

hw5

February 28, 2024

1 Deconvolution

1.1 Part a

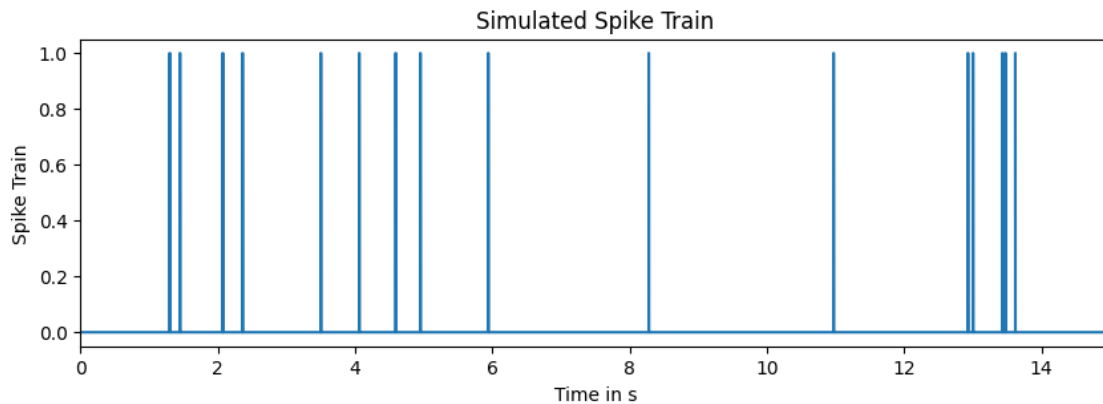
```
[1]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from scipy.fft import fft, fftfreq, ifft

[2]: T = 15
Fs = 2000
p = 0.0003
N = T * Fs

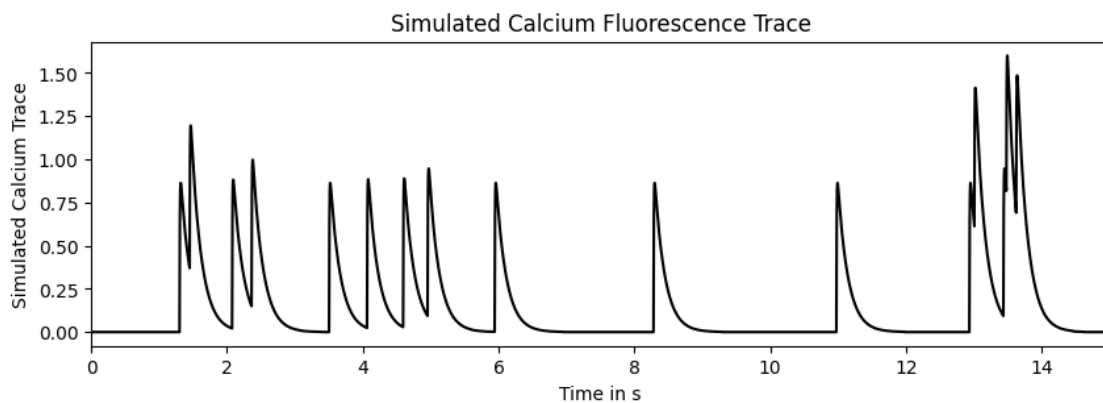
np.random.seed(42)
spike_train = (np.random.rand(N) < p).astype(int)
time_s = np.arange(N) / Fs

plt.figure(figsize = (10, 3))
plt.plot(time_s, spike_train, drawstyle = 'steps-pre')
plt.xlabel('Time in s')
plt.ylabel('Spike Train')
plt.title('Simulated Spike Train')
plt.xlim(0, T)
plt.show()
```



