Supplementary Material to:

Motion Clouds: Model-based stimulus synthesis of natural-like random textures for the study of motion perception

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Computer implementation using Python

In this part, we will briefly describe how Motion Clouds can be implemented while taking into account technical constraints such as discretization and videographic displays. We will also outline the algorithm used to generate our calibrated motion clouds using Python libraries.

Defining Fourier units, discrete units and physical units

In vision research, stimulus parameters depend on experimental conditions such as viewing distance and other properties of the display, such as the refreshing rate. Here, we will define the parameters of interest to implement when computing Motion Clouds based in the parameters showed in Table ?? where give a description of their physical values in one example experimental setup.

Symbol	Magnitude	Value	Unit
\overline{D}	Viewing distance distance	570	[mm]
X, Y	Stimulus size	640 x 480	[px]
VA^1	Stimulus width in degrees of visual angle at viewing distance D	38,1	[deg]
f_{rate}	Frame rate	50	[Hz]
T	Stimulus duration	0.6	[sec]

Table 1: Physical units in an optical imaging set-up.

Both N_X and N_Y are determined by the frame (stimulus) size (X and Y), while N_{frame} is determined by the frame rate (f_{rate}) and the stimulus duration (T). These parameters define the stimulus' spatiotemporal periods. In this example we set $N_{frame}=30$. Additionally, velocities V_x and V_y have arbitrary units with the convention that if $V_x=1$, it means that average motion is equal to an average displacement of one spatial period over one temporal period and the same applies to V_y . (See Figure $\ref{eq:total_state_$

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Defining stimulus and Fourier cubes

Note first that the visual stimulus I is a real-valued function, therefore the inverse Fourier transform of our spectrum must be purely real, and its transform must be Hermitian. This means that the frequency component (f_x, f_y, f_t) is the complex conjugate of the component at frequency $(-f_x, -f_y, -f_t)$. Therefore, there is no information in the negative frequency components that is not already available from the positive frequency components. To ensure that, the envelope will always be symmetric with respect to the origin in the Fourier domain, while the phase spectrum will be Hermitian by construction. An alternative consists in taking the real part of the complex inverse Fourier transform of any envelope (symmetric or not). Note that by construction of the Fourier transform, stimuli are generated in the the 3D toroidal space and they are invariant up to displacement in multiples of the spatiotemporal period. As a consequence, there is no border or center and moreover any given Motion Cloud may be concatenated in space or time: For instance, playing a Motion Clouds movie in a loop is smooth and there is no abrupt transient. This property is useful to create large stimuli with limited resources by "tiling" a stimulus multiple times. Mathematically, a set of Motion Clouds is generated using normalized input arguments. First, we define the quantization of the Fourier space defined above in cubes of size $N_i, j \in X, Y, frame$, respectively for horizontal, vertical and time axis. In practice we will use the Fast Fourier Transform (FFT). As a consequence, the resulting stimulus cube will be of the same size as the frequency cube and N_j , $j \in X, Y, frame$ should be preferentially defined as an integer power of two. Each frequency axis (in Cartesian coordinates $(f_x, f_y \text{ and } f_t)$) belongs always to the interval [-0.5, 0.5]although the number of points is different. The frequency resolution is given by (1/N_X, 1/N_Y, 1/N_{frame}) and $f_x, f_y, f_t = 0.5$ (in cyd_{px}, cyd_{frame}) is the Nyquist frequency, i.e., the maximal frequency that can be represented without having undesirable aliasing effects.

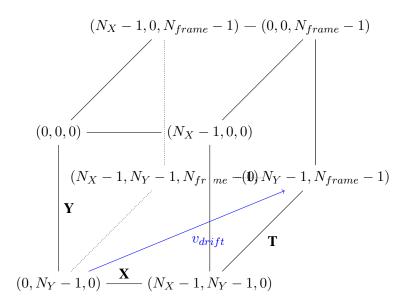


Figure 1

In Figure ?? we show the flow chart of the sequential construction method. We begin by building a three dimensional matrix whose dimensions are given by the input arguments N_X , N_Y and N_{frame} so that $\mathcal{E}(f_x, f_y, f_t) \in \mathbb{R}^{N_X \times N_Y \times N_{frame}}$. The first two define the image size, width and height, respectively. The third dimension is the length of the image-series (number of frames).

