Emergence of complex topologies from flow-weighted optimization of network efficiency

Sebastiano Bontorin, Giulia Cencetti, Riccardo Gallotti, Bruno Lepri and Manlio De Domenico

Networks Days

Bridging micro with macro

Padua, 24-25 October 2024

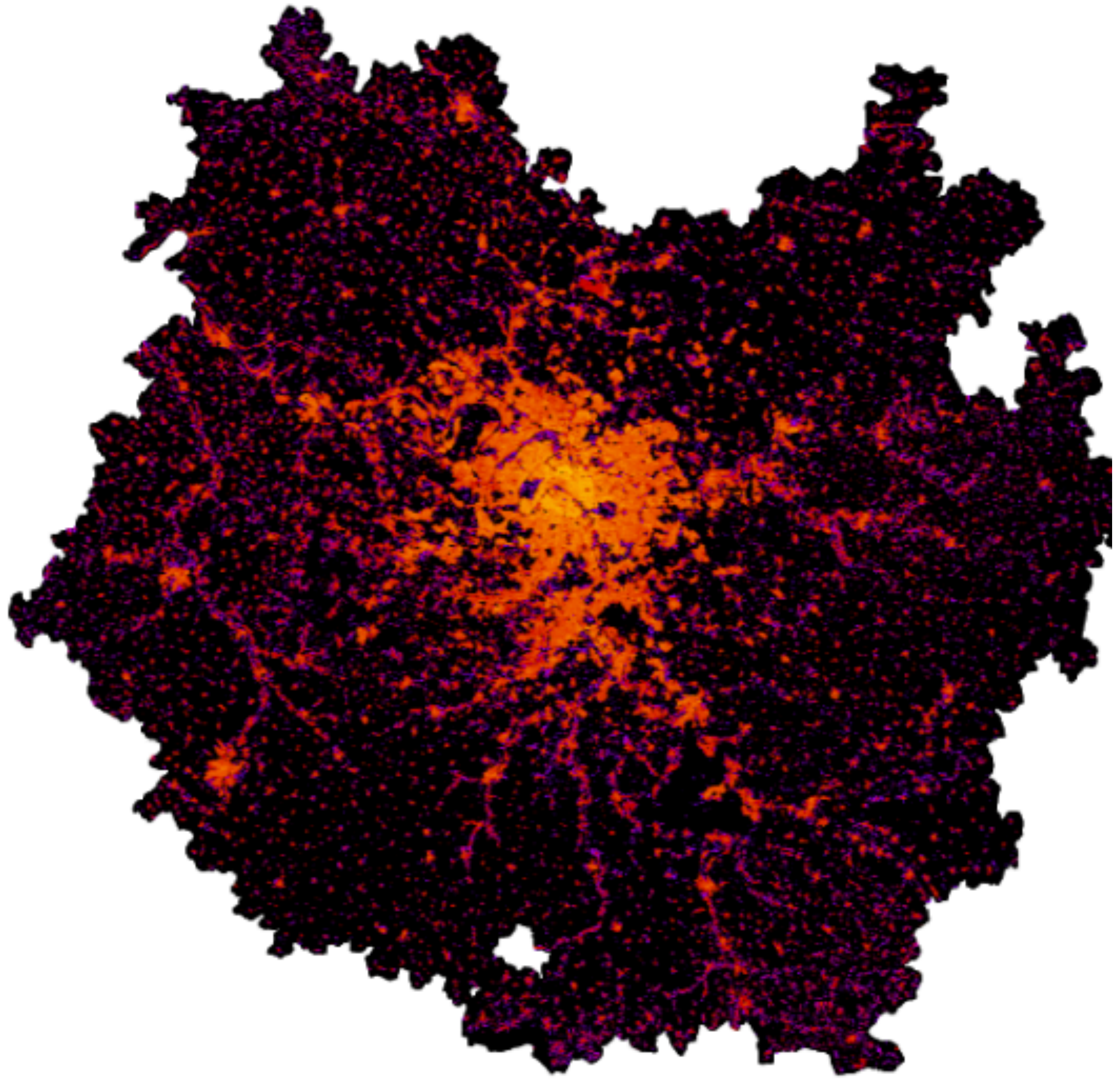
CoMuNe lab

COMPLEX MULTILAYER NETWORKS

CHuB

Cities as complex systems

Population Density

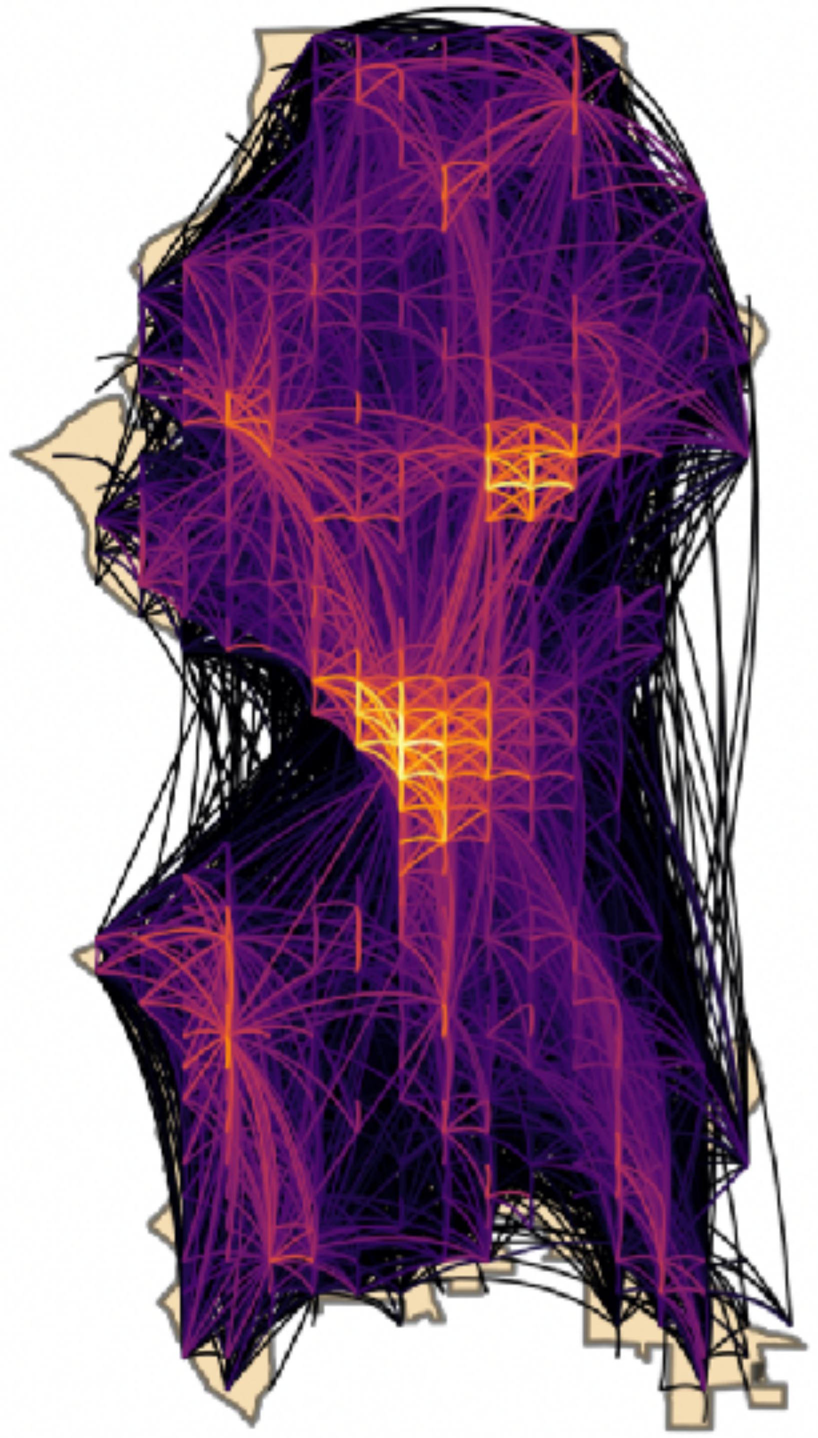


1

Transportation Network

2

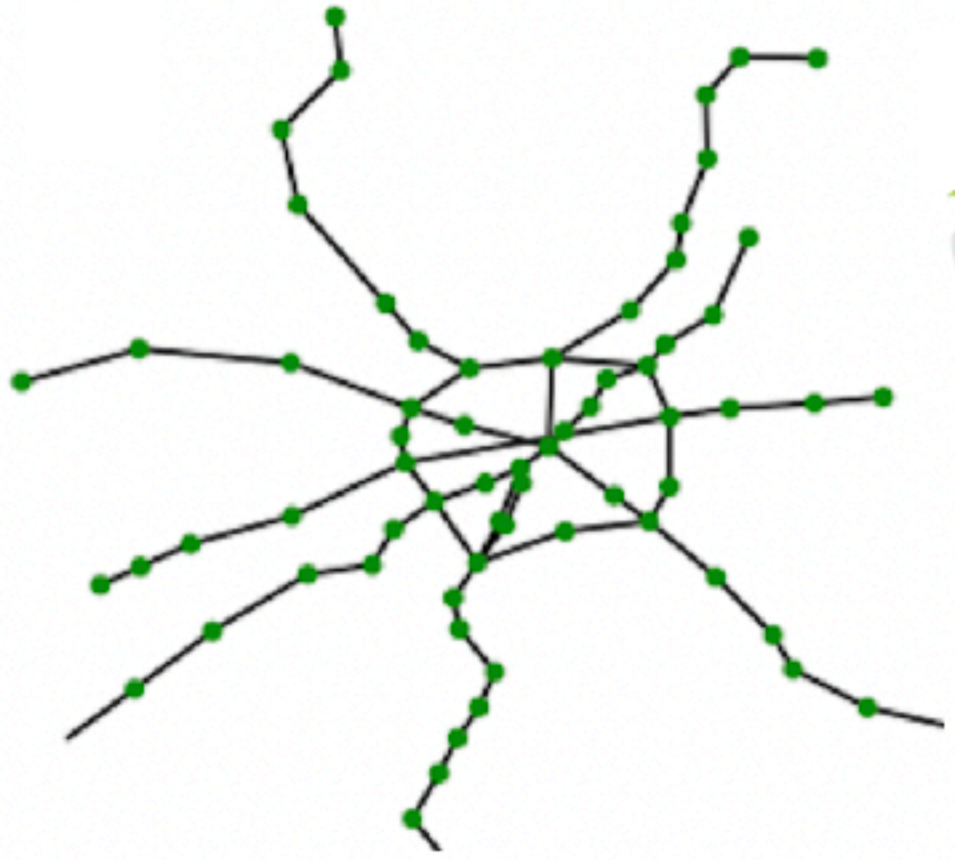
Human Flows

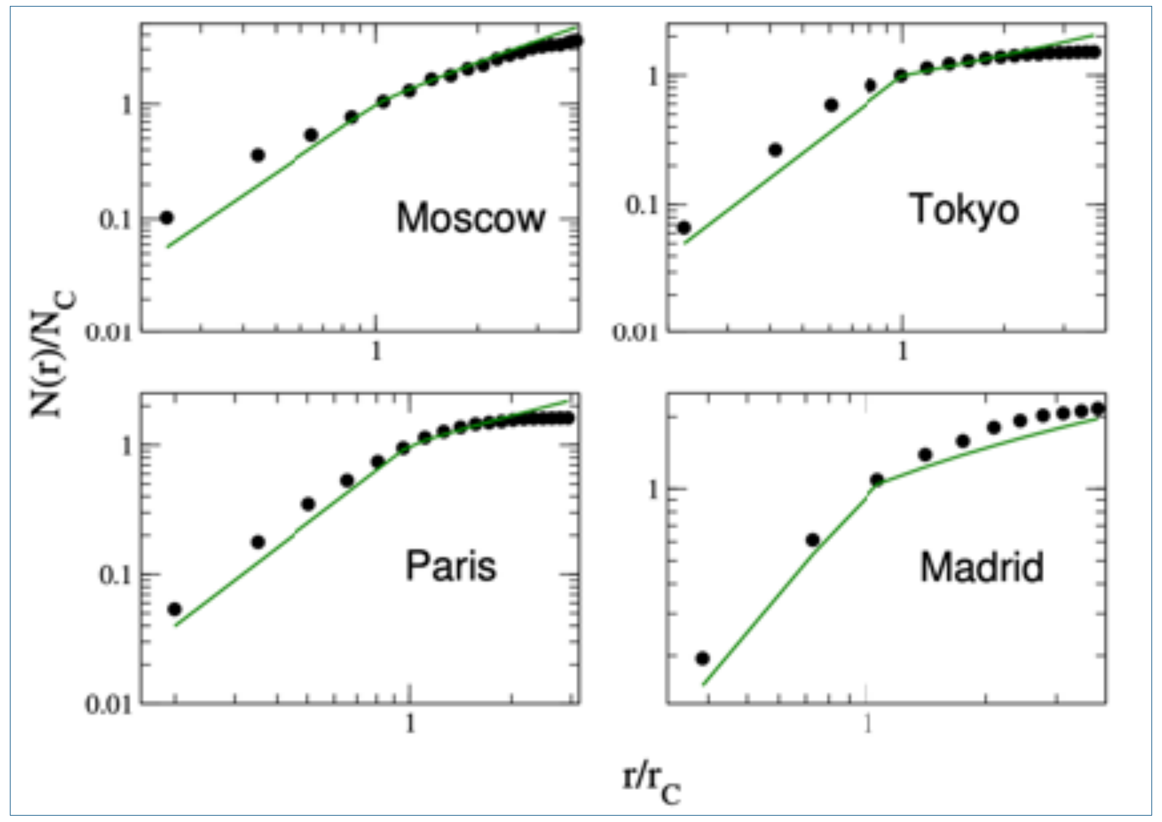


3

1- Barthelemy et al. (2018)
2- Paris Urban Mobility Index Data
3- Bontorin et al. (2024)

