Neutrino Oscillations in Vacuum and Matter [1]

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Notes for neutrino oscillations in vacuum and dense matter.

Vacuum Oscillations

Schrodinger equation is

$$i\partial_t |\Psi\rangle = \mathbf{H} |\Psi\rangle , \tag{1}$$

where for relativistic neutrinos, the energy is

$$\begin{aligned} \mathbf{H} &= \sqrt{p^2 + m^2} \\ &= p\sqrt{1 + \frac{m^2}{p^2}} \\ &\approx p(1 + \frac{1}{2}\frac{m^2}{p^2}). \end{aligned}$$

In general the flavor eigenstates are the mixing of the mass eigenstates with a unitary matrix $\mathbf{U}$, that is

$$|\nu_\alpha\rangle = \mathbf{U}_{\alpha i} |\nu_i\rangle , \tag{2}$$

where the αs are indices for flavor states while the is are indices for mass eigenstates.

To find out the equation of motion for flavor states, plugin in the initary tranformation,

$$i\mathbf{U}_{\alpha i}\partial_t |\nu_i\rangle = \mathbf{U}_{\alpha i}\mathbf{H}^m_{ij} |\nu_j\rangle . \tag{3}$$

I use index ^{m} for representation of Hamiltonian in mass eigenstates. Applying the unitary condition of the transformation,

$$\mathbf{I} = \mathbf{U}^\dagger\mathbf{U}, \tag{4}$$

I get

$$i\mathbf{U}_{\alpha i}\partial_t |\nu_i\rangle = \mathbf{U}_{\alpha i}\mathbf{H}^m_{ij}\mathbf{U}^\dagger_{\mathbf{j\beta}}\mathbf{U}_{\beta k} |\nu_k\rangle , \tag{5}$$

which is simplified to

$$i\partial_t |\nu_\alpha\rangle = \mathbf{H}^f_{\alpha\beta} |\nu_\beta\rangle , \tag{6}$$

since the transformation is time independent.

2 Flavor States

For 2 flavor neutrinos the Hamiltonian in the representation of propagation states,

$$\mathbf{H} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} = \begin{pmatrix} p_1 + \frac{1}{2}\frac{m_1^2}{p_1} & 0 \\ 0 & p_2 + \frac{1}{2}\frac{m_1^2}{p_2} \end{pmatrix}.$$

The equation of motion in matrix form is

$$i\partial_t \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} p_1 + \frac{1}{2}\frac{m_1^2}{p_1} & 0 \\ 0 & p_2 + \frac{1}{2}\frac{m_1^2}{p_2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{7}$$

The flavor eigenstate is a mixing of the propagation eigenstates,

$$\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} \cos\theta_v & \sin\theta_v \\ -\sin\theta_v & \cos\theta_v \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} \tag{8}$$

Denote the rotation matrix using $\mathbf{U}$, the transofmation can be written as

$$|\nu_\alpha\rangle = \mathbf{U}_{\alpha i} |i\rangle , \tag{9}$$

where α is for the flavor eigenstates and i is for the mass eigenstates.

3 Flavor States

Oscillations in Dense Medium