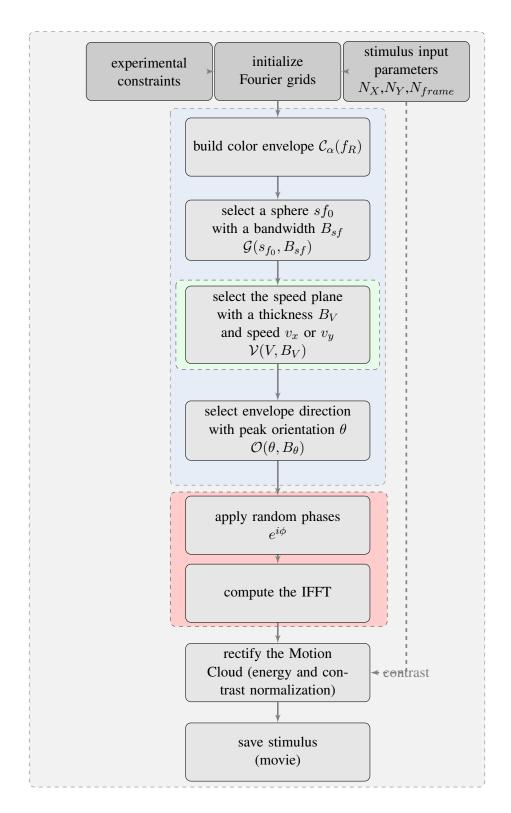


Figure 2

Summary: Flowchart

First, experimental parameters (N_X , N_Y , N_{frame}) are initialized and physical units are normalized (s_{f_0} , V_X , V_Y). Second, the color envelope is generated according to the parameter α . Third, this color envelope (\mathcal{C}_{α}) is multiplied by the global Fourier envelope constructed by the product of the speed (\mathcal{V}), spatial frequency (\mathcal{G}) and orientation envelopes (\mathcal{O}). The last step in the Fourier domain is to multiply the Fourier modulus by a random phase ($e^{i\phi}$). Thus, after computing the 3-dimensional inverse Fourier transform we obtain a dynamic random phase texture, that is the Motion Cloud movie as a numpy array that can further be processed to be for example stored as a sequence of frames.

Code example

Motion Clouds are built using a collection of scripts that provides a simple way of generating complex stimuli suitable for neuroscience and psychophysics experiments. It is meant to be an open-source package that can be combined with other packages such as PsychoPy or VisionEgg.

All functions are implemented in one main script called *MotionClouds.py* that handles the Fourier cube, the envelope functions as well as the random phase generation and all Fourier related processing. Additionally, all the auxiliary visualization tools to plot the spectra and the movies are included. Specific scripts such as *test_color.py*, *test_speed.py*, *test_radial.py* and *test_orientation.py* explore the role of different parameters for each individual envelope (respectively color, speed, radial frequency, orientation). Our aim is to keep the code as simple as possible in order to be comprehensible and flexible. To sum up, when we build a custom Motion Cloud there are 3 simple steps to follow:

1. set the MC parameters and construct the Fourier envelope, then visualize it as iso-surfaces,

2. perform the IFFT and contrast normalization; visualize the stimulus as a 'cube' visualization of the image sequence,

```
movie = mc.random_cloud(envelope)
movie = mc.rectif(movie)
mc.cube(fx, fy, ft, movie, name=name + '_cube') # Visualize the Stimulus
```

3. export the stimulus as a movie (.mpeg format available), as separate frames (.bmp and .png formats available) in a compressed zipped folder, or as a MatlabTM matrix (.mat format).

```
1 mc.anim_save(movie, name, display=False, vext='.mpeg')
```

If some parameters are not given, they are set to default values corresponding to a "standard" Motion Cloud. Moreover, the user can easily explore a range of different Motion Clouds simply by setting an array of values for a determined parameter. Here, for example, we generate 8 MCs with increasing spatial frequency s_{f_0} while keeping the other parameters fixed to default values:

```
for sf_0 in [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6]:
   name_ = 'figures/' + name + '-sf_0-' + str(sf_0).replace('.', '_')
   mc.figures_MC(fx, fy, ft, name_, sf_0=sf_0) # function performing plots for a
        given set of parameters
```

