Problem definition (physics + discretization)

Category	Setting / Value
Problem type	*2D plane-strain Neo-Hookean elasticity (square domain)*
Domain	*([0, L] \times [0, L]), with (L = 1.0)*
Mesh type	*Structured quadrilateral mesh (2D square)*
Mesh resolution	*(N = 42) (42 × 42 grid; used by `create_2D_square_mesh(L, N)`)*
FE element	*4-node quad, Neo-Hookean (`NeoHookeMechanicalLoss2DQuad`)*
Gauss integration	*`num_gp = 2` (2×2 Gauss points per element)*
Material model	*Neo-Hookean, isotropic*
Young's modulus	*(E = 1.0)*
Poisson's ratio	*(\nu = 0.3)*
BC type	*Dirichlet on left/right boundaries for (U_x, U_y)*
BC – left edge	*(U_x = 0.0), (U_y = 0.0)*
BC – right edge	*(U_x = 0.1), (U_y = 0.1)*
Control type	*FourierControl (spatially varying stiffness field (K(x,y)))*
Fourier x-frequencies	*([2, 4, 6])*
Fourier y-frequencies	*([2, 4, 6])*

Category	Setting / Value
Fourier z-frequencies	([0]) (effectively 2D)
Fourier amplitude scale	`beta = 20`
K field range	`min = 1e-1`, `max = 1.0` (clamp of heterogeneity field)
Samples in dictionary	200 Fourier coefficient vectors (when generated via `create_random_fourier_samples`)

PI-FNO model & training hyperparameters (parametric/otf/transfer)

Category	Setting / Value
Operator model	`FourierNeuralOperator2D`
FNO modes (x)	`modes1 = 6`
FNO modes (y)	`modes2 = 6`
Channel width	`width = 8`
Network depth	`depth = 4` residual blocks
Last projection channels	`channels_last_proj = 32`
Output channels	`out_channels = 2` (displacement (u_x, u_y))

Category	Setting / Value
Output scaling	`output_scale = 0.1`
Total number of params	≈ 37,538 (printed in log)
FOL wrapper	`PhysicsInformedFourierParametricOperatorLearning`
Loss	Physics-informed Neo-Hookean FE residual (`NeoHookeMechanicalLoss2DQuad`)
Optimizer	Optax Adam, learning rate `1e-3`
LR scheduler (unused here)	`linear_schedule(1e-4 → 1e-5, transition_steps = 5000)`
Max epochs	`num_epochs = 5000`
Convergence settings	`relative_error = 1e-100`, `absolute_error = 1e-100` (effectively "epoch-based")
Train batch size (param.)	`batch_size = 5`
Train indices	`0–179` → 180 samples
Test indices	`181–199` → 19 samples (sample 180 reserved for OTF)
Training plots	`total_loss`, `residual_rms_batch_mean` (physics residual RMS)
Train checkpoint	Least-loss checkpoint every 100 epochs → `.../flax_train_state`
Final state save	`flax_final_state` in the parametric output folder

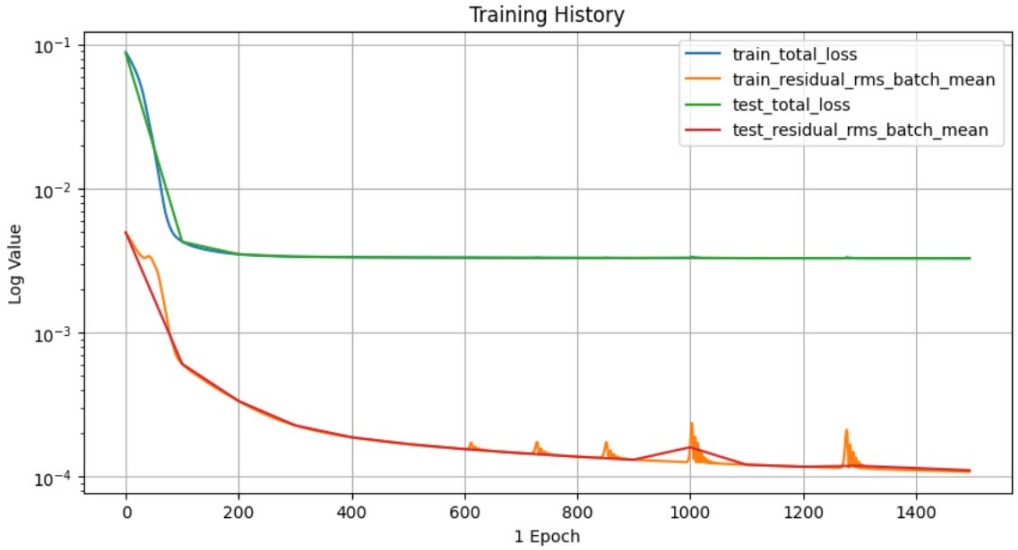
OTF fine-tuning & FE solver hyperparameters (slide 3: “OTF from param”)

