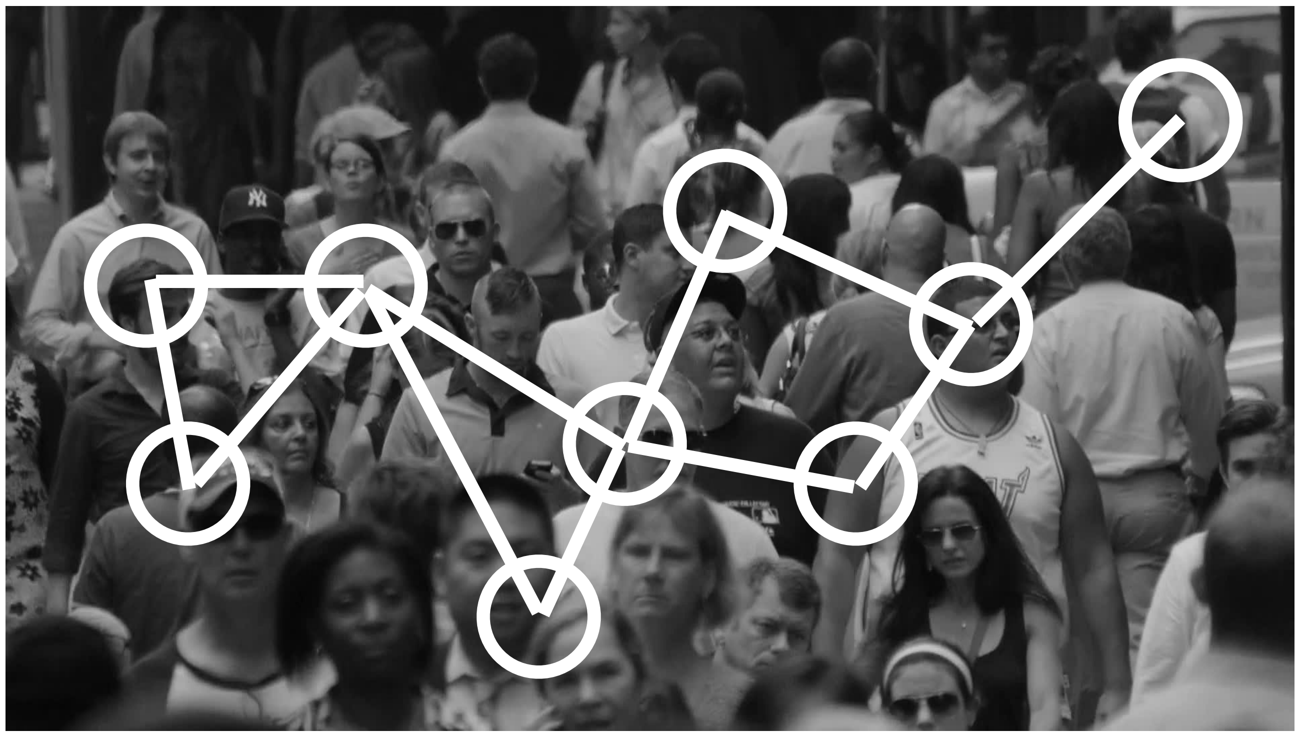
Epidemic graph diagrams as analytics for epidemic control in the data-rich era

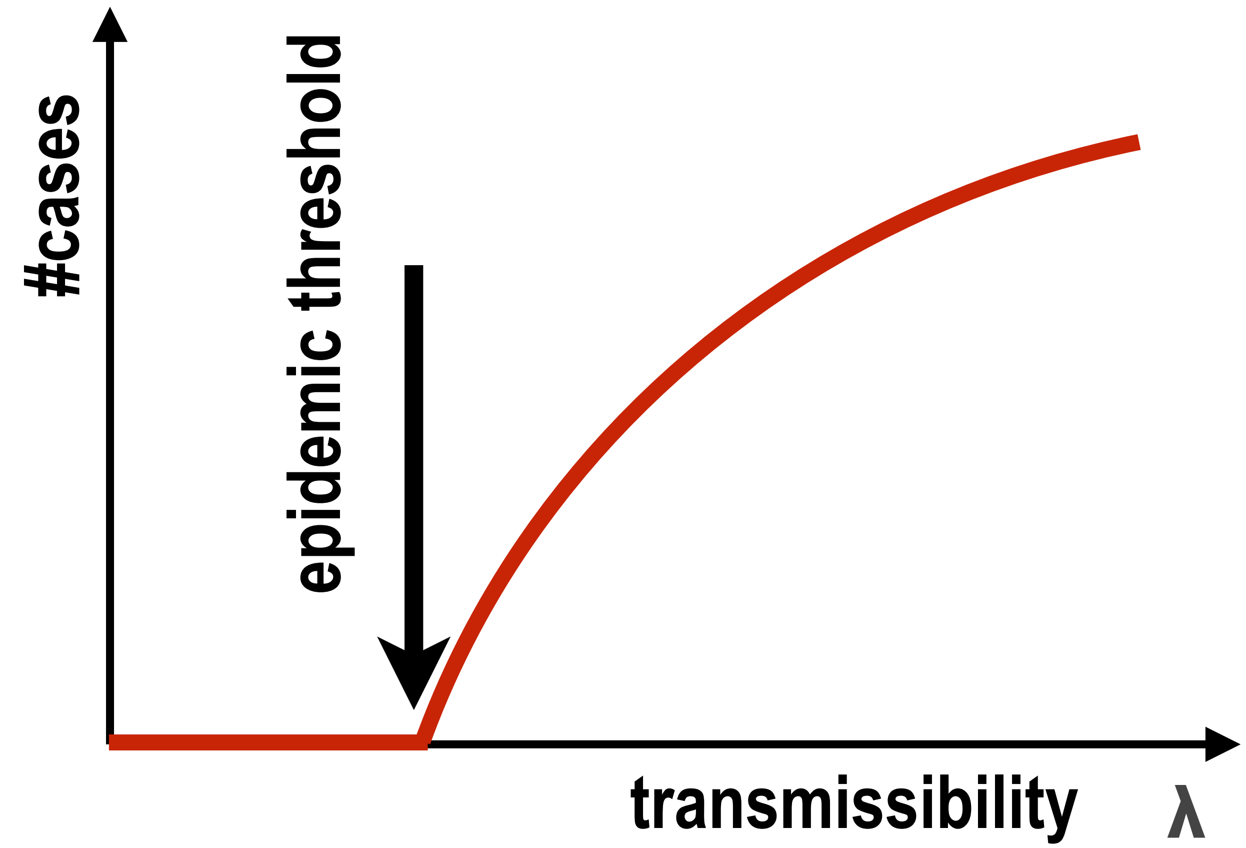
Chiara Poletto
@chpoletto.bsky.social
chiara-poletto.github.io

w/ Eugenio Valdano, Vittoria Colizza, Luca Ferreri, Michele Re Fiorentin, Davide Colombi

Networks days - Oct 24 and 25 2024

epidemic threshold





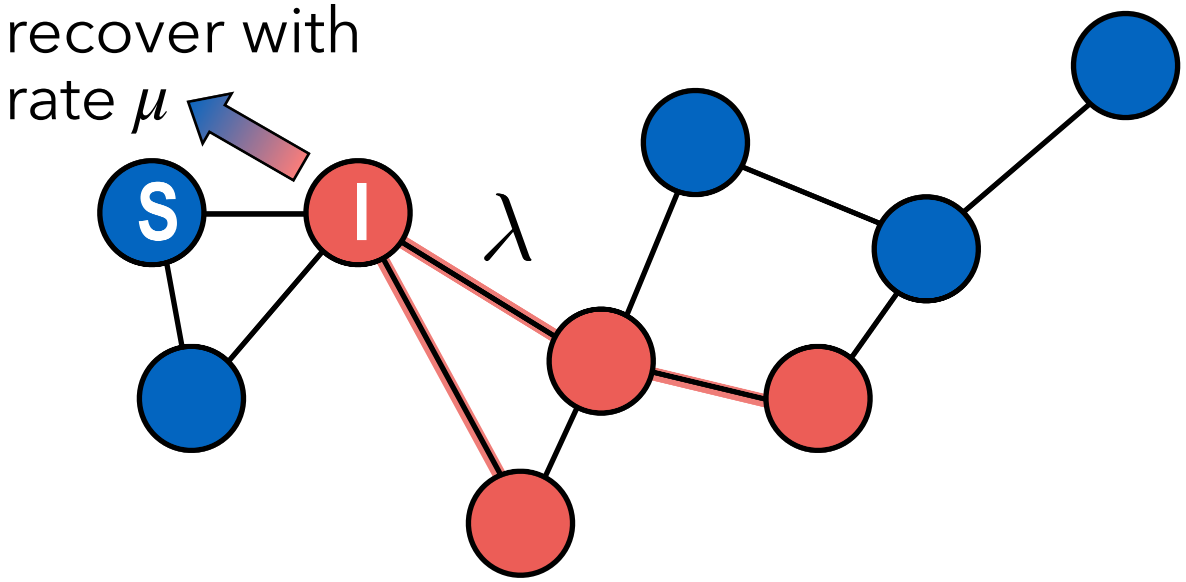
assess the vulnerability to an infection of a given population