Here, we show the source code of *MotionClouds.py*. The test cases are available on request to the corresponding author.

```
#! /usr/bin/env python
  # -*- coding: utf8 -*-
 3
 4
  Main script for generating Motion Clouds
7
  (c) Laurent Perrinet - INT/CNRS
9
  Motion Clouds (keyword) parameters:
10 | size
          — power of two to define the frame size (N_{-}X, N_{-}Y)
          - power of two to define the number of frames (N_frame)
11 size_T
12 N_X
           - frame size horizontal dimension [px]
13 N<sub>-</sub>Y
           - frame size vertical dimension [px]
  N_frame — number of frames [frames] (a full period in time frames)
15 alpha
          - exponent for the color envelope.
  sf_0
           \boldsymbol{-\!\!\!\!-} mean spatial frequency relative to the sampling frequency.
17 ft_0
           - spatiotemporal scaling factor.
18 B_{-}sf
           - spatial frequency bandwidth
19 V<sub>X</sub>
           - horizontal speed component
20 V_Y
          - vertical speed component
          -- speed bandwidth
21 B_V
          - mean orientation of the Gabor kernel
22 theta
23 B_theta — orientation bandwidth
24 loggabor — (boolean) if True it uses a log-Gabor kernel (instead of the traditional
        gabor)
25
  Display parameters:
27
28
              -- movie format. Stimulus can be saved as a 3D (x-y-t) multimedia file:
      .mpg movie, .mat array, .zip folder with a frame sequence.
          - frame image format.
29
30
  T_movie — movie duration [s].
31
  fps
           -- frame per seconds
32
33
34
35
  import os
36 | DEBUG = False
37
  if DEBUG:
38
       size = 5
       size_T = 5
39
40
       figsize = (400, 400) \# faster
41
  else:
42
       size = 7
43
       size_T = 7
       figsize = (800, 800) # nice size, but requires more memory
44
46 import numpy as np
47 \mid N_X = 2 ** size
48 N_{Y} = N_{X}
```

```
49 \mid N_frame = 2 ** size_T
50 ft_0 = N_X/float(N_frame)
51 | alpha = 1.0
52 \mid sf_0 = 0.15
53 \mid B_{-}sf = 0.1
54
   V_X = 1.
55 | \mathbf{V}_{-}\mathbf{Y} = 0.
56 | B_V = .2
57
   theta = 0.
58 B<sub>theta</sub> = np.pi/32.
59 loggabor = True
60 | \text{vext} = '.\text{mpg'}
61 \mid ext = '.png'
62 T_movie = 8. # this value defines the duration of a temporal period
63 \mid fps = int(N_frame / T_movie)
64
65 # display parameters
66 try:
67
        import progressbar
68
        PROGRESS = True
69
   except:
        PROGRESS = False
70
71
72
   # os.environ['ETS_TOOLKIT'] = 'qt4' # Works in Mac
73
   # os.environ['ETS_TOOLKIT'] = 'wx' # Works in Debian
74 MAYAVI = 'Import'
75
   #MAYAVI = 'Avoid' # uncomment to avoid generating mayavi visualizations (and save
       some memory . . . )
76
   def import_mayavi():
77
        global MAYAVI, mlab
78
        if (MAYAVI == 'Import'):
79
            try:
80
                 from mayavi import mlab
81
                MAYAVI = 'Ok : New and shiny'
                 print('Imported Mayavi')
82
83
            except:\\
84
                 try:
                     from enthought.mayavi import mlab
85
86
                     print('Seems you have an old implementation of MayaVi, but things
                         should work')
87
                     MAYAVI = 'Ok but old'
88
                     print('Imported Mayavi')
89
                 except:
90
                     print('Could not import Mayavi')
91
                     MAYAVI = False
92
        elif (MAYAVI == 'Ok : New and shiny') or (MAYAVI == 'Ok but old'):
93
            pass # no need to import that again
94
95
            print('\mbox{We have chosen not to import Mayavi'})
96
   # Trick from http://github.enthought.com/mayavi/mayavi/tips.html : to use offscreen
         rendering, try xvfb: 1-screen 0 1280x1024x24 in one terminal, export DISPLAY
       =:1 before you run your script
97
98 figpath = 'results/'
99 if not(os.path.isdir(figpath)):os.mkdir(figpath)
100
101 def get_grids(N_X, N_Y, N_frame, sparse=True):
102
103
            Use that function to define a reference outline for envelopes in Fourier
                space.
```

```
104
             In general, it is more efficient to define dimensions as powers of 2.
105
        .....
106
107
        if sparse:
108
            fx, fy, ft = np.ogrid[(-N_x//2):((N_x-1)//2 + 1), (-N_y//2):((N_y-1)//2 + 1)]
                 1), (-N_{frame}/2):((N_{frame}-1)/2 + 1)
                                                                  # output is always even.
109
        else:
110
             fx, fy, ft = np.mgrid[(-N_x//2):((N_x-1)//2 + 1), (-N_y//2):((N_y-1)//2 + 1)
                 1), (-N_frame//2):((N_frame-1)//2 + 1)
                                                                   # output is always even.
        fx, fy, ft = fx*1./N_{-}X, fy*1./N_{-}Y, ft*1./N_{-}frame
111
112
        return fx, fy, ft
113
114
   def frequency_radius(fx, fy, ft, ft_0=ft_0):
115
116
         Returns the frequency radius. To see the effect of the scaling factor run
117
         'test_color.py'
118
119
120
        N_X, N_Y, N_f rame = fx.shape[0], fy.shape[1], ft.shape[2]
121
        R2 = fx**2 + fy**2 + (ft/ft_0)**2 # cf . Paul Schrater 00
122
        R2[N_X//2 , N_Y//2 , N_frame//2] = np.inf
123
        return np. sqrt (R2)
124
   \label{eq:def_def} \textbf{def} \ \ envelope\_color(fx\ , \ fy\ , \ ft\ , \ alpha=alpha\ , \ ft\_0=ft\_0\ ):
125
126
127
        Returns the color envelope.
128
        Run 'test_color.py' to see the effect of alpha
129
        alpha = 0 white
        alpha = 1 pink
130
131
        alpha = 2 red/brownian
        (see http://en.wikipedia.org/wiki/1/f_noise)
132
133
134
        f_radius = frequency_radius(fx, fy, ft, ft_0=ft_0)**alpha
135
        return 1. / f_radius
136
