





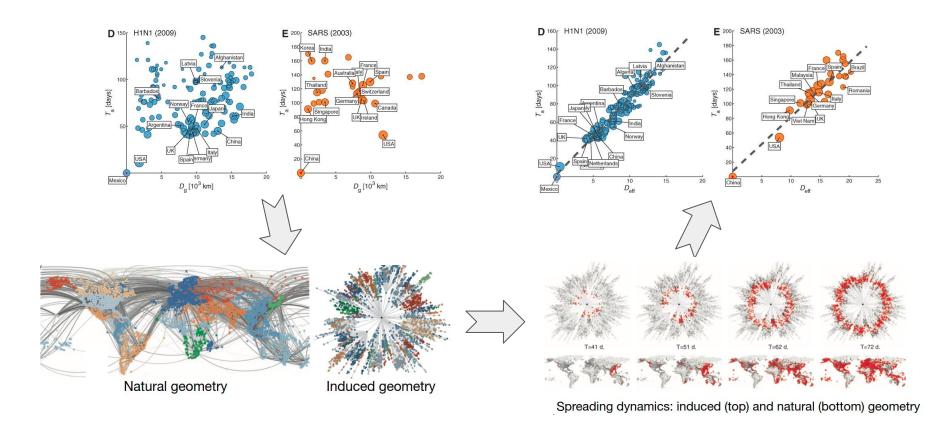


# Unraveling the mesoscale organization induced by network-driven processes

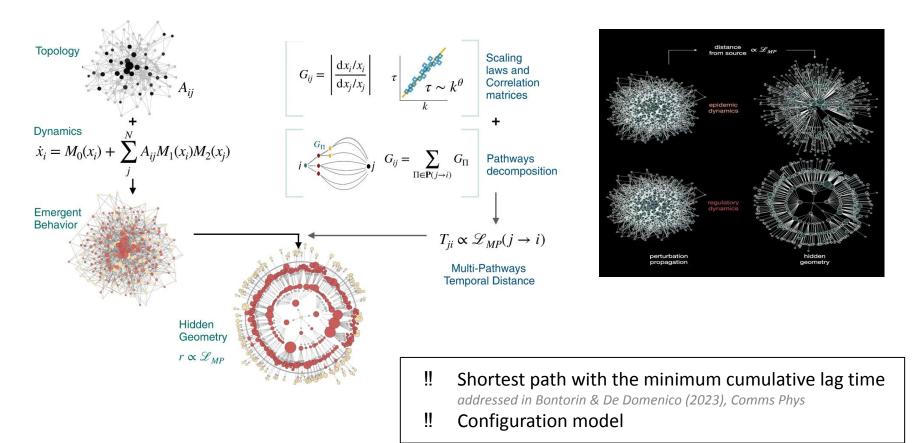
Giacomo Barzon, Oriol Artime, Samir Suweis, Manlio De Domenico

Network Days - 25 October 2024

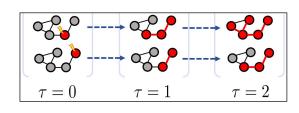
### Network-driven hidden geometry of epidemics spreading