susceptible-infected-susceptible

$$S \xrightarrow{\lambda} I \xrightarrow{\mu} S$$

[Pastor-Satorras Rev Mod Phys 2015]

epidemic threshold on networks: quenched mean-field approach



A_{ij}

$p_i(t) = \mathrm{Prob}(i \text{ is infectious})$

mean field approximation

$\mathrm{Prob}(i \text{ is infectious}, j \text{ is infectious}) = p_i p_j$

Markov chain

[Wang et al. SRDS 2003, Gómez et al. EPL 2010]

$$p_i(t+1) = 1 - [1 - (1-\mu)p_i(t)]\, \Pi_j\, [1 - \lambda A_{ji} p_j(t)]$$

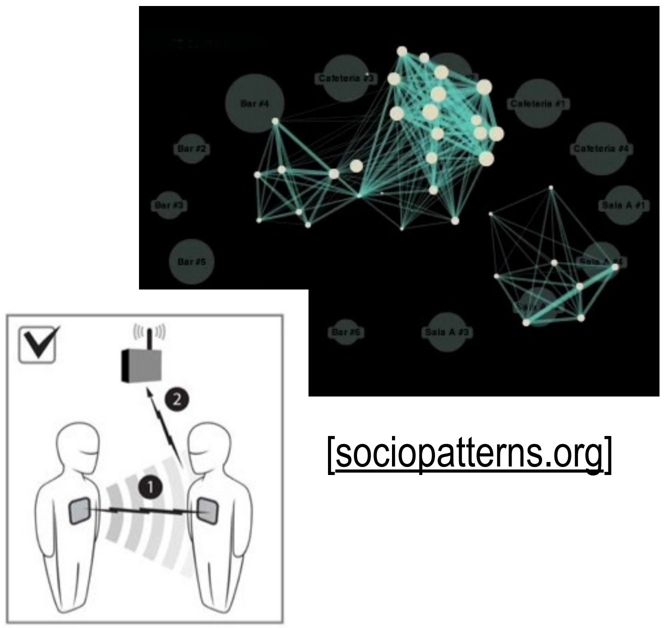
linearize

$$p(t+1) = (1 - \mu + \lambda A^\dagger) p(t) + \mathcal{O}(||p(t)||^2)$$

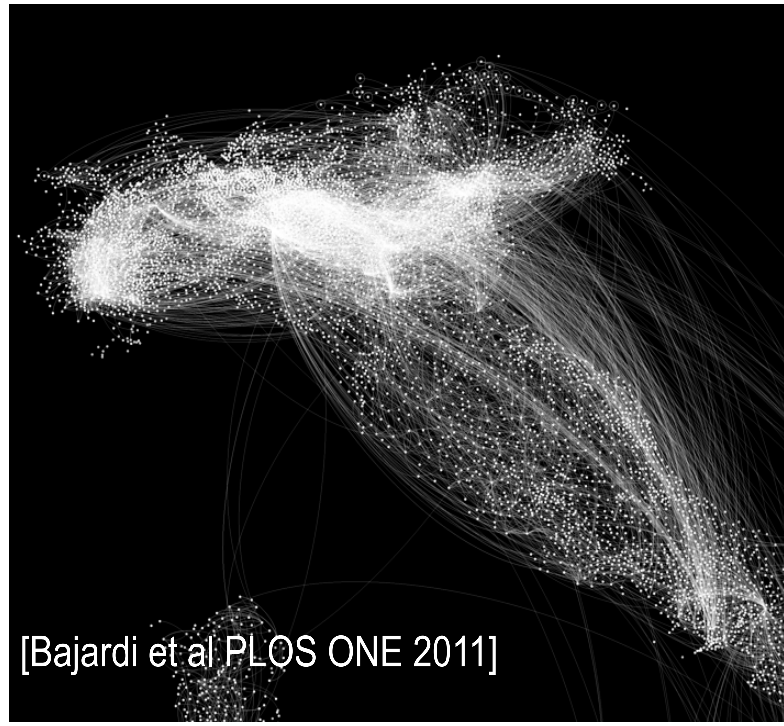
$$\left(\frac{\lambda}{\mu}\right)_{\text{critical}} = \frac{1}{\rho[A]}$$

spectral radius
(largest eigenvalue)

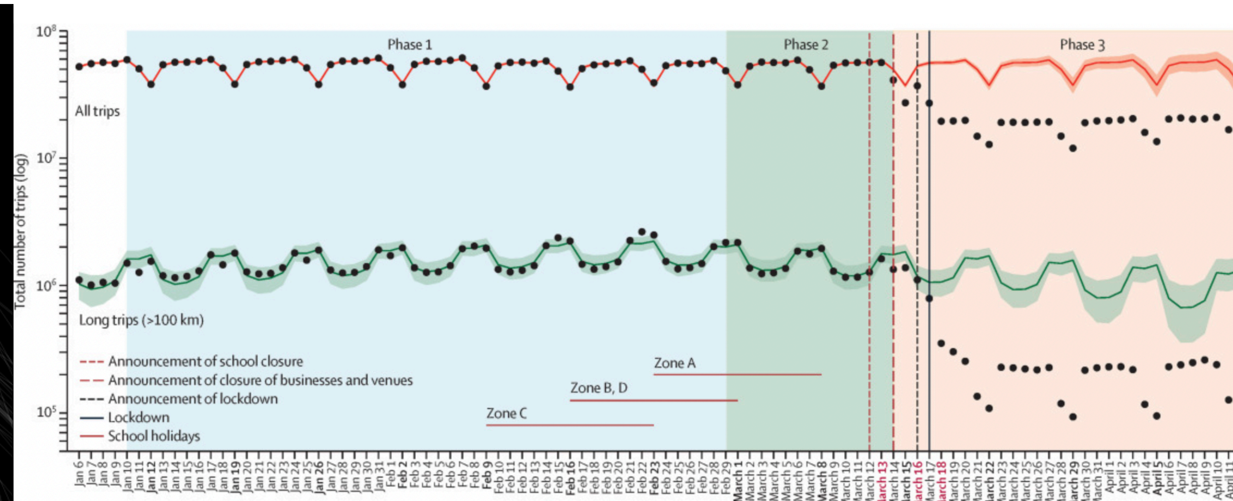
epidemic threshold on temporal networks



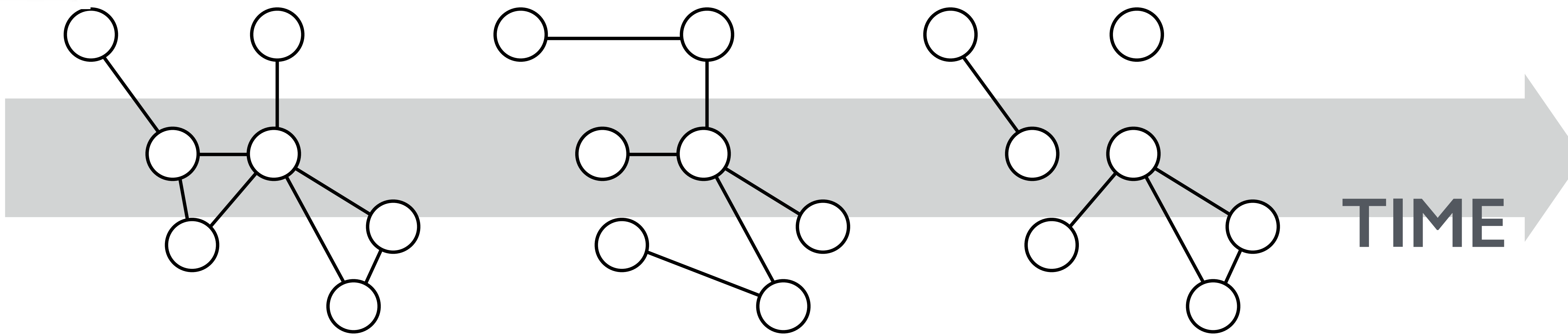
[sociopatterns.org]