137
   \textbf{def} \ \ envelope\_radial(fx\ , \ fy\ , \ ft\ , \ sf\_0=sf\_0\ , \ B\_sf=B\_sf\ , \ ft\_0=ft\_0\ , \ loggabor=loggabor)
138
139
        Radial frequency envelope:
140
        selects a sphere around a preferred frequency with a shell width B_sf.
141
        Run 'test_radial.py' to see the explore the effect of sf_0 and B_sf
142
        if sf_0 == 0: return 1.
143
144
        if loggabor:
145
             # see http://en.wikipedia.org/wiki/Log-normal_distribution
             fr = frequency_radius(fx, fy, ft, ft_0=1.)
146
147
             env = 1./fr*np.exp(-.5*(np.log(fr/sf_0)**2)/(np.log((sf_0+B_sf)/sf_0)**2))
148
             return env
149
        else:
150
             return np.exp(-.5*(frequency\_radius(fx, fy, ft_0 = 1.) - sf_0)**2/B_sf
                 **2)
151
   \boldsymbol{def} \ \ envelope\_speed ( \ fx \ , \ \ fy \ , \ \ ft \ , \ \ V\_X=V\_X \ , \ \ V\_Y=V\_Y \ , \ \ B\_V=B\_V ) :
152
153
154
         Speed envelope:
155
         selects the plane corresponding to the speed (V_X, V_Y) with some thickness
             B_{-}V
156
157
        (V_X, V_Y) = (0,1) is downward and (V_X, V_Y) = (1,0) is rightward in the
```

```
158
                        A speed of V_X=1 corresponds to an average displacement of 1/N_X per frame.
159
                        To achieve one spatial period in one temporal period, you should scale by
                         V_scale = N_X/float(N_frame)
160
161
                         If N_X=N_Y=N_f rame and V=1, then it is one spatial period in one temporal
                         period. it can be seen in the MC cube. Define ft_0 = N_x/N_frame
162
163
164
                     Run 'test_speed.py' to explore the speed parameters
165
                     ,, ,, ,,
166
167
                     env = np.exp(-.5*((ft+fx*V_X+fy*V_Y))**2/(B_V*frequency_radius(fx, fy, ft, ft_0))**2/(B_V*frequency_radius(fx, ft, ft_0))**2/(B_V*frequency_radius(fx, ft_0))**2
                                =1.))**2)
168
                     return env
169
170
         def envelope_orientation(fx, fy, ft, theta=theta, B_theta=B_theta):
171
172
                      Orientation envelope:
173
                      selects one central orientation theta, B_theta the spread
174
                     We use a von-Mises distribution on the orientation.
175
176
                     Run 'test_orientation.py' to see the effect of changing theta and B_theta.
177
178
                      if not(B_theta is np.inf):
179
                                 angle = np.arctan2(fy, fx)
180
                                 envelope_dir = np.exp(np.cos(2*(angle-theta))/B_theta)
181
                                 return envelope_dir
182
                      else: # for large bandwidth, returns a strictly flat envelope
                                 return 1.
183
184
185
         def envelope_gabor(fx, fy, ft, V_X=V_X, V_Y=V_Y,
186
                                                                     B_V = B_V, sf_0 = sf_0, B_s = B_s f, loggabor = loggabor,
                                                                     theta=theta, B_theta=B_theta, alpha=alpha):
187
188
189
                      Returns the Motion Cloud kernel
190
191
192
                     envelope = envelope_color(fx, fy, ft, alpha=alpha)
193
                     envelope \ *= \ envelope\_orientation ( \ fx \ , \ fy \ , \ ft \ , \ theta=theta \ , \ B\_theta=B\_theta )
194
                     envelope \ *= \ envelope\_radial\,(fx \ , \ fy \ , \ ft \ , \ sf\_0 = sf\_0 \ , \ B\_sf=B\_sf \ , \ loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggabor=loggab
195
                      envelope *= envelope\_speed(fx, fy, ft, V\_X=V\_X, V\_Y=V\_Y, B\_V=B\_V)
196
                      return envelope
197
198
         def random_cloud(envelope, seed=None, impulse=False, do_amp=False):
199
200
                      Returns a Motion Cloud movie as a 3D matrix.
201
                      It first creates a random phase spectrum and then it computes the inverse FFT
                                to obtain
202
                     the spatiotemporal stimulus.
203
204
                     - use a specific seed to specify the RNG's seed,
205
                     - test the impulse response of the kernel by setting impulse to True
                     - test the effect of randomizing amplitudes too by setting do_amp to True
206
207
         shape
208
209
                      (N_X, N_Y, N_frame) = envelope.shape
210
                     amps = 1.
211
                      if impulse:
212
                                phase = 0.
213
                      else:
214
                                 np.random.seed(seed=seed)
```

```
215
            phase = 2 * np.pi * np.random.rand(N_X, N_Y, N_frame)
216
            if do_amp:
217
                amps = np.random.randn(N_X, N_Y, N_frame)
218
                # see Galerne, B., Gousseau, Y. & Morel, J.-M. Random phase textures:
                    Theory and synthesis. IEEE Transactions in Image Processing (2010).
                    URL http://www.biomedsearch.com/nih/Random-Phase-Textures-Theory-
                    Synthesis/20550995.html. (basically, they conclude "Even though the
                    two processes ADSN and RPN have different Fourier modulus
                    distributions (see Section 4), they produce visually similar results
                     when applied to natural images as shown by Fig. 11.")
219
220
       Fz = amps * envelope * np.exp(1j * phase)
221
       # centering the spectrum
222
223
       Fz = np.fft.ifftshift(Fz)
224
       Fz[0, 0, 0] = 0.
225
       z = np. fft. ifftn((Fz)). real
226
       return z
```