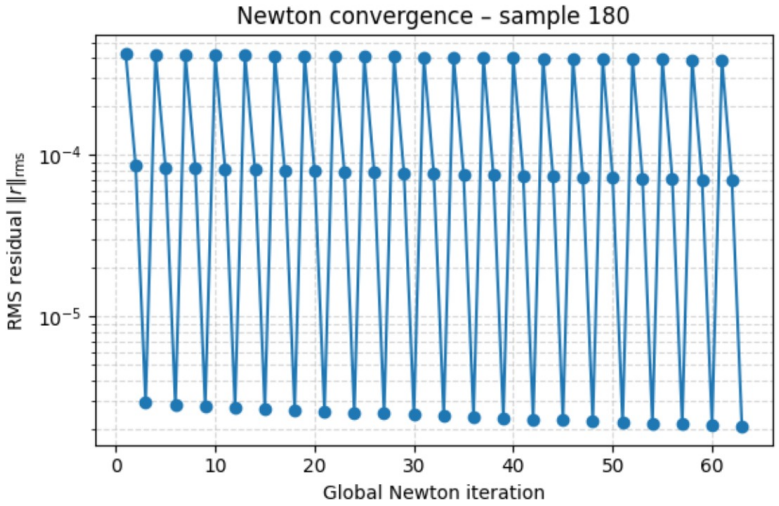
OTF PI-FNO (starting from parametric weights)

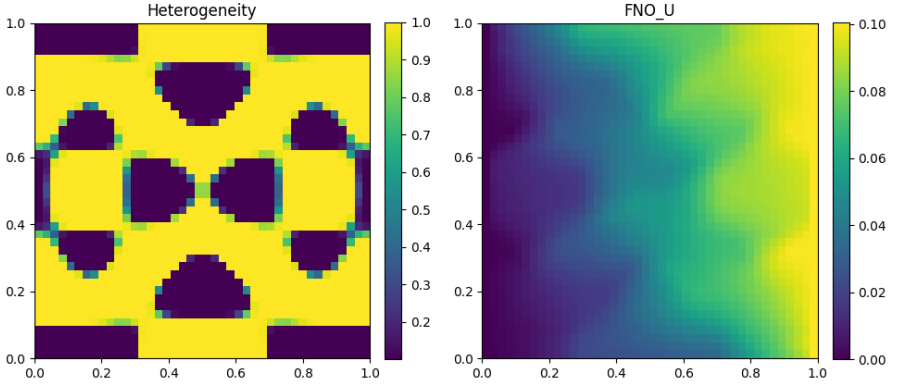
Category	Setting / Value
Initial weights	*Restored from parametric run: `./nn_output_mechanical_2D_neohooke_pi_fno_param/flax_train_state`*
OTF sample ID	*`otf_id = 180`*
Train set (OTF)	*Only coefficient vector of sample 180 (shape (1 \times n_\text{coeffs}))*
Test set (OTF)	*Same as train: sample 180*
`parametric_learning` flag	*`False` (enables true on-the-fly mode)*
Eval cases	*`eval_cases = [180]`*
Batch size (OTF)	*`batch_size = 1`*
Max epochs (OTF)	*`num_epochs = 5000`*
Convergence settings	*Same: `relative_error = 1e-100`, `absolute_error = 1e-100`*
Plots during OTF	*`total_loss`, `residual_rms_batch_mean` saved in `nn_output_mechanical_2D_neohooke_pi_fno_otf_from_param`*
Checkpoint directory (OTF)	*`./nn_output_mechanical_2D_neohooke_pi_fno_otf_from_param/flax_train_state`*