[Bajardi et al PLOS ONE 2011]



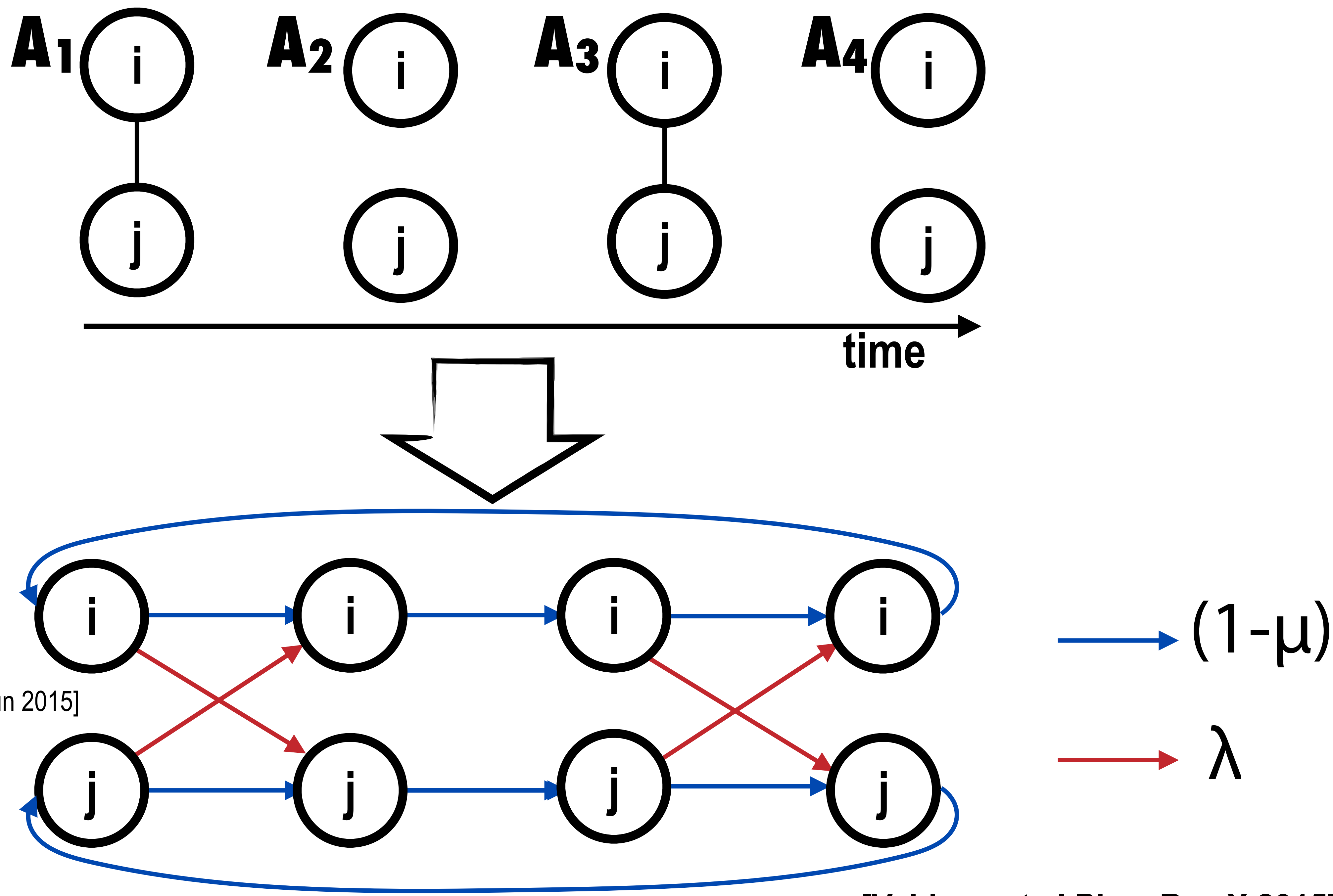
[Pullano et al Lancet Digital Health 2021]



multilayer formalism → quenched mean-field approach

[De Domenico et al PRX 2013]

multilayer representation



[Scholtes et al Nat Commun 2015]

[Valdano et al Phys Rev X 2015]

multilayer representation

threshold on temporal network $\Longleftrightarrow$ threshold on STATIC network

$$M = \begin{pmatrix} 0 & 1-\mu+\lambda A_1 & 0 & \cdots & 0 \\ 0 & 0 & 1-\mu+\lambda A_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-\mu+\lambda A_{T-1} \\ 1-\mu+\lambda A_T & 0 & 0 & \cdots & 0 \end{pmatrix}$$

epi threshold

$\rho[\mathbf{M}(\lambda_{thr},\mu)] = 1$ NT $\longrightarrow$ N $\rho[\mathbf{P}(\lambda_{thr},\mu)] = 1$

[Powell, arXiv, 2011]

infection propagator

$$\mathbf{P} = (1-\mu + \lambda \mathbf{A}_1)\,(1-\mu + \lambda \mathbf{A}_2) \cdots (1-\mu + \lambda \mathbf{A}_T)$$

[Lentz et al, PRL 2013]

[Valdano et al Phys Rev X 2015]

continuous-time limit

infectious propagator $P(s) = \prod_{s'=1}^{s} \left[1 - \mu\Delta t + \lambda\Delta t A_{s'}\right]$

continuous-time limit $\dot{P}(t) = P(t)\left[-\mu + \lambda A(t)\right]$

solution $P(t) = 1 + \sum_{n=1} \mu^n P^{(n)}(t)$, with, for $t = T$,

$$P^{(n)}(T) = \int_0^T dx_1 \int_0^{x_1} dx_2 \dots \int_0^{x_{n-1}} dx_n \left[\frac{\lambda}{\mu} A(x_n) - 1\right] \dots \left[\frac{\lambda}{\mu} A(x_1) - 1\right]$$

Dyson's time-ordering operator: $\mathcal{T} A(t_1) A(t2) = A(t_1)A(t_2)\theta(t_1 - t_2) + A(t_2)A(t_1)\theta(t_2 - t_1)$,

θ is the Heaviside's step function

$$P(t) = \mathcal{T} \exp\left(\int_o^t dx \left[-\mu + \lambda A(x)\right] \right)$$

[Valdano et al Phys Rev Lett 2018]

continuous-time limit

infectious propagator $P(s) = \prod_{s'=1}^{s} \left[1 - \mu\Delta t + \lambda\Delta t A_{s'}\right]$

continuous-time limit $\dot{P}(t) = P(t)\left[-\mu + \lambda A(t)\right]$

solution $P(t) = 1 + \sum_{n=1} \mu^n P^{(n)}(t)$, with, for $t = T$,

$$P^{(n)}(T) = \int_0^T dx_1 \int_0^{x_1} dx_2 \ldots \int_0^{x_{n-1}} dx_n \left[\frac{\lambda}{\mu}A(x_n) - 1\right] \ldots \left[\frac{\lambda}{\mu}A(x_1) - 1\right]$$

Dyson's time-ordering operator: $\mathcal{T} A(t_1)A(t2) = A(t_1)A(t_2)\theta(t_1 - t_2) + A(t_2)A(t_1)\theta(t_2 - t_1)$,

θ is the Heaviside's step function

$$P(t) = \mathcal{T} \exp\left(\int_o^t dx\left[-\mu + \lambda A(x)\right]\right)$$

weak-commutation condition

$$\left[A(t), \int_0^t dx A(x)\right] = 0, \quad \forall\ t \in [0, T] \longrightarrow P(T) = e^{T[-\mu+\lambda\langle A\rangle]}$$

[Valdano et al Phys Rev Lett 2018]

weak-commutation condition

- absence of temporal correlations
- static networks
- annealed networks
- activity driven model
- time-scale separation

weak-commutation condition

$$\left[A(t), \int_0^t dx A(x)\right] = 0, \quad \forall\ t \in [0, T] \longrightarrow P(T) = e^{T[-\mu+\lambda\langle A\rangle]}$$

[Valdano et al Phys Rev Lett 2018]

beyond infectious propagator

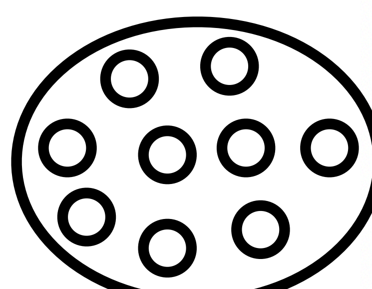
infectious propagator ...