In *MotionClouds.py* additional functions have been written for displaying purposes such as visualization of the Fourier spectrum and saving the stimulus in different formats.

```
2
3
  \mathbf{def} get_size (mat):
4
5
       Get stimulus dimensions
6
7
8
       return [np.size(mat, axis=k) for k in range(np.ndim(mat))]
9
10
  #NOTE: Python uses the first dimension (rows) as vertical axis and this is the Y in
       the spatiotemporal domain. Be careful with the convention of X and Y.
11
12
  def visualize(z, azimuth=290., elevation=45.,
       thresholds = [0.94, .89, .75, .5, .25, .1] \,, \ opacities = [.9, .8, .7, .5, .2, .2] \,,
13
       name=None,\ ext=ext\ ,\ do\_axis=True\ ,\ do\_grids=False\ ,\ draw\_projections=True\ ,\ colorbar=False\ ,\ f\_N=2.\ ,\ f\_tN=2.\ ,\ figsize=figsize\ ):
14
15
16
       """ Visualize the Fourier spectrum """
17
18
       import_mayavi()
19
20
       N_X, N_Y, N_frame = z.shape
       fx, fy, ft = get_grids(N_X, N_Y, N_frame, sparse=False)
2.1
22
       mlab.figure(1, bgcolor=(1, 1, 1), fgcolor=(0, 0, 0), size=figsize)
23
24
       mlab.clf()
25
26
       # Normalize the amplitude.
27
       z /= z.max()
28
       # Create scalar field
29
       src = mlab.pipeline.scalar_field(fx, fy, ft, z)
30
       if draw_projections:
31
           src_x = mlab.pipeline.scalar_field(fx, fy, ft, np.tile(np.sum(z, axis=0)), (
               N_X, 1, 1))
32
           src_y = mlab.pipeline.scalar_field(fx, fy, ft, np.tile(np.reshape(np.sum(z, ft)))
                axis=1), (N_{-}X, 1, N_{-}frame)), (1, N_{-}Y, 1))
           src_z = mlab.pipeline.scalar_field(fx, fy, ft, np.tile(np.reshape(np.sum(z, ft, np.tile)))
33
                axis=2), (N_X, N_Y, 1), (1, 1, N_frame))
```

```
34
35
           # Create projections
36
           border = 0.47
37
           scpx = mlab.pipeline.scalar_cut_plane(src_x , plane_orientation='x_axes' ,
               view_controls=False)
38
           scpx.implicit_plane.plane.origin = [-border, 1/N_Y, 1/N_frame]
39
           scpx.enable_contours = True
40
           scpy = mlab.pipeline.scalar_cut_plane(src_y, plane_orientation='y_axes',
               view_controls=False)
41
           scpy.implicit_plane.plane.origin = [1/N_X, border, 1/N_frame]
42
           scpy.enable_contours = True
43
           scpz = mlab.pipeline.scalar_cut_plane(src_z, plane_orientation='z_axes',
               view_controls=False)
           scpz.implicit\_plane.plane.origin = [1/N\_X, 1/N\_Y, -border]
44
45
           scpz.enable_contours = True
46
47
      # Generate iso-surfaces at differnet energy levels
48
      for threshold, opacity in zip(thresholds, opacities):
49
           mlab.pipeline.iso_surface(src, contours=[z.max()-threshold*z.ptp(),],
50
                                      opacity = opacity)
51
           mlab.outline(extent=[-1./2, 1./2, -1./2, 1./2, -1./2, 1./2],)
52
53
      # Draw a sphere at the origin
54
      x = np.array([0])
55
      y = np.array([0])
56
      z = np.array([0])
57
      s = 0.01
58
      mlab.points3d(x, y, z, extent=[-s, s, -s, s, -s, s], scale_factor=0.15)
59
60
       if colorbar: mlab.colorbar(title='density', orientation='horizontal')
       if do_axis:
61
           ax = mlab.axes(xlabel='fx', ylabel='fy', zlabel='ft',
62
                           extent = [-1./2, 1./2, -1./2, 1./2, -1./2, 1./2],
63
64
65
           ax.axes.set (font_factor = 2.)
66
67
      try:
68
           mlab.view(azimuth=azimuth, elevation=elevation, distance='auto', focalpoint
               ='auto')
69
       except:
70
           print(" You should upgrade your mayavi version")
71
72
       if not(name is None):
73
           mlab.savefig(name + ext, magnification=1, size=figsize)
74
       else:
75
          mlab.show(stop=True)
76
77
      mlab.close(all=True)
78
79
  def cube (im, azimuth = -45., elevation = 130., roll = -180., name=None,
80
            ext=ext, do_axis=True, show_label=True, colormap='gray',
81
            vmin=0., vmax=1., figsize=figsize):
82
83
84
       Visualize the stimulus as a cube
85
86
87
      import_mayavi()
88
89
      N_X, N_Y, N_frame = im.shape
```

```
90
       fx, fy, ft = get_grids(N_X, N_Y, N_frame, sparse=False)
91
92
       mlab.figure(1, bgcolor=(1, 1, 1), fgcolor=(0, 0, 0), size=figsize)
93
       mlab.clf()
94
        src = mlab.pipeline.scalar_field(fx*2., fy*2., ft*2., im)
95
96
       mlab.pipeline.image_plane_widget(src, plane_orientation='z_axes',
97
                                          slice_index = 0, colormap = colormap, vmin = vmin,
                                              vmax=vmax)
       mlab.pipeline.image_plane_widget(src, plane_orientation='z_axes',
98
99
                                          slice_index=N_frame, colormap=colormap,
100
                                          vmin=vmin , vmax=vmax)
101
       mlab.pipeline.image_plane_widget(src, plane_orientation='x_axes', slice_index
           =0.
                                          colormap=colormap , vmin=vmin , vmax=vmax)
102
103
       mlab.pipeline.image_plane_widget(src, plane_orientation='x_axes', slice_index=
           N_X,
104
                                          colormap=colormap , vmin=vmin , vmax=vmax)
105
106
