- Python Setup, Basic Python, numpy, pandas and matplotlib
- Matrix Algebra
- Linear equations and transformations
- Vectors
- Vector Spaces
- Metric Spaces, Normed spaces, Inner Product Spaces
- Orthogonality
- Determinant and Trace Operator
- Matrix Decompositions (Eigen, SVD and Cholesky)
- Symmetric matrices and Quadratic Forms
- Left Inverse, Right Inverse, Pseudo Inverse

Connecting the dots 1
LINEAR REGRESSION

Connecting the dots 2
PRINCIPAL COMPONENT ANALYSIS

$$A = (a_{i,j}) \in \mathfrak{R}^{mxn} \quad 1 \leq i \leq \text{m},\ 1 \leq j \leq \text{n}$$

$$\begin{array}{cl}
A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,(n-1)} & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,(n-1)} & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{(m-1),1} & a_{(m-1),2} & \cdots & a_{(m-1),(n-1)} & a_{(m-1),n} \\ a_{m,1} & a_{m,2} & \cdots & a_{m,(n-1)} & a_{m,n} \end{bmatrix} & \begin{array}{l} \leftarrow \text{Row 1, } a_{1,:} \\ \leftarrow \text{Row 2, } a_{2,:} \\ \\ \leftarrow \text{Row (m-1), } a_{(m-1),:} \\ \leftarrow \text{Row m, } a_{m,:} \end{array}
\end{array}$$

Column 1, $a_{:,1}$ Column 2, $a_{:,2}$ Column (n-1), $a_{:,(n-1)}$ Column n, $a_{:,n}$

$$B = [a_{i,1} \quad a_{i,2} \quad \cdots \quad a_{i,n}] \quad C = \begin{bmatrix} a_{1,j} \\ a_{2,j} \\ \vdots \\ a_{m,j} \end{bmatrix}$$

$$A = \left(a_{i,j}\right) \in \Re^{3x4} \qquad 1 \leq i \leq 3,\ 1 \leq j \leq 4$$

$$A = \begin{array}{c c} \begin{bmatrix} 2 & 7 & 5 & -2 \\ 7 & 6.7 & 5 & 9 \\ 0 & 8 & 0 & 1 \end{bmatrix} & \begin{array}{l} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \end{array} \\ \begin{array}{cccc} \text{Column 1} & \text{Column 2} & \text{Column 3} & \text{Column 4} \end{array} & \end{array}$$

$$A = \begin{bmatrix} a_{1,1} = 2 & a_{1,2} = 7 & a_{1,3} = 5 & a_{1,4} = -2 \\ a_{2,1} = 7 & a_{2,2} = 6.7 & a_{2,3} = 5 & a_{2,4} = 9 \\ a_{3,1} = 0 & a_{3,2} = 8 & a_{3,3} = 0 & a_{3,4} = 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0.5 \\ 2 & 3 \end{bmatrix} \in R^{2x2}$$

$$B = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix} \in R^{3x1}$$

$$C = \begin{bmatrix} 0.31 & 21 & 0 & 3 & -6 & 0 \end{bmatrix} \in R^{1x6}$$

In Class Exercise 1.1

Find $a_{2,1}, a_{4,5}$ and $a_{3,3}$ in A_1

Find $a_{2,1}, a_{4,5}$ and $a_{3,3}$ in A_1

$$A_1 = \begin{bmatrix} -2 & 1 & 2 & 2 \\ 0 & 0 & 3 & 8 \\ 1 & 12 & 3.6 & 1 \\ 7.2 & 7 & 8 & 19 \\ 4 & 0.8 & 33 & 11 \end{bmatrix}$$

In Class Exercise 1.1

SOLUTION

$$\begin{array}{cc} & \begin{array}{cccc} \text{Column 1} & & \text{Column 3} & \end{array} \\ A_1 = \begin{bmatrix} -2 & 1 & 2 & 2 \\ 0 & 0 & 3 & 8 \\ 1 & 12 & 3.6 & 1 \\ 7.2 & 7 & 8 & 19 \\ 4 & 0.8 & 33 & 11 \end{bmatrix} & \begin{array}{l} \\ \text{Row 2} \\ \text{Row 3} \\ \text{Row 4} \\ \\ \end{array} \end{array}$$