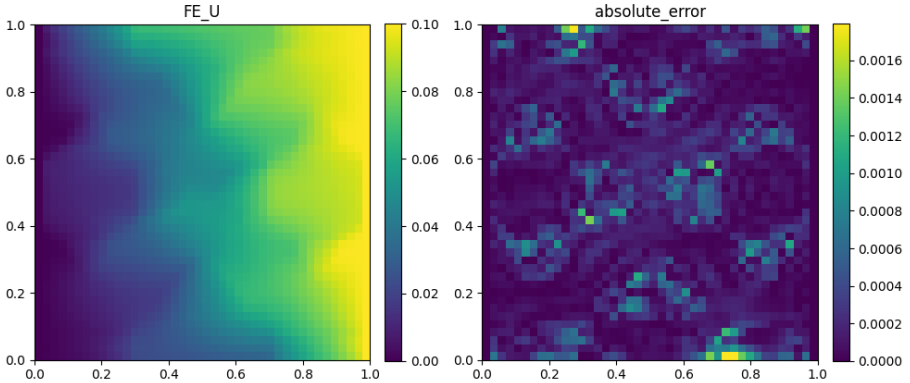
OTF- SLIDE 1

10 mins for training -2000 epochs



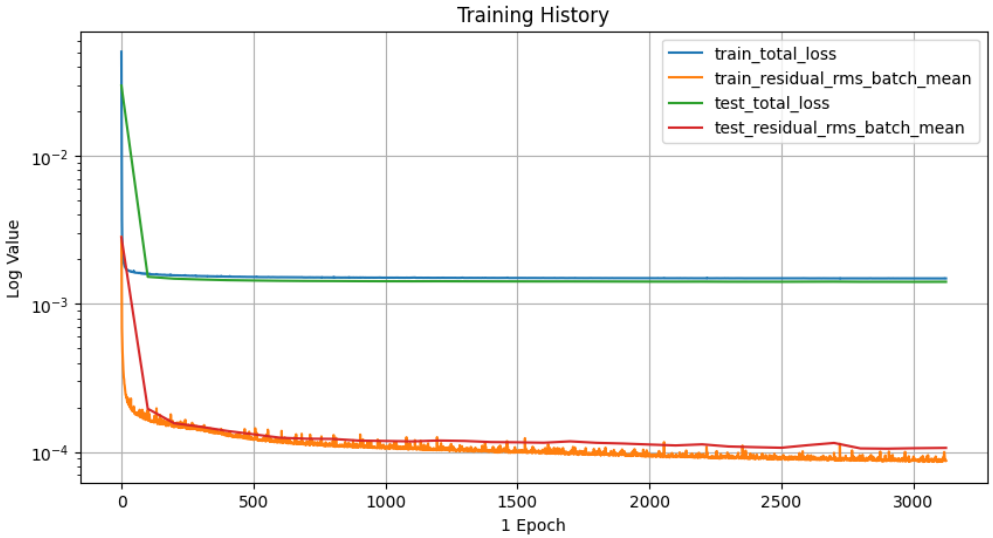


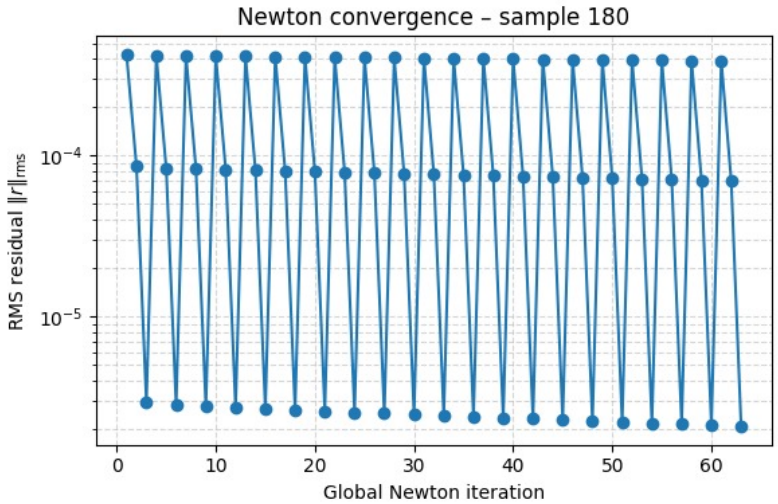


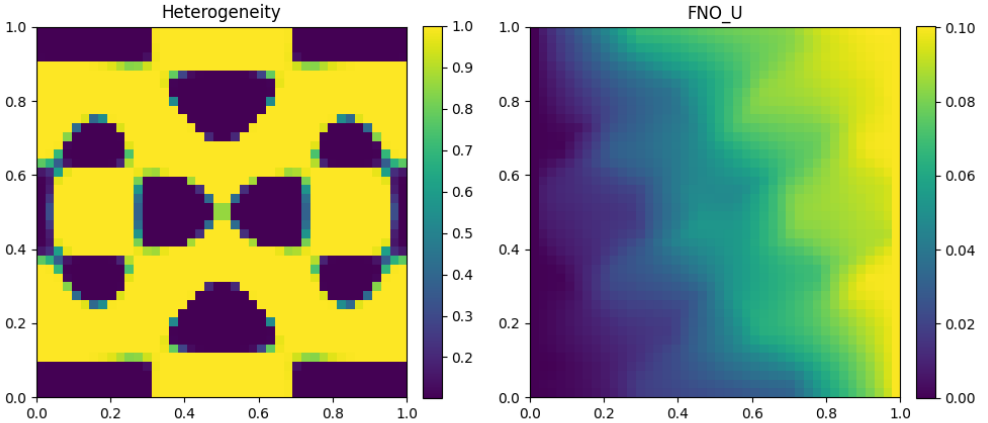


