IMT SCHOOL FOR ADVANCED STUDIES LUCCA

COARSE-GRAINING BIOLOGICAL NETWORKS THROUGH SYMMETRIES AND SYNCHRONIZATION

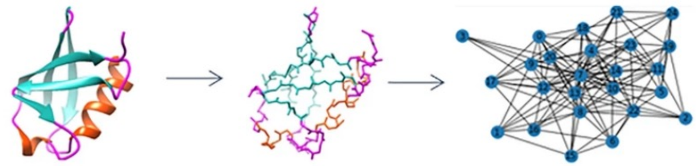
TOMMASO GILI

Networks Days : Bridging micro with macro

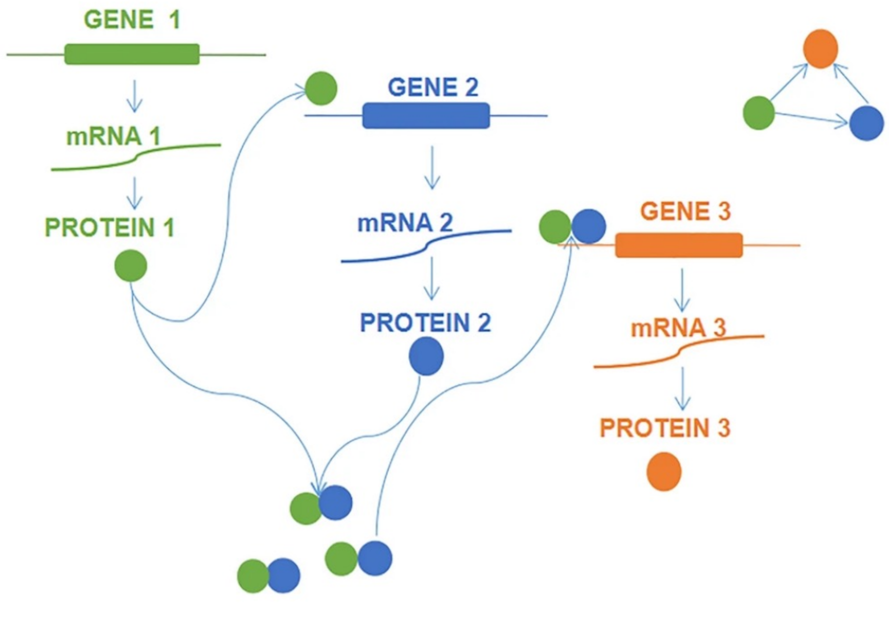
Padua, 25th October 2024

NETWORKS@IMT

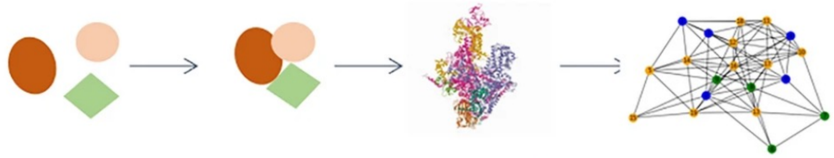
Biological Complex Systems and Networks



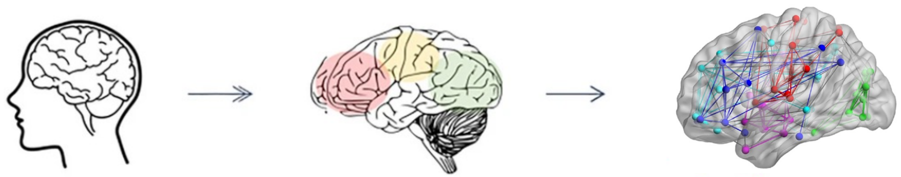
Protein Contact Network



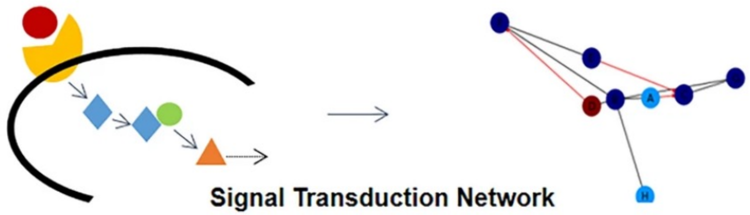
Gene Regulatory Network



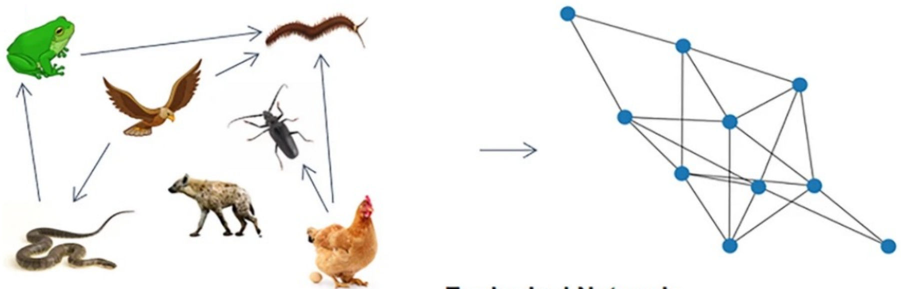
Protein-Protein Interaction Network



Neuronal Network



Signal Transduction Network



Ecological Network

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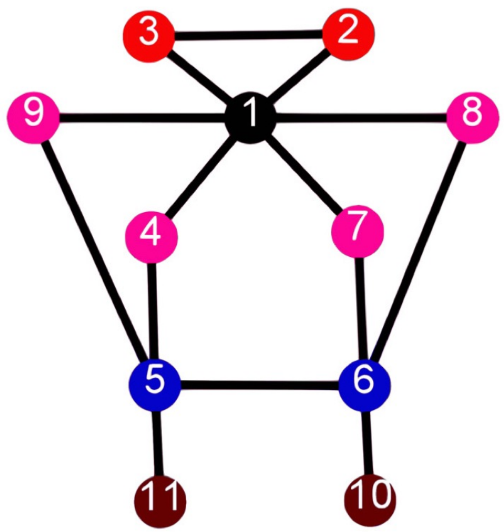
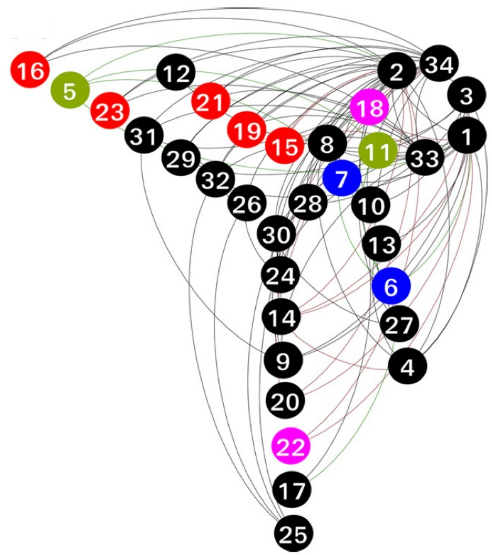
Group Symmetries

Network symmetries (applied to the adjacency matrix) that leave the dynamics of the network unchanged

$$\dot{\boldsymbol{x}}_i = \boldsymbol{F}(x_i) + \sigma \sum_j A_{ij} \boldsymbol{H}(\boldsymbol{x}_j)$$



Toy Model

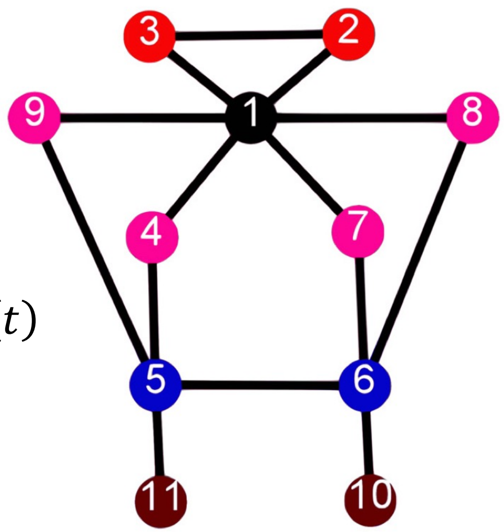


Zachary's Karate Club

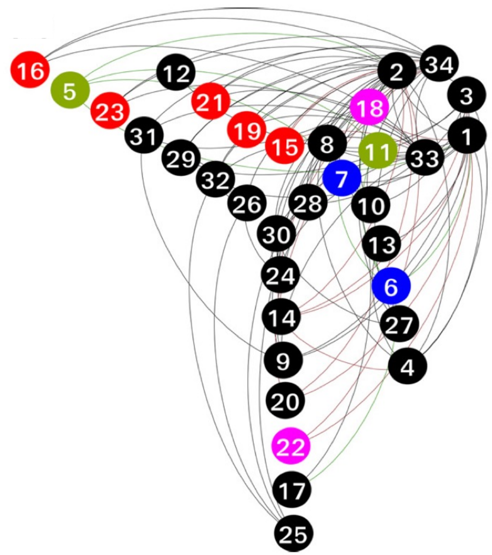
Arenas A et al., Phys. Rep., 469: 93, 2008
Pecora LM et al., Nat. Commun, 5: 4079, 2014
Khanra P et al., Chaos Soliton Fract, 155: 111703, 2022

Global Synchronisation, in which all nodes follow the same trajectory in state space, is a well-studied effect, the conditions for which are related to the network structure and its Group Symmetries.

$$\dot{\theta}_i = \omega_i + K \sum_{j=1}^{N} A_{ij} \, sin\left[\theta_j(t) - \theta_i(t)\right] + \sigma\eta_i(t)$$



Toy Model



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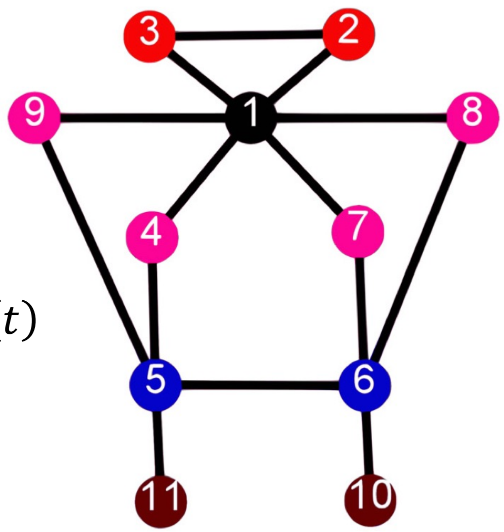
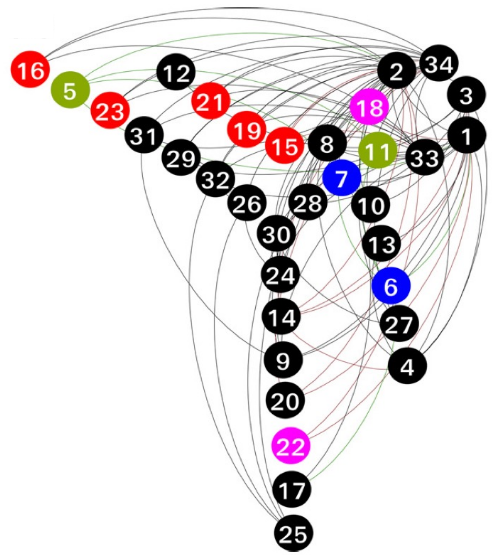
$$\dot{\theta}_i = \omega_i + K \sum_{j=1}^{N} A_{ij} \sin[\theta_j(t) - \theta_i(t)] + \sigma \eta_i(t)$$