       mlab.pipeline.image_plane_widget(src, plane_orientation='y_axes', slice_index
107
                                          colormap=colormap , vmin=vmin , vmax=vmax)
108
       mlab.pipeline.image_plane_widget(src, plane_orientation='y_axes', slice_index=
           N_{-}Y,
109
                                          colormap=colormap , vmin=vmin , vmax=vmax)
110
111
        if do_axis:
112
           ax = mlab.axes(xlabel='x', ylabel='y', zlabel='t',
                           113
114
115
                           x_axis_visibility=True, y_axis_visibility=True,
116
                           z_a x i s_v i s i b i l i t y = True)
117
           ax.axes.set(font_factor=2.)
118
119
           if not(show_label): ax.axes.set(label_format='')
120
121
122
       try:
123
           mlab.view(azimuth=azimuth, elevation=elevation, distance='auto', focalpoint
               ='auto')
124
           mlab.roll(roll=roll)
125
       except:
126
           print(" You should upgrade your mayavi version")
127
128
        if not(name is None):
129
           mlab.savefig(name + ext, magnification=1, size=figsize)
130
        else:
131
           mlab.show(stop=True)
132
133
       mlab.close(all=True)
134
   def anim_exist(filename, vext='.mpg'):
135
136
137
       Check if the movie already exists
138
139
140
       return not(os.path.isfile(filename+vext))
141
142
143 def anim_save(z, filename, display=True, flip=False, vext='.mpg',
```

```
144
                   centered=False, fps=fps):
145
146
        Saves a numpy 3D matrix (x-y-t) to a multimedia file.
147
148
        The input pixel values are supposed to lie in the [0, 1.] range.
149
        ,, ,, ,,
150
151
        import os
                                              # For issuing commands to the OS.
152
        import tempfile
        from scipy.misc.pilutil import toimage
153
154
        def make_frames(z):
155
            N_X, N_Y, N_frame = z.shape
            files = []
156
157
            tmpdir = tempfile.mkdtemp()
158
159
            if PROGRESS:
                 widgets = ["calculating", " ", progressbar.Percentage(), ' ',
    progressbar.Bar(), ' ', progressbar.ETA()]
160
161
162
                 pbar = progressbar.ProgressBar(widgets=widgets, maxval=N_frame).start()
163
            print('Saving sequence ' + filename + vext)
164
            for frame in range(N_frame):
165
                 if PROGRESS: pbar.update(frame)
166
                 fname = os.path.join(tmpdir, 'frame%03d.png' % frame)
167
                 image = np.rot90(z[:, :, frame])
168
                 if flip: image = np.flipud(image)
169
                 toimage(image, high=255, low=0, cmin=0., cmax=1., pal=None,
170
                          mode=None, channel_axis=None).save(fname)
171
                 files.append(fname)
172
                 if PROGRESS: pbar.update(frame)
173
            if PROGRESS: pbar.finish()
174
175
            return tmpdir, files
176
177
        def remove_frames(tmpdir, files):
178
179
            Remove frames from the temp folder
180
181
182
            for fname in files: os.remove(fname)
183
            if not(tmpdir == None): os.rmdir(tmpdir)
184
185
        if vext == '.mpg':
186
            # 1) create temporary frames
187
            tmpdir, files = make_frames(z)
188
            # 2) convert frames to movie
        cmd = 'ffmpeg - v \ 0 - y - sameq - loop\_output \ 0 - r \ ' + str(fps) + \ ' - i \ ' + tmpdir + '/frame\%03d.png \ ' + filename + vext \# + ' 2 > /dev/null')
189
            cmd = 'ffmpeg - v \ 0 - y - sameq - loop_output \ 0 - i ' + tmpdir + '/frame%03d.
190
                png ' + filename + vext # + ' 2>/dev/null')
191
            # print('Doing : ', cmd)
            os.system(cmd) \# + '2 > /dev/null')
192
193
            # To force the frame rate of the output file to 24 fps:
194
            \# ffmpeg - i input.avi - r 24 output.avi
195
            #3) clean up
196
            remove_frames(tmpdir, files)
197
        if vext == '.gif': # http://www.uoregon.edu/~noeckel/MakeMovie.html
198
            # 1) create temporary frames
199
            tmpdir, files = make_frames(z)
200
            # 2) convert frames to movie
201 #
              options = '-pix_fmt \ rgb24 - r' + str(fps) + '-loop_output 0'
```

```
os.system('ffmpeg -i ' + tmpdir + '/frame%03d.png ' + options + filename
202 #
         + vext + '2 > /dev/null'
            options = ' -set delay 8 -colorspace GRAY -colors 256 -dispose 1 -loop 0 '
203
204
            os.system('convert' + tmpdir + '/frame*.png' + options + filename +
                vext)# + '2 > /dev/null'
205
206
            #3) clean up
207
            remove_frames(tmpdir, files)
208
209
        elif vext == '.png':
            toimage(np.flipud(z[:, :, 0]).T, high=255, low=0, cmin=0., cmax=1., pal=1)
210
                None, mode=None, channel_axis=None).save(filename + vext)
211
        elif vext == '.zip':
212
213
            tmpdir, files = make_frames(z)
214
            import zipfile
215
            zf = zipfile.ZipFile(filename + vext, "w")
216
            # convert to BMP for optical imaging
217
            files_bmp = []
218
            for fname in files:
219
                fname_bmp = os.path.splitext(fname)[0] + '.bmp'
220
                # print fname_bmp