1.2 Part b

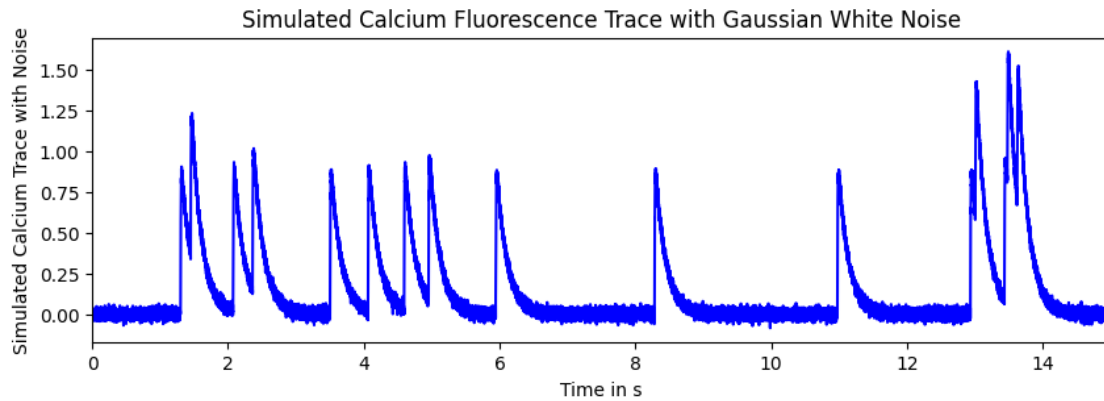
```
[3]: def impulse_response(t, tau1 = 0.005, tau2 = 0.15):  
    double_el_ir_function = (t >= 0) * (1 - np.exp(-t / tau1)) * np.exp(-t /  
    ↪tau2)  
    return double_el_ir_function  
  
delta_t = 1/Fs  
time_impulse_response = np.arange(-1, 1, delta_t)  
h = impulse_response(time_impulse_response)  
simulated_calcium_trace = np.convolve(spike_train, h, mode = 'same')  
  
plt.figure(figsize = (10, 3))  
plt.plot(time_s, simulated_calcium_trace, color = 'black')  
plt.xlabel('Time in s')  
plt.ylabel('Simulated Calcium Trace')  
plt.title('Simulated Calcium Fluorescence Trace')  
plt.xlim(0, T)  
plt.show()
```



1.3 Part c

```
[4]: noise_rms = 0.02  
noise = noise_rms * np.random.randn(N)  
simulated_calcium_trace_noise = simulated_calcium_trace + noise  
  
plt.figure(figsize = (10, 3))  
plt.plot(time_s, simulated_calcium_trace_noise, color = 'blue')  
plt.xlabel('Time in s')  
plt.ylabel('Simulated Calcium Trace with Noise')
```

```
plt.title('Simulated Calcium Fluorescence Trace with Gaussian White Noise')
plt.xlim(0, T)
plt.show()
```