Complex topologies in urban transportation networks

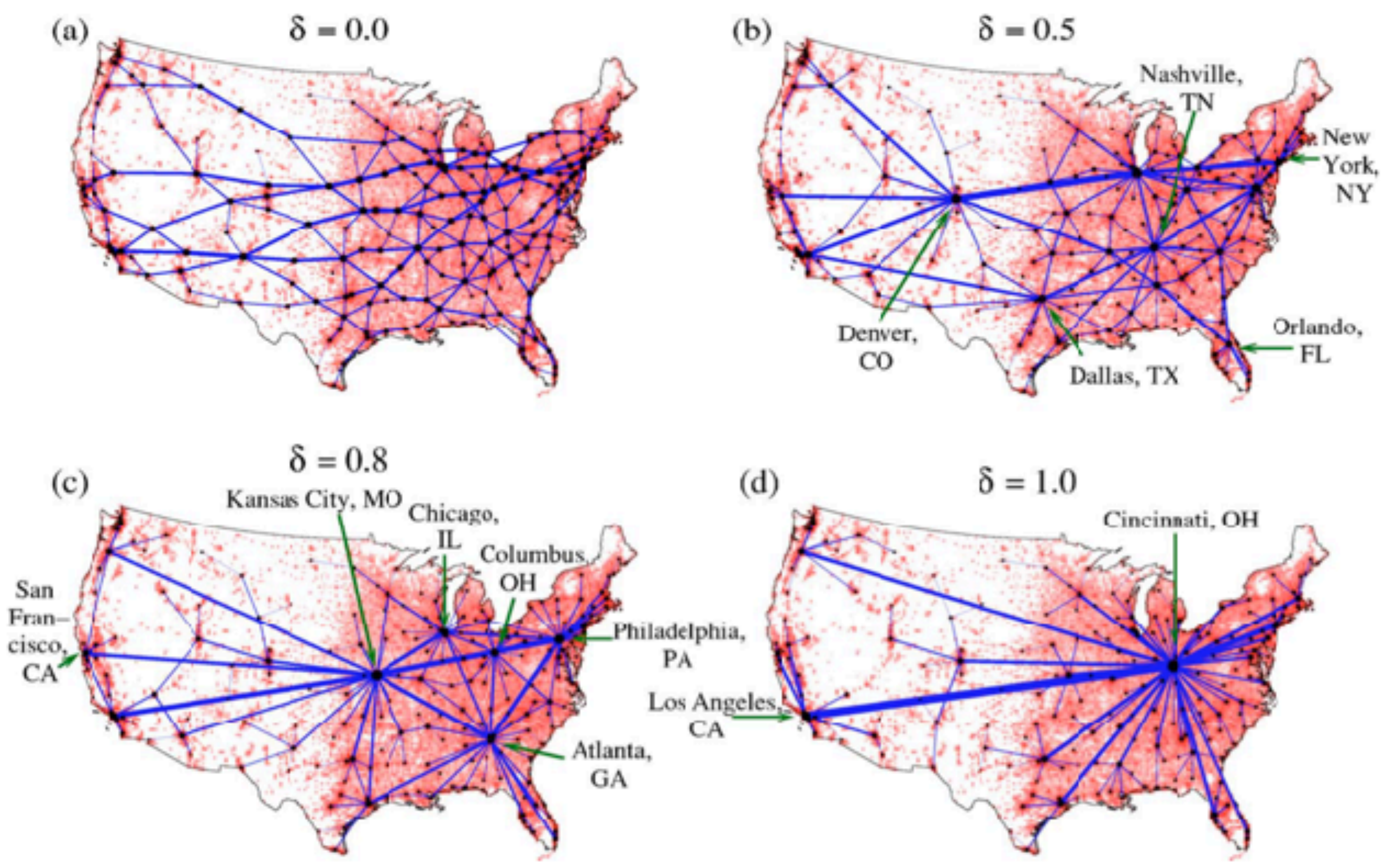




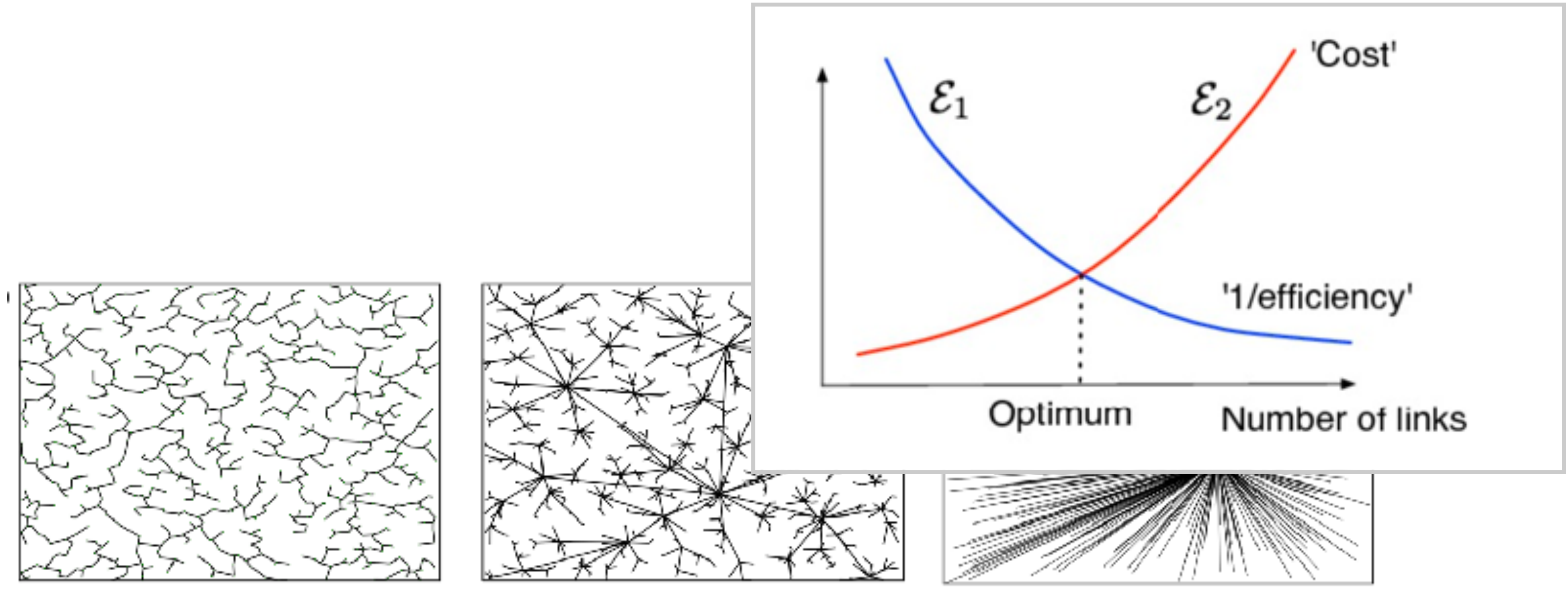
Roth Camille et Al - J. R. Soc. Interface (2012)

Aim:
- Investigate **optimal** spatial network topology given the spatial range of flows
- Reproducing **core + branches** topologies from optimality principles

Optimal spatial networks



Optimal design of spatial distribution networks - Gastner. PRE (2006)

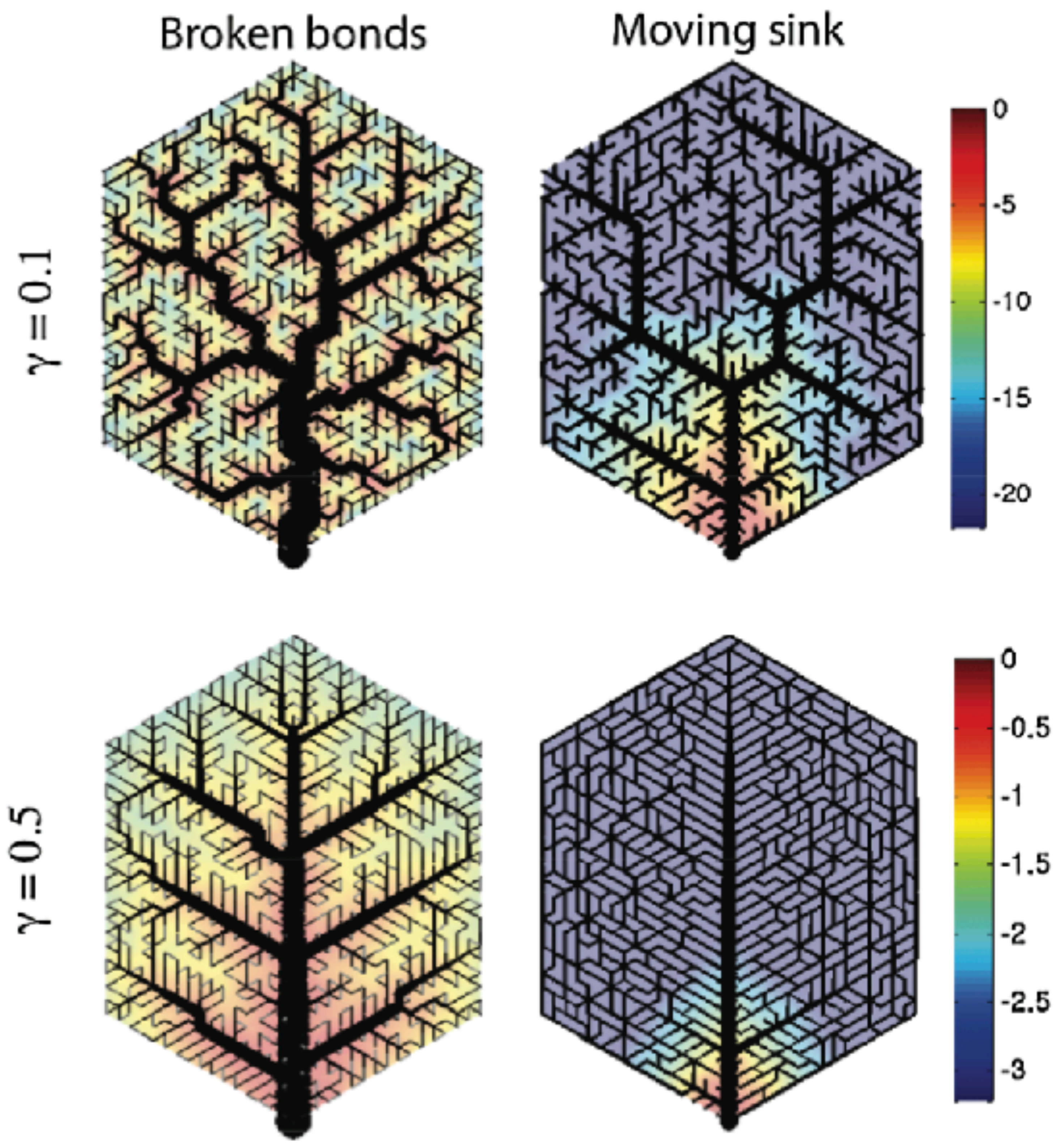


Marc Barthélemy and Alessandro Flammini. J. Stat. Mech. (2006)

- Natural and artificial structures often from self organization processes or local rules

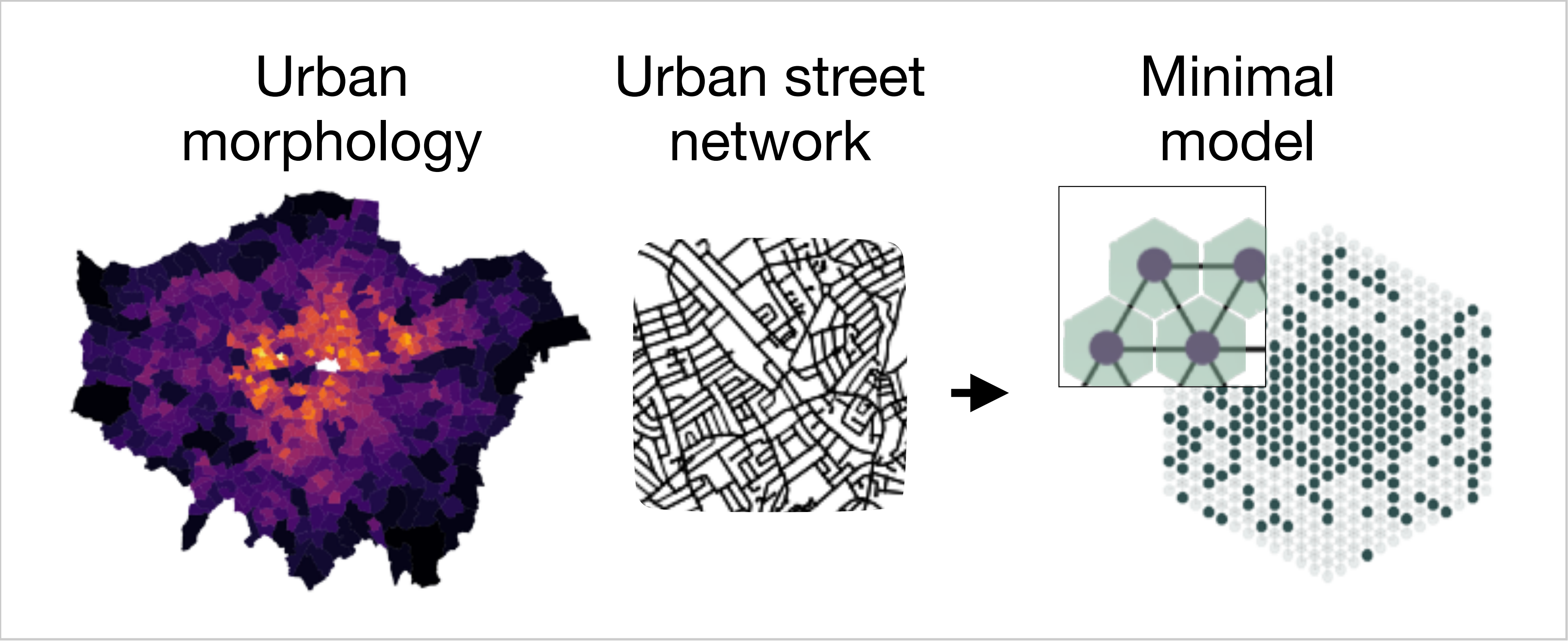
Why optimal models?

