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Network Control Theory in Practice: the Example of Brain Networks

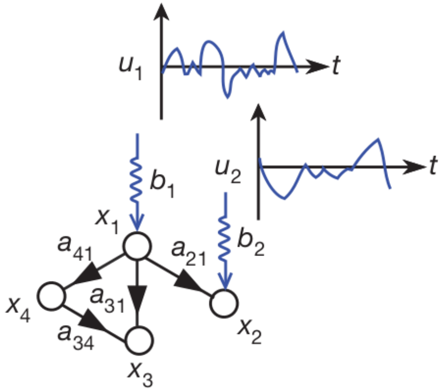
The “Network Control Theory” framework

$$\dot{x}(t) = Ax(t) + Bu(t))$$

A - (N,N) connectivity matrix

B - (N, M) input matrix, M # of drivers

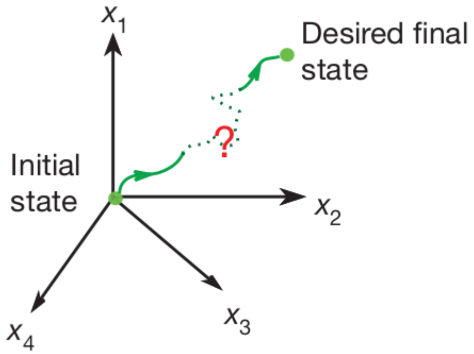
$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & a_{34} \\ a_{41} & 0 & 0 & 0 \end{pmatrix}; B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix};$$



The "Network Control Theory" framework

Control goal:

induce an arbitrary state in the network



Theorems allow computing:

- minimum number of control nodes needed to control the whole network ("structural controllability")
- the control signal **u**(t) to be applied (given the control nodes)

Liu, Y. Y., Slotine, J. J., & Barabási, A. L. (2011). Controllability of complex networks. nature, 473(7346), 167-173.

Network control theory for the brain

brain as network of 100 nodes

A = anatomical connectivity

At macroscopic spatial scales and slow time scales, the dynamics is quasi - linear

1 driver node, 99 target nodes

Gu, S., … & Bassett, D. S. (2015). Controllability of structural brain networks. Nature communications, 6(1), 8414.

“the brain is theoretically controllable through a single region”

And yet, control is difficult

$$u = B^T e^{At_f} W^{-1} (e^{-At_f} x_f - x_0)$$

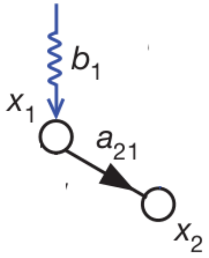
$$W = \int_0^\infty d\tau e^{A\tau} B B^T e^{A^T \tau}$$

Gramian

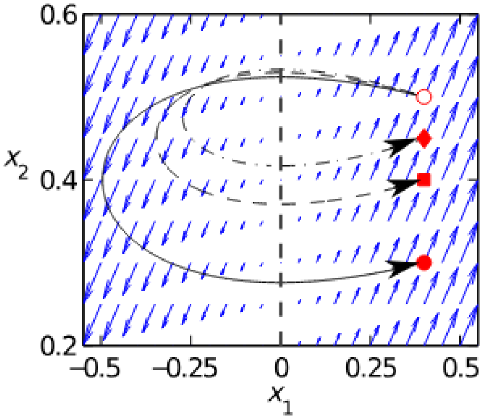
$$E = \int_0^{t_f} dt ||u(t)||^2 = x_f W^{-1} x_0$$

$$E_{max} = x_f \lambda_{min}^{-1} x_0$$

Energy

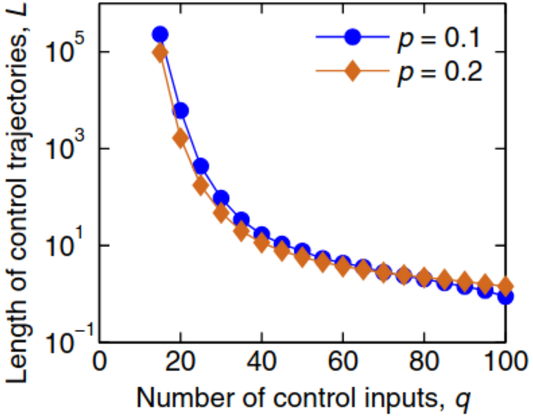


$$A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



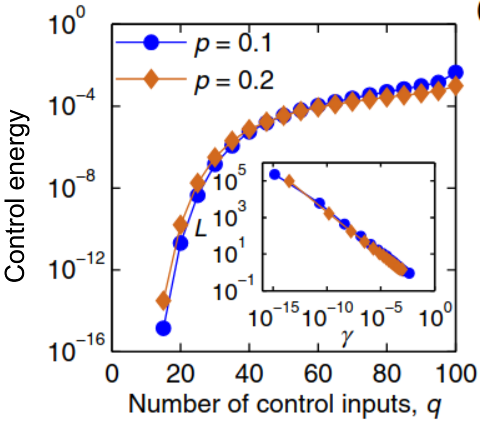
Sun, J., & Motter, A. E. (2013). Controllability transition and nonlocality in network control. PRL 110(20), 208701.