Column 1 Column 3

~~Column 5~~

$a_{2,1}, a_{4,5}, a_{3,3}$

In Class Exercise 1.1
SOLUTION

$$A_1 = \begin{array}{cl} \begin{bmatrix} -2 & 1 & 2 & 2 \\ 0 & 0 & 3 & 8 \\ 1 & 12 & 3.6 & 1 \\ 7.2 & 7 & 8 & 19 \\ 4 & 0.8 & 33 & 11 \end{bmatrix} & \begin{array}{l} \\ \text{Row 2} \\ \text{Row 3} \\ \text{Row 4} \\ \\ \end{array} \\ \begin{array}{cccc} \text{Column 1} & & \text{Column 3} & \end{array} & \end{array}$$

~~Column 5~~

$$a_{2,1}, a_{4,5}, a_{3,3}$$

$$A = (a_{i,j}) \in \Re^{mxn}, B = (b_{i,j}) \in \Re^{mxn}$$

$$C = A + B \rightarrow \forall\ 1 \leq i \leq m, 1 \leq j \leq n, (c_{i,j}) = (a_{i,j}) + (b_{i,j}) \text{ and } C \in \Re^{mxn}$$

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 0 \\ 5 & 3 \\ 9 & 1 \end{bmatrix} \in \Re^{4x2}, B = \begin{bmatrix} 1 & -1 \\ 4 & 2 \\ 7 & 8 \\ 1 & 0 \end{bmatrix} \in \Re^{4x2}$$

$$A+B = \begin{bmatrix} 5 = 4+1 & 6 = 7+(-1) \\ 6 = 2+4 & 2 = 0+2 \\ 12 = 5+7 & 11 = 3+8 \\ 10 = 9+1 & 1 = 1+0 \end{bmatrix} \in \Re^{4x2}$$

In Class Exercise 1.2

Find $A + B$

- $A = \begin{bmatrix} -2 & 1 & 2 & 2 \\ 0 & 0 & 3 & 8 \\ 1 & 12 & 3.6 & 1 \\ 7.2 & 7 & 8 & 19 \\ 4 & 0.8 & 33 & 11 \end{bmatrix}, B = \begin{bmatrix} 20 & -1 & 6 & 5 \\ 0 & 0 & 3 & 4 \\ 7 & 0 & 0 & 1 \\ 9 & 7.1 & 0 & 1 \\ 5 & 0 & 3 & 1 \end{bmatrix}$
- $A = \begin{bmatrix} 5 & 8 & 9 \\ 6 & 6 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

In Class Exercise 1.2
SOLUTION

- A= $\begin{bmatrix} -2 & 1 & 2 & 2 \\ 0 & 0 & 3 & 8 \\ 1 & 12 & 3.6 & 1 \\ 7.2 & 7 & 8 & 19 \\ 4 & 0.8 & 33 & 11 \end{bmatrix}, B = \begin{bmatrix} 20 & -1 & 6 & 5 \\ 0 & 0 & 3 & 4 \\ 7 & 0 & 0 & 1 \\ 9 & 7.1 & 0 & 1 \\ 5 & 0 & 3 & 1 \end{bmatrix}$ → $A + B = \begin{bmatrix} 18 & 0 & 8 & 7 \\ 0 & 0 & 6 & 12 \\ 8 & 12 & 3.6 & 2 \\ 16.2 & 14.1 & 8 & 20 \\ 9 & 0.8 & 36 & 12 \end{bmatrix}$
- A= $\begin{bmatrix} 5 & 8 & 9 \\ 6 & 6 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ $A + B\ is\ invalid$

$$A = \left(a_{i,j}\right) \in \Re^{\mathrm{mx}k}, B = \left(b_{i,j}\right) \in \Re^{\mathrm{kx}n}$$

$$C = A\mathrm{x}B \quad \rightarrow \forall \leq i \leq \mathrm{m}, 1 \leq j \leq \mathrm{n}, \left(c_{i,j}\right) = \sum_{p=1}^{k} \left(a_{i,p}\right) * \left(b_{p,j}\right) \quad and\ \mathrm{C} \in \Re^{mxn}$$

$$\sum_{p=1}^{k} (f(p)) = f(1) + f(2) + \cdots + f((k-1)) + f(k) \qquad \prod_{p=1}^{k} (f(p)) = f(1) * f(2) * \cdots * f\left((k-1)\right) * f(k)$$