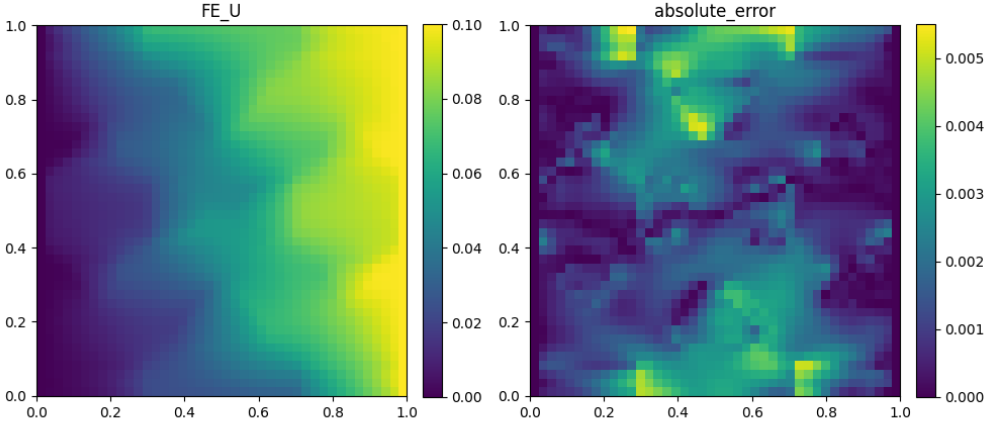
PARAMETRIC- SLIDE 2

30 mins for training – ~~Auto~~ Stopped between 3000-3500 epochs

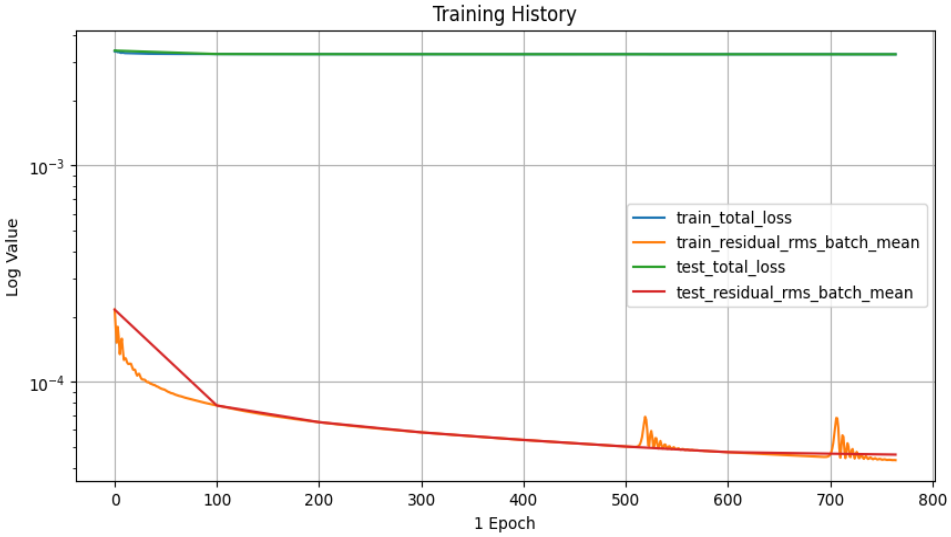








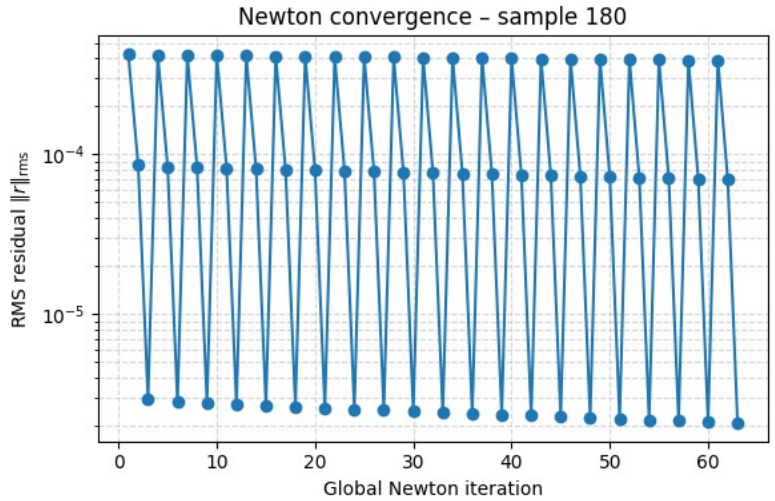