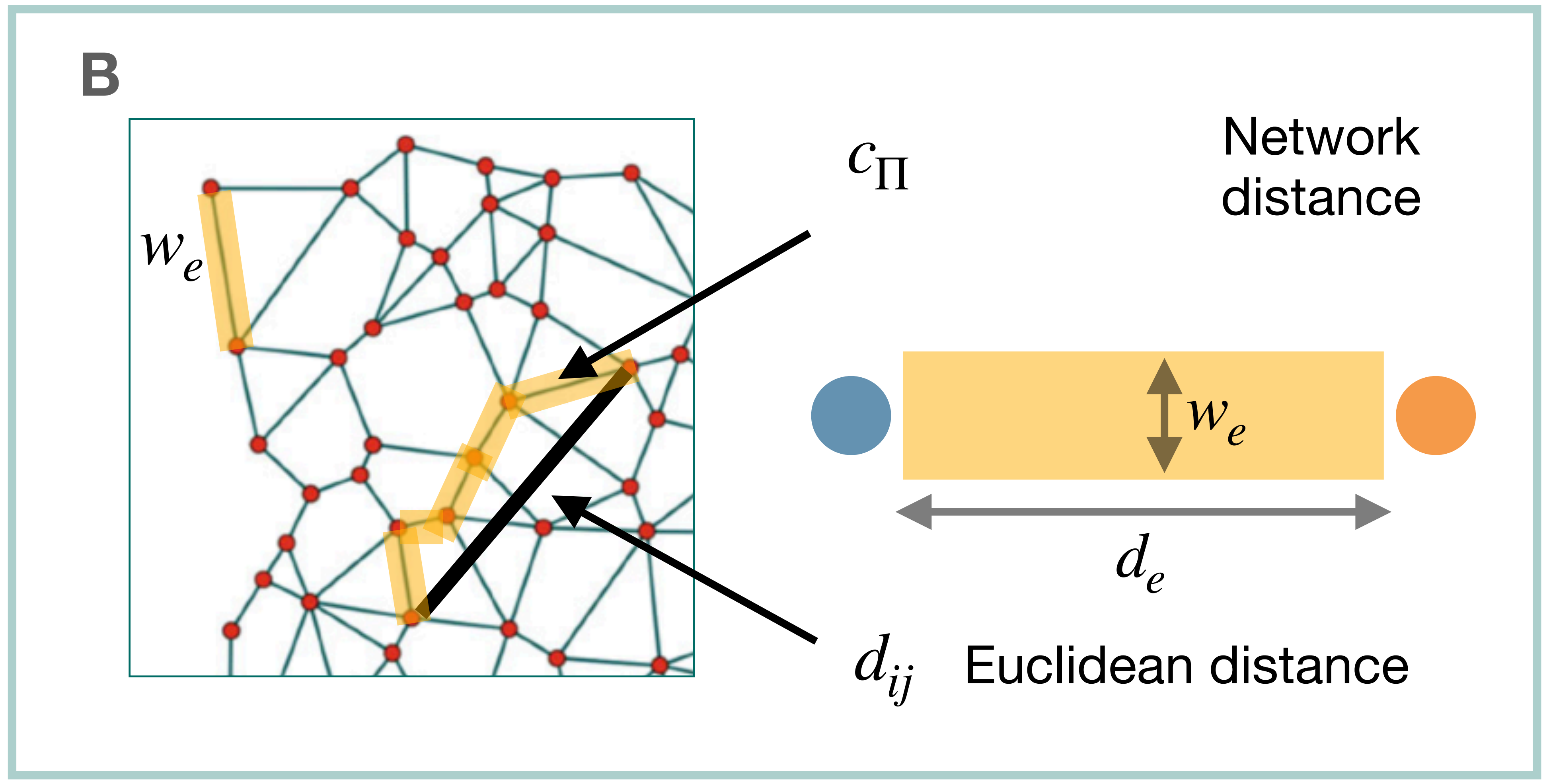
- Optimal network features
- Benchmark topological structures



Damage and Fluctuations Induce Loops in Optimal Transport Networks - Katifori et Al. PRL (2010)