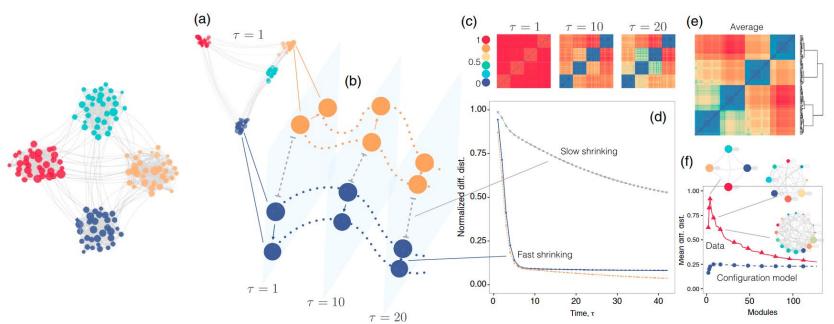
#### Universal temporal distance



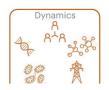
#### Diffusion geometry

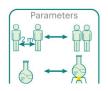


$$d_{\tau}^{2}(i,j) = [\mathbf{p}(\tau|i) - \mathbf{p}(\tau|j)]^{2}$$



#### Jacobian geometry



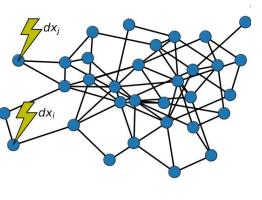


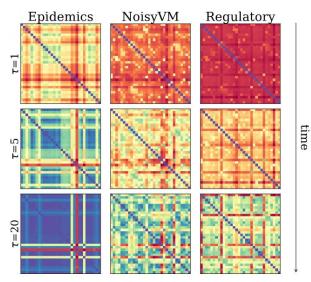


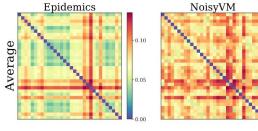
$$\delta \mathbf{x}_{(k)}(0) \equiv dx_k \mathbf{e}_{(k)}$$
$$\delta \dot{\mathbf{x}}_{(k)}(t) \approx J(\mathbf{x}^*) \delta_{(k)} \mathbf{x}(t)$$
$$\delta_{(k)} \mathbf{x}(t) = e^{J(\mathbf{x}^*)t} \delta_{(k)} \mathbf{x}(0)$$

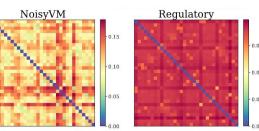
Jacobian distance 
$$d_{\tau}(i,j) = ||e^{J(\mathbf{x}^*)\tau}[dx_i e_{(i)} - dx_j e_{(j)}]||$$

Dynamics	$\partial_{ au}x_i =$
Biochemical	$F - Bx_i - R\sum_j A_{ij}x_ix_j$
Epidemics	$-Bx_i + R\sum_j A_{ij}(1-x_i)x_j$
Mutualistic	$Bx_i(1-x_i) + R\sum_j A_{ij}x_i \frac{x_j^b}{1+x_j^b}$
Neuronal	$-Bx_i + C \tanh x_i + R \sum_j A_{ij} \tanh x_j$
Noisy voter model	$A - Bx_i + \frac{C}{k_i} \sum_j A_{ij} x_j$
Population	$-Bx_i^b + R\sum_j A_{ij}x_j^a$
Regulatory	$-Bx_i^a + R\sum_j A_{ij} \frac{x_j^h}{1+x_j^h}$
Synchronization	$\omega_i + R \sum_j A_{ij} \sin(x_j - x_i)$