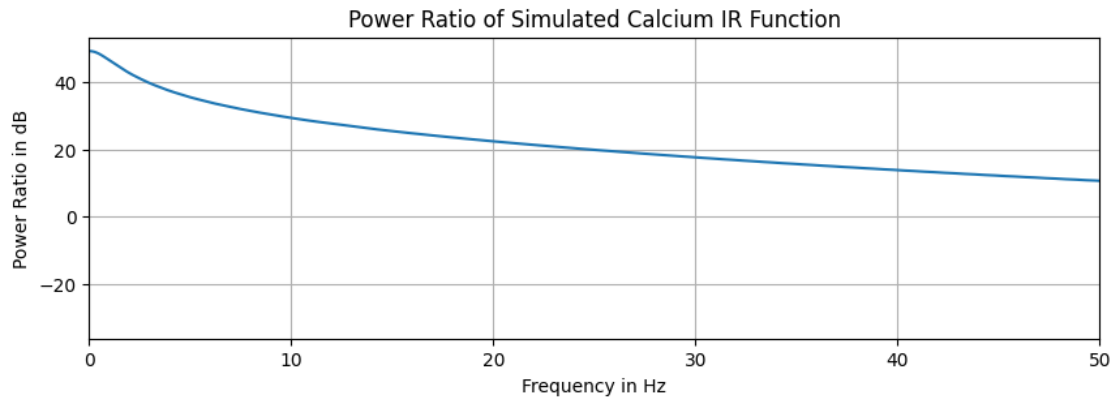


1.4 Part d

low-pass filter

```
[5]: ft_ht = fft(h, n = N)
frequencies = fftfreq(N, d = delta_t)
power_ratio = np.abs(ft_ht)**2
dB_of_power_ratio = 10 * np.log10(power_ratio)

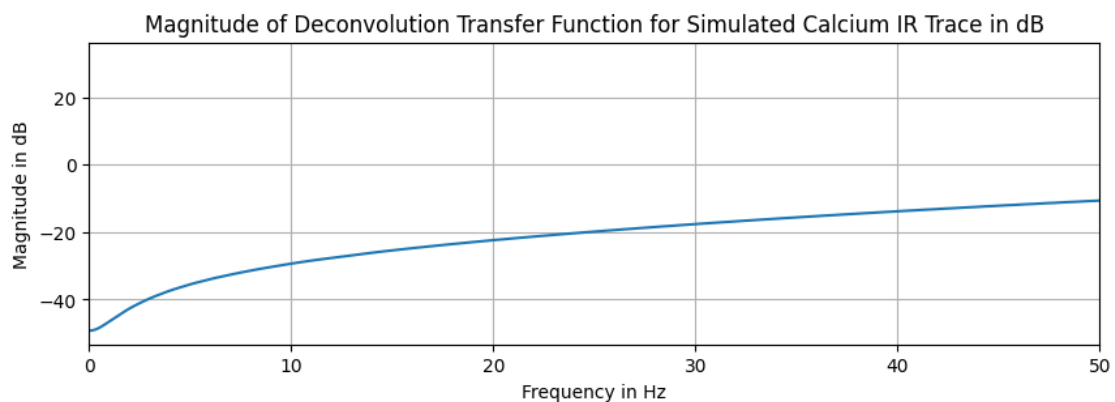
plt.figure(figsize = (10, 3))
plt.plot(frequencies[:N//2], dB_of_power_ratio[:N//2])
plt.xlim(0, 50)
plt.xlabel('Frequency in Hz')
plt.ylabel('Power Ratio in dB')
plt.title('Power Ratio of Simulated Calcium IR Function')
plt.grid(True)
plt.show()
```



1.5 Part e

```
[6]: D_w = 1 / ft_ht
D_w_squared = np.abs(D_w)**2
D_w_squared_dB = 10 * np.log10(D_w_squared)

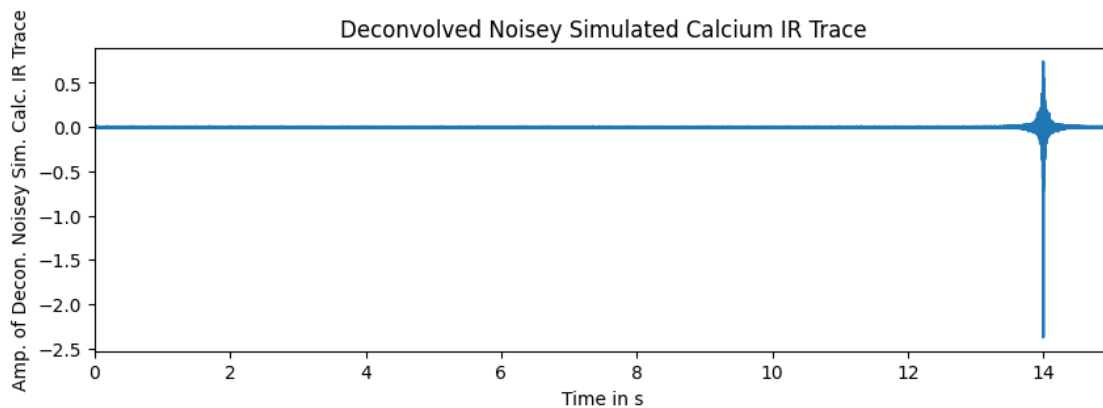
plt.figure(figsize = (10, 3))
plt.plot(frequencies[:N//2], D_w_squared_dB[:N//2])
plt.xlim(0, 50)
plt.xlabel('Frequency in Hz')
plt.ylabel('Magnitude in dB')
plt.title('Magnitude of Deconvolution Transfer Function for Simulated Calcium IR Trace in dB')
plt.grid(True)
plt.show()
```