221
                os.system('convert' + fname + 'ppm:-|convert -size 256x256+0 -
                    colors 256 -colorspace Gray - BMP2: ' + fname_bmp) # to generate 8-
                     bit bmp (old format)
222
                files_bmp.append(fname_bmp)
223
                zf.write(fname_bmp)
224
            zf.close()
225
            remove_frames(tmpdir=None, files=files_bmp)
226
            remove_frames(tmpdir, files)
227
        elif vext == '.mat':
228
229
            from scipy.io import savemat
230
            savemat(filename + vext, {'z':z})
231
        elif vext == '.h5':
232
233
            from tables import openFile, Float32Atom
234
            hf = openFile(filename + vext, 'w')
235
            o = hf.createCArray(hf.root, 'stimulus', Float32Atom(), z.shape)
236
237
            # print o. shape
238
            hf.close()
239
240
   def rectif(z, contrast=.9, method='Michelson', verbose=False):
241
242
        Transforms an image (can be 1,2 or 3D) with normal histogram into
243
        a 0.5 centered image of determined contrast
244
        method is either 'Michelson' or 'Energy'
245
246
247
        # Phase randomization takes any image and turns it into Gaussian-distributed
            noise of the same power (or, equivalently, variance).
        # See: Peter J. Bex J. Opt. Soc. Am. A/Vol. 19, No. 6/June 2002 Spatial
248
           frequency, phase, and the contrast of natural images
249
250
        # Final rectification
251
        if verbose:
252
            print('Before Rectification of the frames')
253
            print( \ '\texttt{Mean='} \ , \ np.mean(z[:]) \ , \ ', \ \texttt{std='} \ , \ np.std(z[:]) \ , \ ', \ \texttt{Min='} \ , \ np.min(z[:]) \ , \ ', \ denote \ )
                [:]) , ', Max=', np.max(z[:]) , ' Abs (Max)=', np.max(np.abs(z[:])))
```

```
254
255
                z -= np.mean(z[:]) # this should be true *on average* in MotionClouds
256
257
                if (method == 'Michelson'):
258
                        z = (.5* z/np.max(np.abs(z[:]))* contrast + .5)
259
260
                        z = (.5* z/np.std(z[:]) * contrast + .5)
261
262
                if verbose:
263
                        import pylab
264
                        pylab.hist(z.ravel())
265
266
                        print('After Rectification of the frames')
267
                        print('\texttt{Mean='}, \ np.mean(z\,[:]) \ , \ ', \ std=' \ , \ np.std(z\,[:]) \ , \ ', \ \texttt{Min='} \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ np.min(z\,[:]) \ , \ ', \ std=' \ , \ , \ std=' \ , \ , \ std=' \ , \ , \ , \ , \ , \ , \ , \ , \
                                [:]), ', Max=', np.max(z[:]))
268
                        print('\texttt{percentage pixels clipped='}, \ np.sum(np.abs(z[:])>1.)*100/z.size)
269
                return z
270
271
       def figures_MC(fx, fy, ft, name, V_X=V_X, V_Y=V_Y, do_figs=True, do_movie=True,
272
                                                  B_V = B_V, sf_0 = sf_0, B_s = B_s f, loggabor = loggabor,
273
                                                  theta=theta, B_theta=B_theta, alpha=alpha, vext=vext,
274
                                                  seed=None, impulse=False, verbose=False):
275
276
                Generates the figures corresponding to the Fourier spectra and the stimulus
                        cubes and
277
                movies.
                The figures names are automatically generated.
278
279
280
                if anim_exist(name, vext=vext):
281
                        z = envelope_gabor(fx, fy, ft, V_X=V_X, V_Y=V_Y,
                                                 B_V=B_V, sf_0=sf_0, B_sf=B_sf, loggabor=loggabor,
282
                                                  theta=theta, B_{theta}=B_{theta}, alpha=alpha)
283
284
                        figures (z, name, vext=vext, do_figs=do_figs, do_movie=do_movie,
285
                                                  seed=seed , impulse=impulse , verbose=verbose)
286
287
       def figures(z, name, vext=vext, do_figs=True, do_movie=True,
288
                                                  seed=None, impulse=False, verbose=False, masking=False):
                if ((MAYAVI == 'Import') or MAYAVI[:2]=='Ok') and do_figs and anim_exist(name,
289
                        vext=ext): visualize(z, name=name)
                                                                                                                       # Visualize the Fourier
290
                if (do_movie and anim_exist(name, vext=vext)) or (MAYAVI and do_figs and
                        anim_exist(name + '_cube', vext=ext)):
                        movie = rectif(random_cloud(z, seed=seed, impulse=impulse), verbose=verbose
291
292
                if (((MAYAVI == 'Import') or MAYAVI[:2]=='Ok') and do_figs and anim_exist(name
                        + '_cube', vext=ext)): cube(movie, name=name + '_cube')
                                                                                                                                                   # Visualize the
                        Stimulus cube
293
                if (do_movie and anim_exist(name, vext=vext)): anim_save(movie, name, display=
                        False, vext=vext)
```