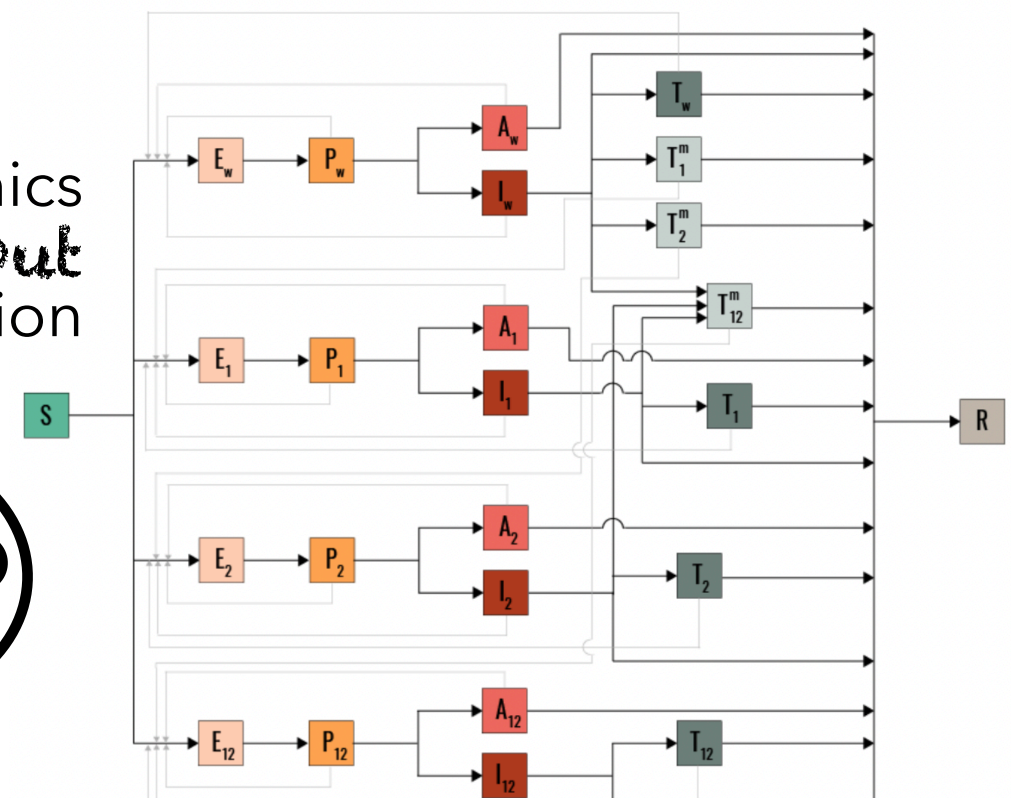
whole complexity of host-to-host interaction
but
oversimplified infection dynamics

$$S \xrightarrow{\lambda} I \xrightarrow{\mu} S$$

... compared with classical mathematical epidemiology

whole complexity of disease dynamics
but
oversimplified host-to-host interaction





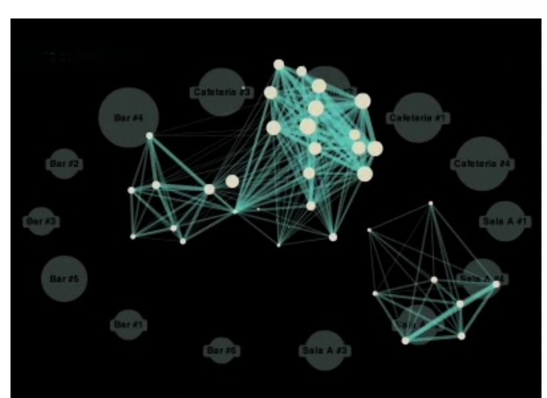
[representation of Wu et al PLOS Med 2009]

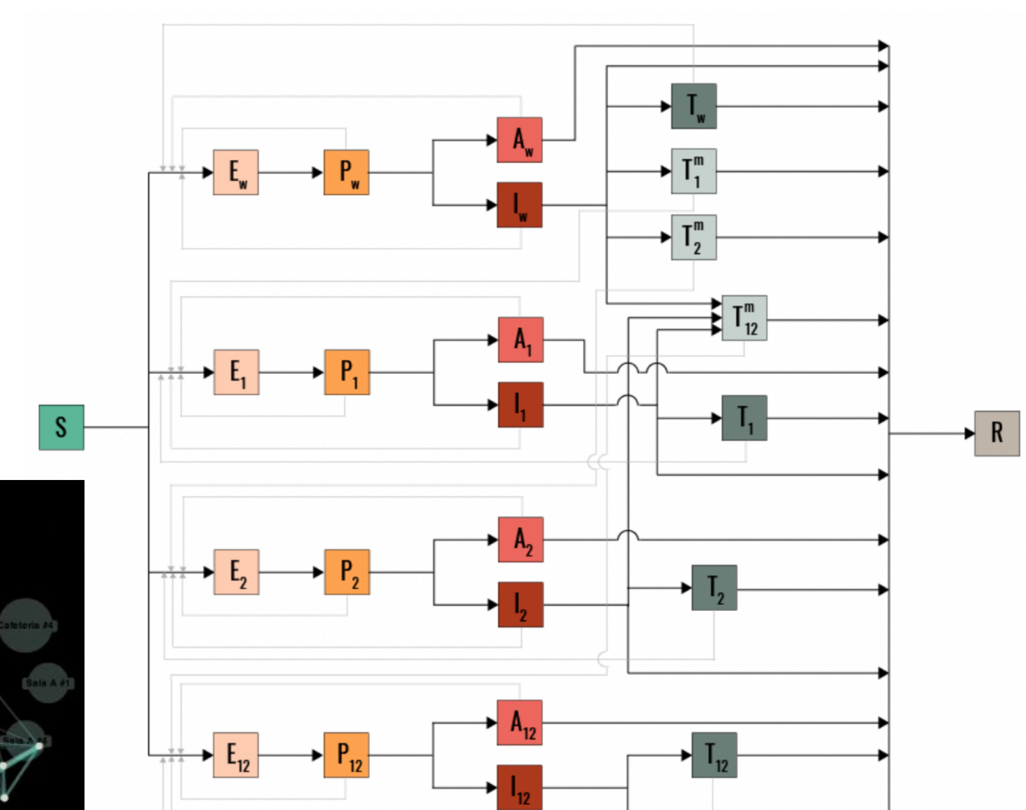
have your cake and eat it too

whole complexity of host-to-host interaction & disease dynamics

identify general rules, building blocks of compartmental models

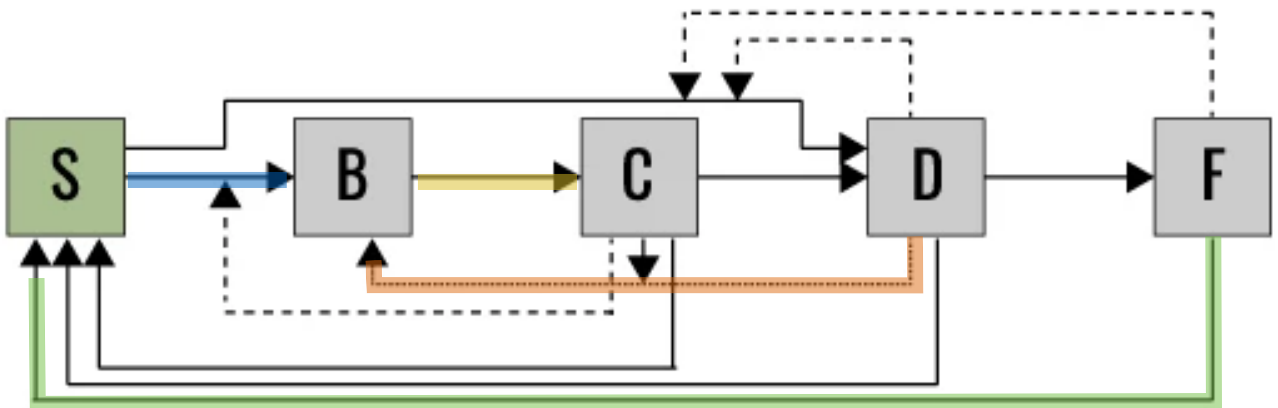
derive the epidemic threshold directly from these rules, obtaining a result valid for any compartmental model