$$\dot{\theta}_i \sim -K \sum_{j=1}^{N} L_{ij} \theta_j(t)$$



Toy Model



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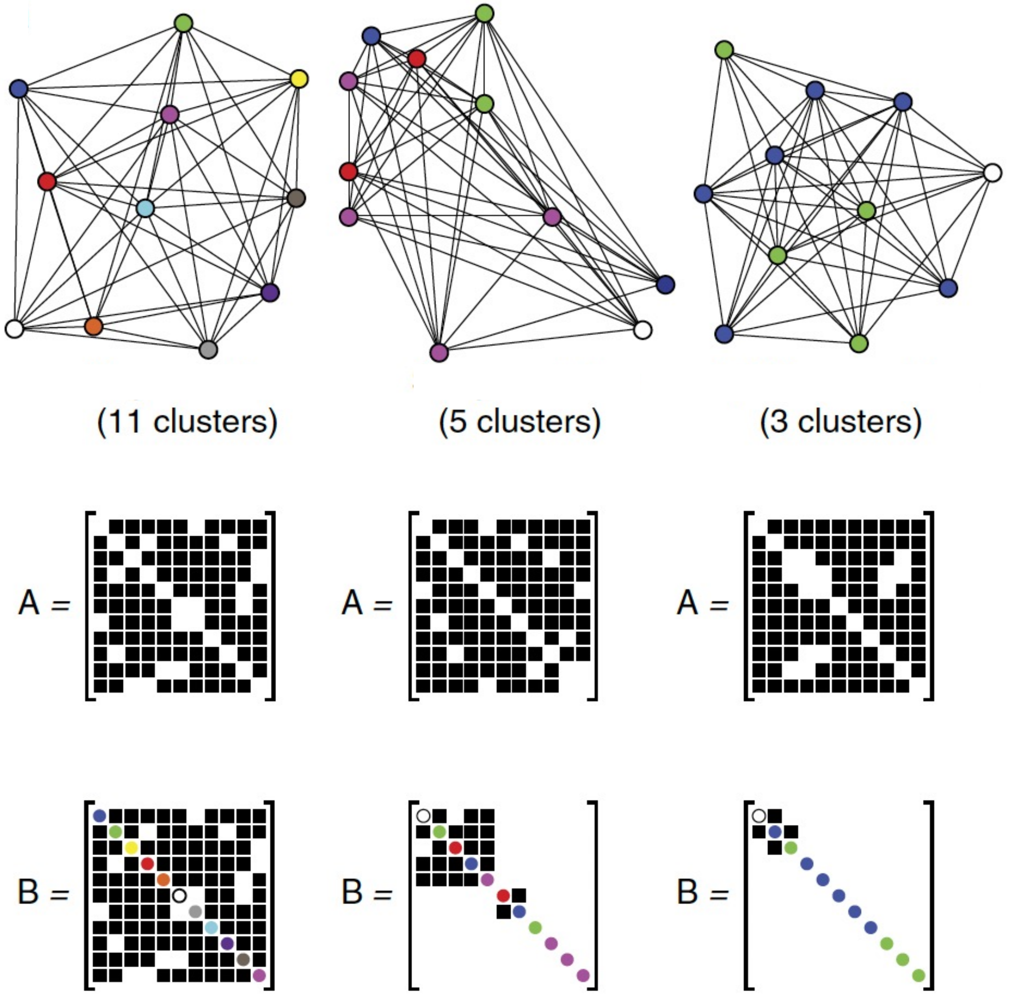
Equally important, and perhaps more ordinary, is Cluster Synchronization, in which patterns or sets of synchronized elements emerge.



Zhou C and Kurths J, Chaos, 16: 015104, 2006.
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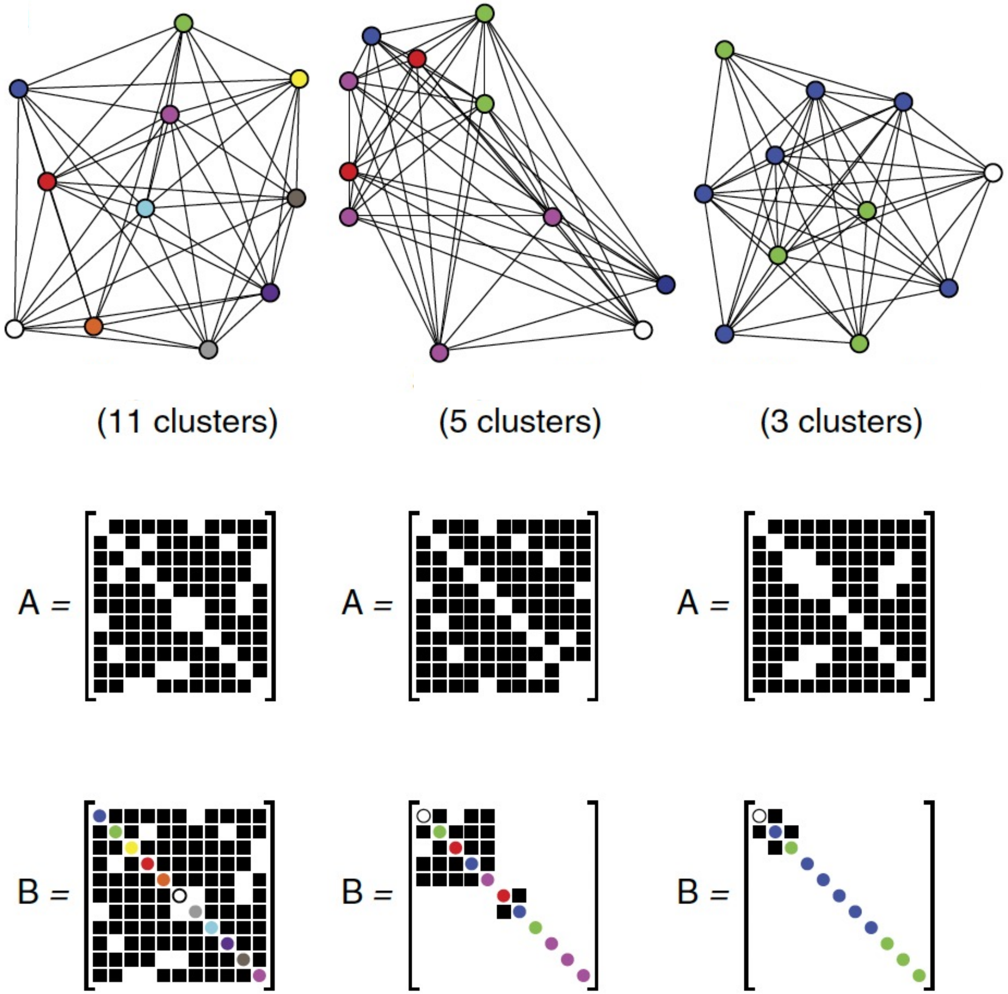
Many real-world networks, such as biological networks, do not admit Group Symmetries but show Cluster Synchronization



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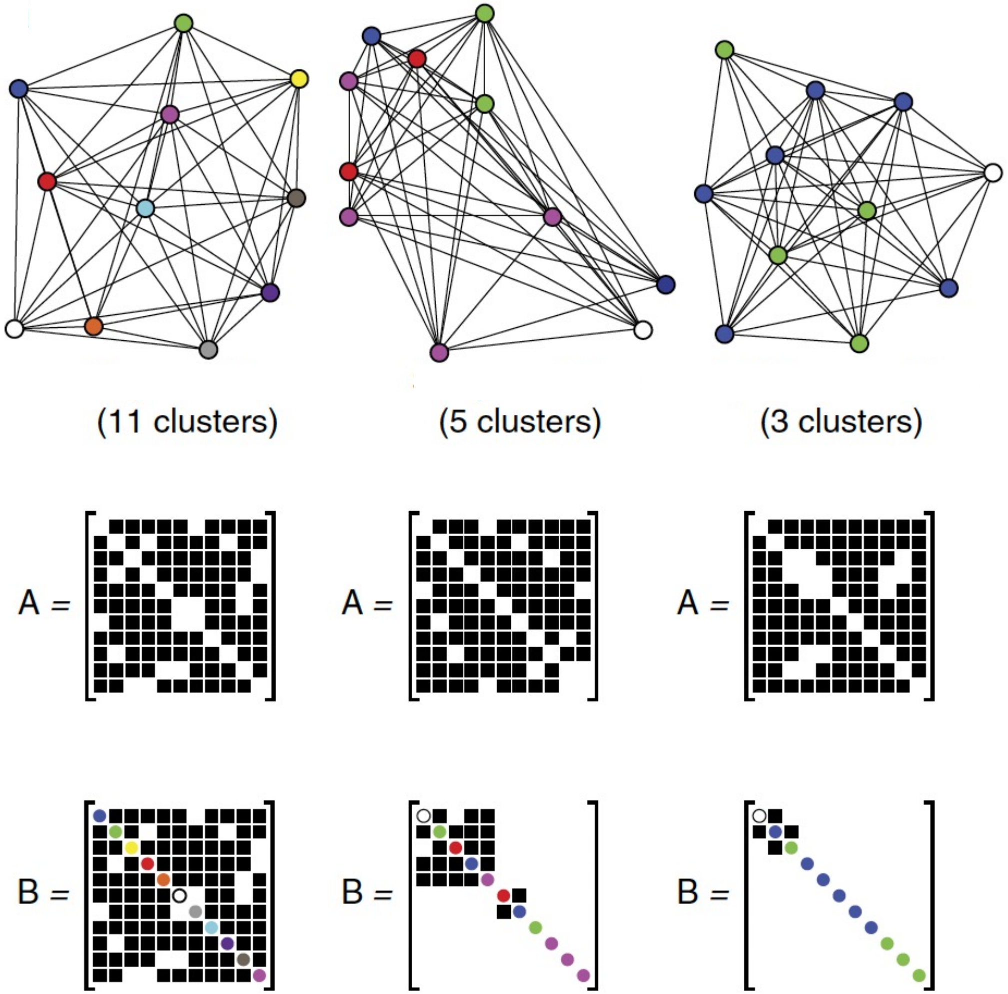
Equally important, and perhaps more ordinary, is Cluster Synchronization, in which patterns or sets of synchronized elements emerge.

Many real-world networks, such as biological networks, do not admit Group Symmetries but show Cluster Synchronization

In such networks Cluster Synchronization is guaranteed by Fibration Symmetries

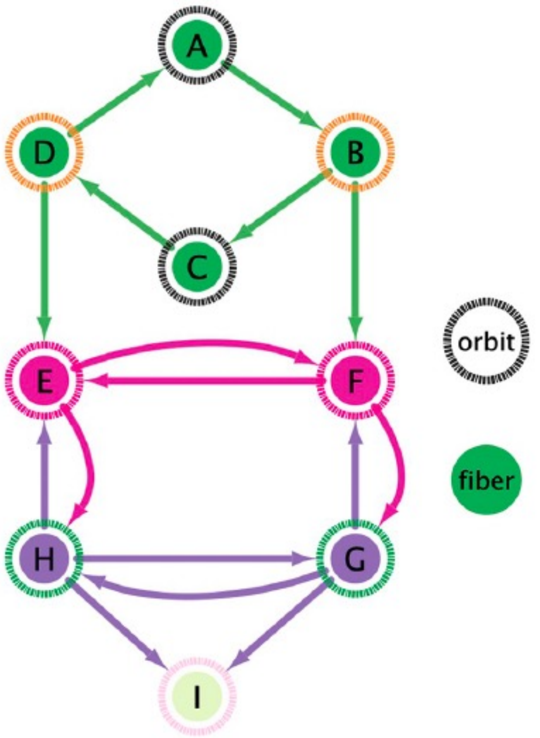


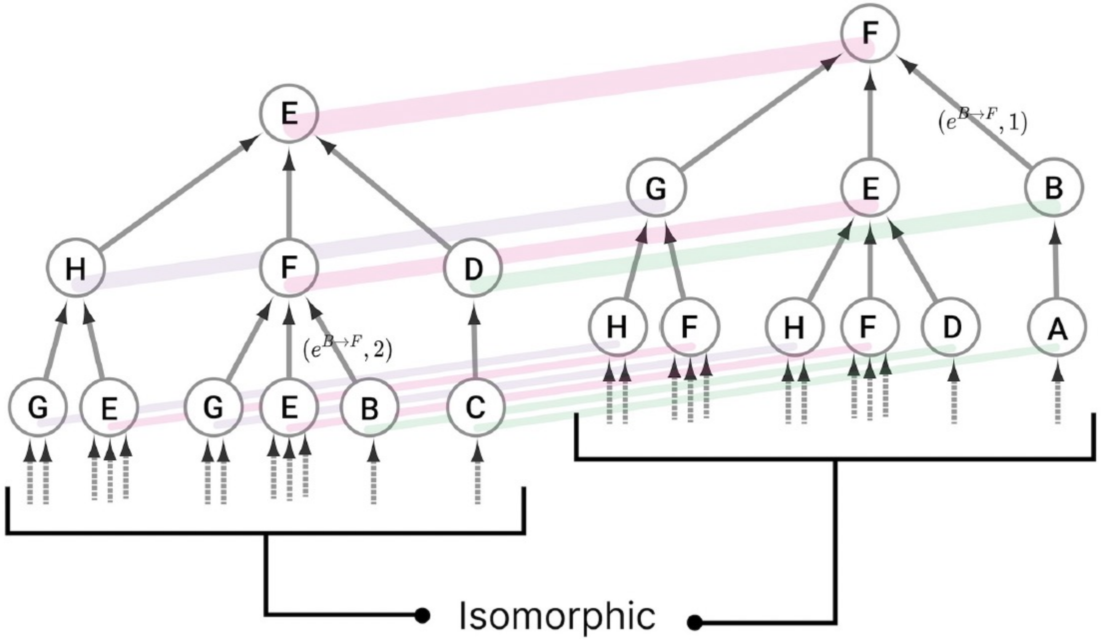
Morone F, and Makse H, Nat Commun, 10 (1): 4961, 2019

