Secant Lines, Average Rate of Change, and Applications

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Summary

This lecture introduces secant lines and the concept of average rate of change. It covers how to visualize secant lines, compute their slope using a given function and two points, and apply these concepts to real-world problems. The lecture emphasizes understanding the units of the average rate of change.

4.1 Secant Lines, Average Rate of Change, and Applications

This section focuses on understanding and calculating the slope of secant lines, also known as the average rate of change, and applying it to various scenarios.

Objectives

- Be able to visualize secant lines.
- Compute the slope of secant lines.
- Apply secant lines to real-world problems.

Definition(s) 4.1: The Slope of the Secant Line

- The slope of the secant line of y = f(x) through points P(x_1, f(x_1)) and Q(x2, f(x2)) is given by:

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

- Alternatively, this is referred to as the average rate of change on the interval [x_1, x2].

Visualizing a Secant Line

[Diagram]: A graph illustrating a function f(x) with two specified points, P and Q, on the curve. A straight line, the secant line, passes through these two points. It is noted that a secant line is defined by two specified points, but may pass through additional points on the function curve by

coincidence.

- To visualize and interact with secant lines and average rates of change, an applet can be used. The link is https://www.desmos.com/calculator/kktiplmtbk

Example 4.2: Calculating the Slope of a Secant Line

- Let f(x) = sqrt(x + 1). Find the slope of the secant line joining the points (3, f(3)) and (8, f(8)).
- Alternate Phrasing: Calculate the average rate of change of f(x) = sqrt(x + 1) on the interval [3, 8].

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$m = \frac{f(8) - f(3)}{8 - 3} = \frac{\sqrt{9} - \sqrt{4}}{5} = \frac{3 - 2}{5} = \frac{1}{5}$$

- The choice of x1 and x2 does not affect the final slope value, as (f(3) - f(8))/(3 - 8) also yields 1/5.

Example 4.3: Average Rate of Change from Tabular Data

- In the year 2010, Ryan started a business, Ryan's Friendly Board Game Emporium. To the left is a table of the number of board games sold each year. Find the average rate of change during 2010 to 2016 and interpret the result.

Year	Board Games Sold
2010	512
2011	601
2012	943
2013	1120
2014	1342
2015	1854
2016	2612

$$m = \frac{f(2016) - f(2010)}{2016 - 2010} = \frac{2612 - 512}{6} = \frac{2100}{6} = 350$$

- Interpretation:
  - On average, from 2010 to 2016, Ryan's number of board games sold increased by 350 each year.

Remark 4.4: Units of Average Rate of Change

- The average rate of change of f(x) has units of: units of f / units of x.