A minimal model for urban morphology

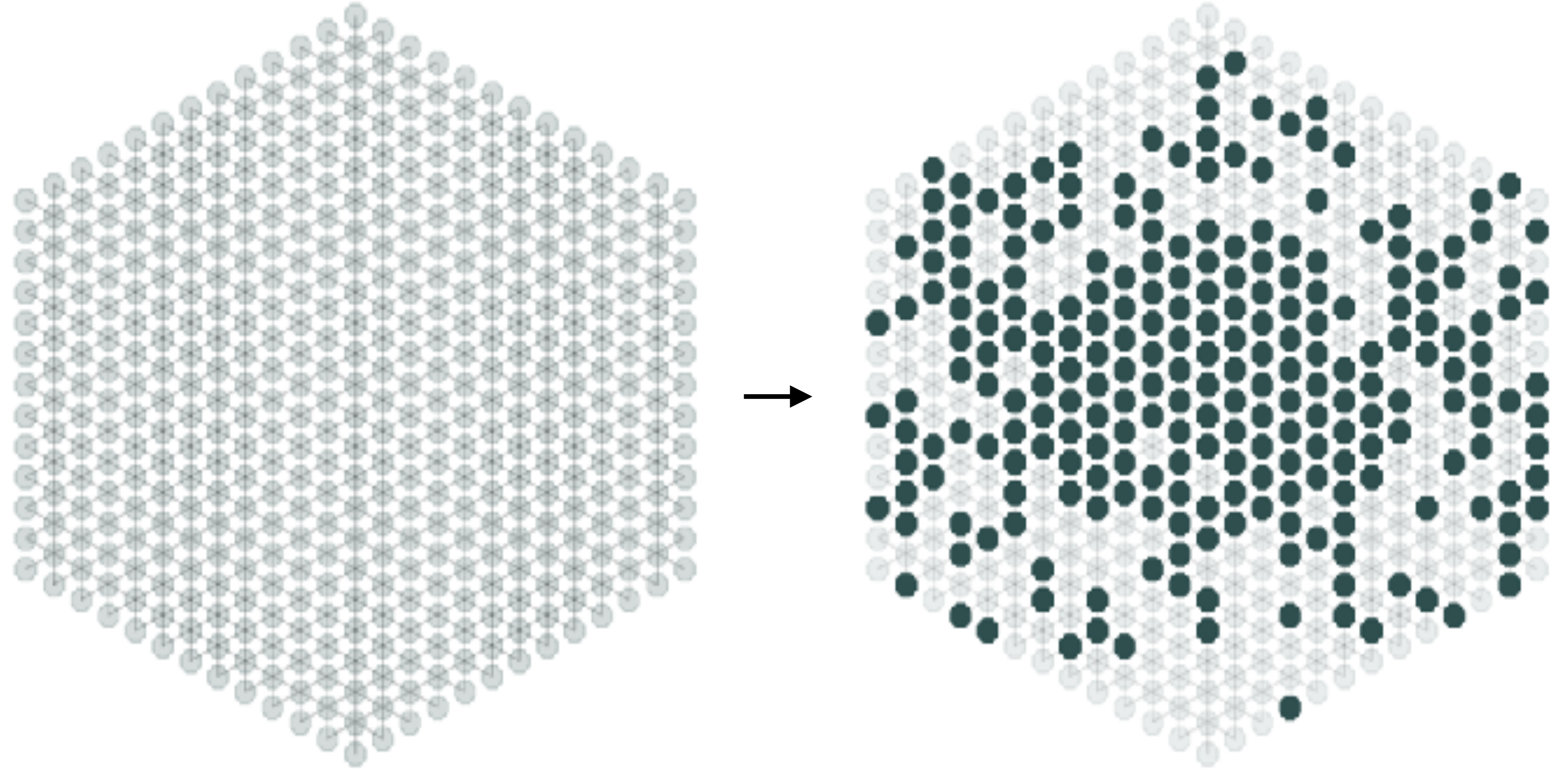




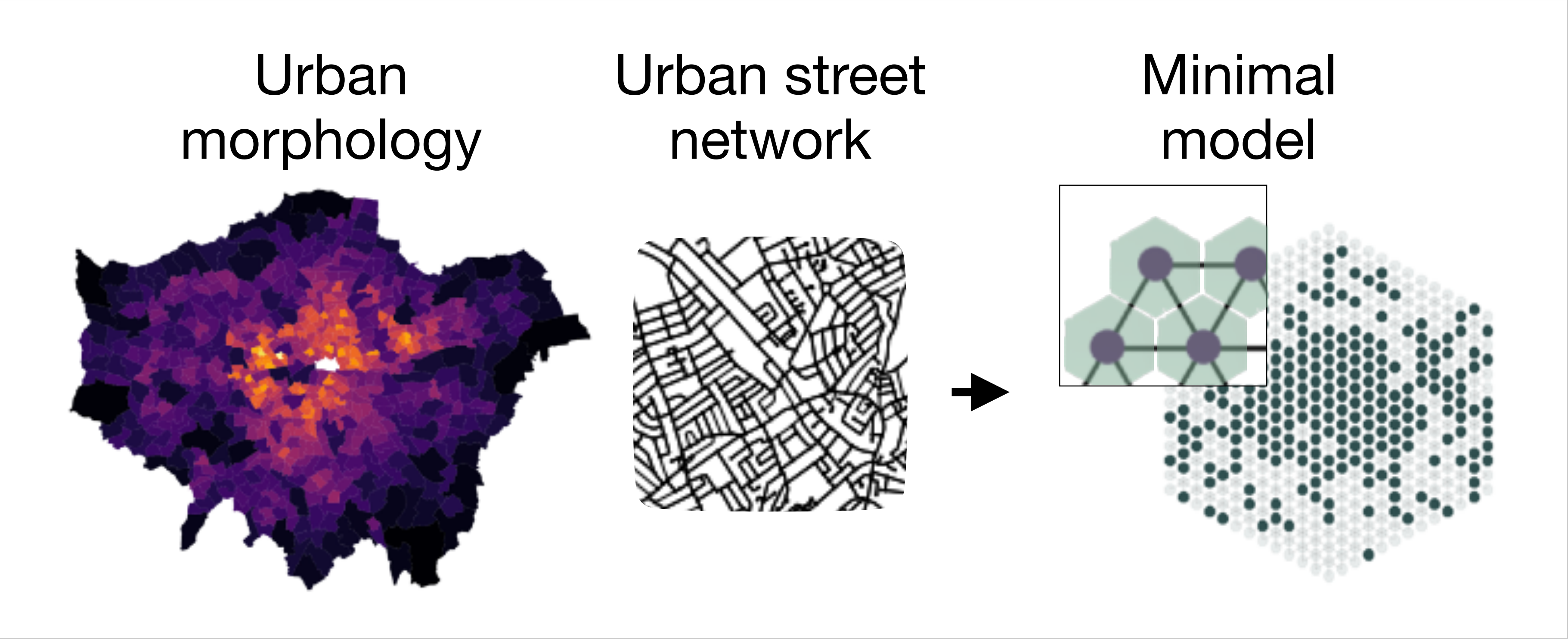
$w_e =$ **edge width (velocity)**

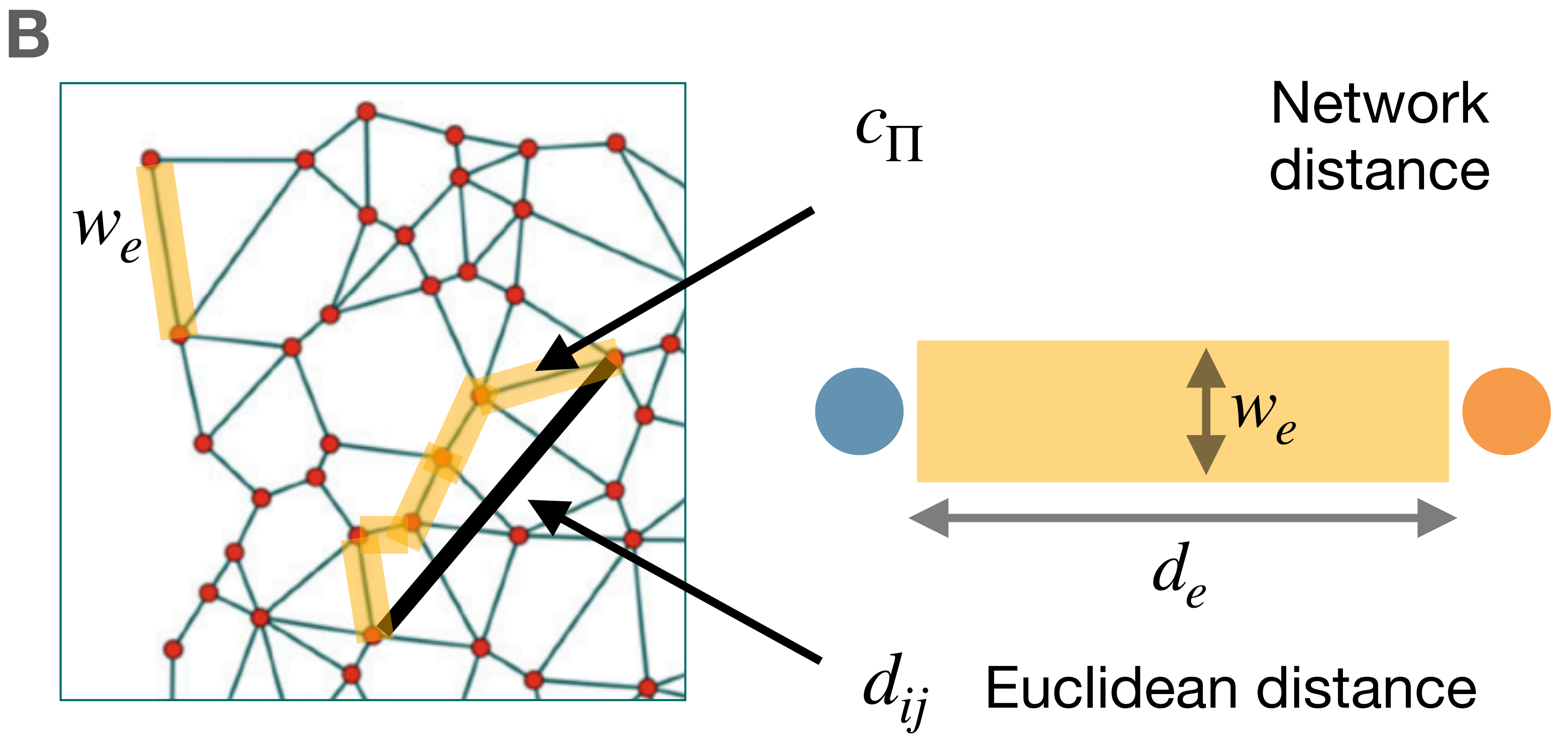
Lattice **Planar** Substrate

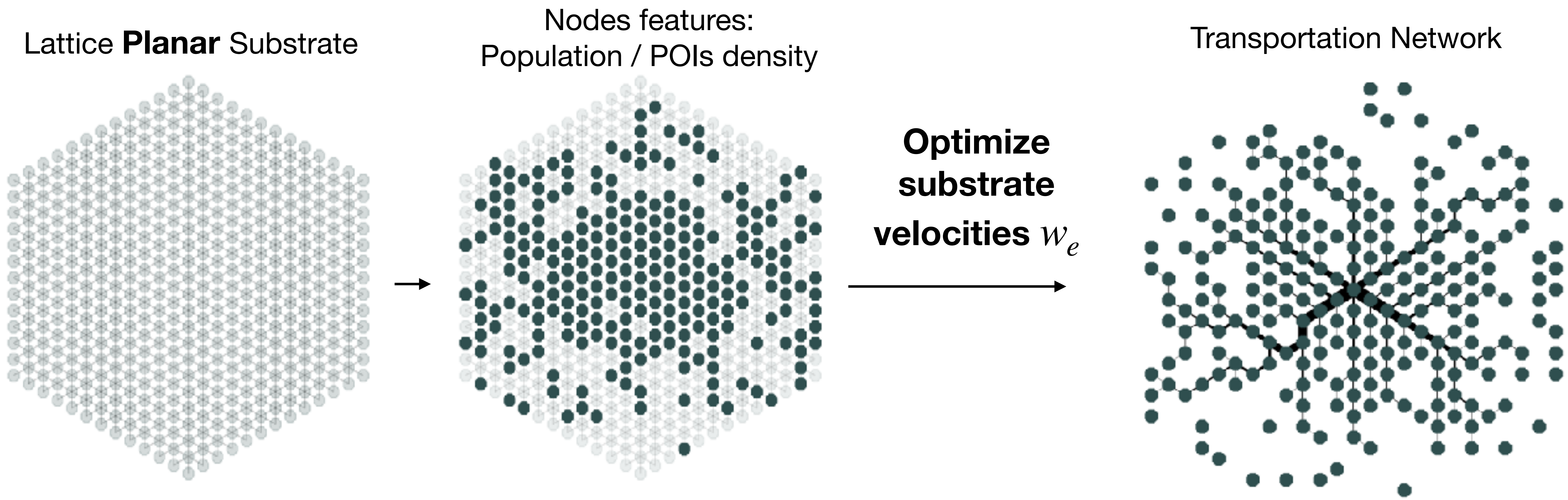
Nodes features:
Population / POIs density



A minimal model for urban morphology



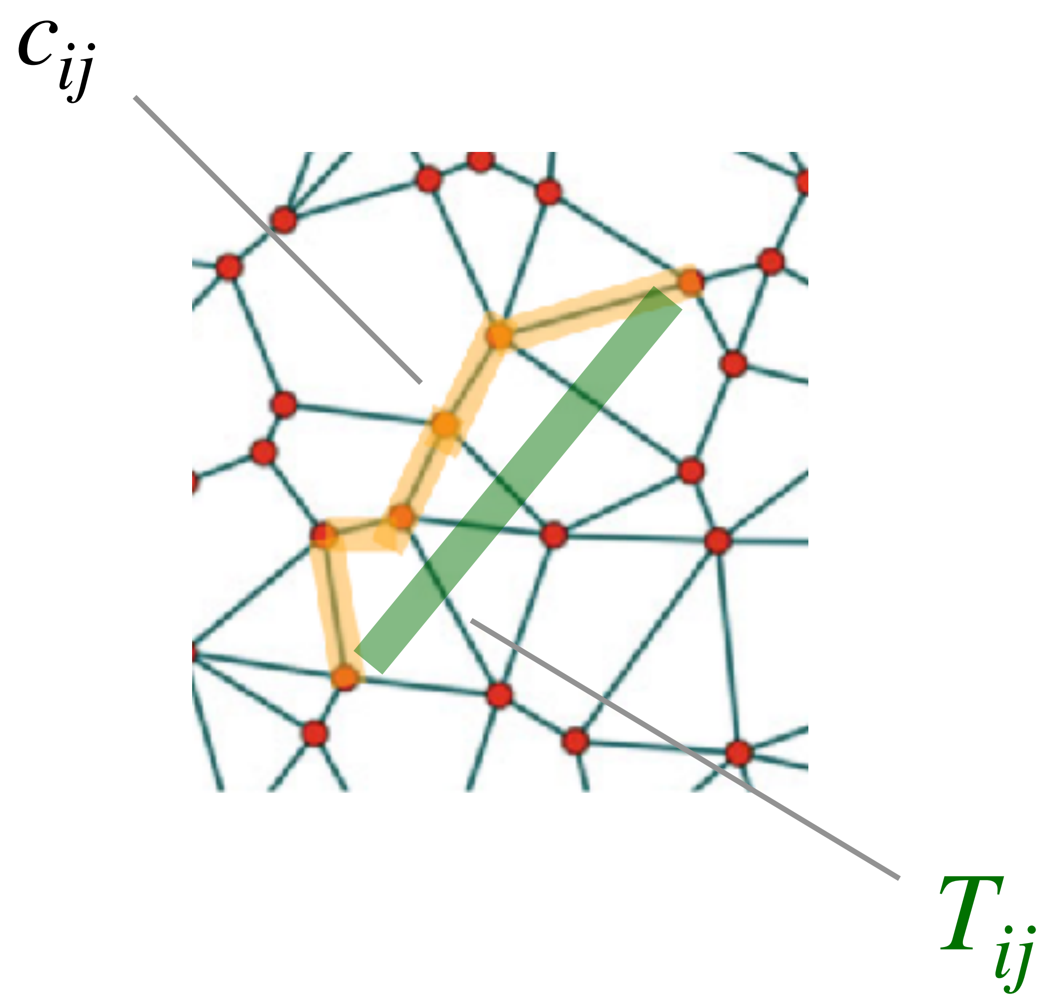




Flow weighted optimization of temporal effective distance

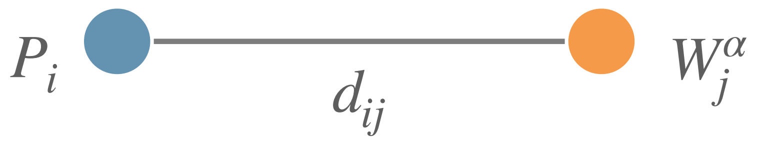
Effective temporal distance

$$c_{ij} = \min_{\Pi \in \Omega_{\Pi_{ij}}} \left[\sum_{e \in \Pi_{ij}} \frac{d_e}{w_e} \right]$$



Modeling human flows: [1]
(flows between *i* and *j*, gravity-like)

$$T_{ij} \propto P_i W_j^{\alpha} \exp\left(-\beta d_{ij}\right)$$



[1] - A Wilson. Entropy in Urban and Regional Modelling (2013)

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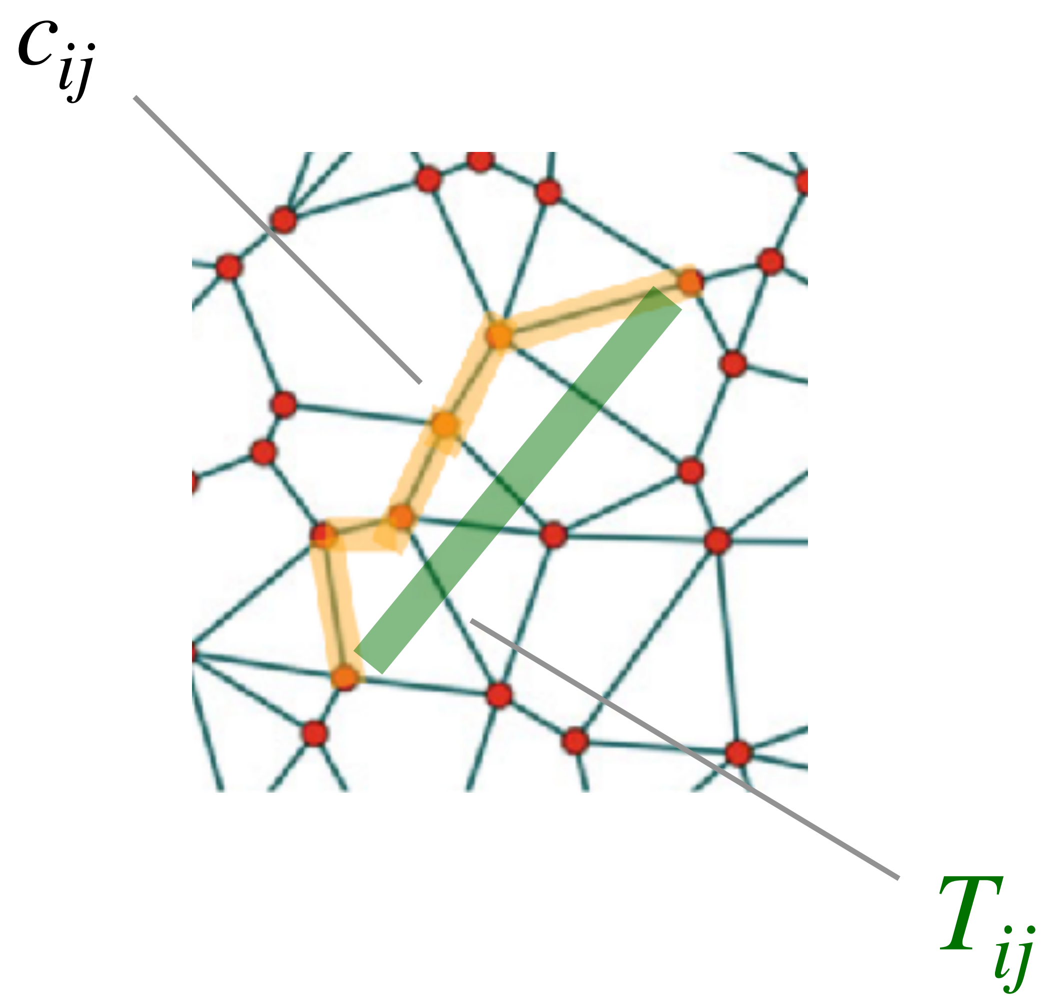

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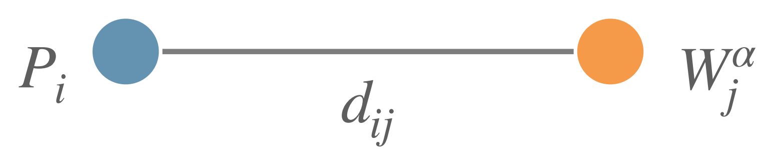
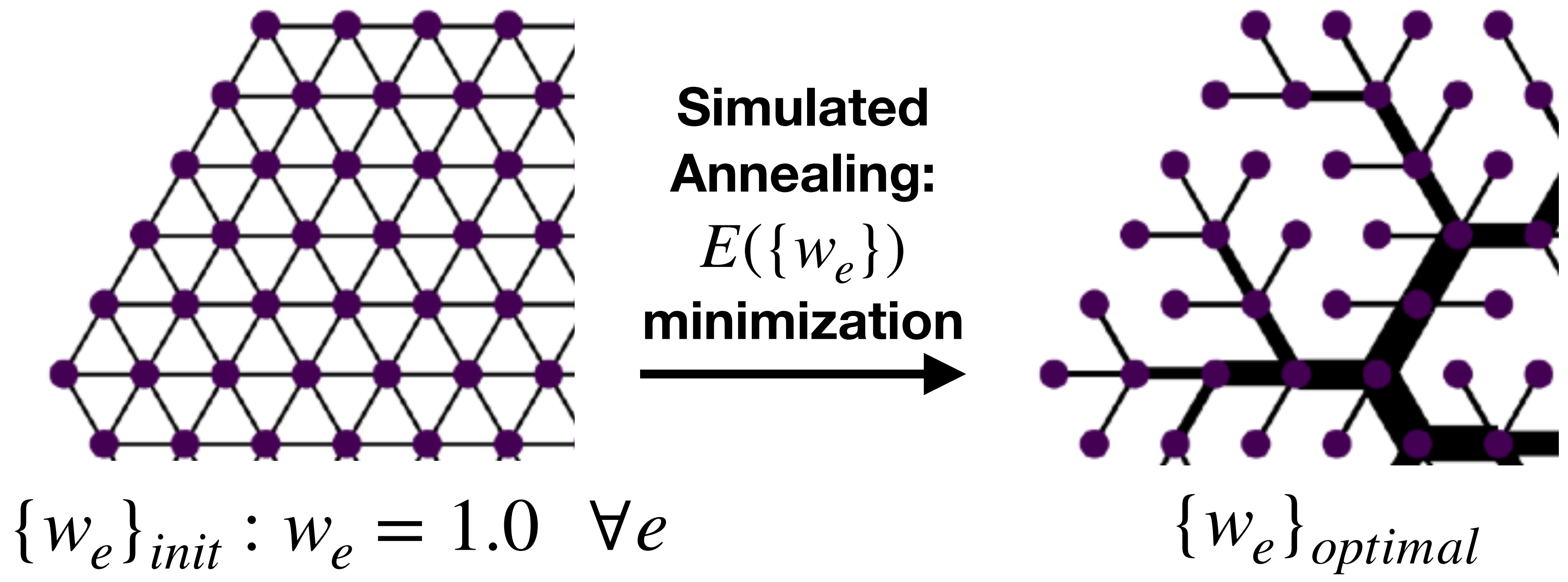


Flux-weighted Efficiency

$$E(G) = \sum_{i \in \text{sources}} \sum_{j \in \text{targets}} T_{ij} \cdot c_{ij}$$

find $\{w_e\}$ such that $E(G)$ is minimum



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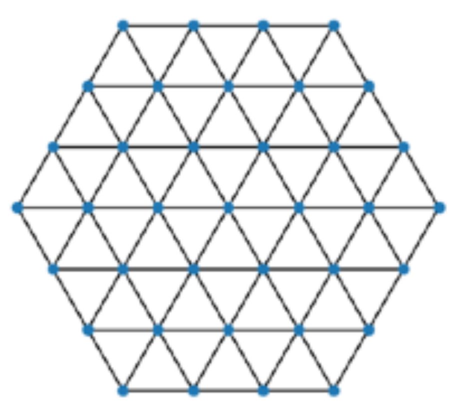
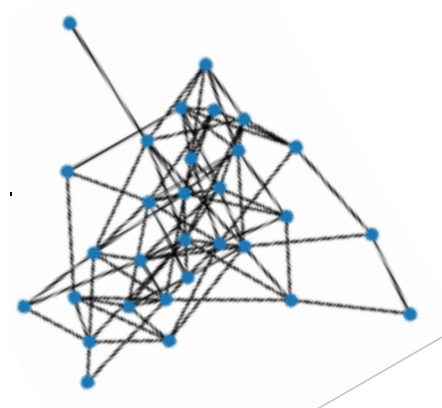
Simple network models



spatial
network
(HEX)