Morone F, Leifer I, Makse H, PNAS, 17 (15): 8306-8314, 2020

I Leifer, et al., J. Stat. Mech.: Theory Exp, (7): 073403, 2022

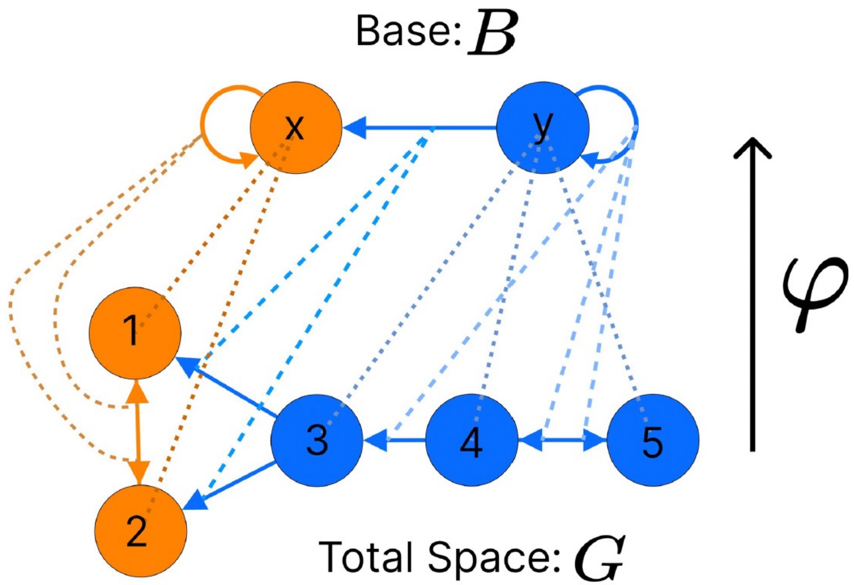
Fibration Symmetries are morphisms between networks that identify clusters of synchronized nodes (called fibers) with isomorphic input trees.





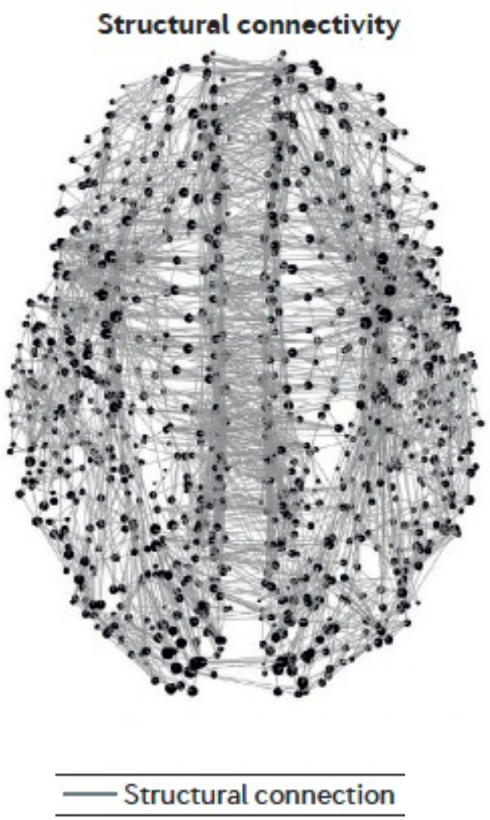
Boldi, P. and Vigna, S. Discrete Math. 243, 21–66, 2002
Avila B et al., Plose One, Plos one 19 (4), e0297669, 2024

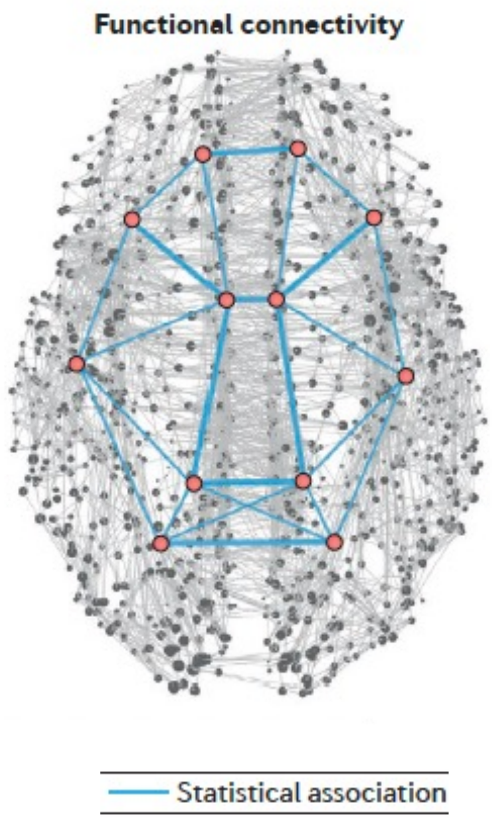
Nodes in a fiber can be collapsed by a symmetry fibration into a single representative node called the base.



The fibers are then the synchronized building blocks of the network and symmetry fibrations are transformations that preserve the dynamics of information flow in the network.

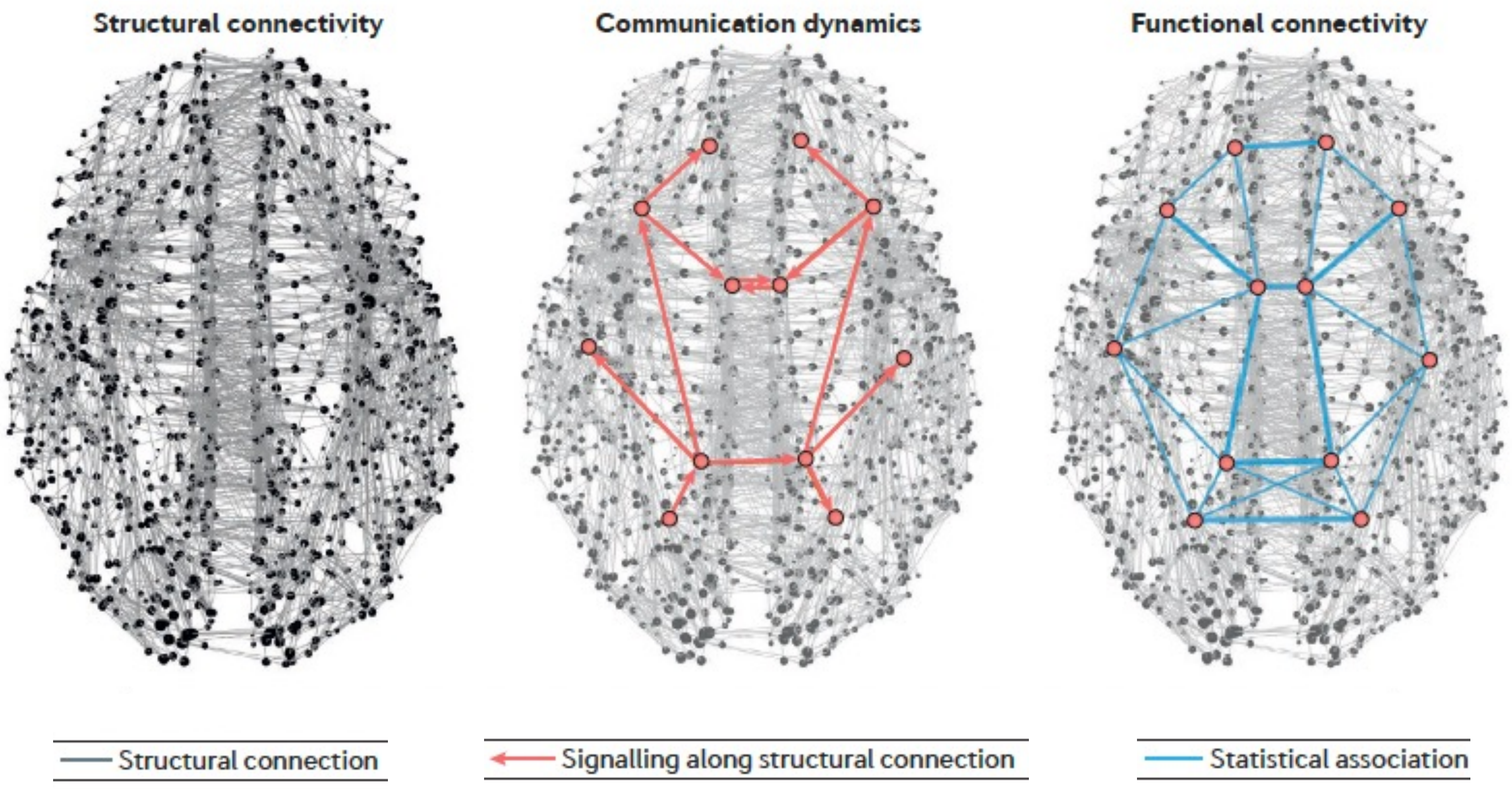
Communication in the brain





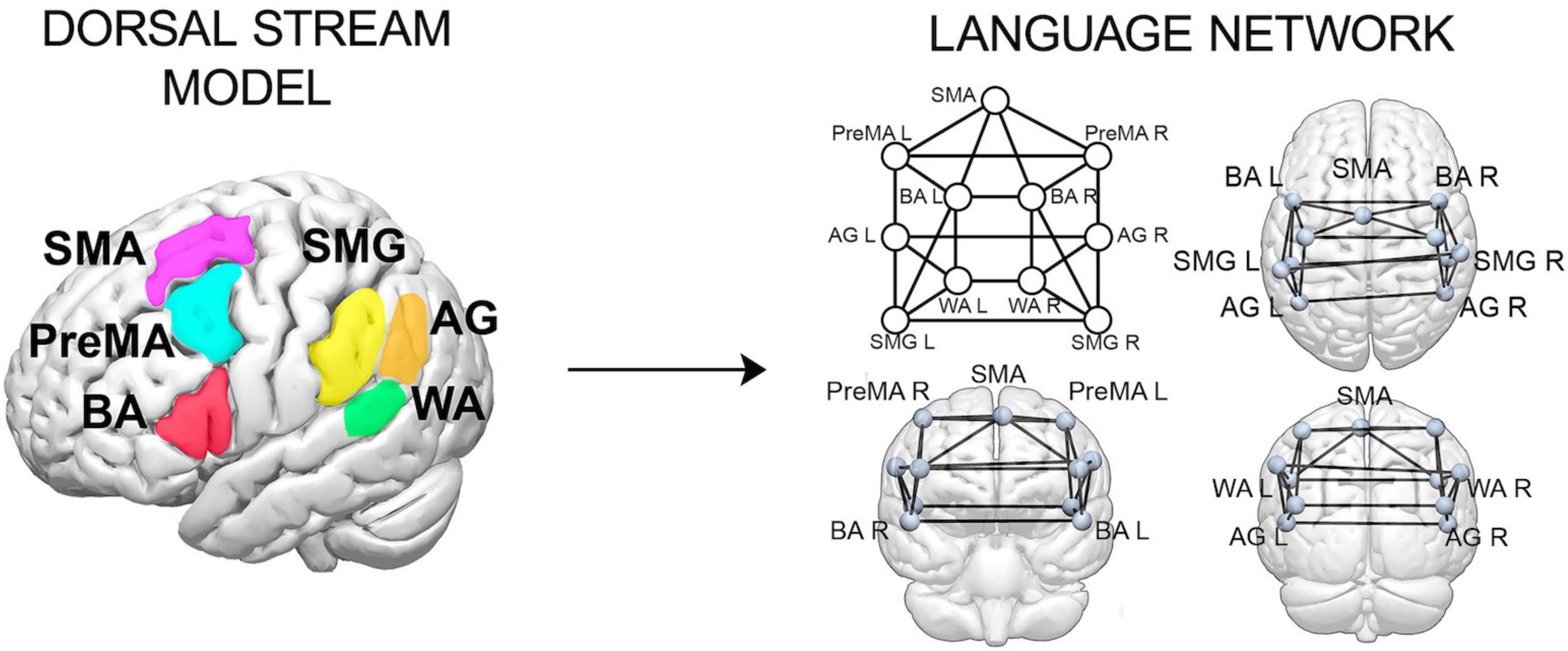
Park H-J & Friston K. Structural and functional brain networks: from connections to cognition. Science 342: 1238411 (2013)

Communication in the brain



Park H-J & Friston K. Structural and functional brain networks: from connections to cognition. Science 342: 1238411 (2013)

