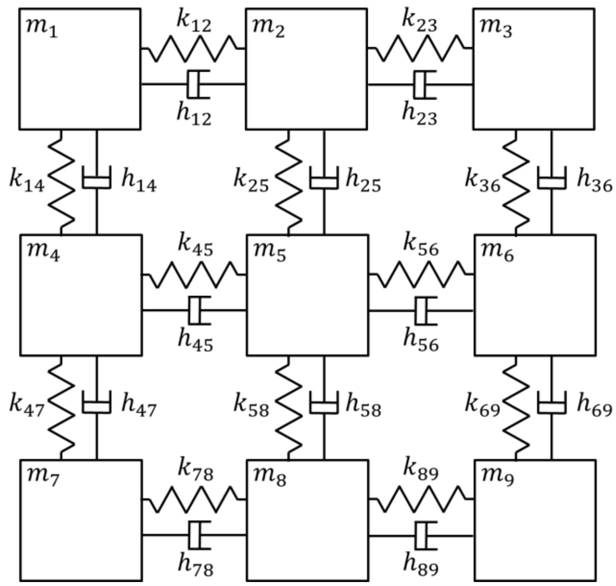
Control of an array of masses

Consider the problem of controlling the array of nine masses depicted in the following figure.



where force u_i^x and u_i^y can be exerted on each cart on the horizontal and vertical, respectively, directions. The corresponding parameter values are specified in the corresponding MATLAB file. The mathematical model related to the position of each mass on the plane (p_i^x, p_i^y) (i.e., the displacement of its position with respect to the rest position) is

$$m_i \ddot{p}^x{}_i = u_i^x + \sum_{j \in \mathcal{N}_i} \left(k_{i,j}(p_j^x - p_i^x) + h_{i,j}(\dot{p}^x{}_j - \dot{p}^x{}_i) \right)$$

$$m_i \ddot{p}^y{}_i = u_i^y + \sum_{j \in \mathcal{N}_i} \left(k_{i,j}(p_j^y - p_i^y) + h_{i,j}(\dot{p}^y{}_j - \dot{p}^y{}_i) \right)$$

where $\mathcal{N}_i$ is the set of other masses with which the i-th mass interacts through springs and dumpers. We can finally write the decomposed model as

$$\dot{x} = Ax + Bu$$

where $x = [x_1^T, \dots, x_9^T]^T$ and $u = [u_1^T, \dots, u_9^T]^T$, being $x_i = \begin{bmatrix} p_i^x \\ v_i^x \\ p_i^y \\ v_i^y \end{bmatrix}$ (note that $v_i^x = \dot{p}_i^x$ and $v_i^y = \dot{p}_i^y$) and $u_i = \begin{bmatrix} u_i^x \\ u_i^y \end{bmatrix}$, respectively, for all $i = 1, \dots, 9$. The matrices are specified in the corresponding MATLAB file. Note that all state variables are measurable.

Problem:

1. Decompose the state and input vectors into subvectors, consistently with the physical description of the system. Obtain the corresponding decomposed model.
2. Generate the system matrices (both continuous-time and discrete-time, the latter with a sampling time selected compatibly with the continuous-time dynamics). Perform the following analysis:
    a. Compute the eigenvalues and the spectral abscissa of the (continuous-time) system. Is it open-loop asymptotically stable?
    b. Compute the eigenvalues and the spectral radius of the (discrete-time) system. Is it open-loop asymptotically stable?
3. For different state-feedback control structures (i.e., centralized, decentralized, and different distributed schemes) perform the following actions
    a. Compute the continuous-time fixed modes
    b. Compute the discrete-time fixed modes
    c. Compute, if possible, the CONTINUOUS-TIME control gains using LMIs to achieve the desired performances. Apply, for better comparison, different criteria for computing the control laws.
    d. Compute, if possible, the DISCRETE-TIME control gains using LMIs to achieve the desired performances. Apply, for better comparison, different criteria for computing the control laws.
    e. Analyze the properties of the so-obtained closed-loop systems (e.g., stability, eigenvalues) and compute the closed-loop trajectories (generated both in continuous-time and in discrete-time) of the mass positions p_i^x and p_i^y starting from a common random initial condition.