Interaction uncertainty in financial networks

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Interbank network minimal model

The book value of a bank i fluctuates over time due to price variations of its external assets, and these fluctuations will then affect the book value of another bank j if it is a counterparty of i,

$$dx_i = \sigma\, dW_i + \gamma M_{ij} h^j\, dt, \tag{1}$$

where the adapted stochastic process $dh_i = -\beta h_i dt + \sqrt{\beta}\, dx_i$ is the recent variation of x_i over the timescale β^{-1}, which models the finite speed of the market reaction.

Stochastic integral

Define $\hat{A} \equiv \beta\hat{\delta} - \sqrt{\beta}\gamma\hat{M}$ where $\hat{\delta}$ denotes the identity matrix. We obtain

$$x_i(t) = \sigma W_i(t) + \sigma\sqrt{\beta}\gamma S_i(t), \tag{2}$$

where $W_i(t) = \int_0^t dW_i(t')$ is standard Brownian motion and $S_i(t)$ is the stochastic integral

$$S_i(t) \equiv \int_0^t dt' \int_0^{t'} M_{ij} \left(e^{-\hat{A}(t'-t'')}\right)^{jk} dW_k(t''), \tag{3}$$

which sums the propagation onto node i of the fluctuations in all nodes k during the time interval $[0, t)$.

The stress observable

Let us consider the sample variance of the banks states as a quantifier of stress in financial networks,

$$y \equiv \frac{1}{N-1} u^i x_i \left(x_i - \frac{1}{N} u^j x_j \right) . \tag{4}$$

It sounds reasonable from the regulators perspective to ask that exposures between banks, as quantified by the interaction matrix $\hat{M}$, do not destabilize the financial system.

On the short-medium term

Let us assume that exposures can be considered approximately fixed on the short-medium term, and study the problem in an expansion,

$$\left(e^{-\hat{A}\tau}\right)_{ij} = \delta_{ij} - A_{ij}\tau + \mathcal{O}(\tau^2), \tag{5}$$

which limits the analysis to timescales τ satisfying $\tau \lesssim \beta^{-1}$ and $\tau \lesssim \beta^{-1/2}\gamma^{-1}$.

The conditional stress expectation $\mathbb{E}_{\hat{M}} y$ establishes the conditions on $\hat{M}$ under which interactions are beneficial to stabilize the financial system.

Conditional stress expectation

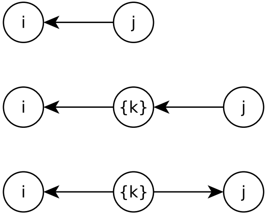
$$\mathbb{E}_{\hat{M}} y = \sigma^2 t + \frac{\sigma^2 \sqrt{\beta}\gamma}{N-1} u^i \left(M_{ii} - \frac{1}{N} u^j M_{ij} \right) \left(1 - \frac{\beta}{3} t \right) t^2 + \frac{\sigma^2 \beta \gamma^2}{3(N-1)} M_{ik} \left[\widetilde{M}^{ik} - \frac{1}{N} u^i u_j \widetilde{M}^{jk} \right] t^3, \quad (6)$$

where by $\widetilde{M}_{ij} \equiv M_{ij} + M_{ji}$ we denote the symmetrized interaction matrix elements.

The first order term $\sigma^2 t$ is the standard statistics coming from the uncorrelated Brownian motions.

The effect of direct interactions appears at the second order where a negative correction occurs if off-diagonal terms are overall larger than diagonal terms, meaning when $u^i M_{ii} < u^i u^j M_{ij}/N$. For the financial network this means that positive exposures reduce the expectation of stress, as indeed positive correlations imply more homogeneous returns over the banks.

Interpretation of t^3 terms.



The third order term in the second line is the effect of indirect interactions occurring in the two forms

- the noise on node j propagates to the other nodes $\{k\}$ and then on to node i, giving the term $u^i u_j M_{ik} M^{kj}$.
- the noise on the other nodes $\{k\}$ affects directly both nodes i and j thereby creating a correlation, giving the term $u^i u_j M_{ik} M^{jk}$.

Random interaction matrix.

Assume now that regulators do not have detailed knowledge of exposures between banks, but only of the average squared exposure in the network γ^2, so that we can take $\hat{M}$ to be a random matrix with independent Gaussian entries of unit variance, and obtain

$$\mathbb{E}y = \sigma^2 t \left[1 + \frac{\beta\gamma^2}{3} (N+1)\, t^2 \right], \tag{7}$$

meaning that, on average, **interactions destabilize the network**.

Test of the expansion.

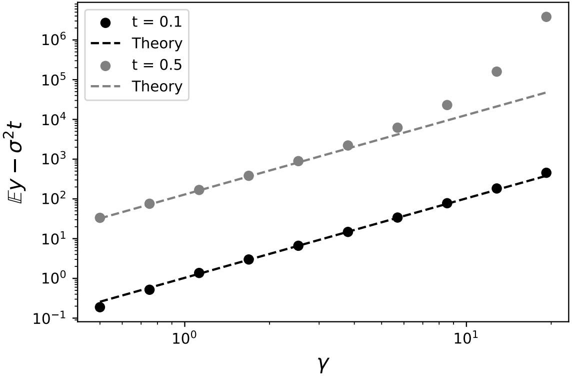


Figure: Quadratic scaling of the interaction correction with the interaction strength γ according to the theoretical estimate of Eq. (7).

Questions?

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