Brain Network Topology and Synchronization



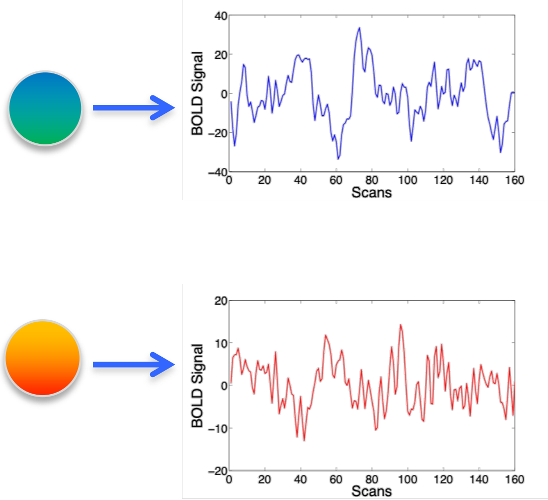
Gili T et al. 2024, arXiv:2409.02674

Brain Network Synchronization

RESTING STATE
VERB GENERATION
PHONEMIC FLUENCY

fMRI timeseries from anatomical regions of interest

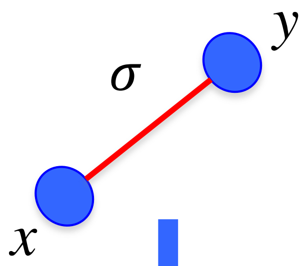
- Angular Gyrus L (AG L)
- Angular Gyrus R (AG R)
- Broca's Area L (BA L)
- Broca's Area R (BA R)
- Premotor Area L (PreMA L)
- Premotor Area R (PreMA R)
- Supplementary Motor Area (SMA)
- Supramarginal Gyrus L (SMG L)
- Supramarginal Gyrus R (SMG R)
- Wernicke's Area L (WA L)
- Wernicke's Area R (WA R)



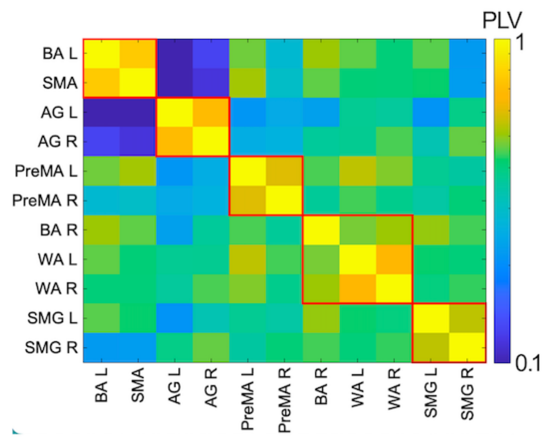
Phase Locking Value (PLV)

$$\sigma(x_i(t), x_j(t)) = \left| \left\langle e^{-j(x_i(t)-x_j(t))} \right\rangle_t \right|$$

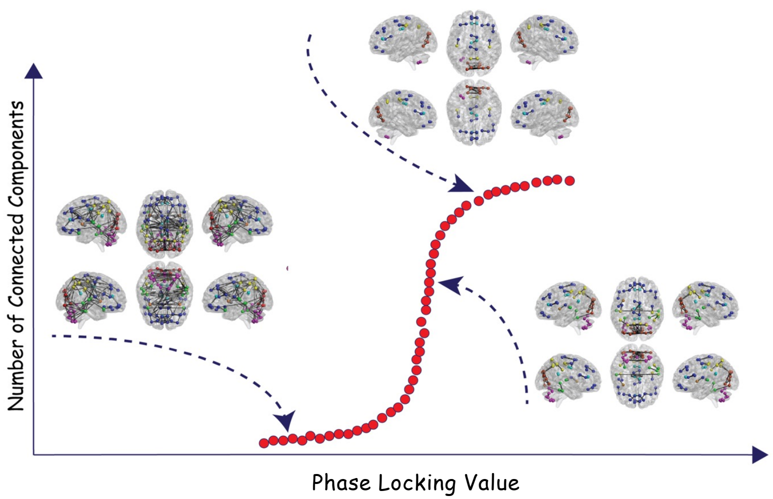
Synchronization



Language Functional Network



Percolation Process



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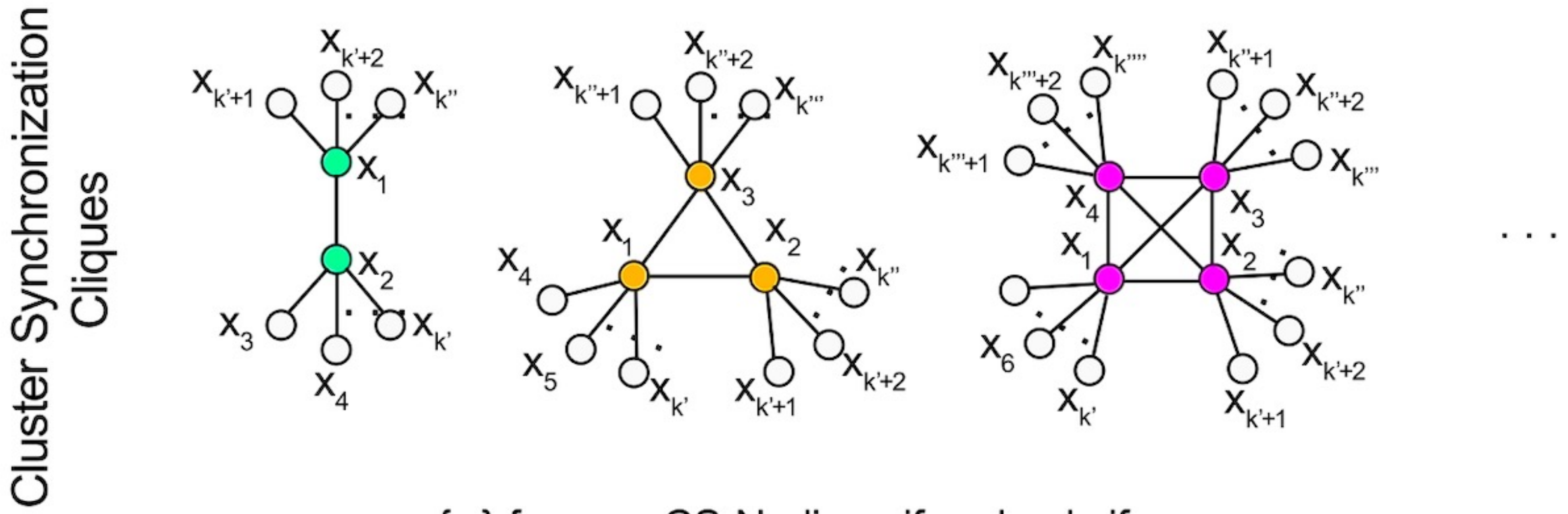
Cluster Synchronization Cliques



$\{x_k\}$ forms a CS N-clique if and only if

$$\sum_{i<j}^{1,N} \sigma(x_i(t), x_j(t)) \geq \frac{N(N-1)}{2} \sigma(x_k(t), x_{k'}(t)) \qquad \forall k = 1, \ldots, N \text{ and } k' \in \mathcal{M}_k$$

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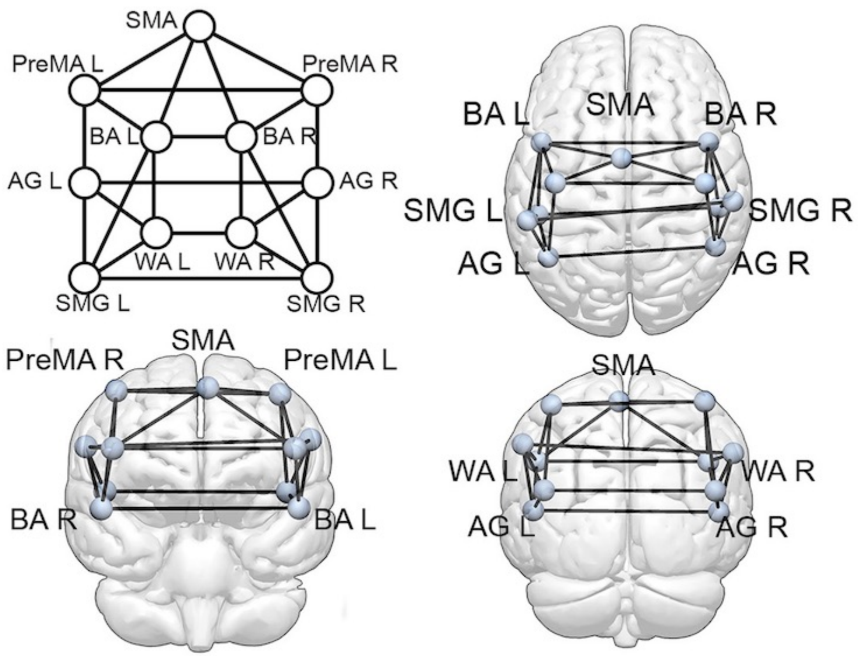
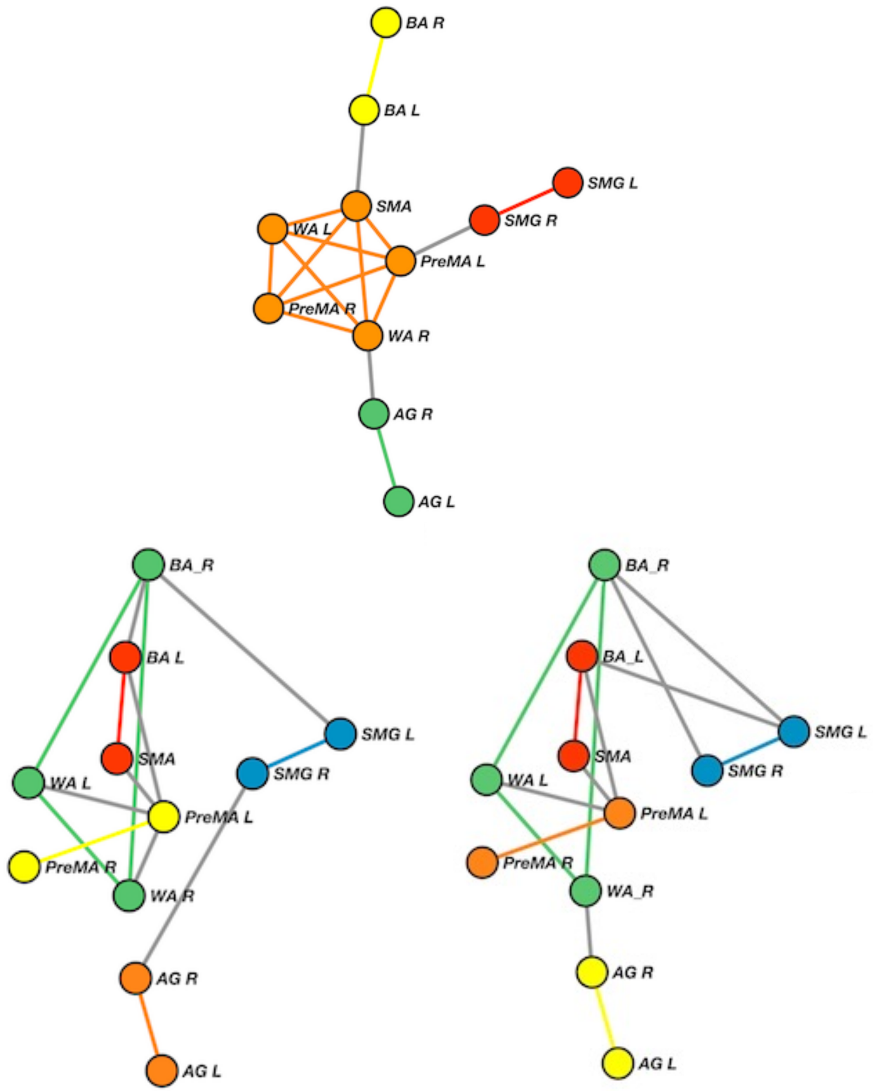
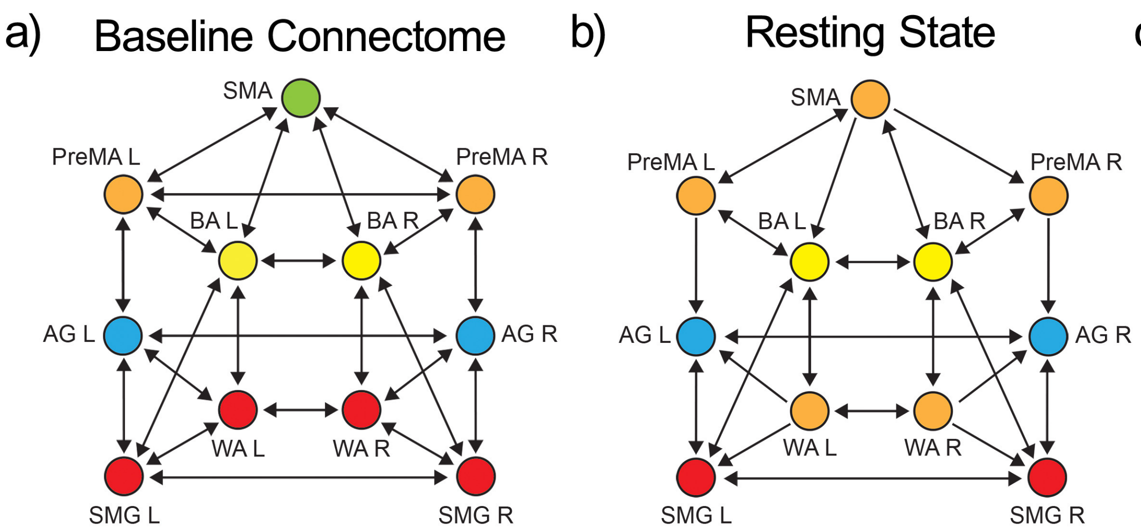
SYNCHRO COLOURING

