Distributions

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Contents

1 Normal Distribution

1.1 Basic Form

- X: random variable.
- μ: mean or expectation.
- σ: standard deviation.
- σ^2: variance.
- $X \sim \mathcal{N}(\mu, \sigma^2)$: X is distributed normally with μ and σ^2,

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2}). \tag{1}$$

1.2 Multivariate Form

- $\mathbf{X}$: random vector, $\mathbf{X} = (X_1, \ldots, X_k)$.
- $\boldsymbol{\mu}$: mean or expectation vector, $\boldsymbol{\mu} = \mathrm{E}[\mathbf{X}] = (\mathrm{E}[\mathbf{X_1}], \mathrm{E}[\mathbf{X_2}], \ldots, \mathrm{E}[\mathbf{X_k}])$.
- $\boldsymbol{\Sigma}$: $k \times k$ convariance matrix, $\Sigma_{i,j} = \mathrm{E}[(X_i - \mu_i)(X_j - \mu_j)] = \mathrm{Cov}[X_i, X_j]$, where $1 \leq i, j \leq k$.

- Q: precision matric $\mathbf{\Sigma}^{-1}$
- $X \sim \mathcal{N}_{\|}(\boldsymbol{\mu}, \mathbf{\Sigma})$: X is distributed normally with $\boldsymbol{\mu}$ and $\mathbf{\Sigma}$ and k indicates k-dimension.

For **non-degenerate case:**

$$f_{\mathbf{X}}(x_1, \ldots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\mathbf{\Sigma}|}} \exp(-\frac{(\mathbf{X}-\boldsymbol{\mu})^{\mathrm{T}} \mathbf{\Sigma}^{-1} (\mathbf{X}-\boldsymbol{\mu})}{2}) \tag{2}$$

The descriptive statistic $\sqrt{(\mathbf{X}-\boldsymbol{\mu})^{\mathrm{T}} \mathbf{\Sigma}^{-1} (\mathbf{X}-\boldsymbol{\mu})}$ is known as the Mahalanobis distance.

2 Poisson Distribution

$$f(k;\lambda) = \mathrm{P}(X=k) = \frac{\lambda^k e^- - \lambda}{k!} \tag{3}$$

where the positive real number $\lambda = \mathrm{E}(X) = \mathrm{Var}X$ is the average number of events per intervel, and $k = 0, 1, 2, \ldots$.