[representation of Wu et al PLOS Med 2009]

from the compartmental model to the epidemic graph diagram

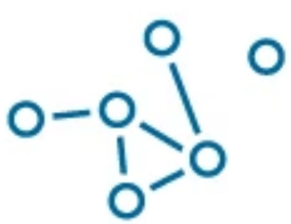


$B \to C$: spontaneous transition with rate γ_{BC}

$F \to S$: spontaneous transition to S with rate μ_{FS}

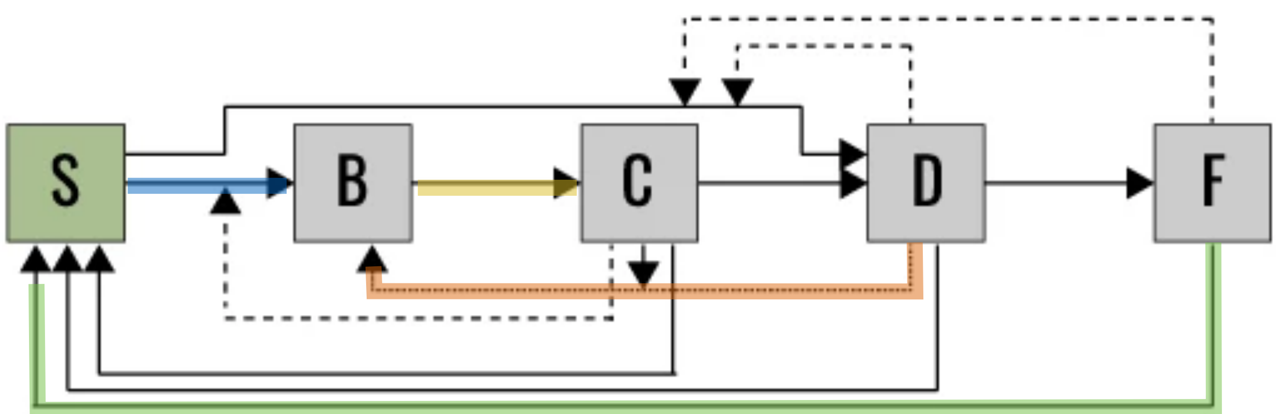
$S + C \to B + C$: transmission from S with rate λ_{CB}

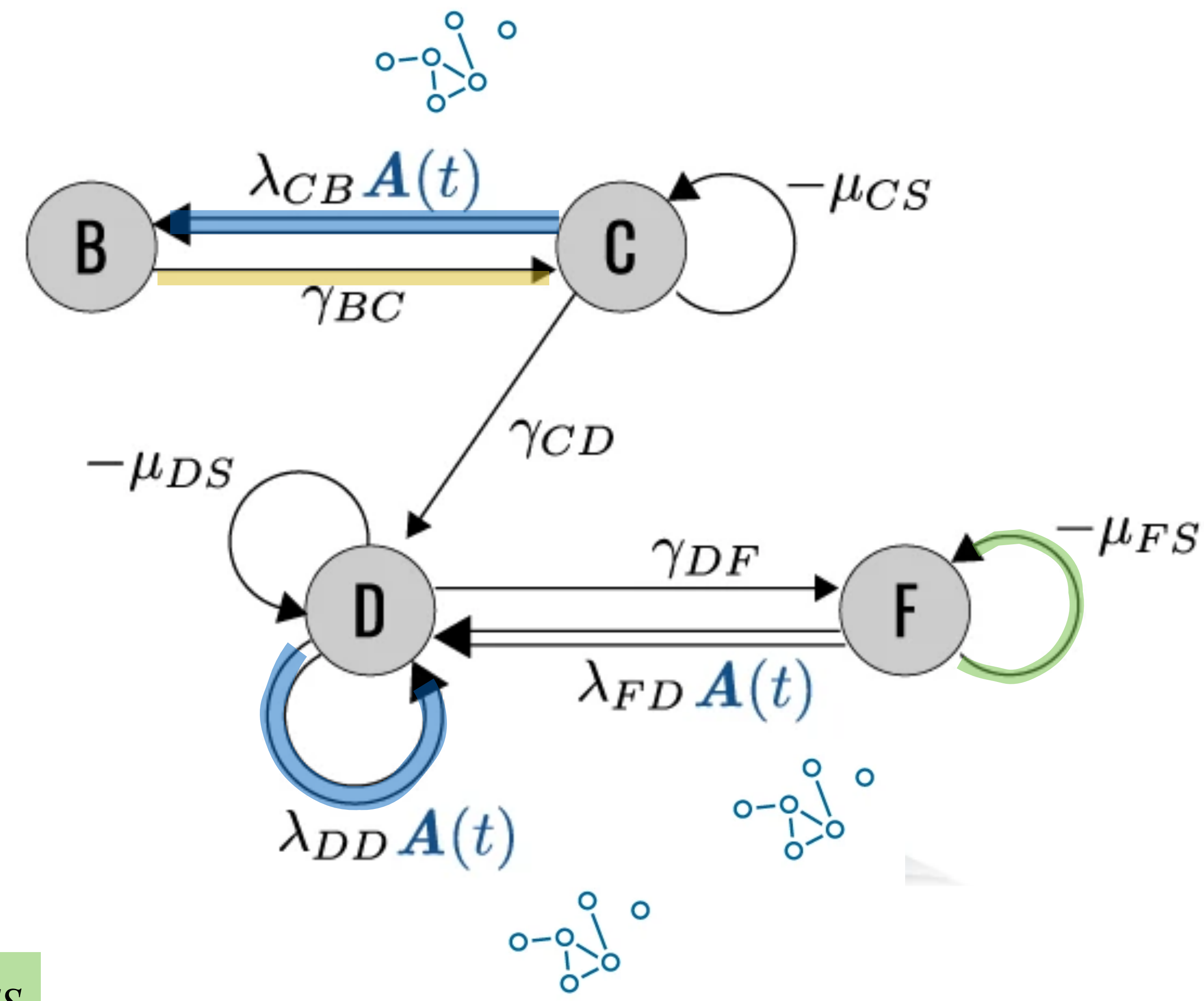
$D + C \to B + C$: transmission from $D \neq S$ with rate ω_{DCB}



[Valdano et al Nat Commun 2023]

from the compartmental model to the epidemic graph diagram





$B \to C$: spontaneous transition with rate γ_{BC}

$F \to S$: spontaneous transition to S with rate μ_{FS}

$S + C \to B + C$: transmission from S with rate λ_{CB}

~~$D + C \to B + C$: transmission from $D \neq S$ with rate ω_{DCB}~~

omitted

[Valdano et al Nat Commun 2023]