LANGUAGE NETWORK







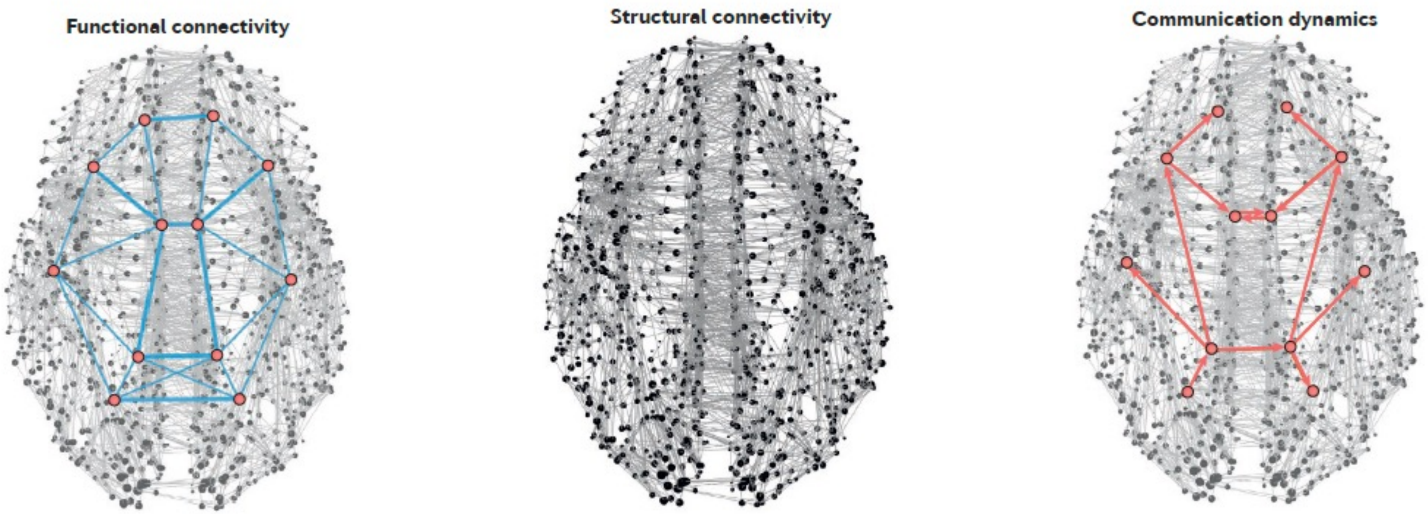
Fibration Symmetry
and
Group Symmetry

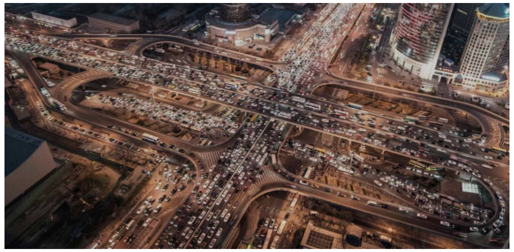
Fibration Symmetry
and
Broken Group Symmetry

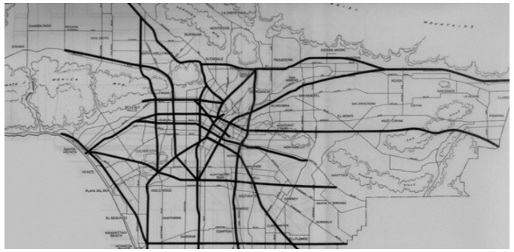
Broken Fibration Symmetry
and
Broken Group Symmetry

PreMA L: Premotor Left
PreMA R: Premotor Right
SMA: Supplementary Motor Area
BA L: Broca's Area Left
BA R: Broca's Area Right
AG L: Angular Gyrus Left
AG R: Angular Gyrus Right
WA L: Wernicke's Area Left
WA R: Wernicke's Area Right
SMG L: Supramarginal Gyrus Left
SMG R: Supramarginal Gyrus Right

Take Home Message







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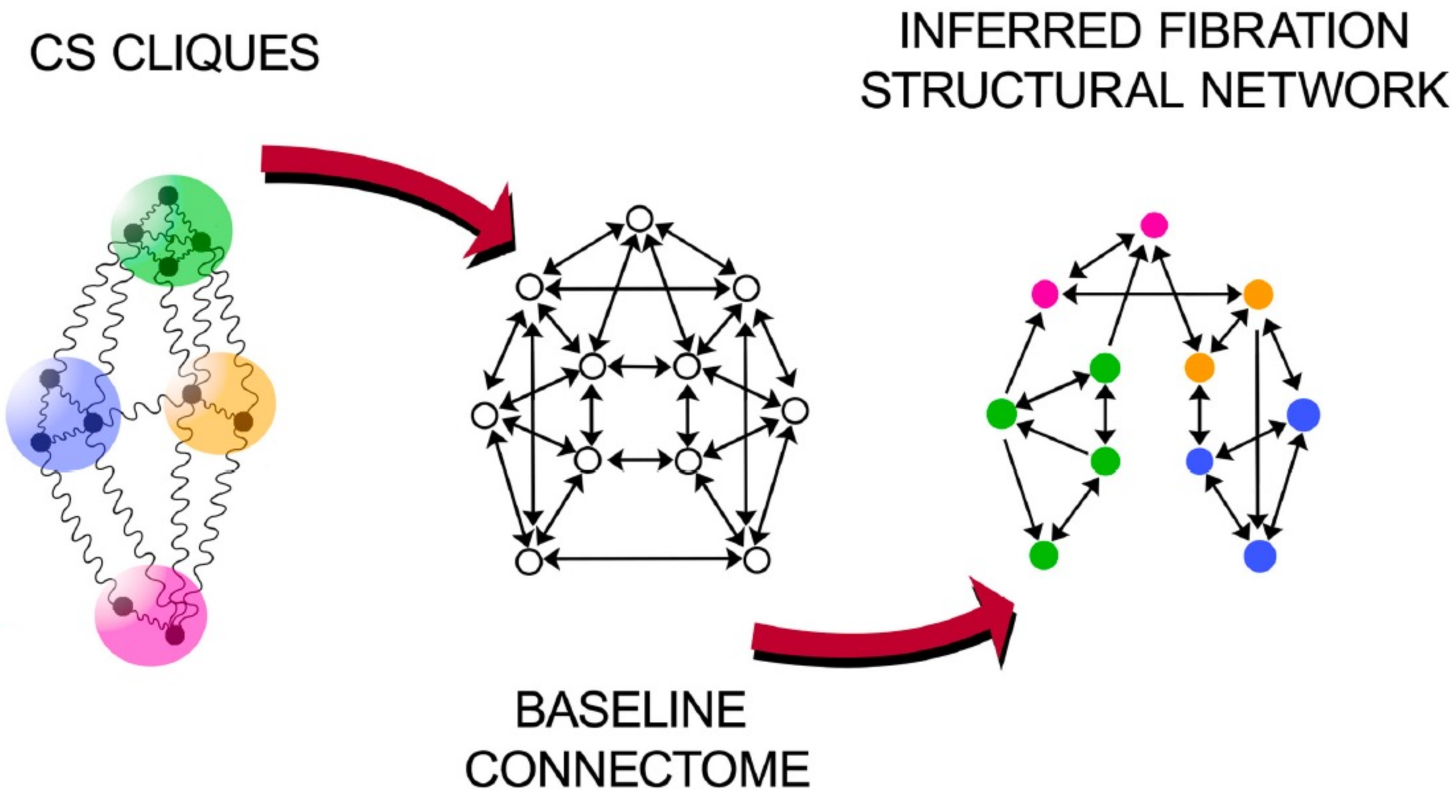
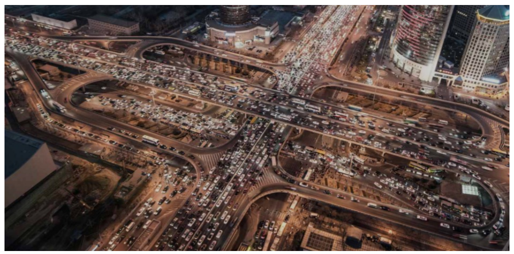
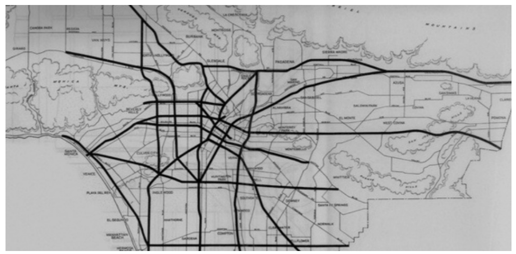
Take Home Message







Thanks to

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CCNY 175 Years

Memorial Sloan Kettering Cancer Center

CENTRO RICERCHE ENRICO FERMI

ROMA TRE UNIVERSITÀ DEGLI STUDI

Università Ca' Foscari Venezia

CNR Istituto dei Sistemi Complessi

UNIVERSITÀ DEGLI STUDI DI MILANO

THE UNIVERSITY OF NEW MEXICO

IMP Research Institute of Molecular Pathology

Hernan Makse

Andrea Gabrielli

Guido Caldarelli

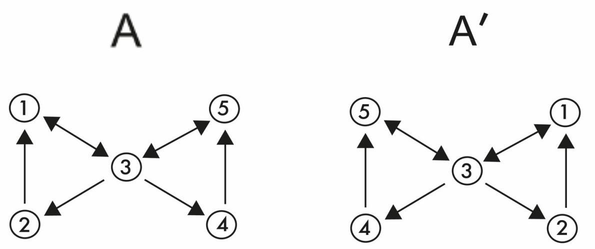
Paolo Boldi

David Phillips

Andrei Holodny

Manuel Zimmer

Network with a symmetry group

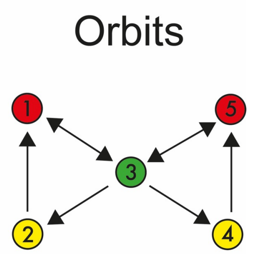


A

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

A′

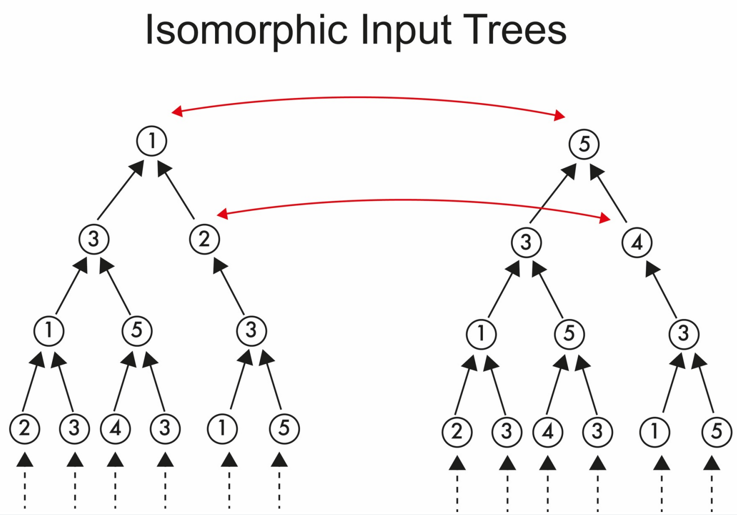
$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

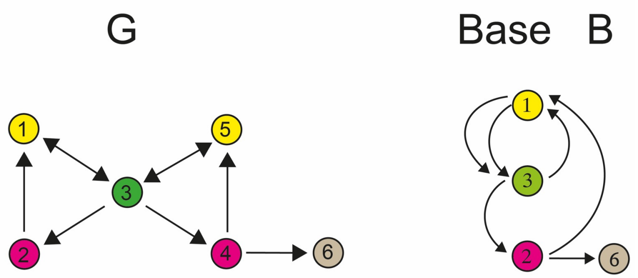


$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix} \longleftrightarrow P = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A = P^{-1} A' P$$