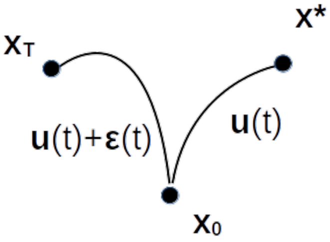
And yet, control is difficult

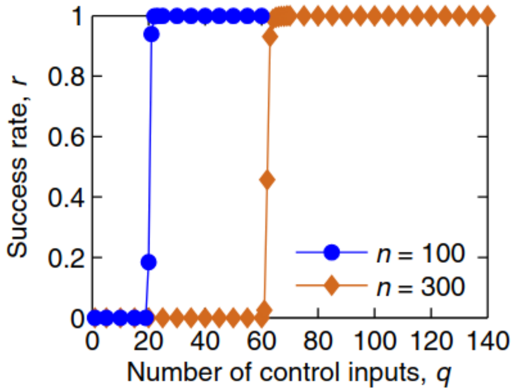


Control trajectories are long, energy large …

… unless a sizable number of control nodes is used

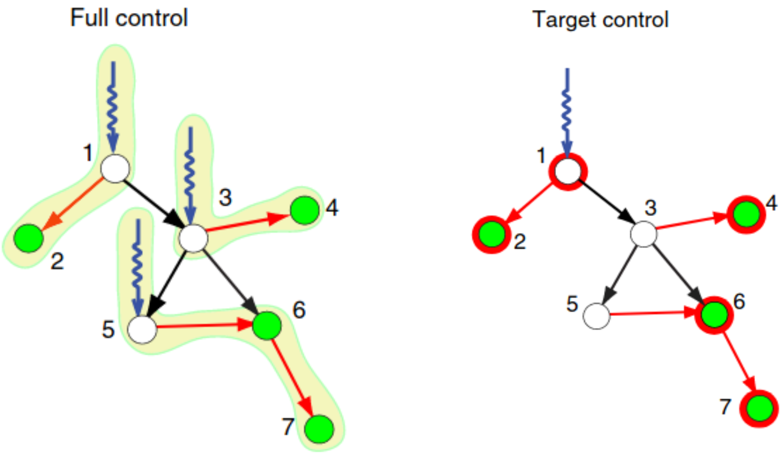






Control numerically fails unless a sizable number of control nodes is used

A step back: target control



Gao, J., Liu, Y. Y., D'souza, R. M., & Barabási, A. L. (2014). Target control of complex networks. Nature communications, 5(1), 1-8.

Control goal:

induce an arbitrary state in *a selected subnetwork*

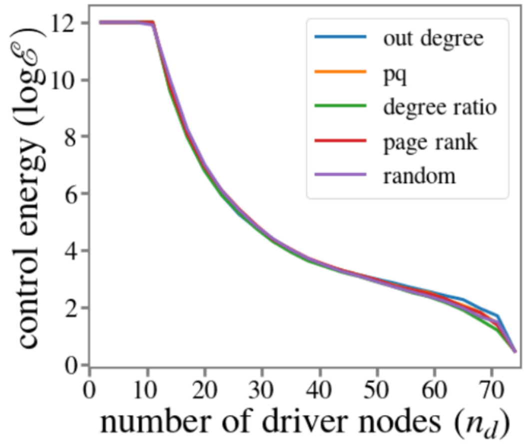
Significantly reduce number of control nodes needed, as well as control energy

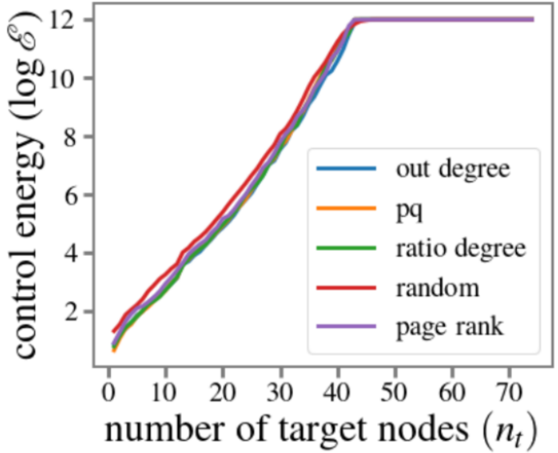
Target control of brain networks

Control energy is astronomically large unless many nodes are controlled

Control energy scales *exponentially* with number of target nodes

We should restrict attention to a few target nodes at most





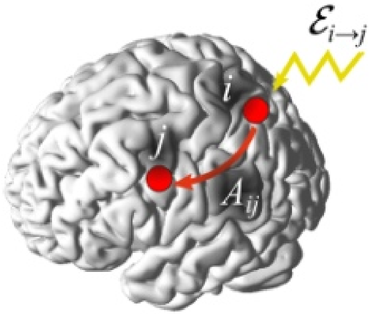
Manjunatha, K. K. H., Baron, G., Benozzo, D., Silvestri, E., Corbetta, M., Chiuso, A., ... & Allegra, M. (2024). Controlling target brain regions by optimal selection of input nodes. PLOS Computational Biology, 20(1), e1011274.