from the compartmental model to the epidemic graph diagram

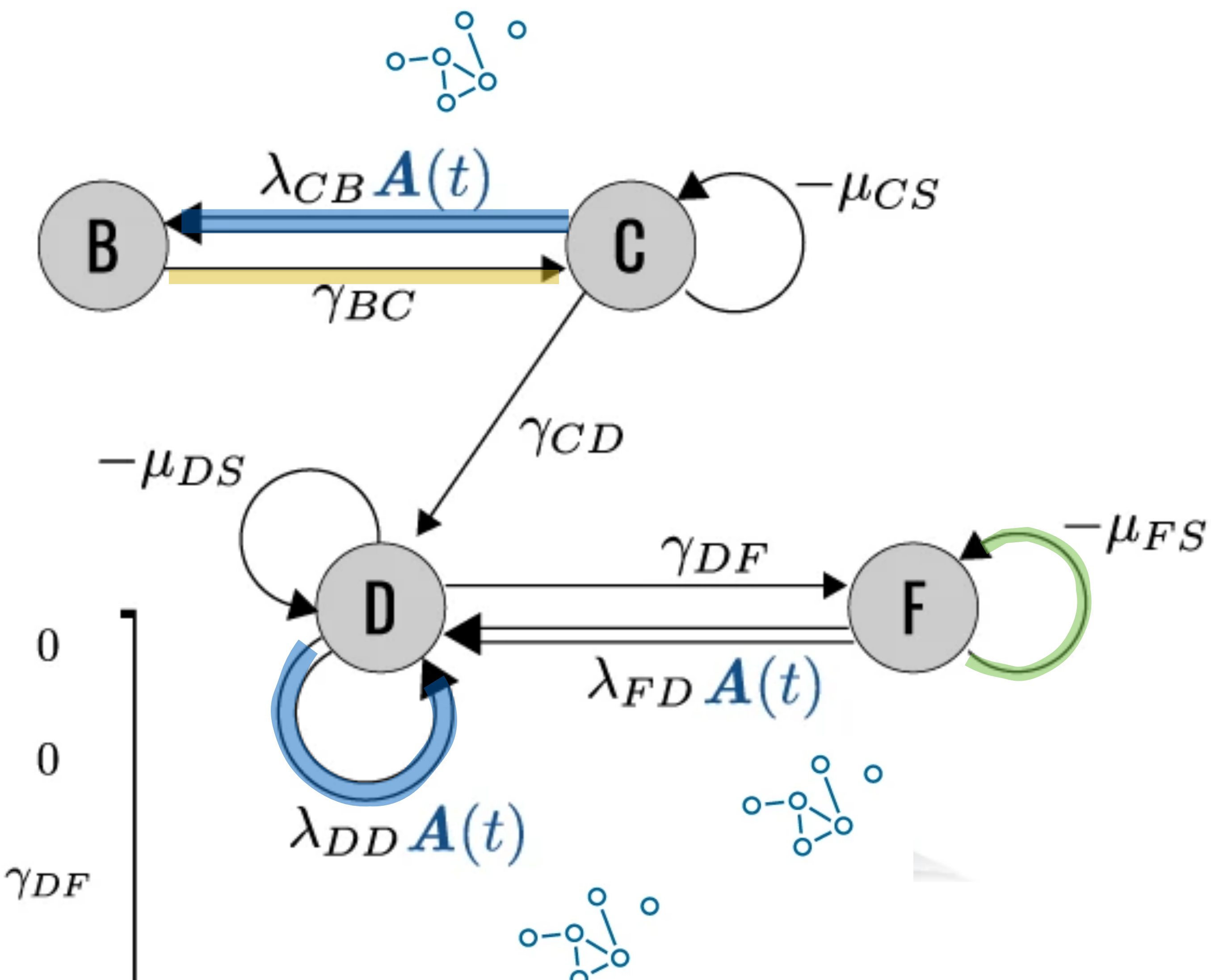
$$\dot{p}(t) = \left[\boldsymbol{\gamma} - \boldsymbol{\mu} + \boldsymbol{\lambda} \boldsymbol{A}(\boldsymbol{t})\right] p(t) = \mathbf{J}(t)p(t)$$

$$\boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{\lambda} \ \in R^{N_c}$$

$$A(t) \in \mathbb{R}^N$$

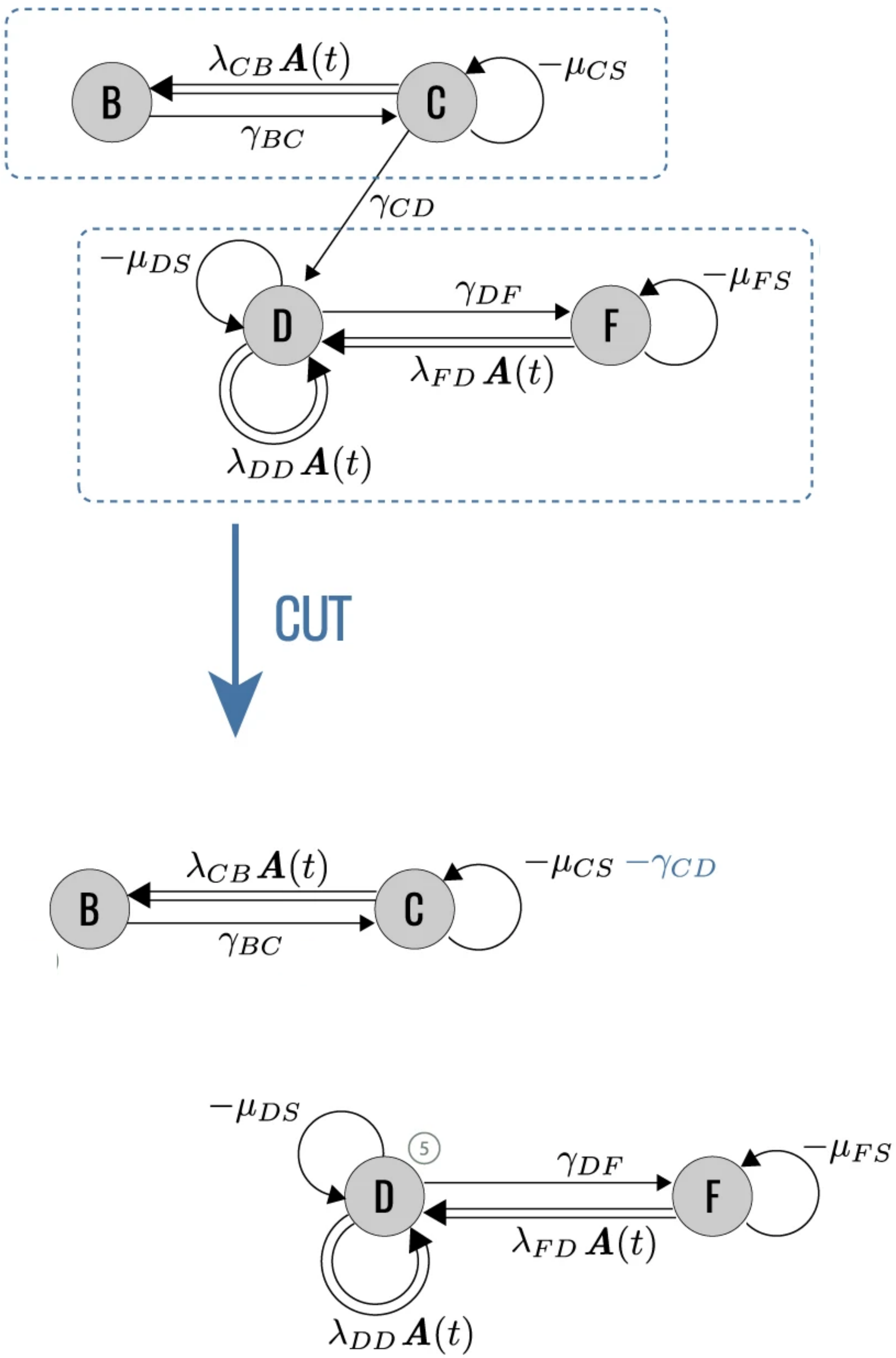
$$p(t), \mathbf{J}(t) \in \mathbb{R}^N \otimes R^{N_c}$$

$$\boldsymbol{J}(t) = \begin{bmatrix} -\gamma_{BC} & \gamma_{BC} & 0 & 0 \\ \lambda_{CB}\boldsymbol{A}(t) & -\mu_{CS} - \gamma_{CD} & \gamma_{CD} & 0 \\ 0 & 0 & \begin{matrix} -\mu_{DS} - \gamma_{DF} + \\ +\lambda_{DD}\boldsymbol{A}(t) \end{matrix} & \gamma_{DF} \\ 0 & 0 & \lambda_{FD}\boldsymbol{A}(t) & -\mu_{FS} \end{bmatrix}$$