Network with no symmetry group and a fibration symmetry





$$\phi : G \rightarrow B$$

Cluster Synchronization Cliques

Hilbert Transform

$n_i(t)$ 

$n_j(t)$ 

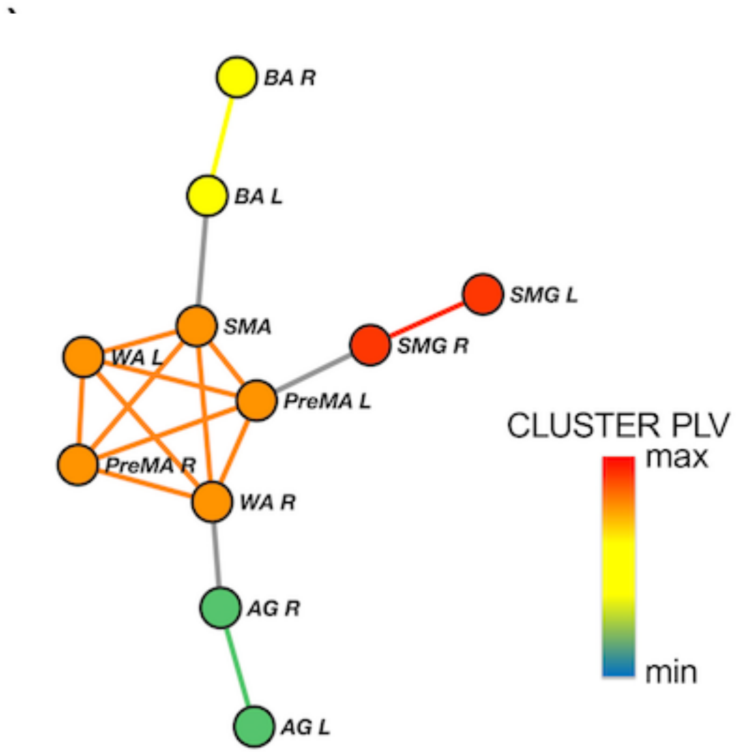
$$\hat{n}_i(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{n_i(t)}{t-\tau} d\tau \qquad \hat{n}_j(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{n_j(t)}{t-\tau} d\tau$$

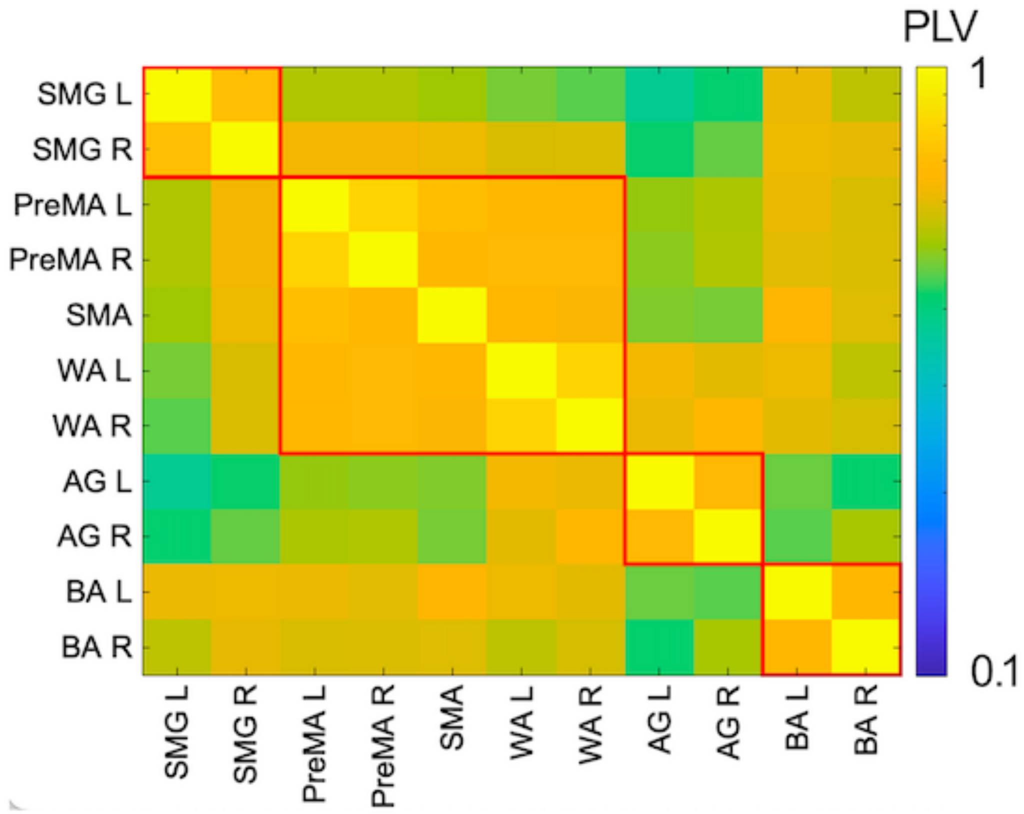
Phase Coupling

$$x_i(t) = arg(n_i(t) + j\hat{n}_i(t)) \qquad x_j(t) = arg(n_j(t) + j\hat{n}_j(t))$$

$$\sigma(x_i(t), x_j(t)) = \left| \left\langle e^{-j(x_i(t) - x_j(t))} \right\rangle_t \right|$$

RESTING STATE





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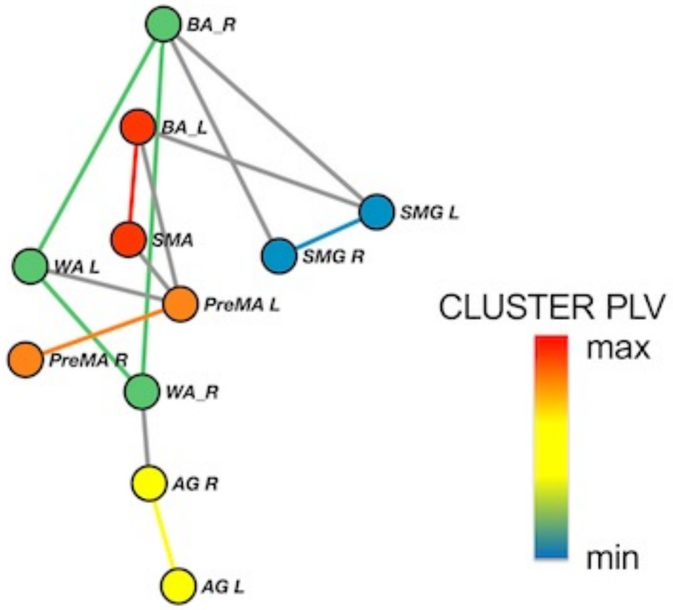
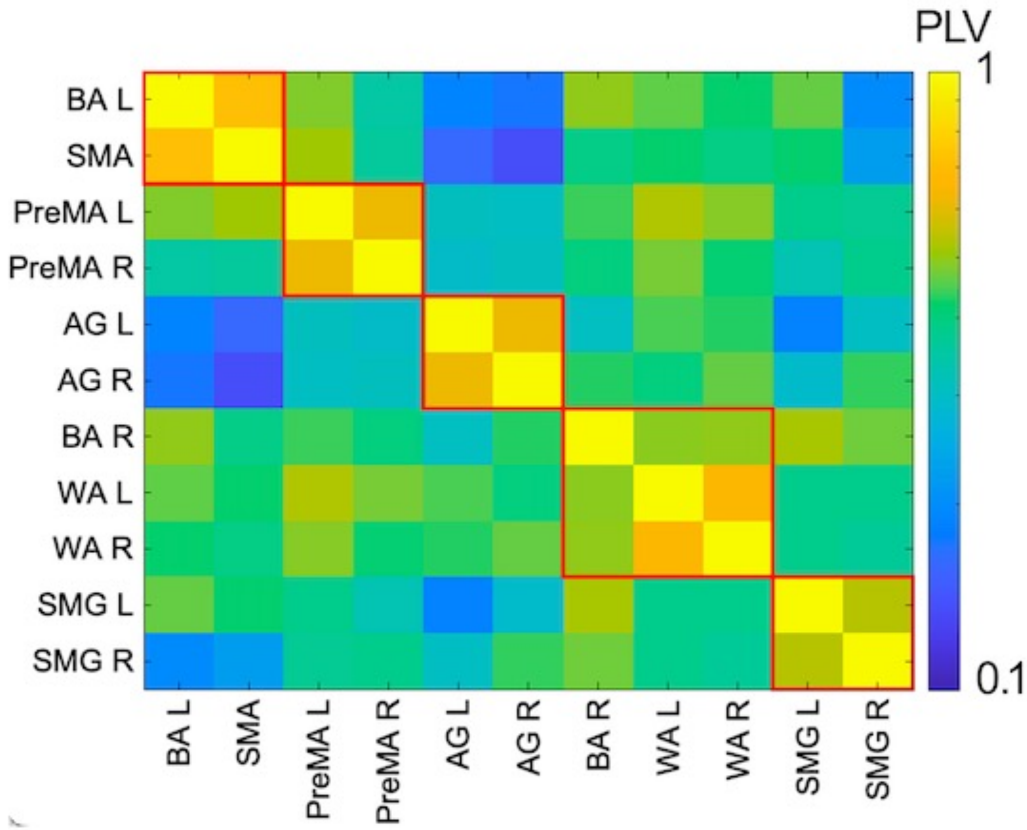
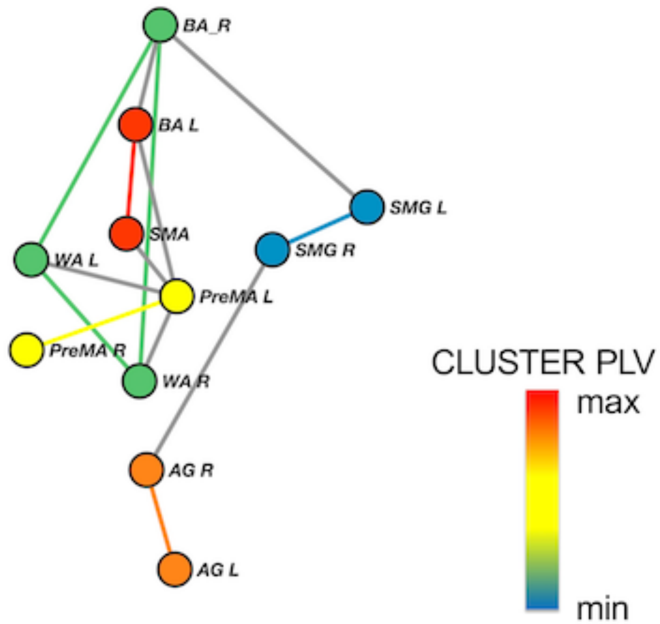
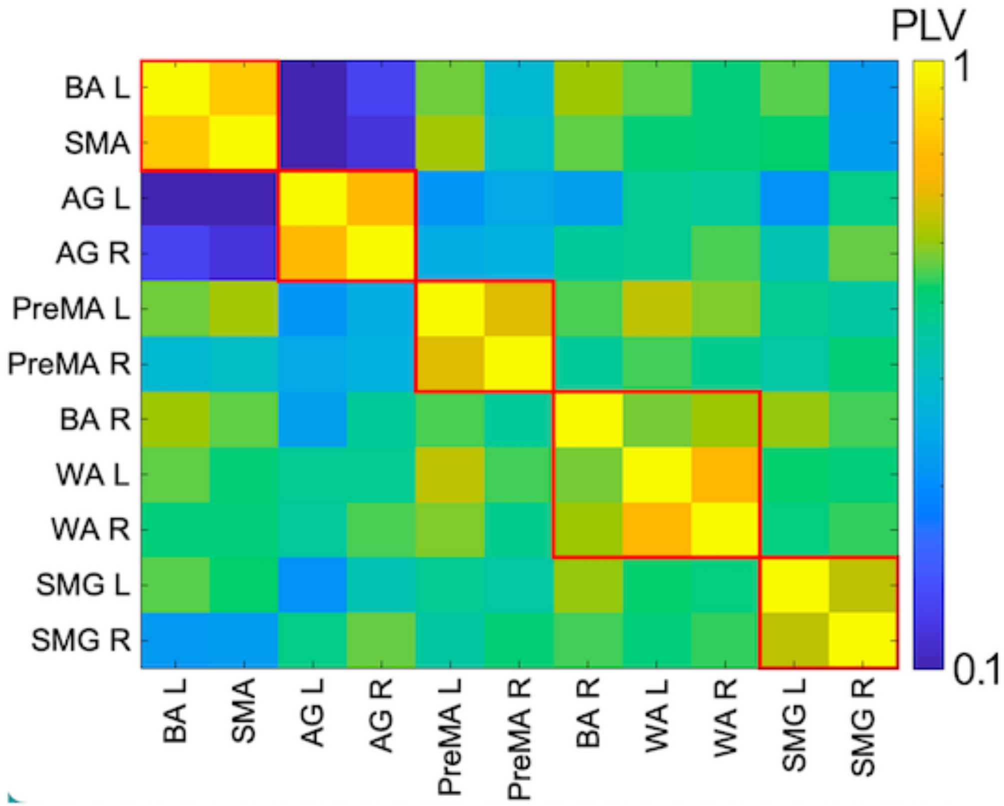
VERB GENERATION TASK