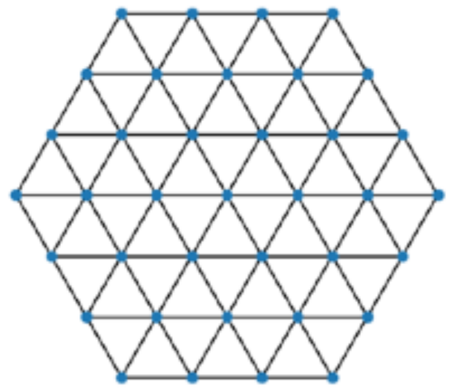


non-spatial
network
(ER)

Simple network models

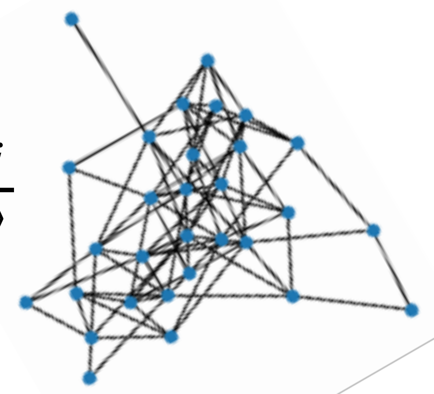
T_{ij}

$$p_{ij} \propto e^{-\beta \frac{r_{ij}}{\langle r \rangle}}$$



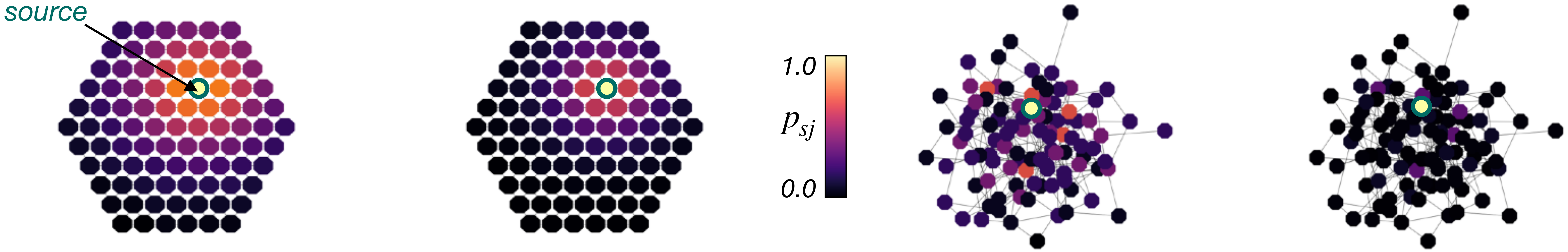
spatial network (HEX)

$$p_{ij} \propto e^{-\beta \frac{L_{ij}}{\langle L \rangle}}$$



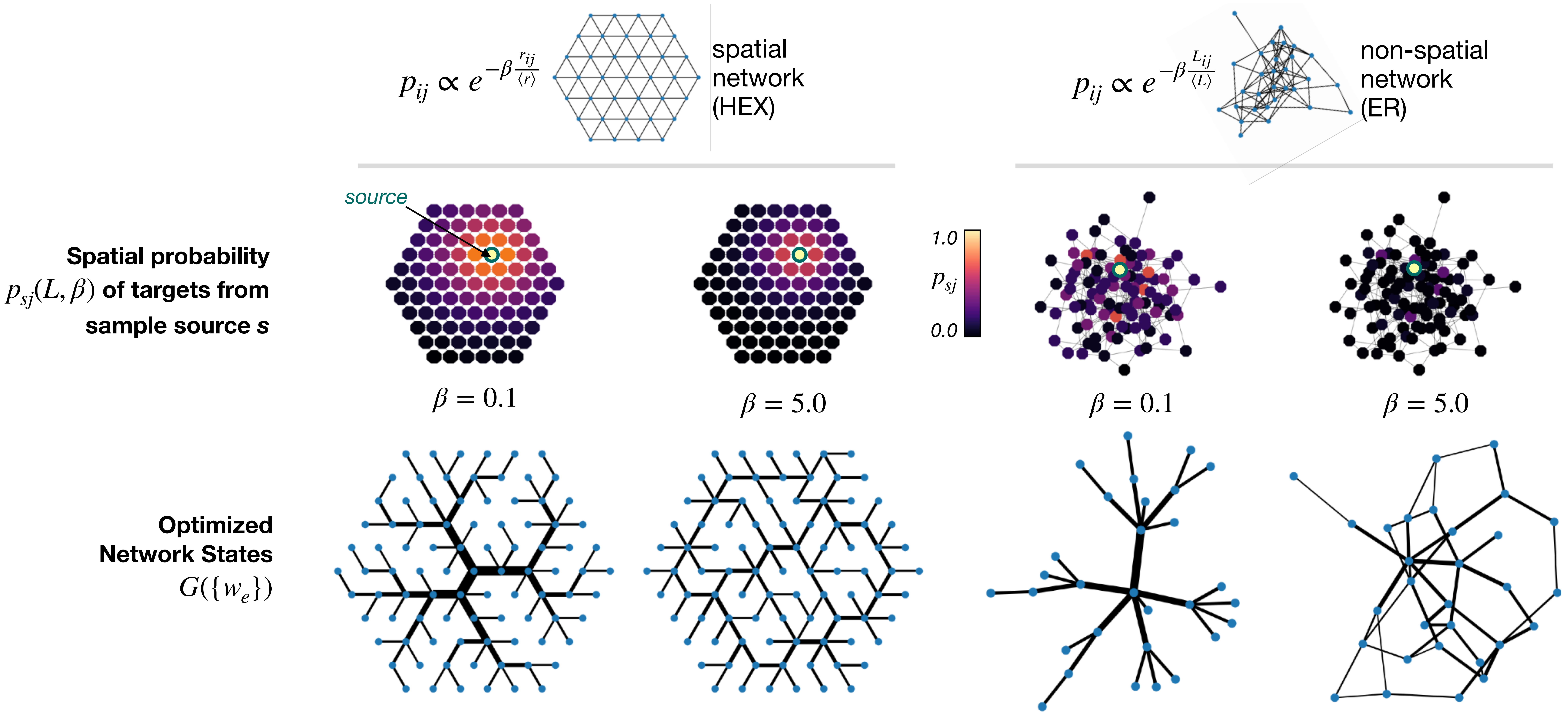
non-spatial network (ER)

Spatial probability $p_{sj}(L, \beta)$ of targets from sample source *s*



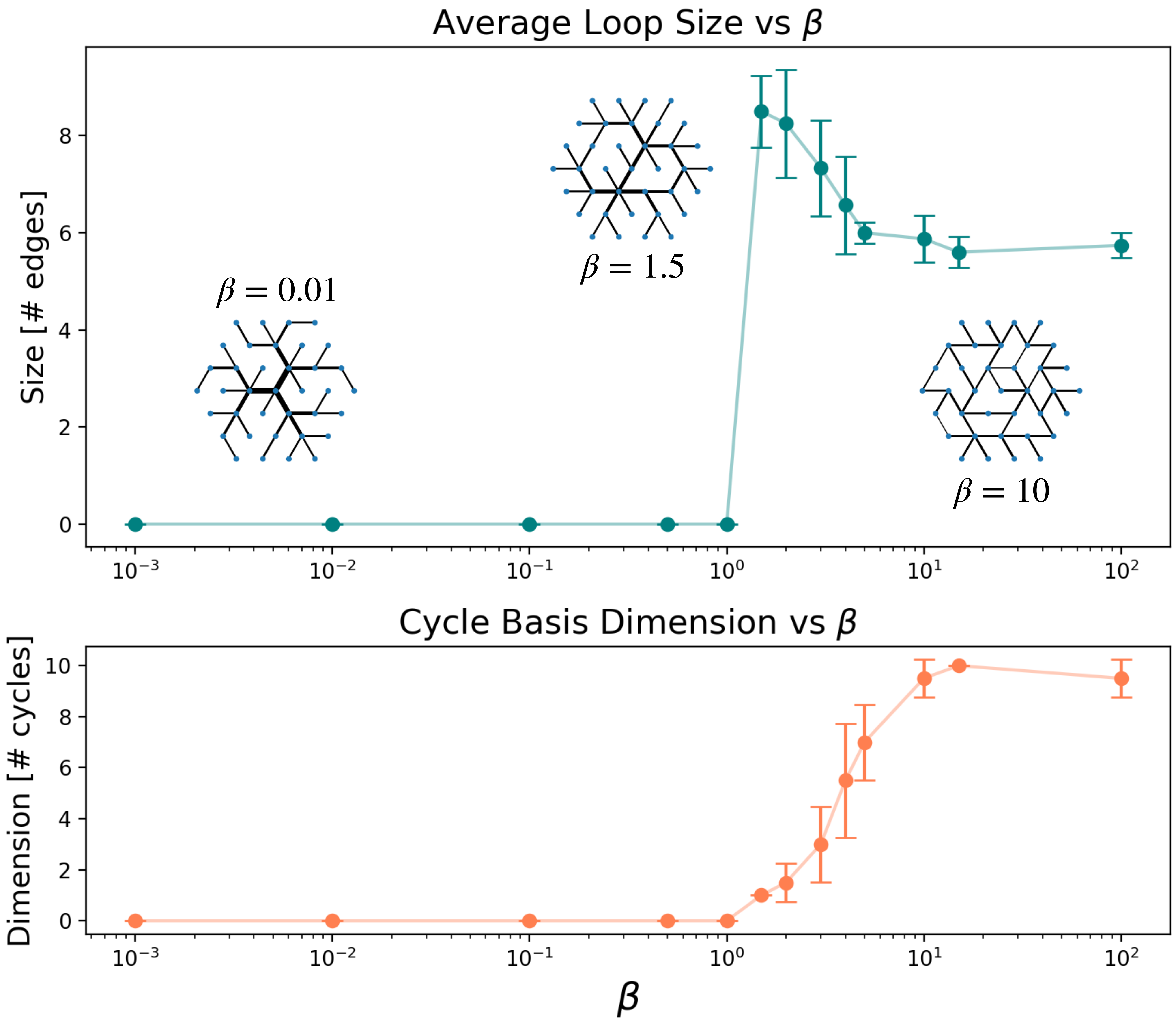
$\beta = 0.1$ $\beta = 5.0$ $\beta = 0.1$ $\beta = 5.0$

Simple network models



Simple network models: results

- **Loops presence**: cycle basis dimension
- Optimal topologies ranging from trees to lattice-like with loops

