



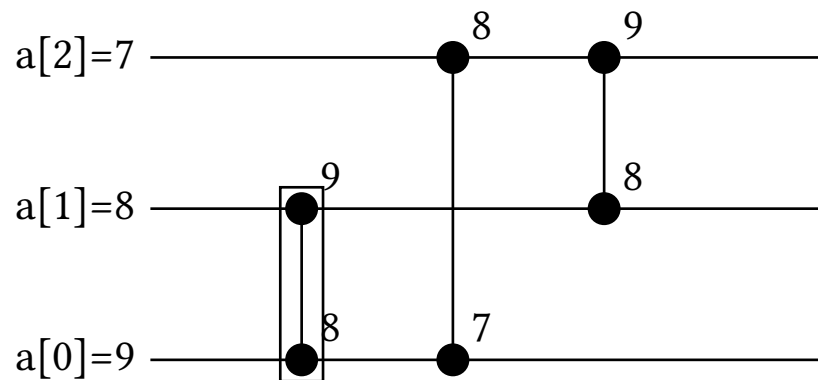
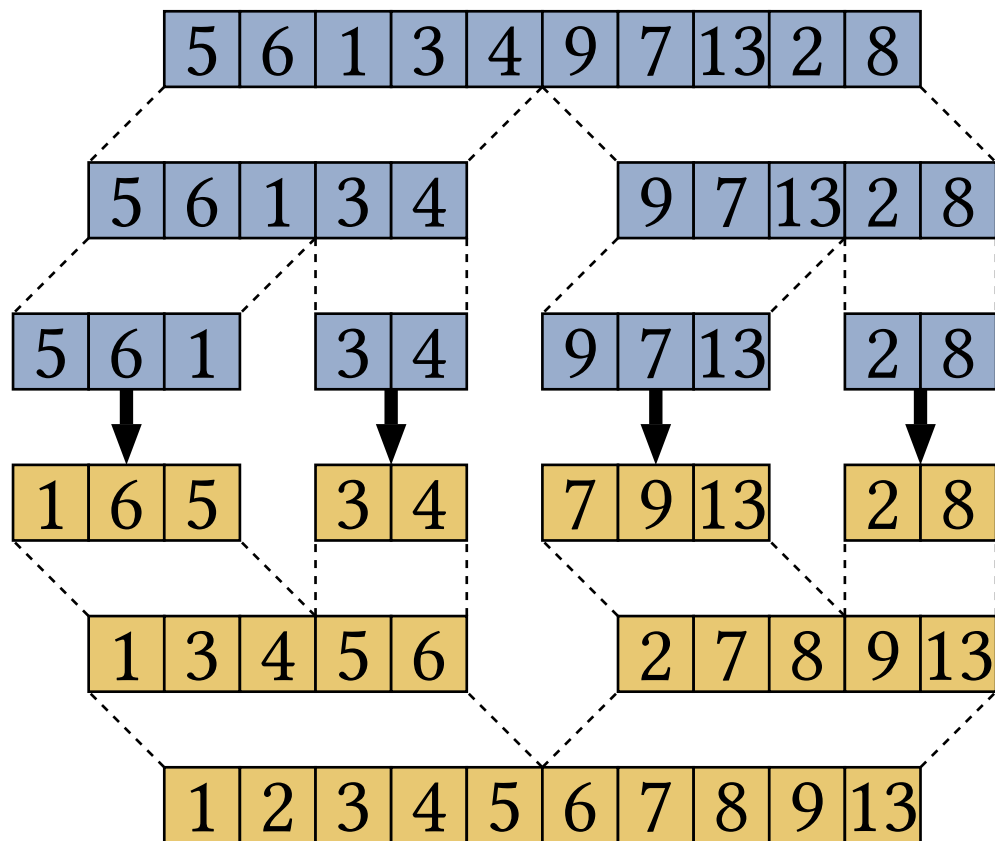
Synthesis of Sorting Kernels

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Sorting Kernels



```
mov rdi, rax  
cmp rbx, rax  
cmovl rax, rbx  
cmovl rbx, rdi
```



Model

			register	swap	lt	gt
mov	s1	r2	13	9	-	-
cmp	r1	r2	13	9	9	-
cmovg	r2	r1	13	9	9	-
cmovg	r1	s1	13	13	9	-
			9	13	9	-

sample from
{mov, cmovl, cmovg, cmp}





TSNE-embedding of solutions

n	Program Length	Search Space
3	11	$10^{19.9}$
4	20	10^{40}
5	≈ 33	$10^{71.2}$
6	≈ 45	$10^{108.4}$