PARAMETRIC TRANSFER LEARN- SLIDE 3.1

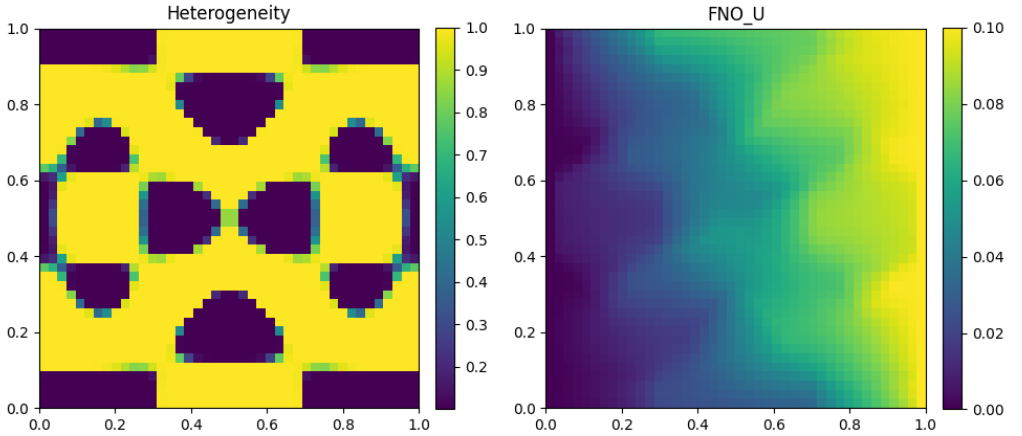


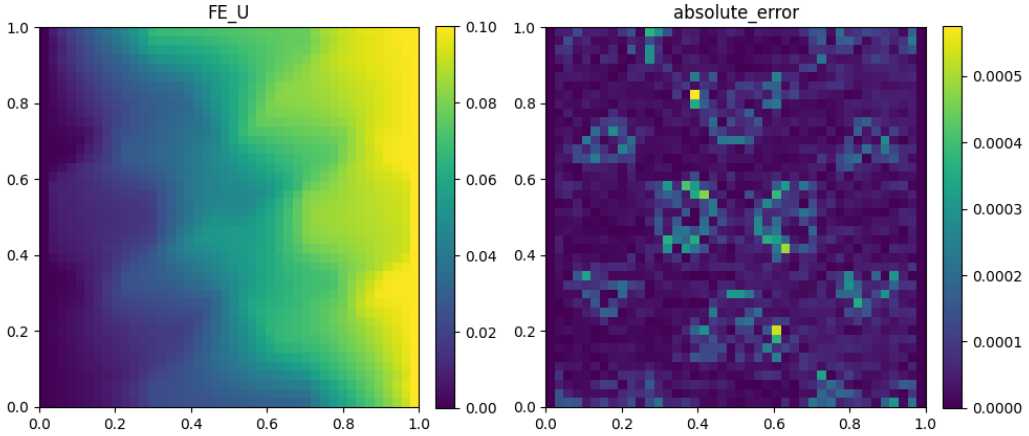
5 mins for training ***~~Auto~~ Stopped between 700-800 epochs***