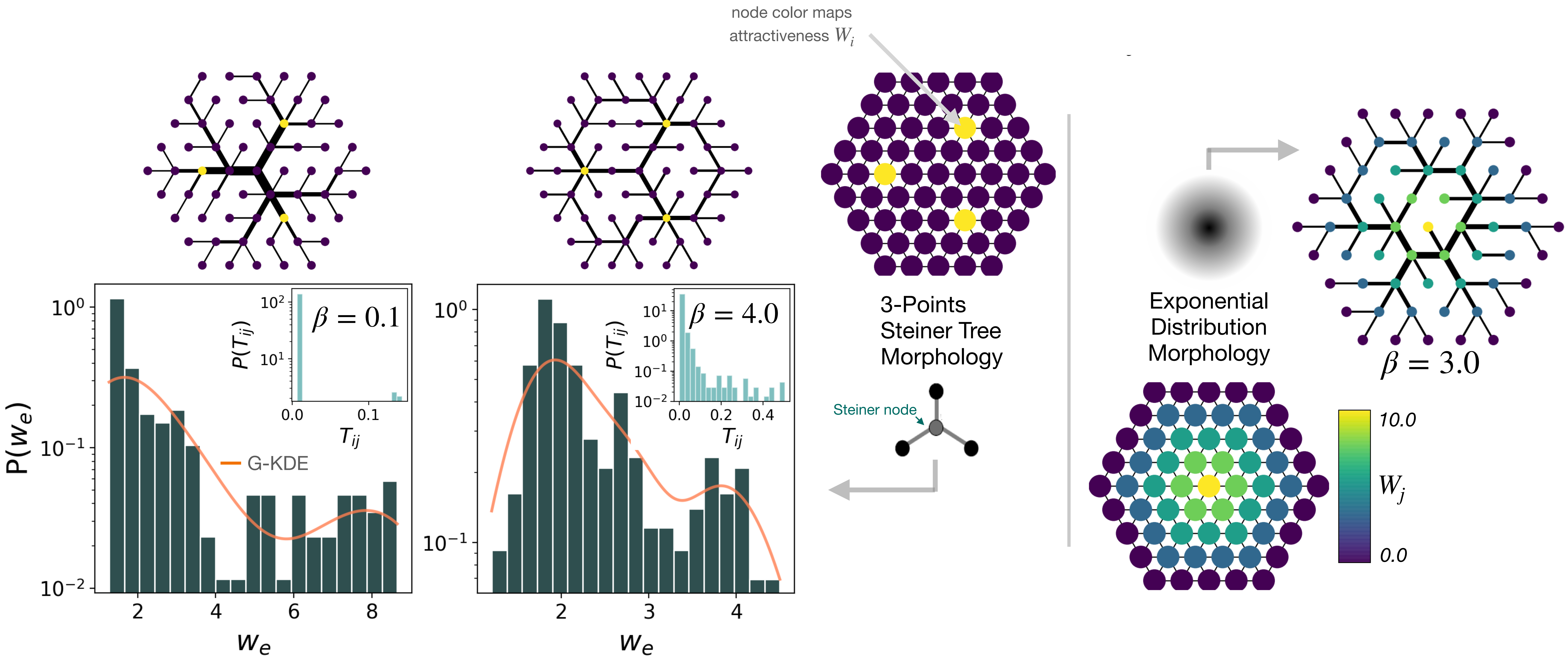


Introducing spatial attractiveness of nodes

- Attractiveness W_j biases traffic towards specific nodes

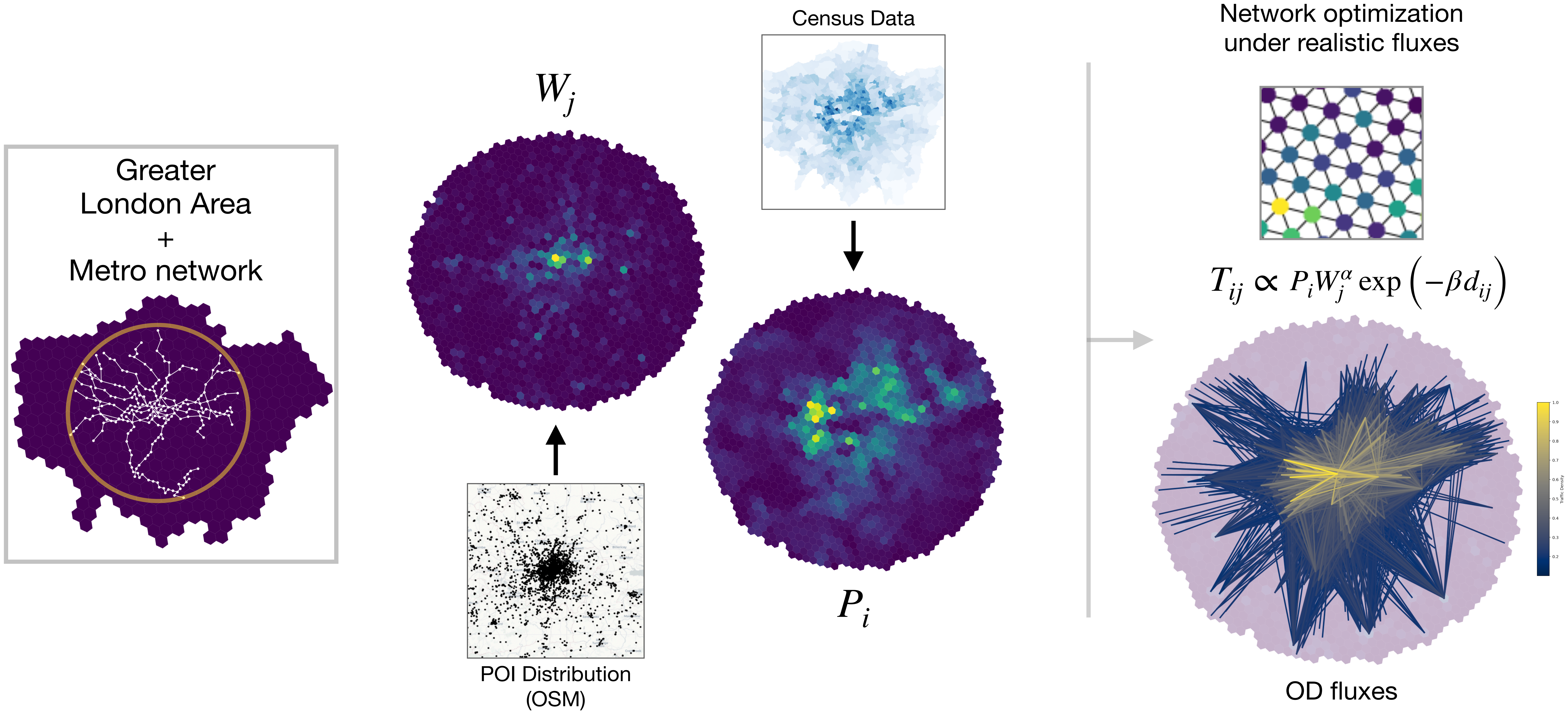
$$T_{ij} \propto \frac{W_j \cdot e^{-\beta d_{ij}/\langle d \rangle}}{\sum_k W_k \cdot e^{-\beta d_{ik}/\langle d \rangle}}$$

- Second mode in the optimal weights distribution, multi-layer as optimal in heterogeneous flows



Application on real urban morphologies:

Greater London Area

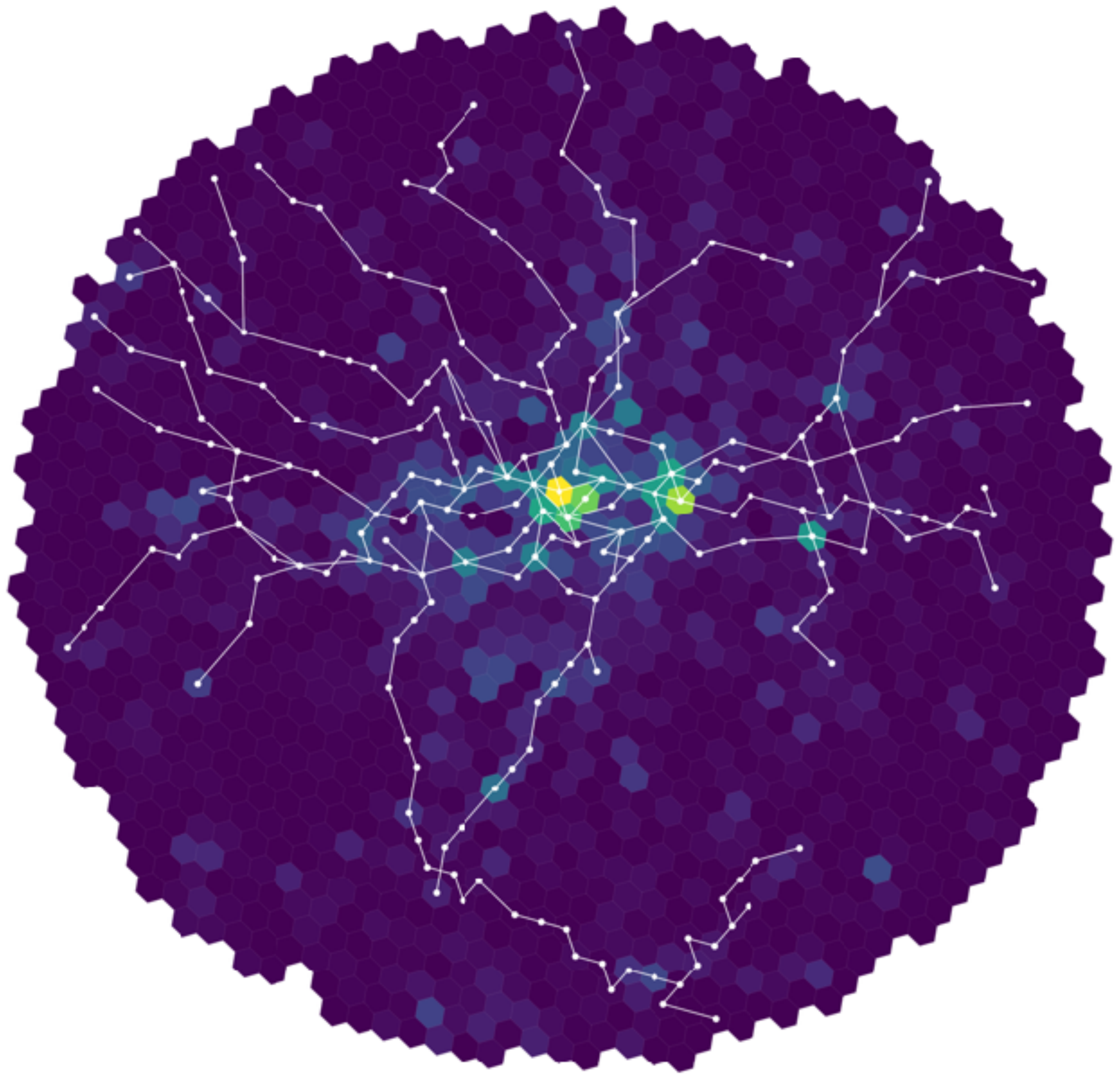


- Population and attractiveness data are recovered from Census 2011 and amenities distribution from OSM

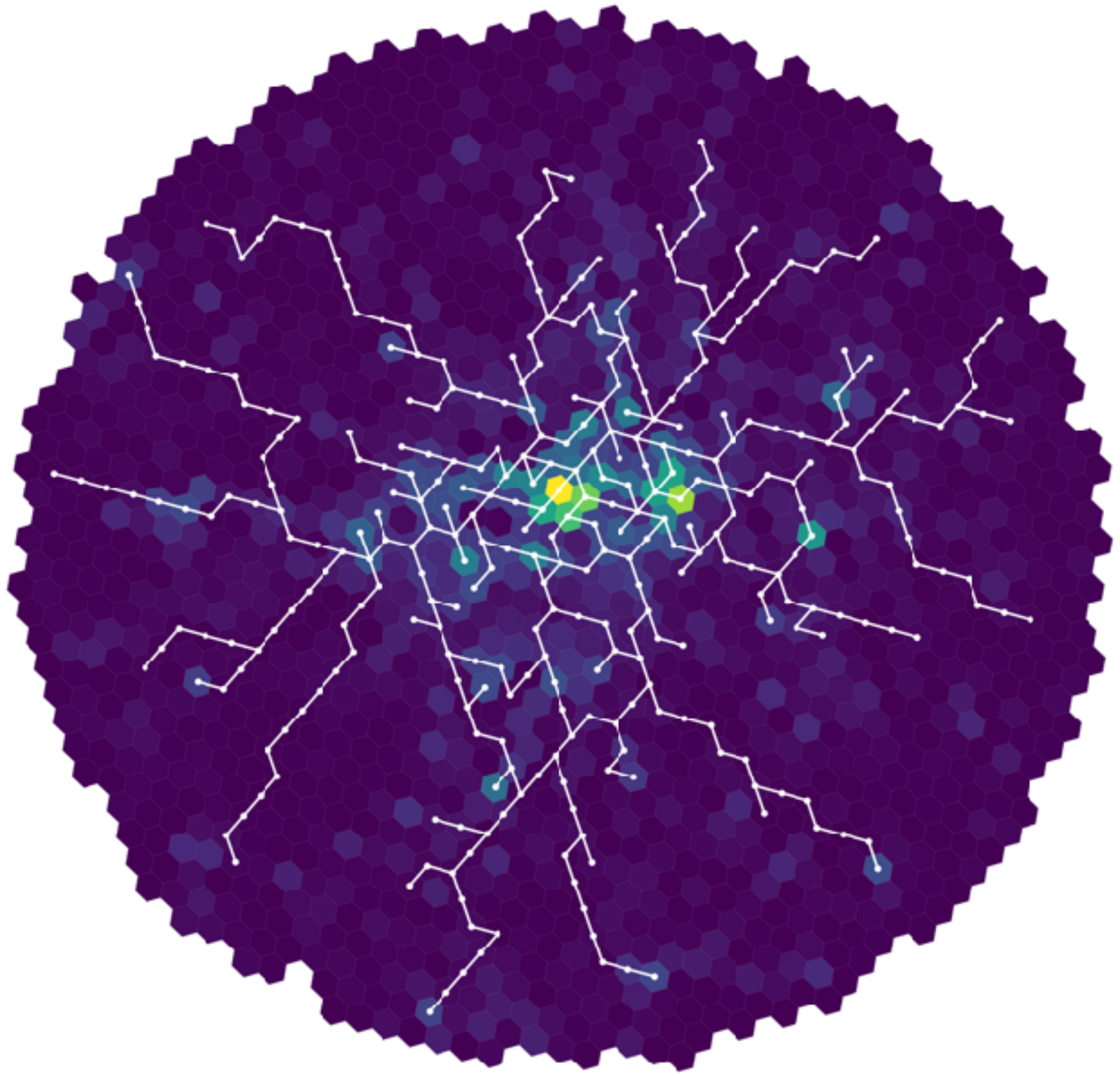
- Traffic T_{ij} simulated via spatial interaction model