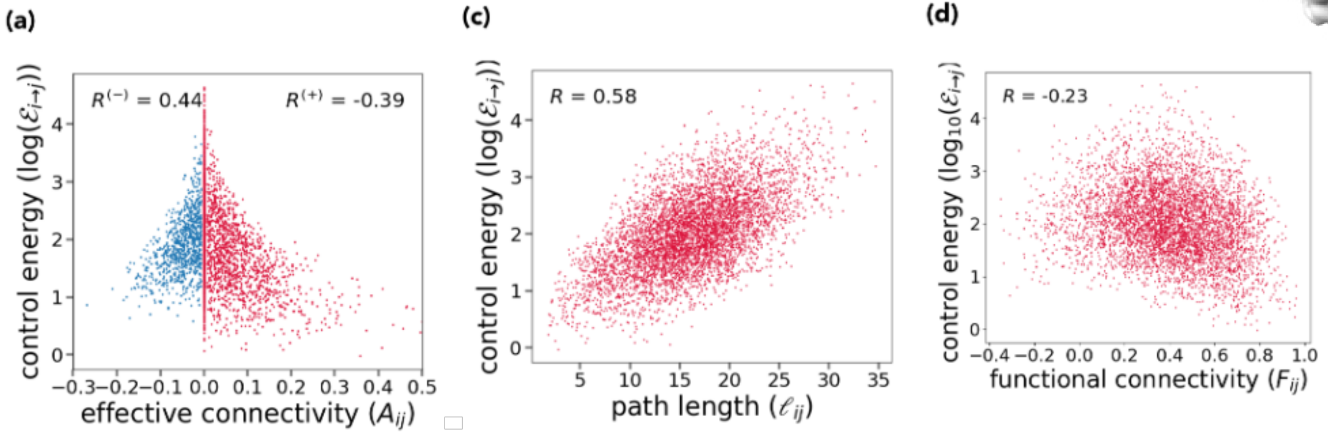
Target control of brain networks

Energy decreases with direct and indirect effective connections

Brain networks are signed

Both positive and negative connections contribute to control





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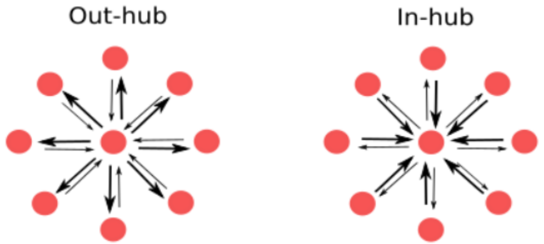
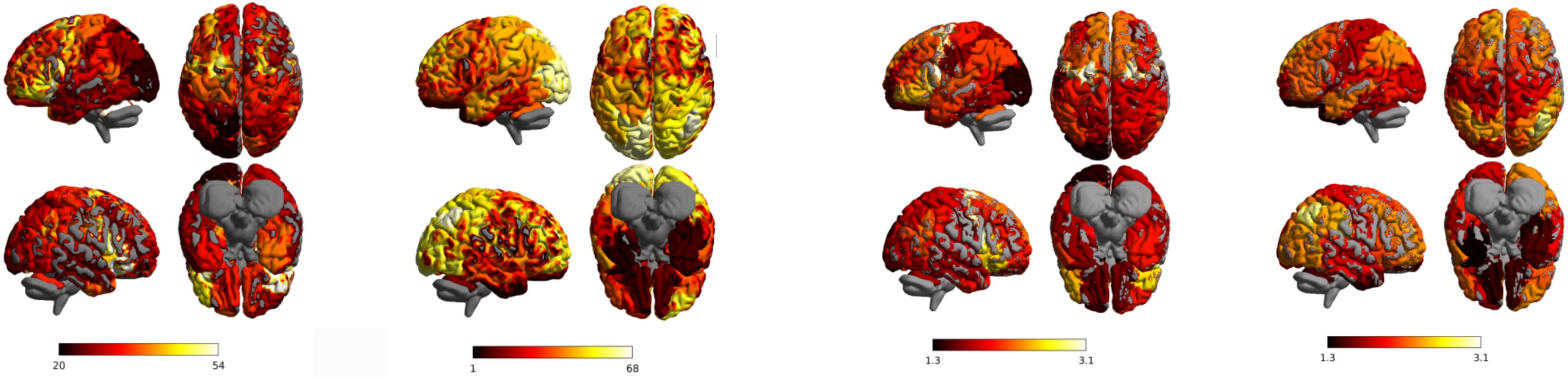
Target control of brain networks

Brain networks are asymmetric

Driver energy (energy to control other nodes from a nodes)
correlates with EC out-strength

Target energy (energy to control a node)
correlates with EC in-strength





driver energy   target energy   out-strength   in-strength

Conclusions

- Elegant mathematical framework to control networks with linear dynamics

- Fine-grained control of the whole network requires acting on many nodes

- Adopting a Target control approach makes the problem more feasible

- Indirect and negative links are crucial for control

- Other *control goals?*

- *Coarse-grained approach?*

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