[Valdano et al Nat Commun 2023]

grammar operations



decompose the diagram in its strongly connected components

the epidemic threshold is the smallest of the components

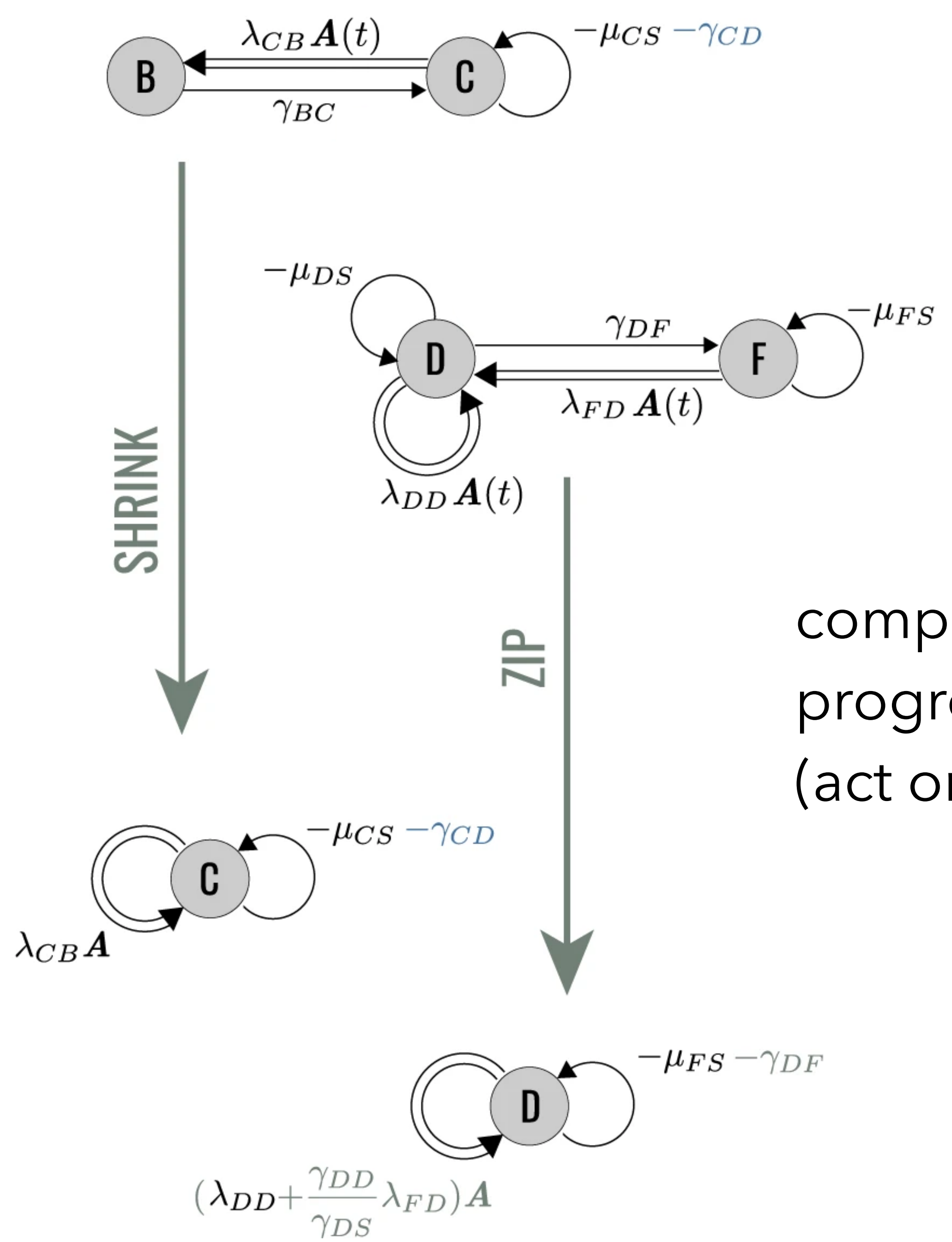
[Valdano et al Nat Commun 2023]

grammar operations

Under the weak commutation condition

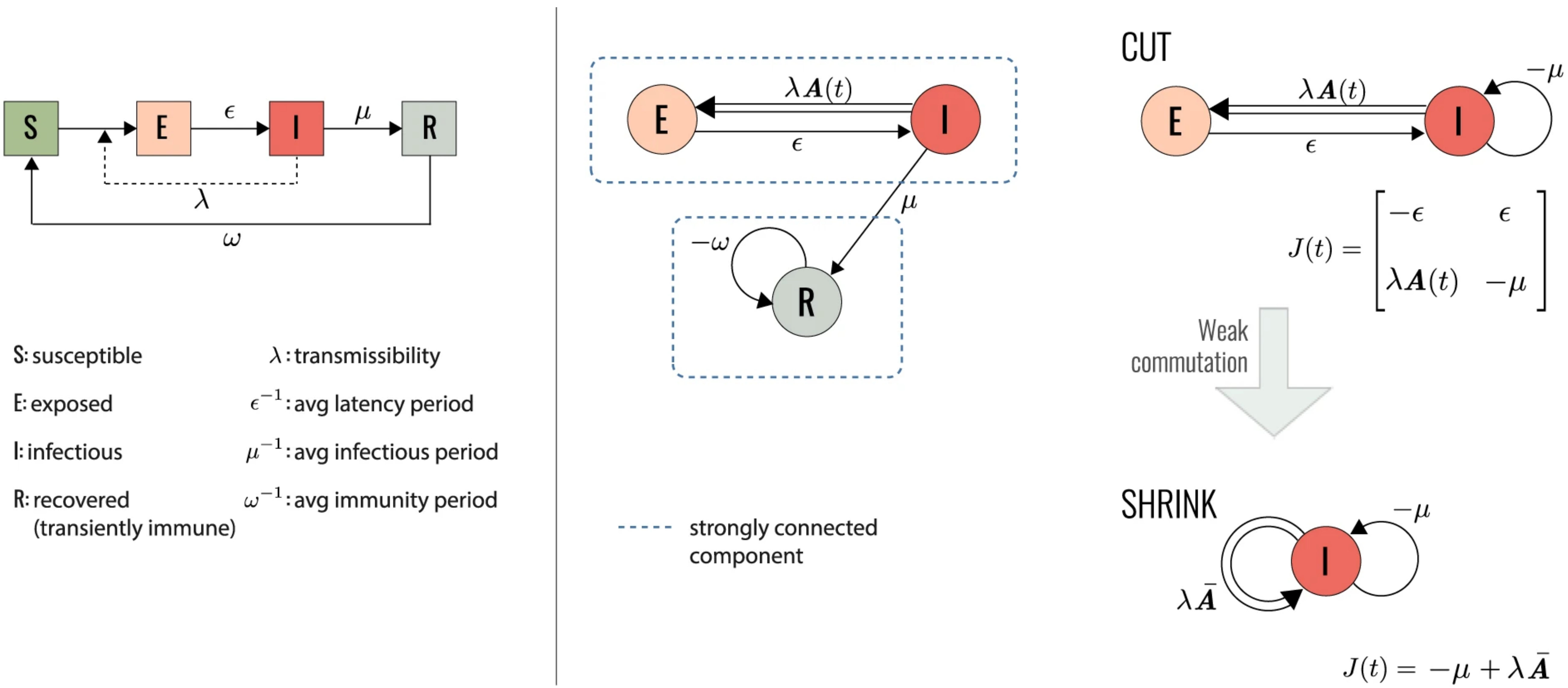
Simplify the diagram

merge pair of nodes
(act locally)



compress complex disease progressions
(act on the global topology)