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 8 & 3 & 1 \\ 2 & 2 & 2 \end{bmatrix} \in \Re^{3x3}, B = \begin{bmatrix} 4 & 5 \\ 2 & 3 \\ 1 & 1 \end{bmatrix} \in \Re^{3x2}$$

$$(c_{i,j}) = \sum_{p=1}^{3} (a_{i,p}) * (b_{p,j}) \qquad AXB = \begin{bmatrix} 19 & 24 \\ 39 & 50 \\ 14 & 18 \end{bmatrix}$$

In Class Exercise 1.3

True or False?

- $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \longrightarrow A * B = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$
- $A = \begin{bmatrix} 1 & 0 & -9 \\ 6 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix} \longrightarrow A * B = \begin{bmatrix} 11 & 7 \\ 6 & 8 \end{bmatrix}$

$$\text{AXB=} \begin{bmatrix} 19 = (1*4)+(4*2)+(7*1) = \sum_{p=1}^{3} (a_{1,p}) * (b_{p,1}) & 24 = (1*5)+(4*3)+(7*1) = \sum_{p=1}^{3} (a_{1,p}) * (b_{p,2}) \\ 39 = (8*4)+(3*2)+(1*1) = \sum_{p=1}^{3} (a_{2,p}) * (b_{p,1}) & 50 = (8*5)+(3*3)+(1*1) = \sum_{p=1}^{3} (a_{2,p}) * (b_{p,2}) \\ 14 = (2*4)+(2*2)+(2*1) = \sum_{p=1}^{3} (a_{3,p}) * (b_{p,1}) & 18 = (2*5)+(2*3)+(2*1) = \sum_{p=1}^{3} (a_{3,p}) * (b_{p,2}) \end{bmatrix}$$

$\in \Re^{3x2}$

In Class Exercise 1.3
SOLUTION

- A= $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \dashrightarrow A * B = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$
- A= $\begin{bmatrix} 1 & 0 & -9 \\ 6 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix} \dashrightarrow A * B = \begin{bmatrix} 11 & 7 \\ 6 & 8 \end{bmatrix}$

- **True**
- **False, due to invalidity**

$$B = \left(b_{i,j}\right) \in \Re^{mxn}, a \in \Re$$

$$C = a(B) \rightarrow \forall \leq i \leq m,\ 1 \leq j \leq n,\ \left(c_{i,j}\right) = a * \left(b_{i,j}\right) \quad and\ C \in \Re^{mxn}$$

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 8 & 9 \\ 0 & 2 & 1 \\ 2 & 4 & 1 \end{bmatrix} \in \Re^{4x3}, a = 4 \dashrightarrow aA = \begin{bmatrix} 4*1 = 4 & 4*4 = 16 & 4*7 = 28 \\ 4*3 = 12 & 4*8 = 32 & 4*9 = 36 \\ 4*0 = 0 & 4*2 = 8 & 4*1 = 4 \\ 4*2 = 8 & 4*4 = 16 & 4*1 = 4 \end{bmatrix} = \begin{bmatrix} 4 & 16 & 28 \\ 12 & 32 & 36 \\ 0 & 8 & 4 \\ 8 & 16 & 4 \end{bmatrix} \in \Re^{4x3}$$

$$B = \begin{bmatrix} 2 & 2 & 9 \\ 1 & 3 & 0 \\ 7 & 5 & -6 \\ 2 & 5 & 4 \end{bmatrix} \in \Re^{4x3}, \text{A-B = A+(-1(B))} = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 8 & 9 \\ 0 & 2 & 1 \\ 2 & 4 & 1 \end{bmatrix} + (-1) \begin{bmatrix} 2 & 2 & 9 \\ 1 & 3 & 0 \\ 7 & 5 & -6 \\ 2 & 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 8 & 9 \\ 0 & 2 & 1 \\ 2 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -9 \\ -1 & -3 & 0 \\ -7 & -5 & 6 \\ -2 & -5 & -4 \end{bmatrix} = \begin{bmatrix} -1 & 2 & -2 \\ 2 & 5 & 9 \\ -7 & -3 & 7 \\ 0 & -1 & -3 \end{bmatrix}$$

$$A = (a_{i,j}) \in \Re^{mxn}, B = (b_{i,j}) \in \Re^{mxn}$$

$$C = A \odot B \rightarrow \forall \leq i \leq m, 1 \leq j \leq n, (c_{i,j}) = a_{i,j} * b_{i,j} \quad and\ C \in \Re^{mxn}$$