Results on real urban morphologies:
Greater London Area

GLA Tube



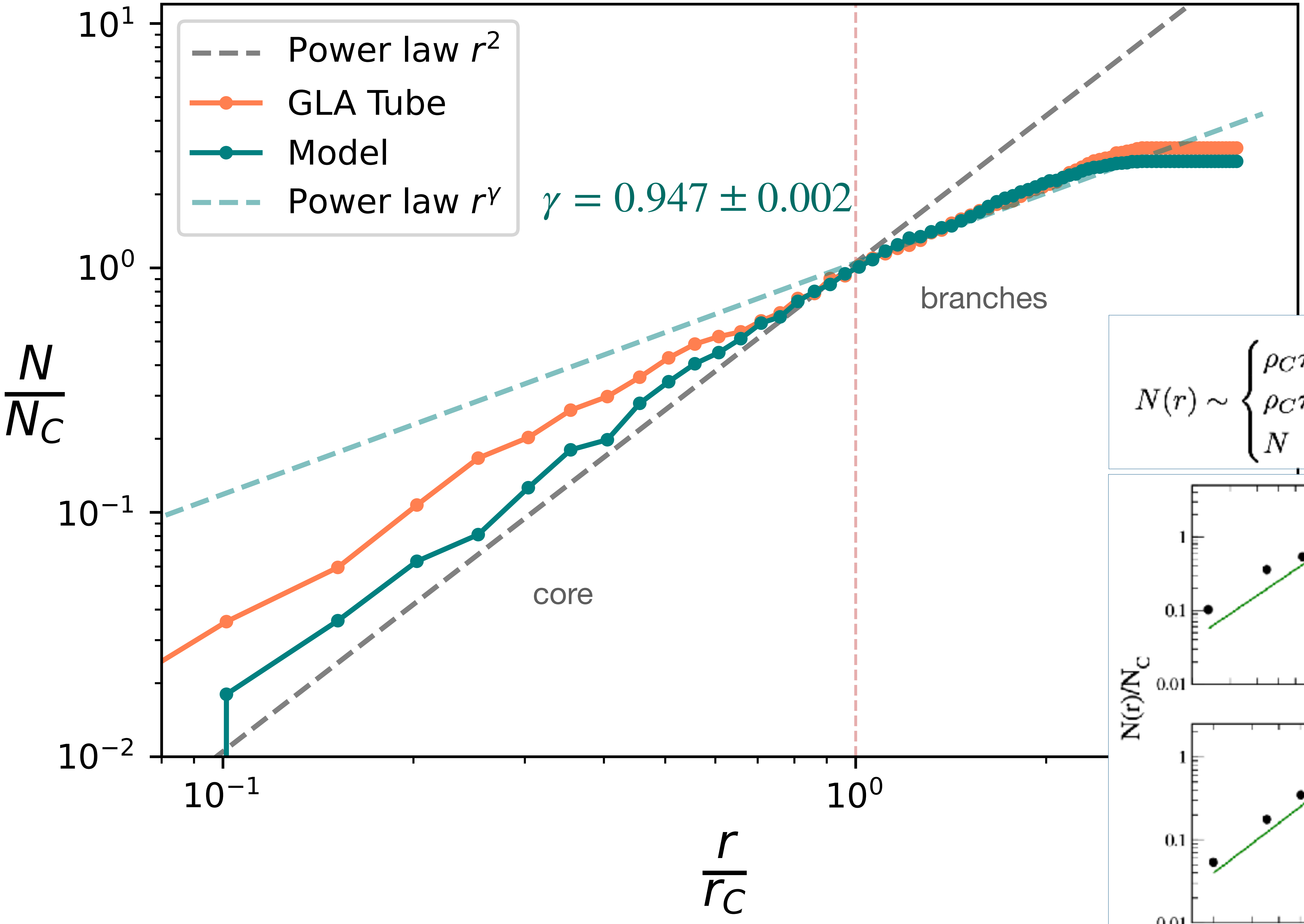
Model



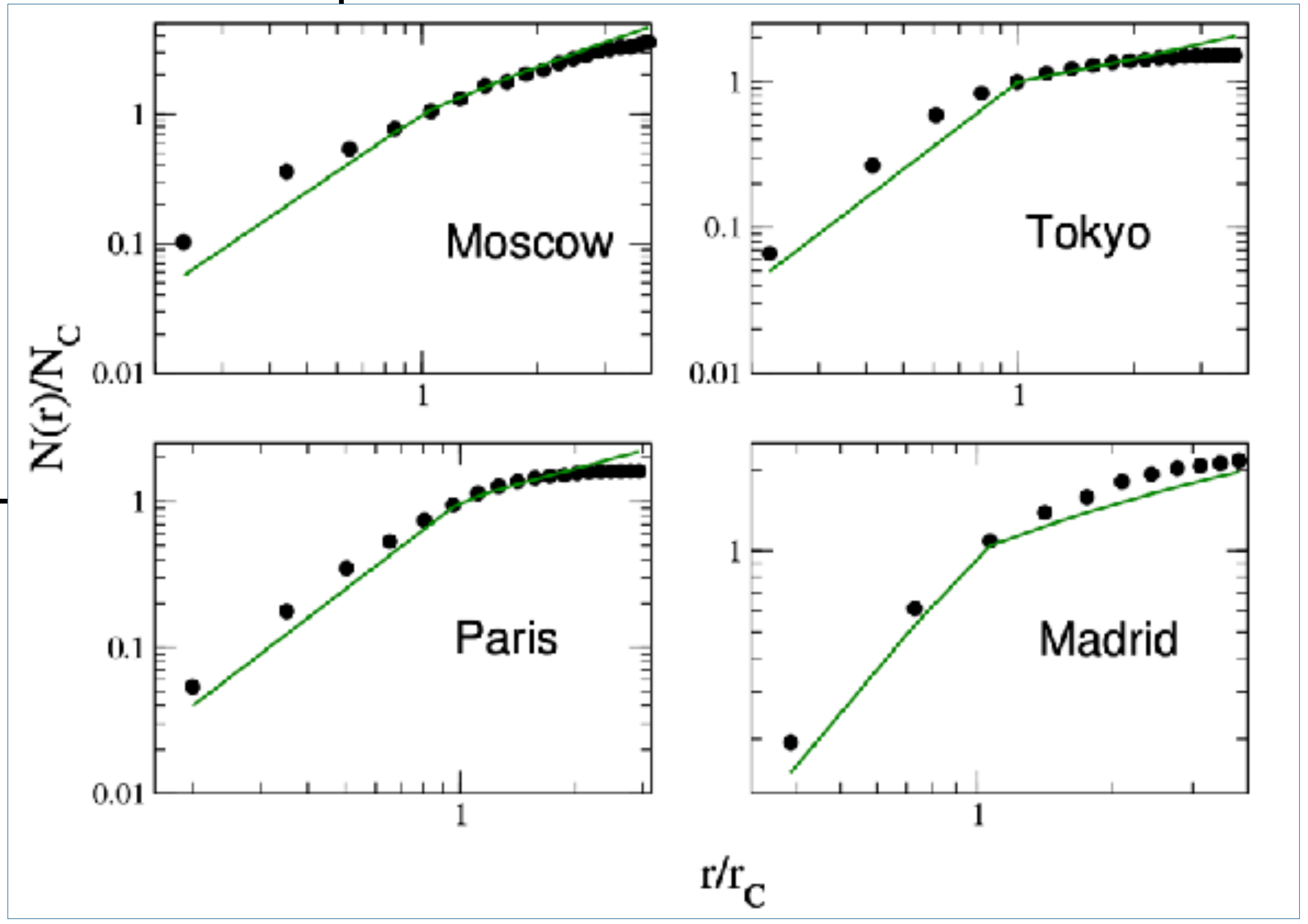
- Statistical and topological properties of the real tube network are recovered: **core with loops + branches**

Results on real urban morphologies:

Scaling properties: core + loops

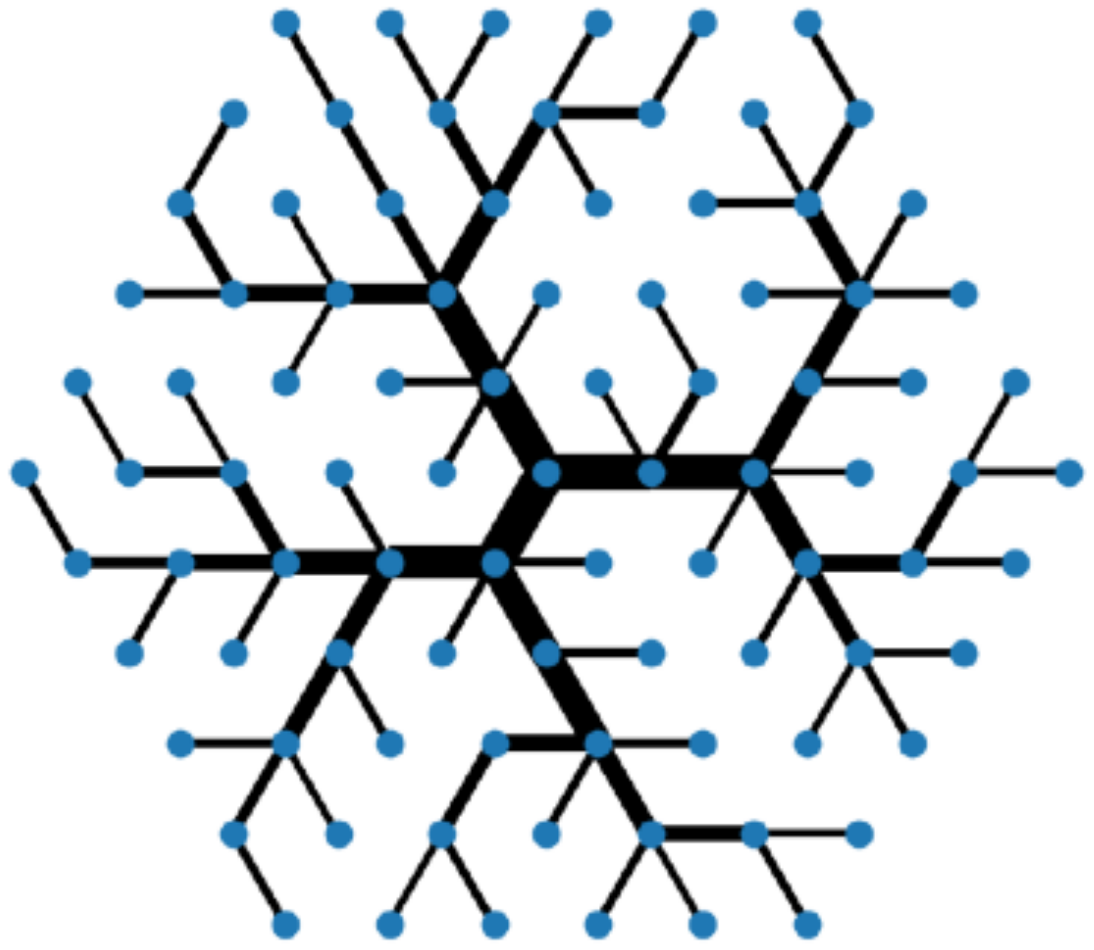


$$N(r) \sim \begin{cases} \rho_C \pi r^2 & \text{for} \quad r < r_C \\ \rho_C \pi r_C^2 + N_B \int_{r_C}^{r} \frac{dr}{\Delta(r)} & \text{for} \quad r_C < r < r_{\max} \\ N & \text{for} \quad r > r_{\max} \end{cases}$$

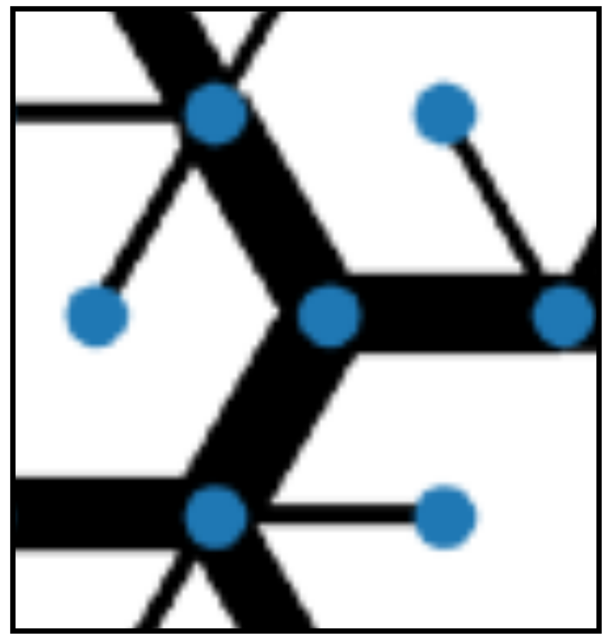


Roth Camille et Al - J. R. Soc. Interface (2012)

Adding user equilibrium: beyond single path routing

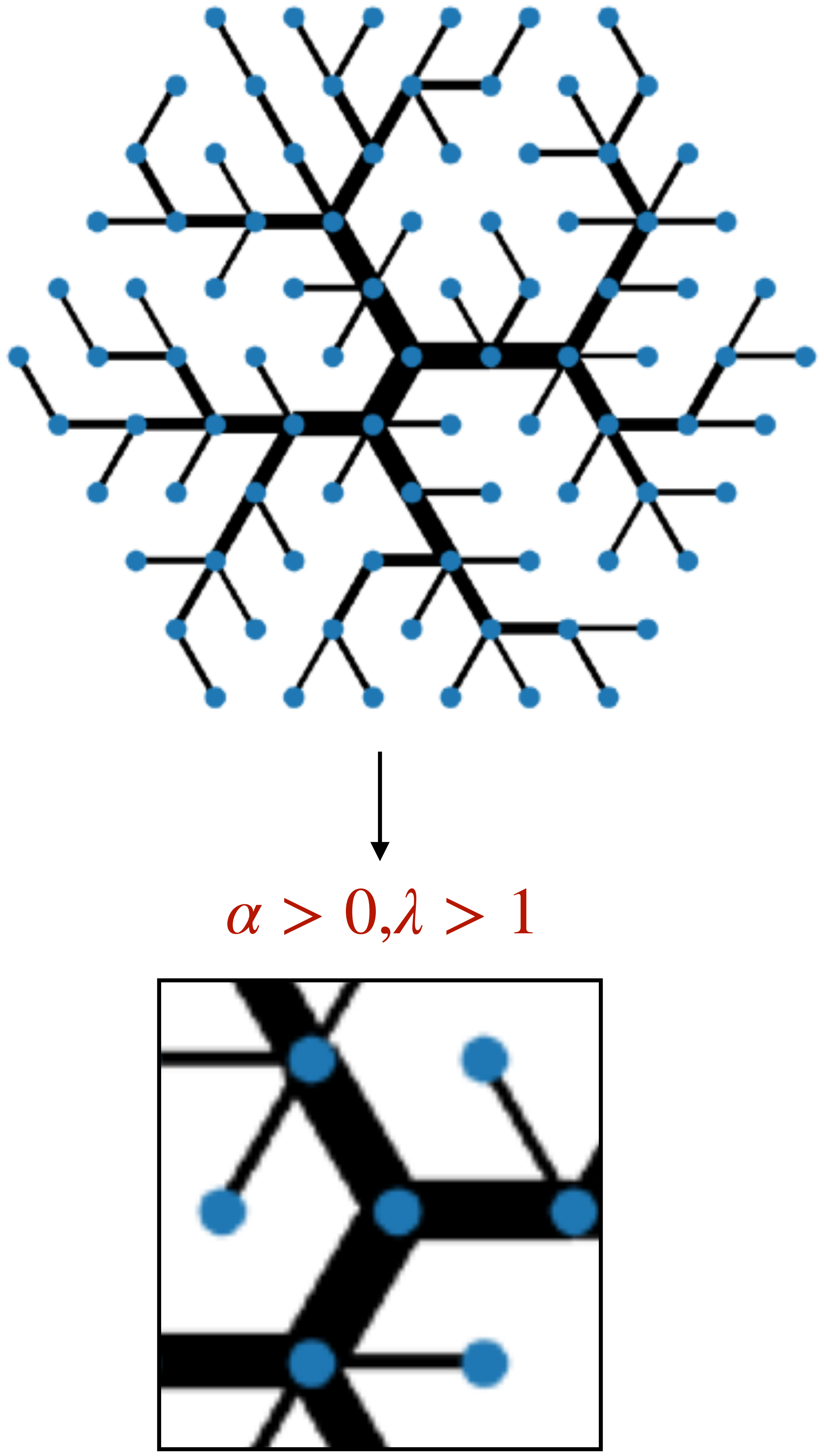


$\alpha > 0, \lambda > 1$



- Routing ODs following the shortest path paradigm is not realistic, we add **congestion** effects via User Equilibrium

Adding user equilibrium: beyond single path routing



- Routing ODs following the shortest path paradigm is not realistic, we add **congestion** effects via User Equilibrium

- Delay function (BPR) with c_e constant:

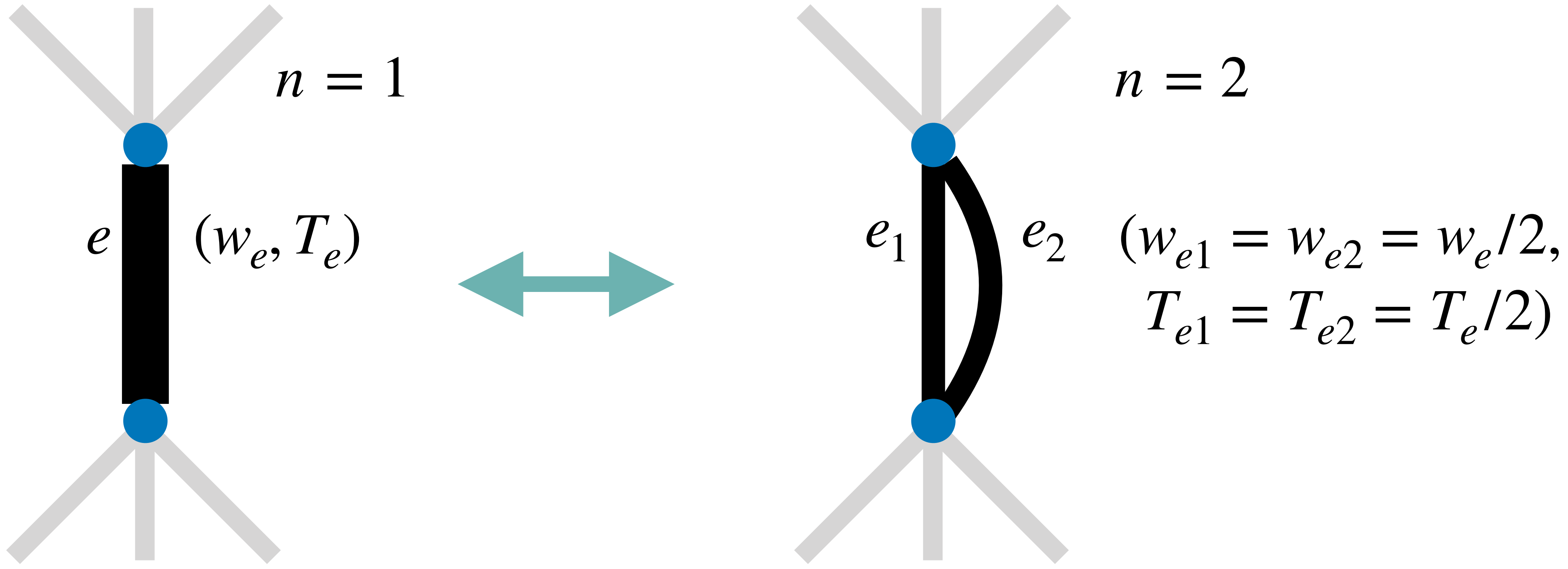
$$t_e = t_0 \cdot \left[1 + \alpha \left(\frac{T_e}{c_e}\right)^{\lambda}\right]$$

- Study the **collapse of the tree structure given the congestion intensity**

Transition tree to multi-pathways: analytical

- Comparison of the flow-weighted temporal efficiency in the two critical scenarios

$$E_e\left[T_e, w_e, (\lambda, \alpha)\right] \leq \sum_{e \in \mathcal{N}_e} E_e\left[\frac{T_e}{n}, \frac{w_e}{n}, (\lambda, \alpha)\right]$$



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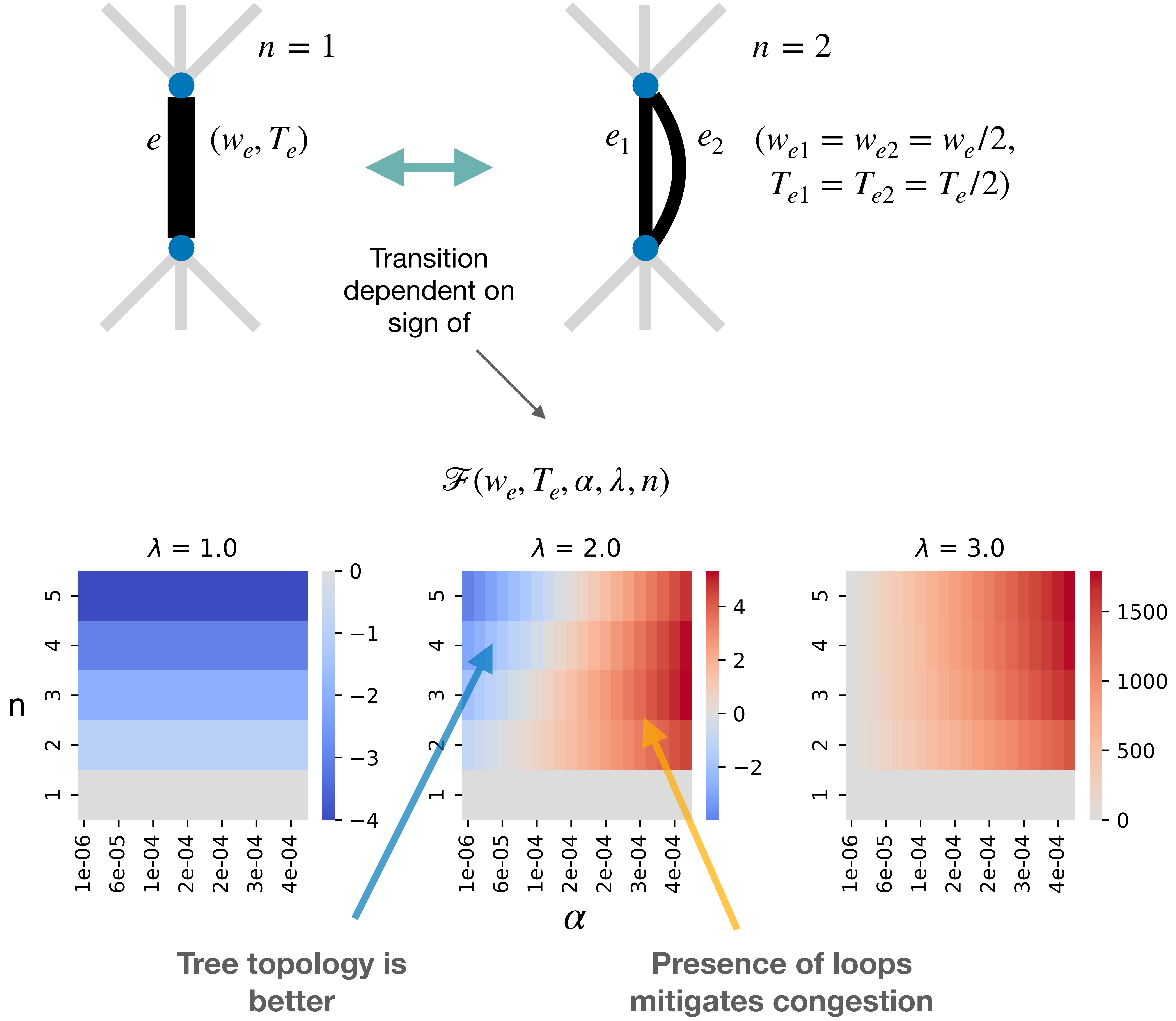
- Analytical condition given the congestion parameters

$$\frac{T_e}{w_e} \cdot \left[(1-n) + \alpha T_e^{\lambda}\left(1 - \frac{1}{n^{\lambda-1}}\right)\right] \le 0$$

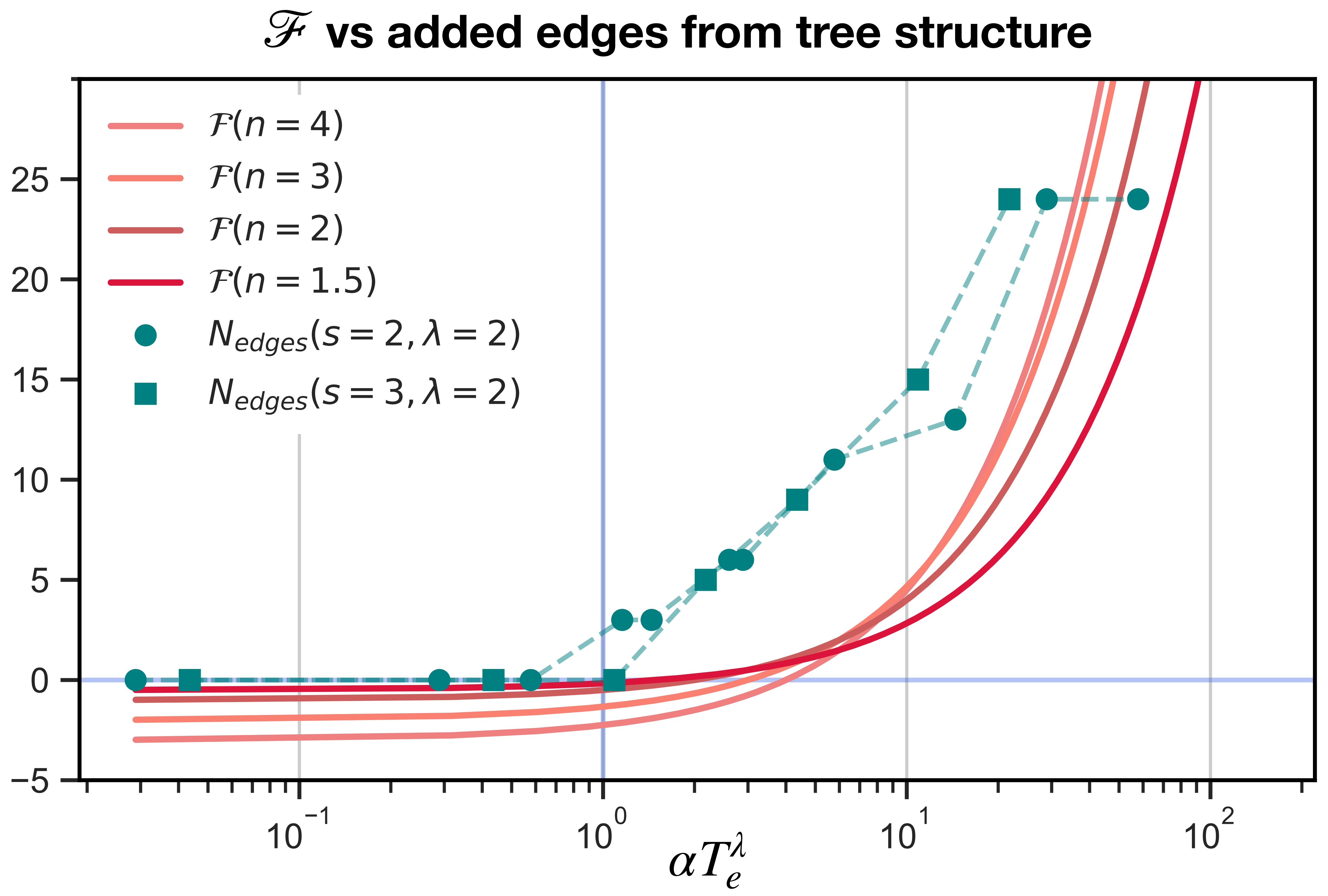
$$\frac{T_e}{w_e} \cdot \mathcal{F}\left(n, \alpha, T_e, \lambda\right) \le 0.$$

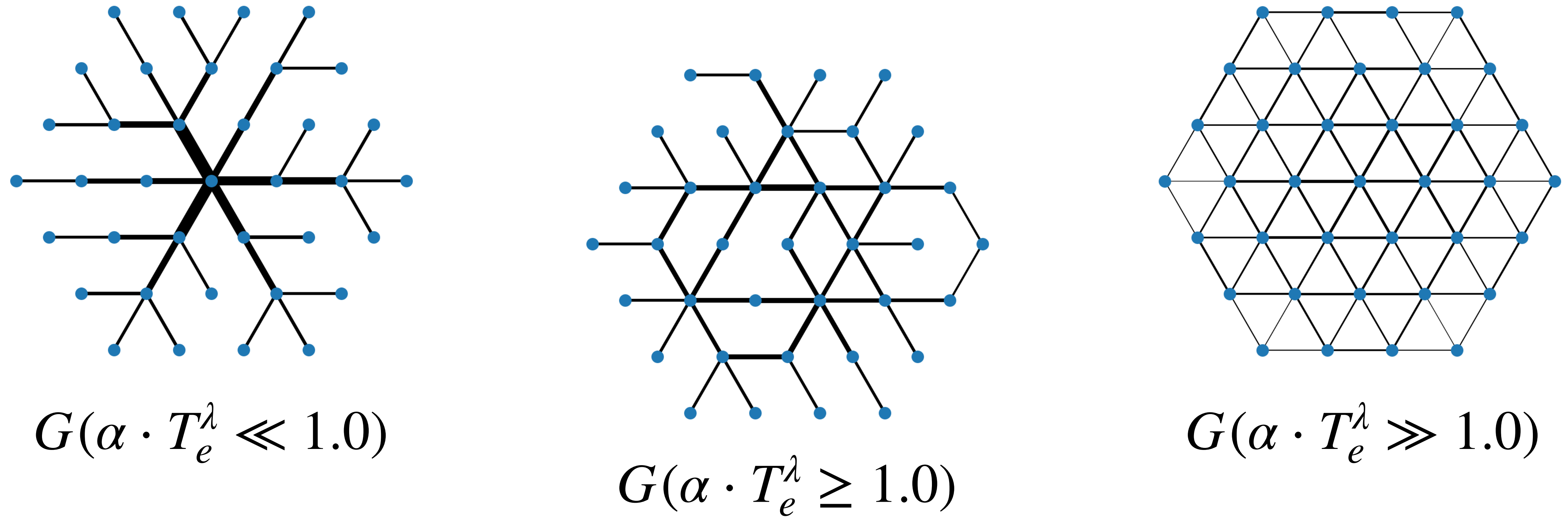


Adding user equilibrium:
Simulations vs analytical prediction

- Compare simulated topologies against analytical prediction: emergence of new edges from tree
  $N_{edges} = N_{edges}(G) - N_{edges}(G^{tree})$

- Transition from tree to a complete lattice to mitigate congestion





Take home messages

- Transition between complex spatial topologies driven by the range of flows
- Additional paths and loops are required to mitigate congestion processes
- Core + branches topology emerges as optimal when flows are characterized by patterns resembling human mobility in cities

PHYSICAL REVIEW X **14,** 021050 (2024)

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Emergence of Complex Network Topologies from Flow-Weighted Optimization of Network Efficiency

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Thank you!

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