SEIRS



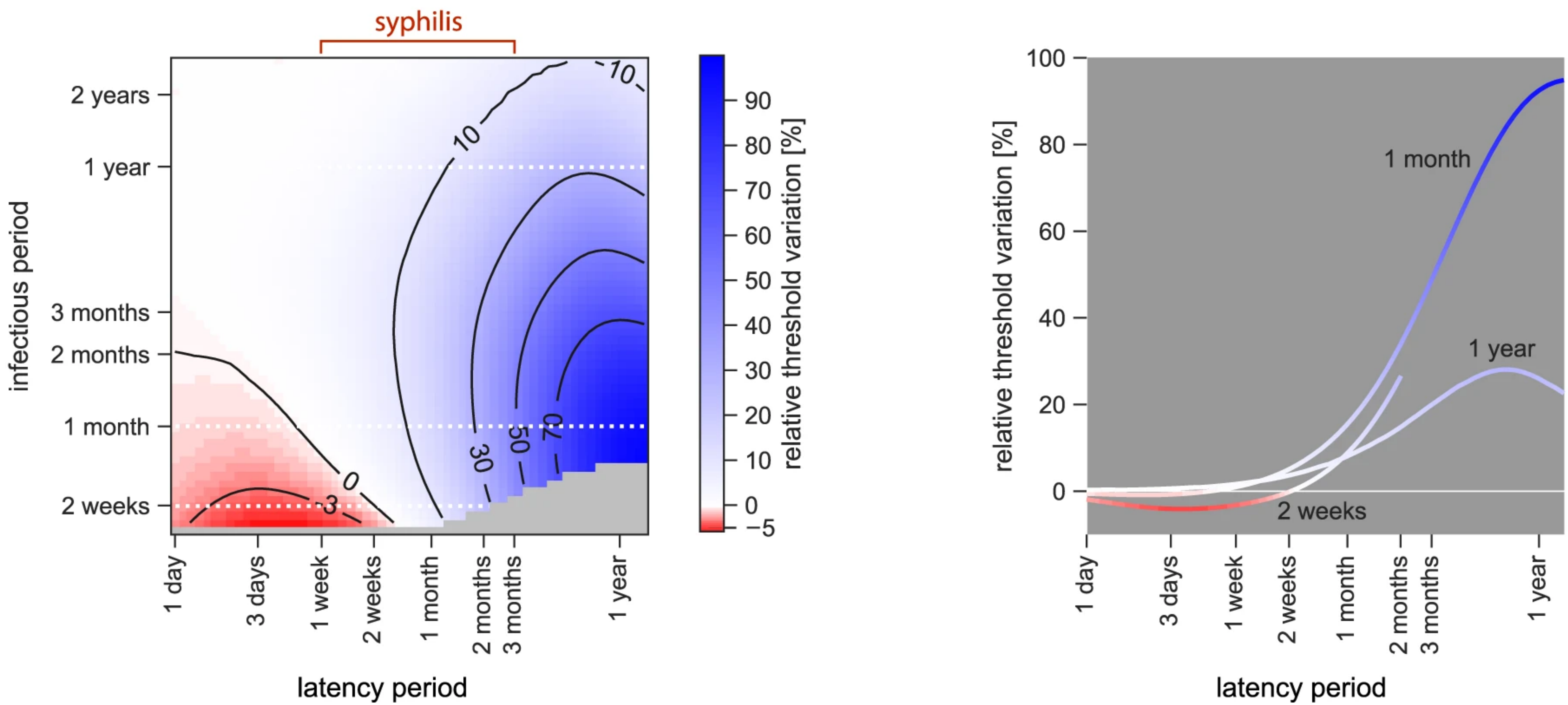
under the weak commutation condition, SEIRS simplifies to SIS …

[Valdano et al Nat Commun 2023]

SEIRS

under the weak commutation condition, SEISR simplifies to SIS …

… in a generic temporal network latency matters



[Valdano et al Nat Commun 2023]

ack:

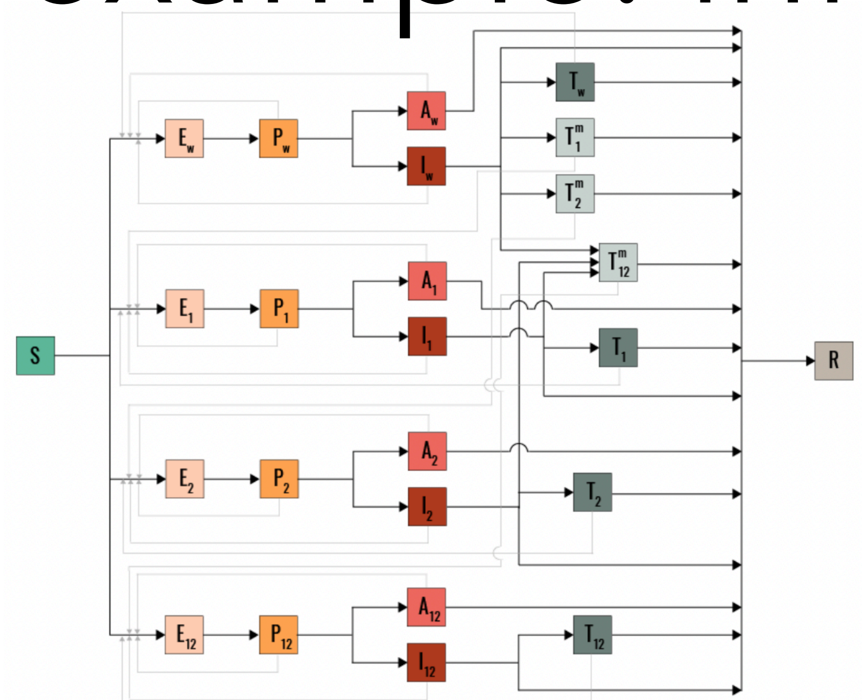
Eugenio Valdano
Vittoria Colizza
Luca Ferreri
Michele Re Fiorentin
Davide Colombi

ref:

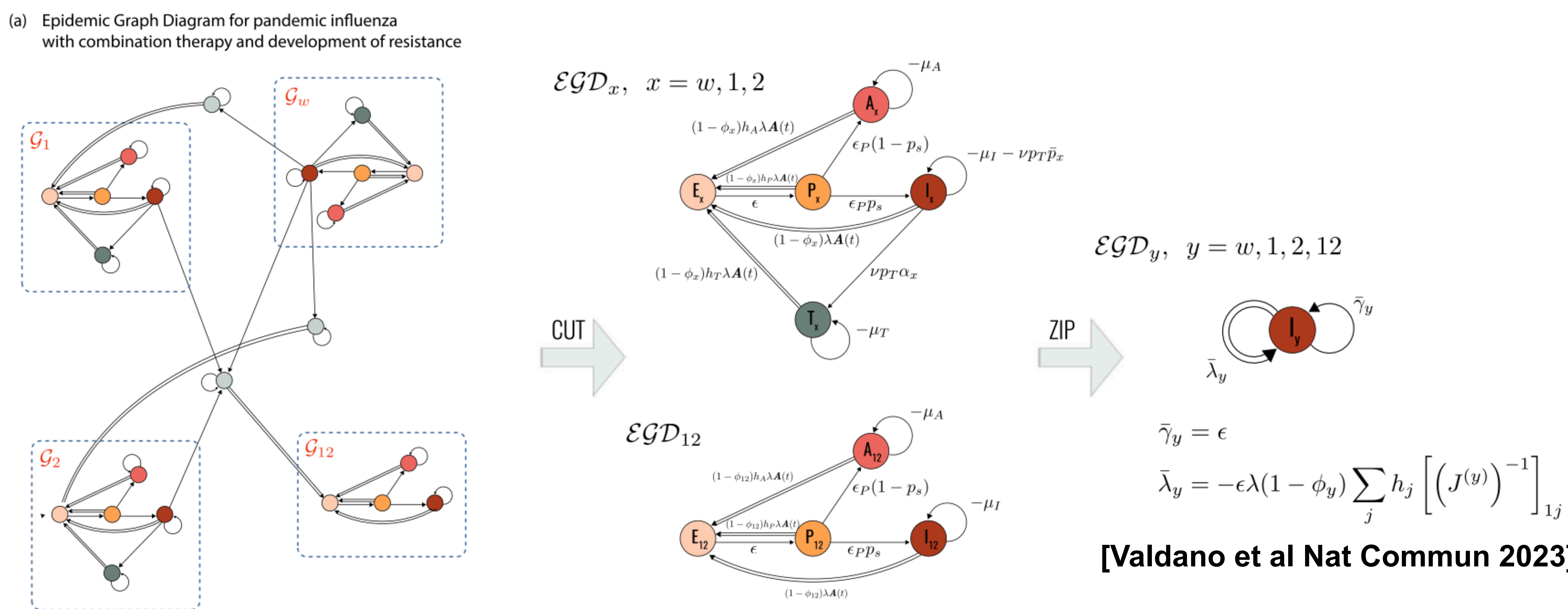
E Valdano, L Ferreri, C Poletto, V Colizza Phys Rev X (2015)
E Valdano, M Re Fiorentin, C Poletto, V Colizza Phys Rev Lett (2018)
E Valdano, D Colombi, C Poletto, V Colizza Nat Commun (2023)

@chpoletto.bsky.social
chiara-poletto.github.io

example: influenza



[representation of Wu et al PLOS Med 2009]



[Valdano et al Nat Commun 2023]