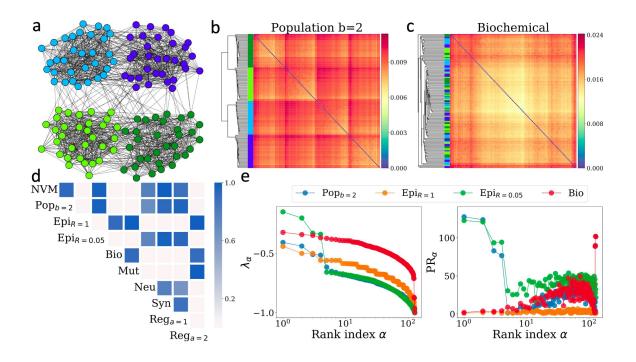








#### Structural vs functional hierarchy

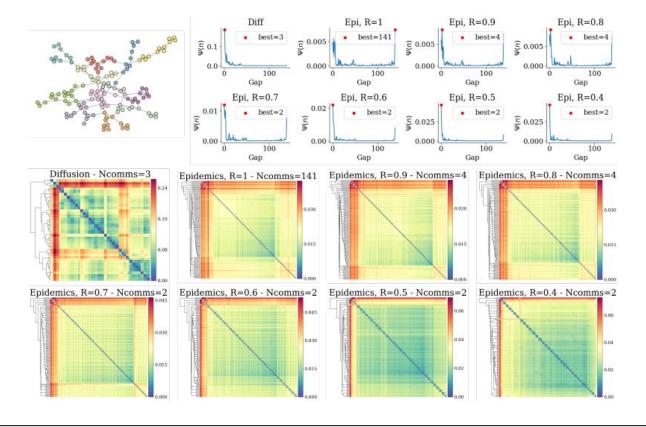


$$\delta \mathbf{x}_{(k)}(t) = dx_{(k)} \sum_{\alpha} \frac{\mathbf{e}_{(k)} \cdot \mathbf{u}_{\alpha}}{\mathbf{u}_{\alpha} \cdot \mathbf{v}_{\alpha}} e^{\lambda_{\alpha} t} \mathbf{v}_{\alpha}$$

$$PR_{\alpha} = \frac{\left(\sum_{i} v_{i,\alpha} u_{i,\alpha}\right)^{2}}{\sum_{i} \left(v_{i,\alpha} u_{i,\alpha}\right)^{2}}$$

- Effective hierarchy may diverge from structural hierarchy
- Functional regimes unraveled by the Jacobian spectrum

#### Transitions in emergent communities



Environmental conditions may lead to a continuous transition between regimes

#### Linking brain structure and function

Published: 14 December 2017

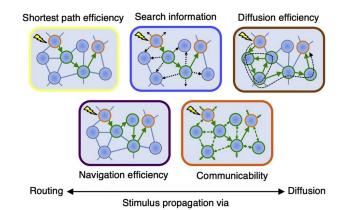
#### Communication dynamics in complex brain networks

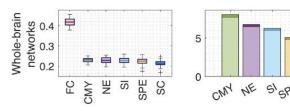
Andrea Avena-Koenigsberger, Bratislav Misic & Olaf Sporns

Nature Reviews Neuroscience 19, 17-33 (2018) Cite this article

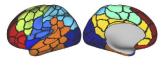
Network communication models narrow the gap between the modular organization of structural and functional brain networks

Caio Seguin <sup>a b c</sup> ○ ⊠, Sina Mansour L <sup>a g</sup>, Olaf Sporns <sup>c d e f</sup>, Andrew Zalesky <sup>1 a g</sup>, Fernando Calamante <sup>1 b h i</sup>

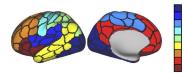












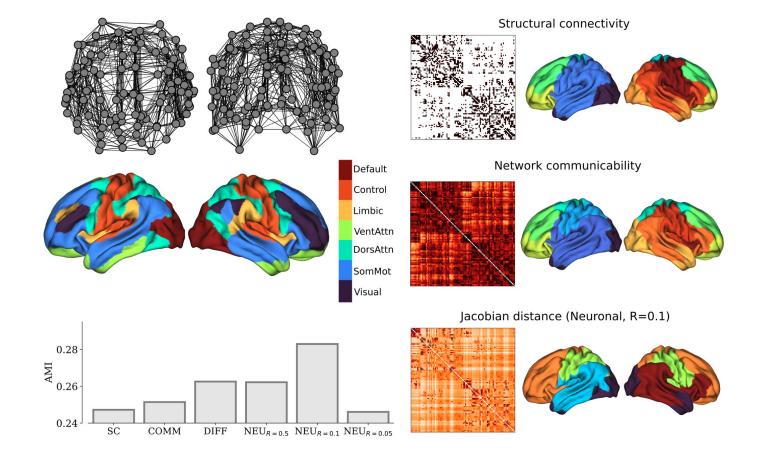


Reference functional partition (Yeo7)

SC partition ( $\gamma_{492}$ )

CMY partition ( $\gamma_{492}$ )

### Process-driven geometry of brain networks



#### Summary

- Perturbation-based geometry of any nonlinear processes unfolding on a network
- Interplay between physical dynamical rules, environmental parameters and network topology
- Effective emergent interactions explained by the spectrum of the Jacobian
- Relax the linearization around a steady-state

## Thank you for your attention!