1.6 Part f

```
[7]: sc_deconvolved_signal = ifft(simulated_calcium_trace_noise * D_w).real

plt.figure(figsize = (10, 3))
plt.plot(time_s, sc_deconvolved_signal)
plt.xlim(0, T)
plt.xlabel('Time in s')
plt.ylabel('Amp. of Decon. Noisy Sim. Calc. IR Trace')
plt.title('Deconvolved Noisy Simulated Calcium IR Trace')
plt.show()
```



1.7 Part g

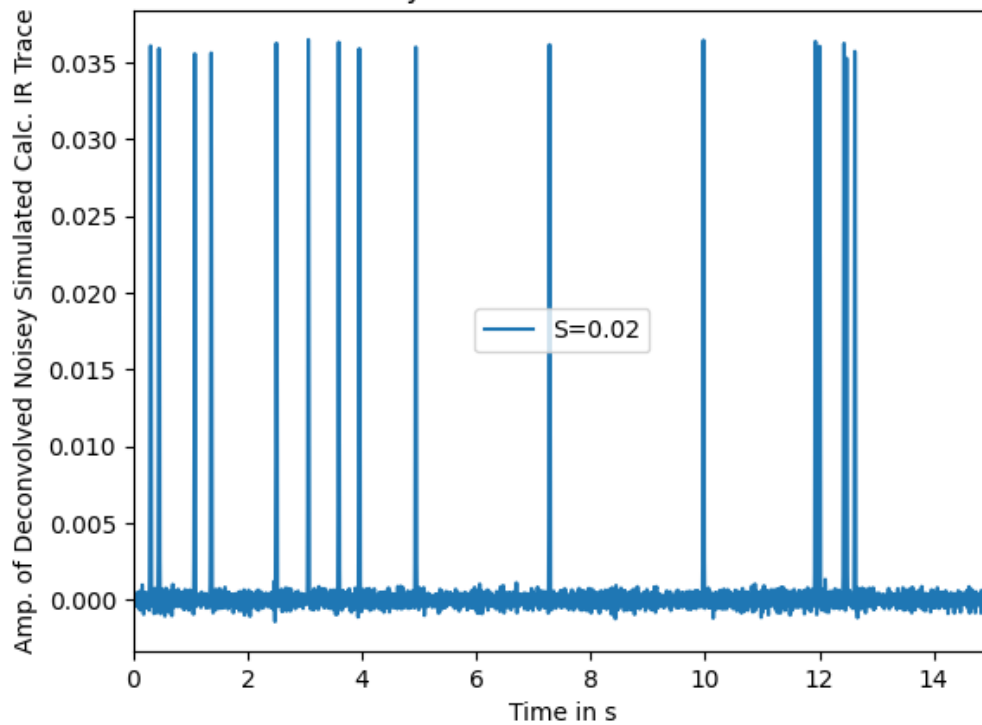
```
[8]: snr = [0.02]

for snr_index in snr:

    D_wiener = np.conj(ft_ht) / (np.abs(ft_ht)**2 + 1/snr_index)
    sc_deconvolved_signal_wiener_fft = fft(simulated_calcium_trace_noise, n = N)
    ↪N) * D_wiener
    sc_deconvolved_signal_wiener = ifft(sc_deconvolved_signal_wiener_fft).real
    plt.plot(time_s, sc_deconvolved_signal_wiener, label = f"S={snr_index}")

plt.xlabel('Time in s')
plt.ylabel('Amp. of Deconvolved Noisy Simulated Calc. IR Trace')
plt.title('Wiener Deconvolved Noisy Simulated Calcium IR Trace for 0.02 SNR_↪
    ↪Value')
plt.legend()
plt.xlim(0, T)
plt.show()
```