5602 solutions for $n = 3$



State of the Art

- sorting network 
- handoptimized 
- $n = 3$: 6min
- $n = 4$: 30min
- $n = 5$: 17.5h



faster synthesis



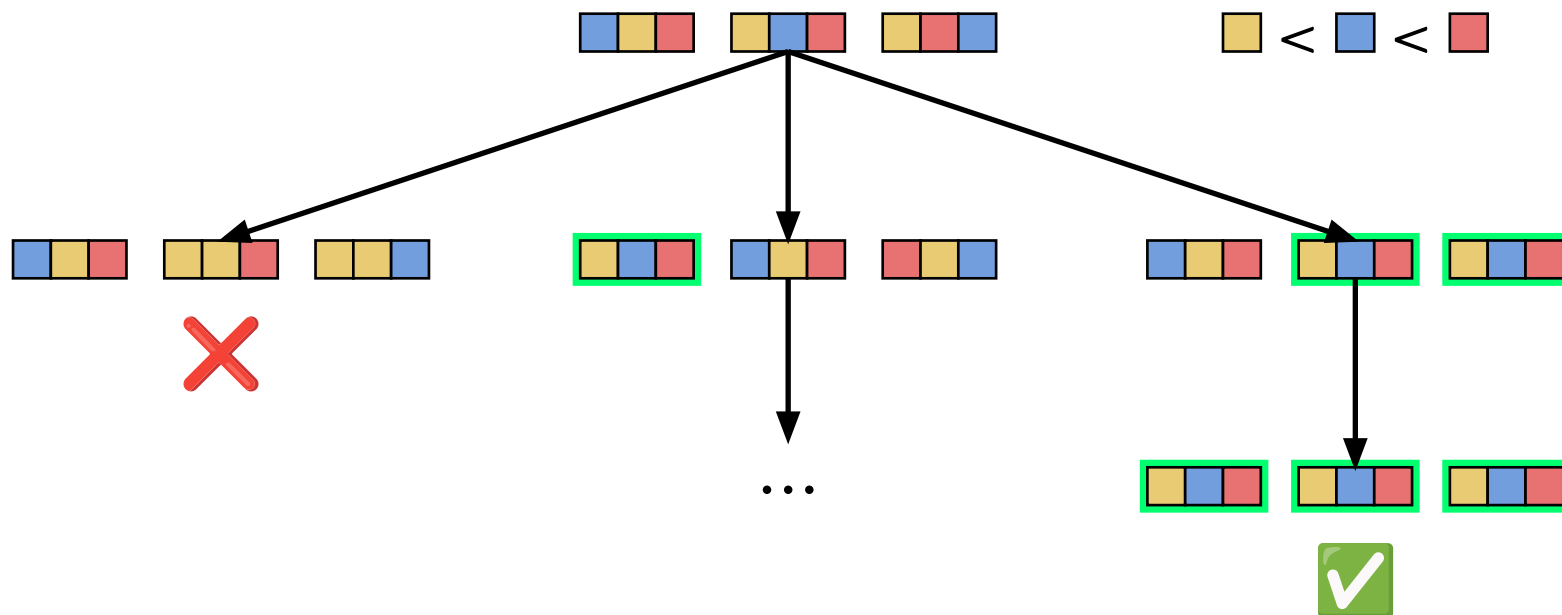
faster sorting kernels









minimality proof



Enumerative Synthesis



Enumerative Synthesis







1.  Select open state
2.  Apply Instruction
3.  Check for viability
4.  Check for solution
5.  Cut non-promising
6.  Deduplicate states

A★ with heuristics:

- permutations
- permutations + scratch register
- delete-relaxed
(maximum per permutation)



Enumerative Synthesis

1.  Select open state
2.  Apply Instruction
3.  Check for viability
4.  Check for solution
5.  Cut non-promising
6.  Deduplicate states

Remove redundant/non-sensical:







- `cmp r1 r2; cmp r1 r3`
- `cmp r1 r1`

Restrict to beneficial:

- `delete-relaxed`
- `cmp r2 r1 → cmp r1 r2`



Enumerative Synthesis







1.  Select open state
2.  Apply Instruction
3.  Check for viability
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Cut programs:

- number eliminated
- longer than bound/solution
- can not be completed in time









Enumerative Synthesis

1.  Select open state
2.  Apply Instruction
3.  Check for viability
4.  Check for solution
5.  Cut non-promising
6.  Deduplicate states

All permutations already sorted



Enumerative Synthesis







1.  Select open state
2.  Apply Instruction
3.  Check for viability
4.  Check for solution
5.  Cut non-promising
6.  Deduplicate states

Cut if
permutation count $> k \times \text{best}$

Cut	Solutions
$k = 1$	222
$k = 1.5$	838
$k = 2$	5602
$k = \infty$	5602



Enumerative Synthesis

1.  Select open state
2.  Apply Instruction
3.  Check for viability
4.  Check for solution
5.  Cut non-promising
6.  Deduplicate states

Hashset-based deduplication
of states



Solver-Based Techniques

$$\begin{aligned} & \forall r : P(r) = o \rightarrow \\ & \underbrace{(\forall 1 \leq i \leq |r| : o_i \leq o_{i+1})}_{\text{ascending}} \wedge \\ & \underbrace{(\forall x : |\{i : r_i = x\}| = |\{i : o_i = x\}|)}_{\text{same elements}} \end{aligned}$$



Solver-Based Techniques

$$r \in \text{Perm}(1..n)$$

$$\forall r : P(r) = o \rightarrow \forall 1 \leq i \leq r : o_i = i$$



Solver-Based Techniques

$$\bigwedge_{r \in \text{Perm}(1..n)} \bigwedge_{1 \leq i \leq n} P(r)_i = i$$

Heuristics:

- `cmp r1 r2; cmp r2 r3` \rightarrow `cmp r2 r3`
- `cmp r1 r1` \rightarrow `noop`
- `cmp r3 r2` \rightarrow `cmp r2 r3`
- only read initialized
- do not make uncompleteable



Solver-Based Synthesis $n = 3$

SMT	Approach
97min	CEGIS, arbitrary inputs
25min	CEGIS, 1.. n
44min	all permutations
—	SyGuS (CVC5, Metalift)

Planning	Approach
679s	Scorpion planner
216s	Lama planner grounded
3.54s	Lama Planner

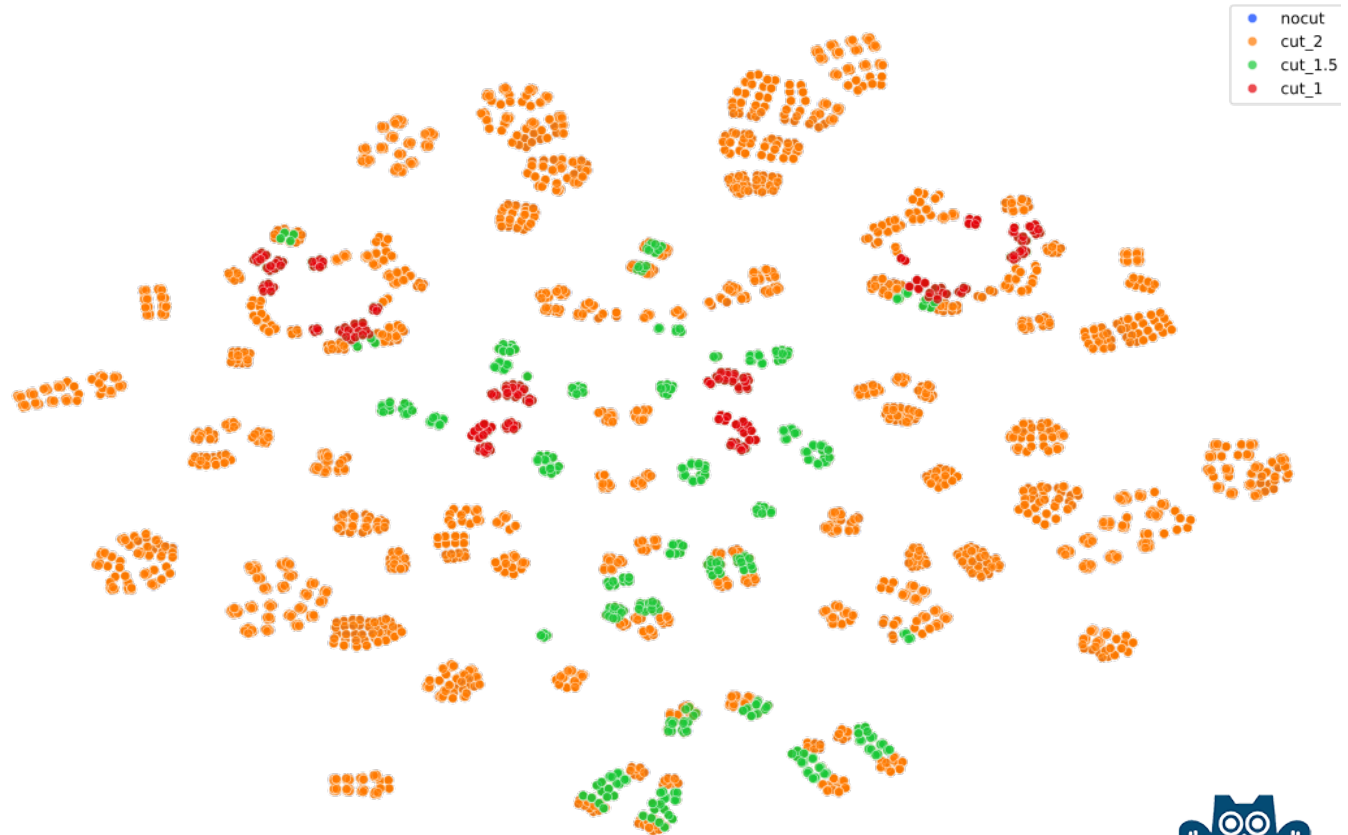
CP	Approach
—	ILP, MIP
—	CP (MiniZinc other)
232s	chuffed, no heuristic
70s	chuffed, $h, = 1..n$
30s	chuffed, $h, \leq, 1..3$