Both functions **visualize** (line 37) and **cube** (line 100) generate isometric views of a cube. The first one displays isosurfaces enclosing volumes at 6 different energy values with respect to the peak amplitude of the Fourier spectrum. The Cartesian coordinate system is represented by 3 orthogonal grid planes going through the origin. The origin is represented by a black dot where the three 3 orthogonal axes converge. In addition to that, it is also possible to obtain the orthogonal projections onto the corresponding normal planes to the Cartesian axes, illustrated by 10 contour level curves. We enable the projection onto the $f_x - f_t$ and $f_y - f_t$ planes in order to observe the changes in the tilt of the speed plane (reflecting

respectively a change in V_X or V_Y), as well as its thickness. Furthermore, the projection onto the $f_x - f_y$ plane allows us to see the average orientation θ and the spread of the orientation envelope. The outlines delineate the frequency domain extension in Fourier units as described in ??. The second function draws the isometric view of the movie cube. The first frame of the movie lies on the plane x-y, motion direction is seen as diagonal trajectories on the top face (x-t) plane) and on the right face (y-t plane), reflecting respectively a change in V_X or V_Y .

Annex

Approximating normal and log-normal distributions

In our implementation we can choose whether to use the log-normal derived function or simply approximate it by a Gaussian envelope. We demonstrate here that:

$$\frac{\ln(f) - \mu}{\sigma} \approx \frac{f - sf_0}{B_{sf}}$$

The log-Gabor envelope is approximately Gaussian in a neighborhood of sf_0 , for $sf - sf_0 \ll B_{sf}$ (for small values of σ , $\ln(1+x)$ is approximately x that is to say the log-normal is approximately Gaussian). Since,

$$\frac{-\log^2\left(\frac{f}{sf_0}\right)}{2\cdot\log^2\left(\frac{1+B_{sf}}{sf_0}\right)} = -\frac{1}{2}\cdot\left(\frac{\log\left(\frac{f}{sf_0}\right)}{\log\left(\frac{1+B_{sf}}{sf_0}\right)}\right)^2 \tag{1}$$

and

$$\frac{\log\left(\frac{f}{sf_0}\right)}{\log\left(1 + \frac{B_{sf}}{sf_0}\right)} = \frac{\log\left(1 + \frac{f - sf_0}{sf_0}\right)}{\log\left(1 + \frac{B_{sf}}{sf_0}\right)} \tag{2}$$

with $\frac{f}{sf_0}=1+\frac{f-sf_0}{sf_0}$. Then, near sf_0 , i.e. in the neighborhood of sf_0 , and for $f-sf_0 << B_{sf}$, this function can be represented by the first order Taylor expansion

$$\frac{\log\left(1 + \frac{f - sf_0}{sf_0}\right)}{\log\left(1 + \frac{B_s f}{sf_0}\right)} = \frac{\frac{f - sf_0}{sf_0}}{\frac{B_s f}{sf_0}} = \frac{f - sf_0}{B_s f}$$
(3)

so in the sf_0 neighborhood, the pdf (of f) is:

$$p(f) = \exp\left(\frac{-\log^2\left(\frac{f}{sf_0}\right)}{2 \cdot \log^2\left(\frac{1 + B_s f}{sf_0}\right)}\right)$$
(4)

$$= \exp\left(-\frac{1}{2} \cdot \left(\frac{\log\left(\frac{f}{sf_0}\right)}{\log\left(\frac{1+B_{sf}}{sf_0}\right)}\right)^2\right)$$
 (5)

$$= \exp\left(-\frac{1}{2}\left(\frac{f - sf_0}{B_{sf}}\right)^2\right) \tag{6}$$

that identifies to the desired normal distribution $\mathcal{N}(f;sf_0,B_{sf})$.