Absolute Error reduction = 0.005 - 0.0005 = 0.0045

Relative improvement = 0.0045 / 0.005 = 0.9 = 90%



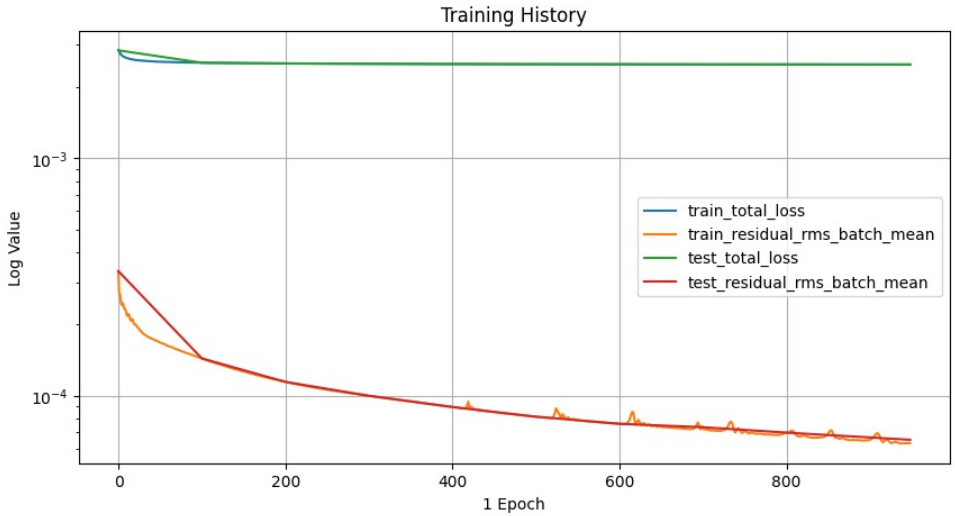


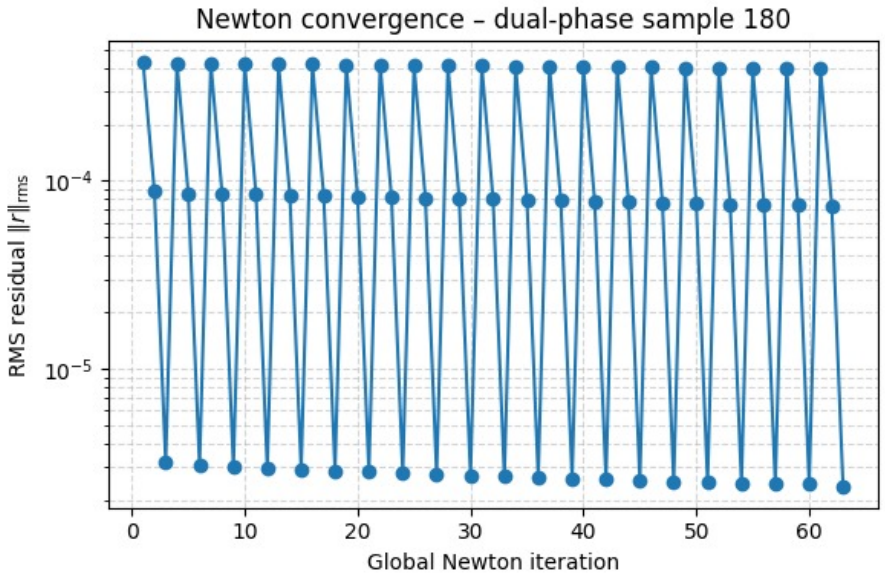


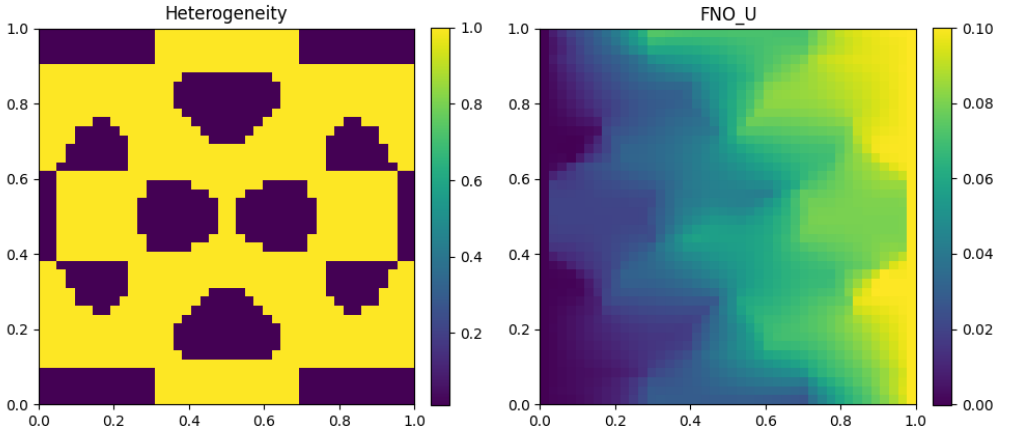
PARAMETRIC - TRANSFER LEARN - SLIDE 3.2