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$$A = \begin{bmatrix} 7 & 8 & 2 \\ 3 & 1 & 1 \end{bmatrix} \in \Re^{2x3}$$

$$B = \begin{bmatrix} 7 & 10 & 5 \\ 7 & 6 & 7 \end{bmatrix} \in \Re^{2x3}$$

$$\dashrightarrow A \odot B = \begin{bmatrix} 7 * 7 = 49 & 8 * 10 = 80 & 2 * 5 = 10 \\ 3 * 7 = 21 & 1 * 6 = 6 & 1 * 7 = 7 \end{bmatrix} = \begin{bmatrix} 49 & 80 & 10 \\ 21 & 6 & 7 \end{bmatrix} \in \Re^{2x3}$$

Properties of Matrix Operations

$$a, b \in \mathfrak{R}, A, B \in \mathfrak{R}^{mxn}$$

$$0_{R^{mxn}} = \begin{bmatrix} b_{1,1} = 0 & b_{1,2} = 0 & \cdots & b_{1,(n-1)} = 0 & b_{1,n} = 0 \\ b_{2,1} = 0 & b_{2,2} = 0 & \cdots & b_{2,(n-1)} = 0 & b_{2,n} = 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b_{(m-1),1} = 0 & b_{(m-1),2} = 0 & \cdots & b_{(m-1),(n-1)} = 0 & b_{(m-1),n} = 0 \\ b_{m,1} = 0 & b_{m,2} = 0 & \cdots & b_{m,(n-1)} = 0 & b_{m,n} = 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $A + 0_{R^{mxn}} = A$
- $a(A + B) = aA + aB$
- $(a + b)A = aA + bA$
- $a(bA) = (ab)A$
- $A(BC) = (AB)C$
- $A(B + C) = AB + AC$
- $(A + B)C = AC + BC$
- $AB \neq BA$
- $AB = AC \nRightarrow B = C$
- $AB = 0 \nRightarrow A = 0 \text{ or } B = 0$

$$A = (a_{i,j}) \in \Re^{mxn}$$

$$\rightarrow \forall \le i \le m,\ 1 \le j \le n, \quad A^T = (a_{j,i}) \in \Re^{nxm}$$

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,(n-1)} & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,(n-1)} & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{(m-1),1} & a_{(m-1),2} & \cdots & a_{(m-1),(n-1)} & a_{(m-1),n} \\ a_{m,1} & a_{m,2} & \cdots & a_{m,(n-1)} & a_{m,n} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{1,1} & a_{2,1} & \cdots & a_{(m-1),1} & a_{m,1} \\ a_{1,2} & a_{2,2} & \cdots & a_{(m-1),2} & a_{m,2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{1,(n-1)} & a_{2,(n-1)} & \cdots & a_{(m-1),(n-1)} & a_{m,(n-1)} \\ a_{1,n} & a_{2,n} & \cdots & a_{(m-1),n} & a_{m,n} \end{bmatrix}$$

- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$
- $(ABC)^T = C^T B^T A^T \quad - (A_1 A_2 \ldots A_n)^T = A_n^T \ldots A_2^T A_1^T$

In Class Exercise 1.4

Find A^T, B^T, AB, $(AB)^T$ and $B^T A^T$.

Confirm that $(AB)^T = B^T A^T$

- $A = \begin{bmatrix} 5 & 7 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 0 & -3 \\ 0 & 2 & 2 \end{bmatrix}$

In Class Exercise 1.4
SOLUTION

$A = \begin{bmatrix} 5 & 7 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 0 & -3 \\ 0 & 2 & 2 \end{bmatrix}$

- $A^T = \begin{bmatrix} 5 & 3 \\ 7 & 1 \\ 2 & 4 \end{bmatrix}$
- $B^T = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 0 & 2 \\ 1 & -3 & 2 \end{bmatrix}$
- $AB = \begin{bmatrix} 5 & 29 & -12 \\ 3 & 23 & 8 \end{bmatrix}$
- $(AB)^T = \begin{bmatrix} 5 & 3 \\ 29 & 23 \\ -12 & 8 \end{bmatrix}$
- $B^T A^T = \begin{bmatrix} 5 & 3 \\ 29 & 23 \\ -12 & 8 \end{bmatrix}$

THE IDENTITY OF A MATRIX AND THE INVERSE MATRIX
THE IDENTITY MATRIX HAS ALL ELEMENTS 0, EXCEPT THOSE OF THE LEADING DIAGONAL. But this is not why its called the identity matrix.