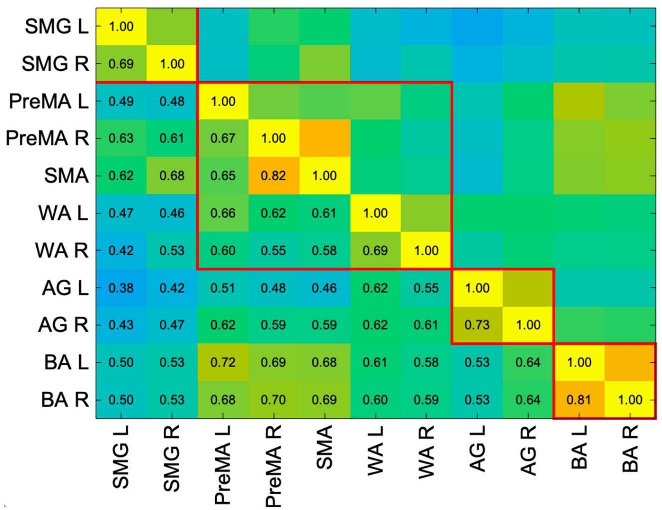


PHONEMIC FLUENCY TASK

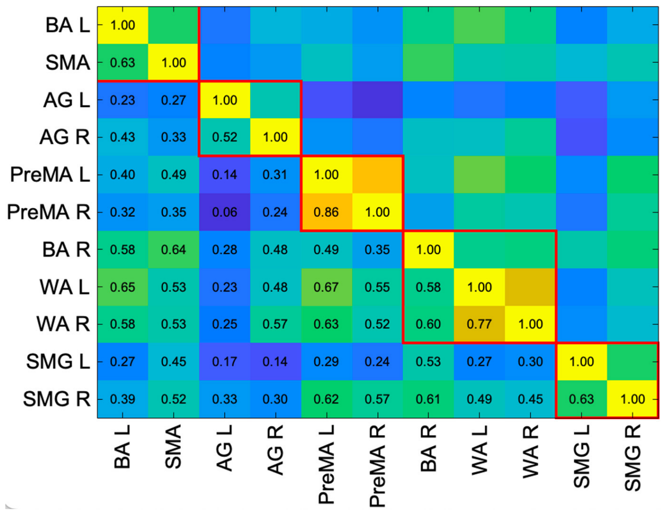




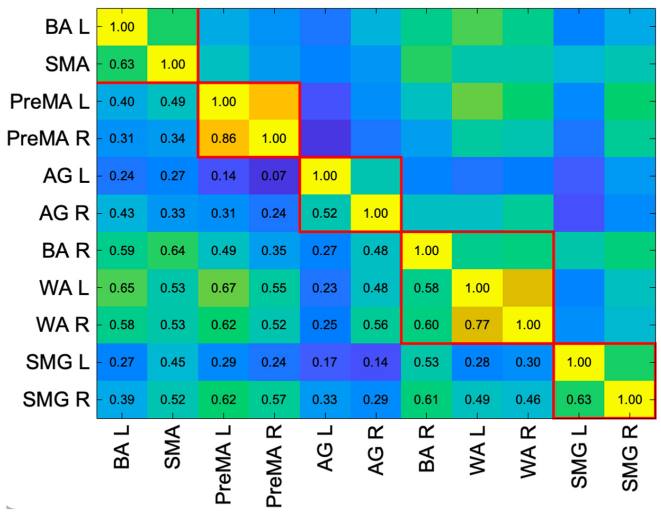
Resting State



Phonemic Fluency Task



Verb Generation Task



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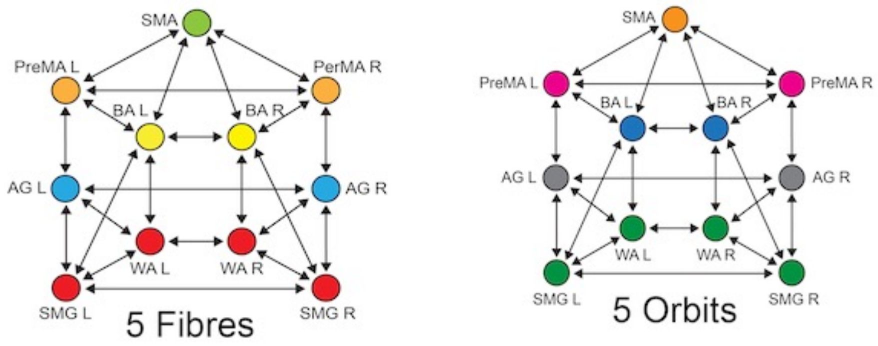
Symmetry analysis of dorsal stream baseline connectome

Fibration symmetry = Group symmetry

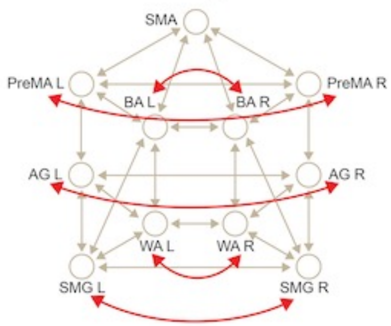


1: SMA
2: PreMA L
3: PreMA R
4: BA L
5: BA R
6: AG L
7: AG R
8: WA L
9: WA R
10: SMG L
11: SMG R

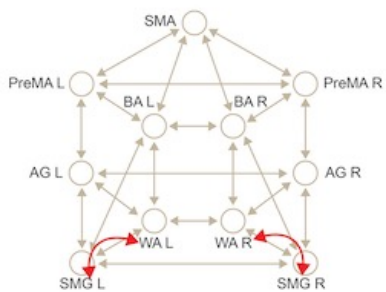
Automorphisms

$$\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 3 & 2 & 5 & 4 & 7 & 6 & 9 & 8 & 11 & 10 \end{pmatrix} \quad \pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 10 & 11 & 8 & 9 \end{pmatrix} \quad \pi_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 3 & 2 & 5 & 4 & 7 & 6 & 11 & 10 & 9 & 8 \end{pmatrix}$$

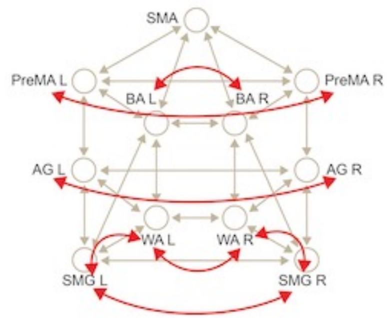
π_1



π_2



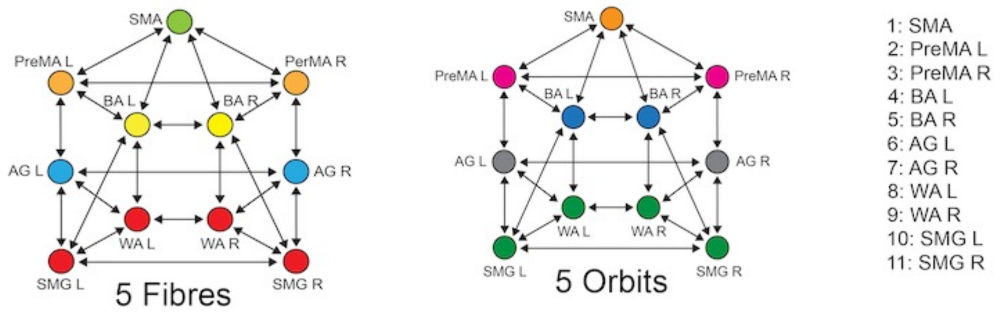
π_3



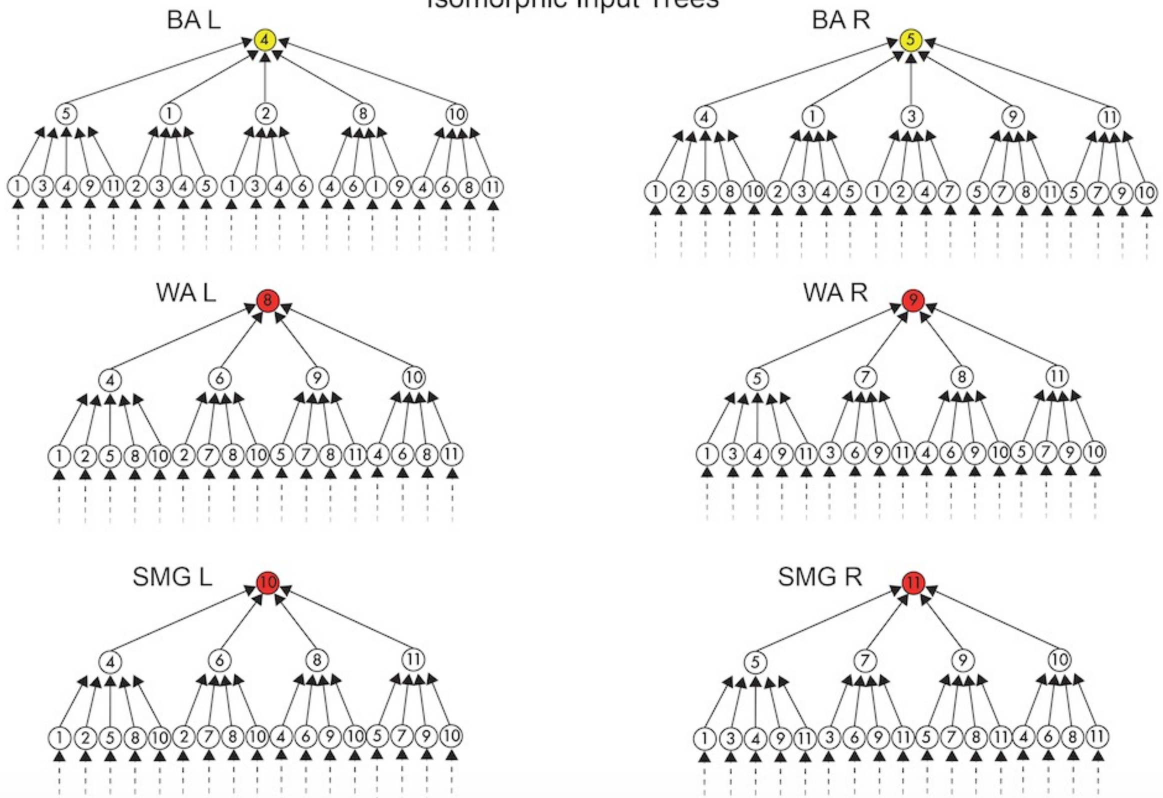
IMT SCHOOL FOR ADVANCED STUDIES LUCCA

Symmetry analysis of dorsal stream baseline connectome

Fibration symmetry = Group symmetry



Isomorphic Input Trees

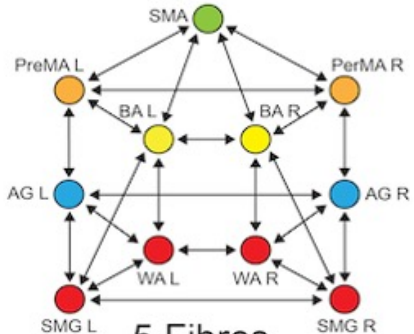


IMT SCHOOL FOR ADVANCED STUDIES LUCCA

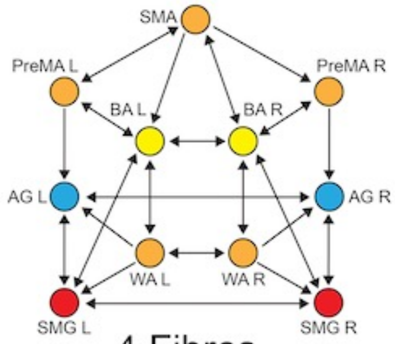
Symmetry analysis of inferred resting state structural network