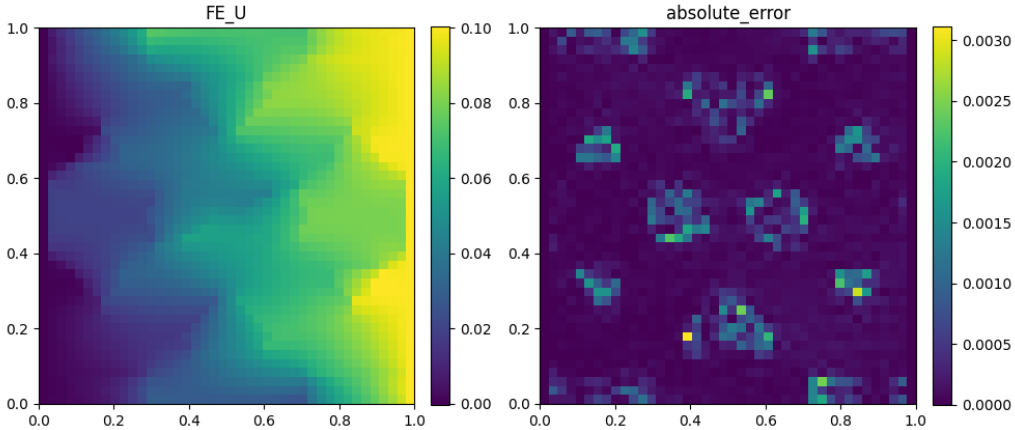
-Dual phase heterogeneous sample

7 mins for training ~~– Auto~~ Stopped between 700-800 epochs









PARAMETRIC TRANSFER LEARN- SLIDE 3.3

-Dual phase heterogeneous sample with Circular inclusion

4 mins for training ~~– Auto~~ Stopped between 500-600 epochs