag{15}$$

$$e_{mean} = \sqrt{\frac{1}{\sum \frac{1}{e_n^2}}} = \sqrt{\frac{1}{\frac{1}{0.02^2} + \frac{1}{0.02^2} + \frac{1}{0.02^2}}} = 0.01 \tag{16}$$

Hence, $\bar{c} = 3.02 \pm 0.01$

B. Part B: Pulsed Laser Method Analysis

	Distance (D)/m	Time (t)/ns	Speed of light (c)/10^8 m s^{-1}	$\%e_D$	$\%e_t$	$\%e_c$	e_c/10^8 m s^{-1}
Trial 1	54.12	198	2.73	0.06%	2%	2%	0.05
Trial 2	57.32	202	2.84	0.05%	1%	1%	0.03
Trial 3	65.68	220	2.99	0.05%	1%	1%	0.03

Fig. 15.

Similar to part A, the same set of equations are applied in part B, as shown below:

$$c_1 = \frac{D_1 \pm e_D}{t_1 \pm e_t} = \frac{l_1}{t_1} \pm \sqrt{(\%e_D)^2 + (\%e_t)^2} = c_1 \pm e_1 \tag{17}$$

e_D is roughly the same as in previous section, i.e. 3 cm since the same sources of error such as sagging tape and thick mirror apply in the same manner. e_t is inherent in the determination of crest or trough position in the oscilloscope display. The tip of trough appears to spread over the range of about 3 ns, which roughly coincides with the vertical crosshair thickness of about 3 to 4 ns. The values of e_D and e_t are the same in all trials.

$$\bar{c} = \frac{\sum \frac{c_n}{e_n^2}}{\sum \frac{1}{e_n^2}} = \frac{\frac{2.73}{0.02^2} + \frac{2.84}{0.02^2} + \frac{2.99}{0.02^2}}{\frac{1}{0.05^2} + \frac{1}{0.03^2} + \frac{1}{0.03^2}} = 2.89 \tag{18}$$

$$e_{mean} = \sqrt{\frac{1}{\sum \frac{1}{e_n^2}}} = \sqrt{\frac{1}{\frac{1}{0.05^2} + \frac{1}{0.03^2} + \frac{1}{0.03^2}}} = 0.02 \tag{19}$$

Hence, $\bar{c} = 2.89 \pm 0.02$

	$\bar{c}$ (10^8 m/s)	% theoretical discrepancy from c = $3 \cdot 10^8$ m/s	% uncertainty of experiment
Method A	3.02 $\pm$ 0.01	0.7%	0.3%
Method B	2.89 $\pm$ 0.02	4%	0.9%

Fig. 16.

Michelson-Foucault in part A is more accurate and precise than the use of pulsed laser in part B. One reason is the larger number of data points in part A which decreases the random error in magnitude and significance.

V. CONCLUSION

This experiment effectively served its purpose of successfully measuring the speed of light using two methods: Michelson-Foucault method and Pulsed laser method. For the Michelson-Foucault method, we did three trials and the speed of light was found to be $2.95 \pm 0.2(10^8 m/s)$, $3.12 \pm 0.2(10^8 m/s)$ and $2.98 \pm 0.2(10^8 m/s)$. The average speed of light was found to be $3.02 \pm 0.01(10^8 m/s)$ with an error difference of 0.7% from the accepted value of the speed of light. For the Pulsed Laser method, we did also three trials where we varied the distance between the mirror and the pulsed laser source. The speed of light was found to be $2.73 \pm 0.02(10^8 m/s)$, $2.84 \pm 0.01(10^8 m/s)$ and $2.99 \pm 0.01(10^8 m/s)$. The average speed of light was found to be $3.02 \pm 0.01(10^8 m/s)$ with an error difference of 4% from the accepted value of the speed of light.

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