Control of mass-spring system

The system is depicted in the following figure.

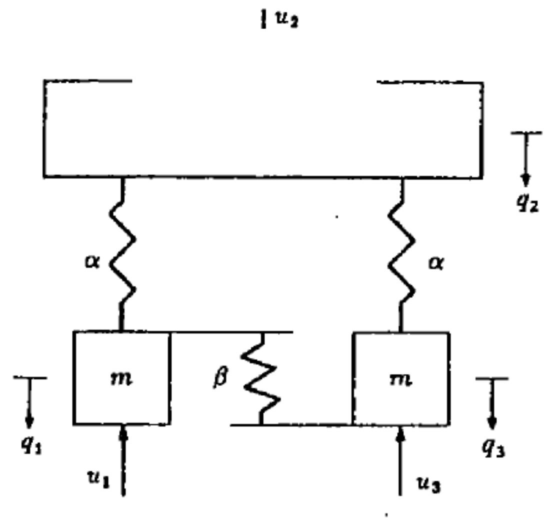

Fig. 8.1. Mass–spring system.

The equations of motion are

$$m\ddot{q}_1 = -\alpha(q_1 - q_2) - \beta(q_1 - q_3) - u_1,$$

$$M\ddot{q}_2 = -\alpha(q_2 - q_1) - \alpha(q_2 - q_3) + u_2,$$

$$m\ddot{q}_3 = -\alpha(q_3 - q_2) - \beta(q_3 - q_1) - u_3.$$

Defining $x = [q_1, v_1, q_2, v_2, q_3, v_3]^T$, $v_i = \dot{q}_i$, and $u = [u_1, u_2, u_3]^T$, the "centralized" system matrices are specified in the corresponding MATLAB file. We assume that all state variables are measurable.

Problem:

1. Decompose the state and input vectors into subvectors, consistently with the physical description of the system. Obtain the corresponding decomposed model.
2. Generate the system matrices (both continuous-time and discrete-time, the latter with a sampling time of choice, compatible with the continuous-time system dynamics). Perform the following analysis:

    a. Compute the eigenvalues and the spectral abscissa of the (continuous-time) system. Is it open-loop asymptotically stable?
    b. Compute the eigenvalues and the spectral radius of the (discrete-time) system. Is it open-loop asymptotically stable?
3. For different state-feedback "control structures" (i.e., centralized, decentralized, and different distributed schemes) perform the following actions:
    a. Compute the continuous-time fixed modes
    b. Compute the discrete-time fixed modes
    c. Compute, if possible, the CONTINUOUS-TIME control gains using LMIs to achieve the desired performances. Apply, for better comparison, different criteria for computing the control laws.
    d. Compute, if possible, the DISCRETE-TIME control gains using LMIs to achieve the desired performances. Apply, for better comparison, different criteria for computing the control laws.
    e. Analyze the properties of the so-obtained closed-loop systems (e.g., stability, eigenvalues) and compute the closed-loop system trajectories (generated both in continuous-time and in discrete-time) of the variables q_1, q_2, q_3 starting from a common random initial condition.