Wiener Deconvolved Noisy Simulated Calcium IR Trace for 0.02 SNR Value

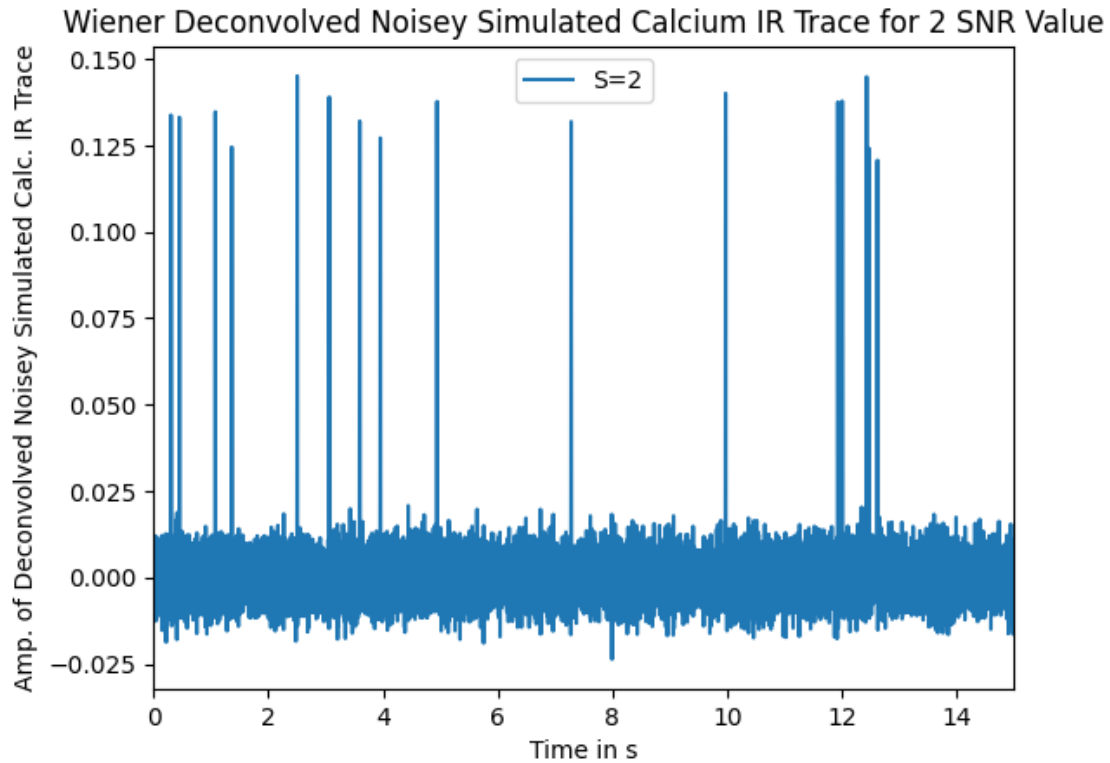


```
[9]: snr = [2]

for snr_index in snr:

    D_wiener = np.conj(ft_ht) / (np.abs(ft_ht)**2 + 1/snr_index)
    sc_deconvolved_signal_wiener_fft = fft(simulated_calcium_trace_noise, n = N)
    ↪N) * D_wiener
    sc_deconvolved_signal_wiener = ifft(sc_deconvolved_signal_wiener_fft).real
    plt.plot(time_s, sc_deconvolved_signal_wiener, label = f"S={snr_index}")

plt.xlabel('Time in s')
plt.ylabel('Amp. of Deconvolved Noisy Simulated Calc. IR Trace')
plt.title('Wiener Deconvolved Noisy Simulated Calcium IR Trace for 2 SNR_↪
    ↪Value')
plt.legend()
plt.xlim(0, T)
plt.show()
```