Fibration symmetry ≠ Group symmetry

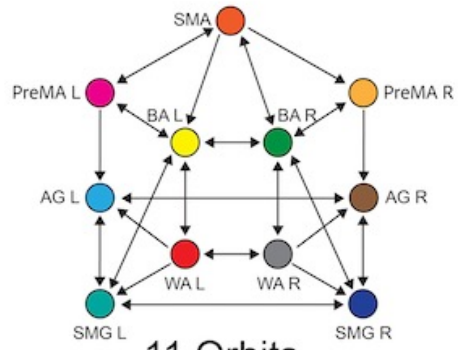
BASELINE CONNECTOME



5 Fibres



4 Fibres



11 Orbits

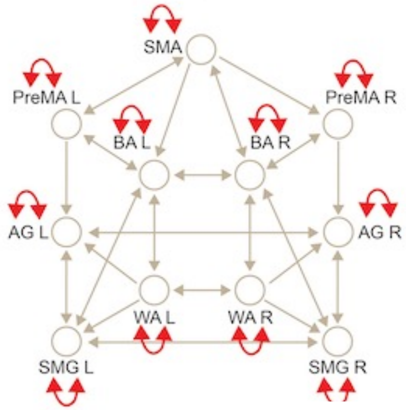
1: SMA
2: PreMA L
3: PreMA R
4: BA L
5: BA R
6: AG L
7: AG R
8: WA L
9: WA R
10: SMG L
11: SMG R

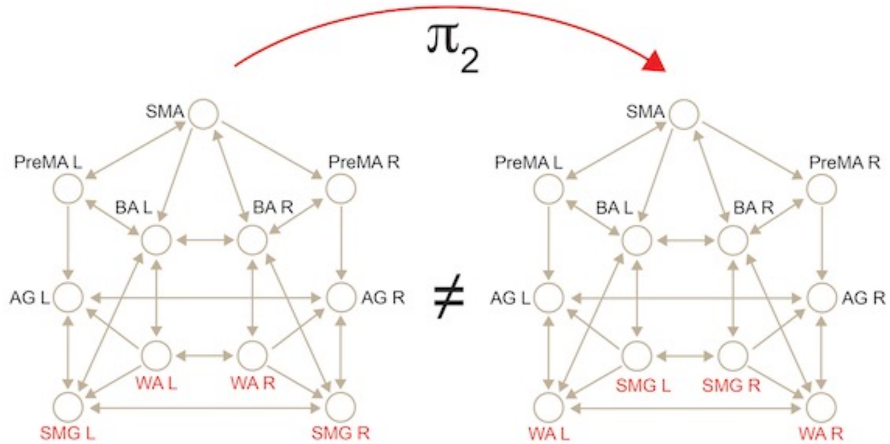
Automorphisms

$$\pi_0 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{pmatrix}$$

$$\pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 10 & 11 & 8 & 9 \end{pmatrix}$$

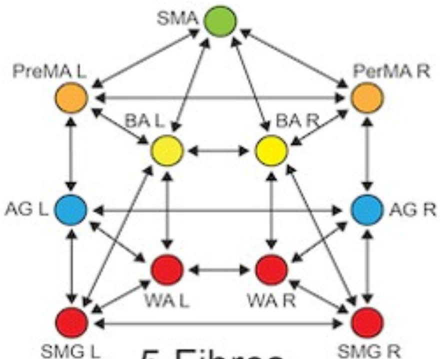
π_0





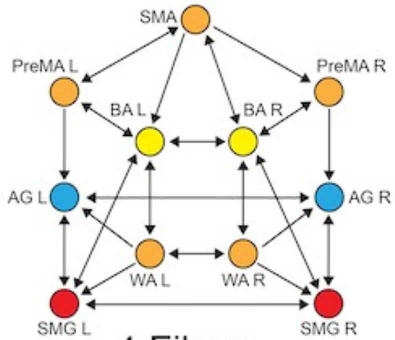
Symmetry analysis of inferred resting state structural network

BASELINE CONNECTOME

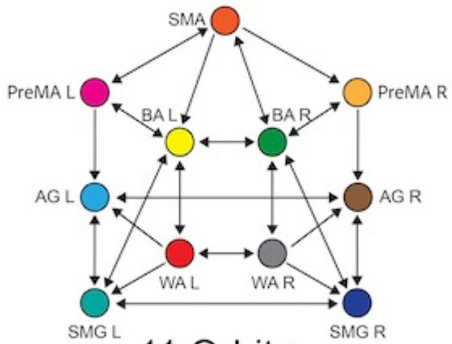


5 Fibres

Fibration symmetry ≠ Group symmetry



4 Fibres



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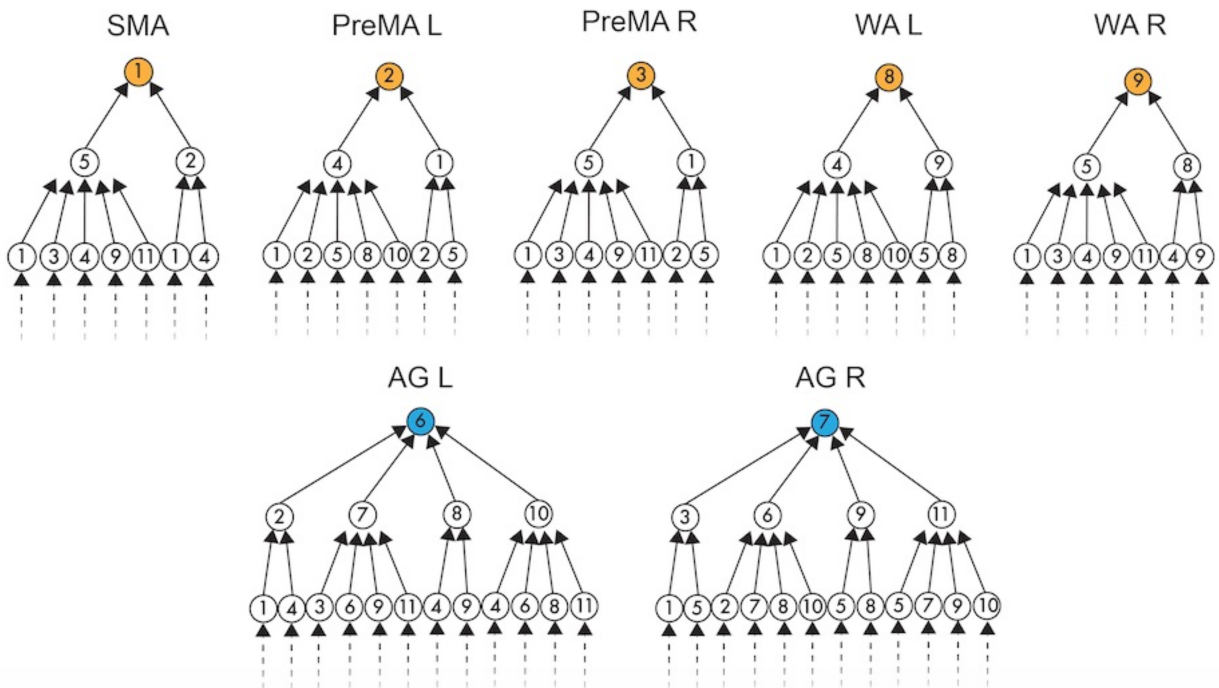
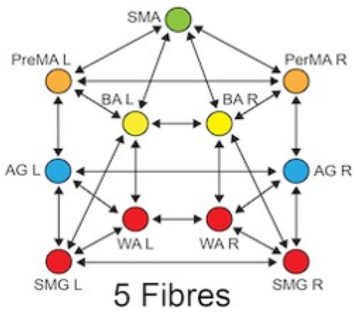
Isomorphic Input Trees



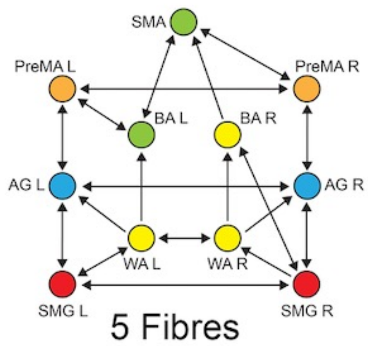
IMT SCHOOL FOR ADVANCED STUDIES LUCCA

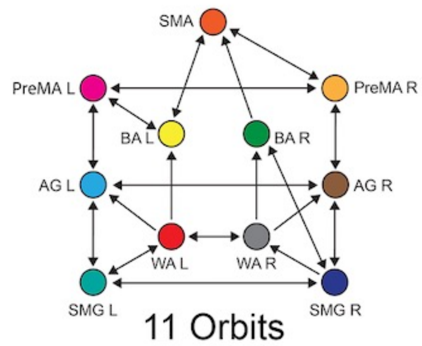
Symmetry analysis of inferred language structural network

BASELINE CONNECTOME



Fibration symmetry ≠ Group symmetry





1: SMA
2: PreMA L
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4: BA L
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6: AG L
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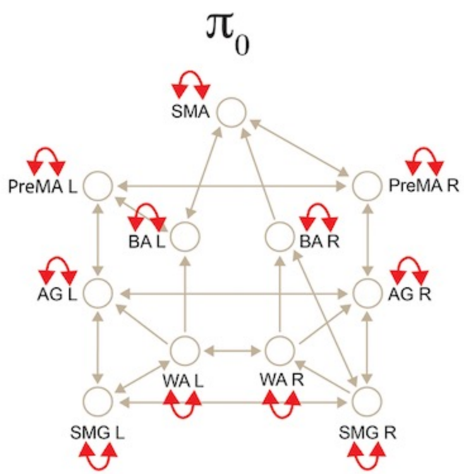
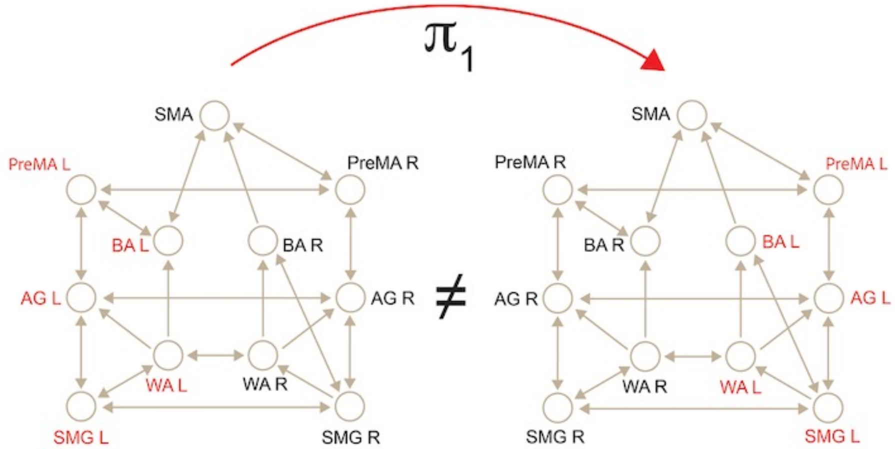
Automorphisms

$$\pi_0 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{pmatrix}$$

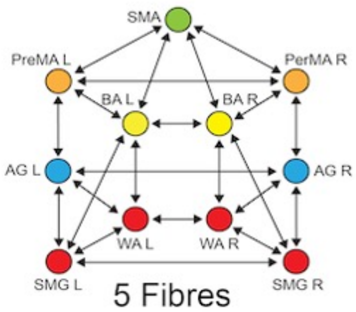
$$\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 3 & 2 & 5 & 4 & 7 & 6 & 9 & 8 & 11 & 10 \end{pmatrix}$$



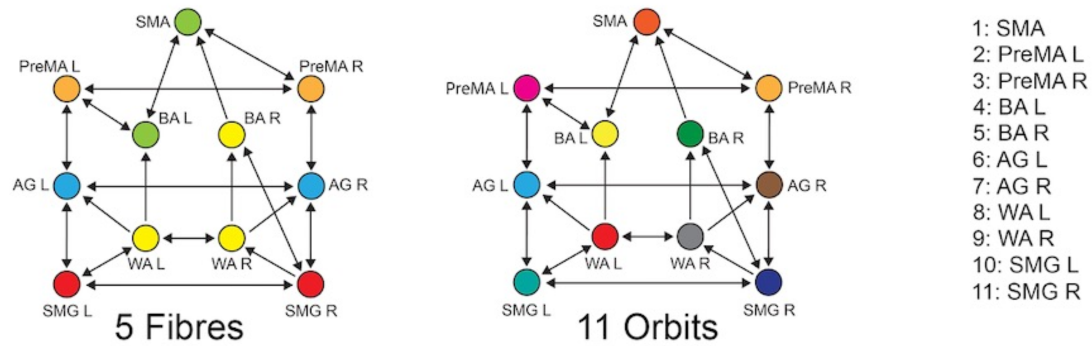


Symmetry analysis of inferred language structural network

BASELINE CONNECTOME



Fibration symmetry ≠ Group symmetry



Isomorphic Input Trees