Enum: 97ms



Evaluation Enumeration $n = 3$

Approach	Time
Dijkstra	56s
Dijkstra parallel	17s
Dedup, viable	8.6s
Dedup, A★	1.7s
+viable, instr	0.7s
+cut $k = 1$	0.1s



Evaluation Enumeration $n \geq 3$

	$l = 11$	$l = 20$	$l = 33$
Approach	$n = 3$	$n = 4$	$n = 5$
Enumeration	97ms	2.4s	11min
AlphaDev-RL	6min	30min	17.5h
AlphaDev-S	0.4s	0.6s	5.75h

- All solutions for $n = 3$: 10min
- Optimality for $n = 4$: 2weeks



Evaluation Kernels

Kernel	$n = 3$	$n = 4$	$n = 5$
<u>Enumeration</u>	5.8ms	9.4ms	14.8ms
Mimicry ¹	8.0ms	8.8ms	—
AlphaDev	6.7ms	10.4ms	16.2ms
Sorting Network (Cmp)	7.1ms	14.8ms	19.4ms
<u>MinMax</u>	4.6ms	7.0ms	10.7ms
Sorting Network	5.3ms	8.1ms	12.2ms

¹Mimicry. 2023. Faster Sorting Beyond DeepMind's AlphaDev. <https://www.mimicry.ai/faster-sorting-beyond-deepminds-alphadev> Accessed: 2023-09-20

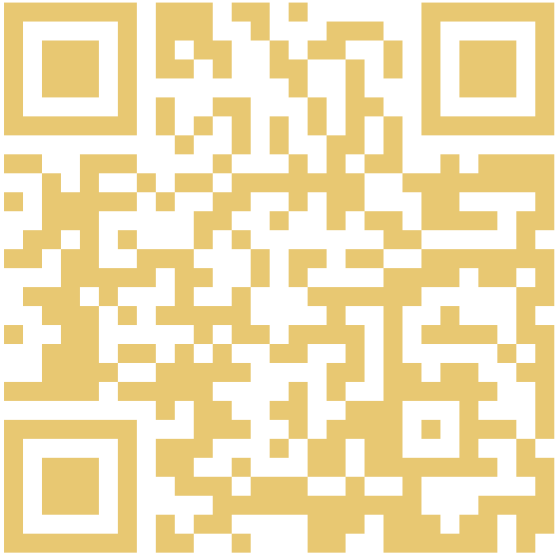


Evaluation Kernels MinMax

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Conclusion



faster synthesis



faster sorting kernels



minimality proof



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↑ Project on GitHub