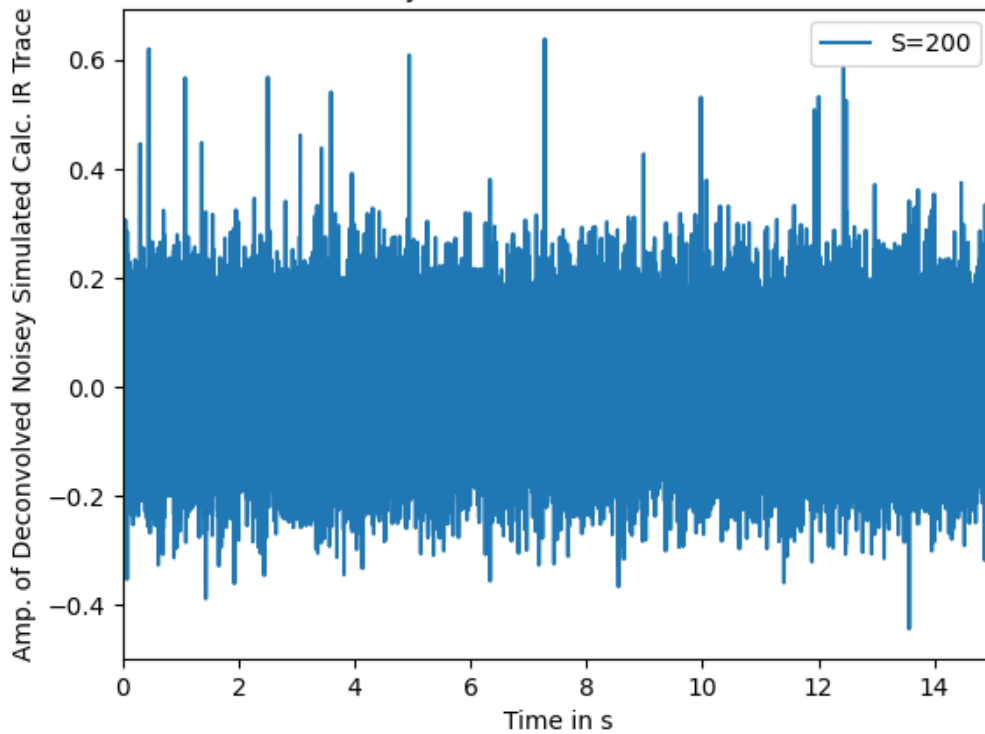
```
[10]: snr = [200]

for snr_index in snr:

    D_wiener = np.conj(ft_ht) / (np.abs(ft_ht)**2 + 1/snr_index)
    sc_deconvolved_signal_wiener_fft = fft(simulated_calcium_trace_noise, n = N)
    ↪N) * D_wiener
    sc_deconvolved_signal_wiener = ifft(sc_deconvolved_signal_wiener_fft).real
    plt.plot(time_s, sc_deconvolved_signal_wiener, label=f"S={snr_index}")

plt.xlabel('Time in s')
plt.ylabel('Amp. of Deconvolved Noisy Simulated Calc. IR Trace')
plt.title('Wiener Deconvolved Noisy Simulated Calcium IR Trace for 200 SNR_
↪Value')
plt.legend()
plt.xlim(0, T)
plt.show()
```

Wiener Deconvolved Noisy Simulated Calcium IR Trace for 200 SNR Value



```
[11]: snr = [0.02, 2, 200]

for snr_index in snr:
    D_wiener = np.conj(ft_ht) / (np.abs(ft_ht)**2 + 1/snr_index)
    sc_deconvolved_signal_wiener_fft = fft(simulated_calcium_trace_noise, n = N)
    ↪N) * D_wiener
    sc_deconvolved_signal_wiener = ifft(sc_deconvolved_signal_wiener_fft).real
    plt.plot(time_s, sc_deconvolved_signal_wiener, label=f"S={snr_index}")

plt.xlabel('Time in s')
plt.ylabel('Amp. of Deconvolved Noisy Simulated Calc. IR Trace')
plt.title('Wiener Deconvolved Noisy Simulated Calcium IR Trace for Several SNR_
    ↪Values')
plt.legend()
plt.xlim(0, T)
plt.show()
```


Wiener Deconvolved Noisy Simulated Calcium IR Trace for Several SNR Values