Complete Linear algebra: (beginner to expert) theory and practice in python, with concrete applications in machine learning and other fields.

$$A = (a_{i,j}) \in \Re^{nxn}, I_n \in \Re^{nxn}, I_n = \begin{cases} 1, if\ i = j \\ 0, if\ i \neq j \end{cases}$$

$$\rightarrow \forall \leq i \leq n, 1 \leq j \leq n, \quad A^{-1}A = AA^{-1} = I_n \in \Re^{nxn}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{23} & a_{33} & a_{34} \\ a_{41} & a_{24} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{23} & a_{33} & a_{34} \\ a_{41} & a_{24} & a_{43} & a_{44} \end{bmatrix}$$

$$AI_n = A = I_nA$$

$*$: $e = 1, x(e) = x(1) = 1(x), x^{-1} = \frac{1}{x}$ $since\ x\left(\frac{1}{x}\right) = e = 1.$

$+$: $e = 0, x + e = x + 0 = x, \ x^{-1} = -x \ \ since\ x + (-x) = 0$==e

- $\left(A^{-1}\right)^{-1} = A$
- $(AB)^{-1} = B^{-1}A^{-1}$

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In Class Exercise 1.5

Show that

- $AB(AB)^{-1} = I_n, If\ A, B \epsilon\ \mathfrak{R}^{\mathrm{nx}n}$

In Class Exercise 1.5
SOLUTION

$$- (AB)(AB)^{-1} = (AB)(B^{-1}A^{-1})$$
$$= (ABB^{-1}A^{-1})$$
$$= (AI_nA^{-1})$$
$$= (AA^{-1})$$
$$= I_n$$

COMPLETE LINEAR ALGEBRA

LINEAR ALGEBRA > **MATRIX ALGEBRA** > Echelon Form(EF)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

All rows which are not completely zeros are above any rows made of only zeros.

The left most non-zero value of a row(leading entry) is in a column to the right of the leading entry of the row above it & all entries in a column below a leading entry are zeros.

If the leading entry in each nonzero row is 1 and each leading 1 is the only nonzero entry in its column, then the matrix is in its row reduced echelon form (RREF)

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$$\begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,1} & x_{3,2} & x_{3,3} \end{bmatrix} \begin{matrix} l_1 \\ l_2 \\ l_3 \end{matrix} \sim \begin{bmatrix} kx_{1,1} & kx_{1,2} & kx_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ \left(\frac{1}{b}\right)x_{3,1} & \left(\frac{1}{b}\right)x_{3,2} & \left(\frac{1}{b}\right)x_{3,3} \end{bmatrix} \begin{matrix} l_{1=kl_1} \\ l_2 \\ l_3 = \left(\frac{1}{b}\right)l_3 \end{matrix}$$

A row can be replaced by its scalar multiple

$$\begin{bmatrix} 0 & 2 & 6 & 1 & 2 \\ 3 & 0 & 0 & 1 & 0 \\ 3 & 4 & 7 & 4 & 0 \\ 8 & 2 & 6 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{array} \sim \begin{bmatrix} 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 6 & 1 & 2 \\ 3 & 4 & 7 & 4 & 0 \\ 8 & 2 & 6 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} r_1 = r_2 \\ r_2 = r_1 \\ r_3 \\ r_4 \\ r_5 \end{array}$$

$$\sim \begin{bmatrix} 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 6 & 1 & 2 \\ 0 & 4 & 7 & 3 & 0 \\ 0 & 2 & 6 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} r_1 \\ r_2 \\ r_3 = r_3 - r_1 \\ r_4 = r_4 - \frac{8}{3} r_1 \\ r_5 \end{array}$$

$$\sim \begin{bmatrix} 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 6 & 1 & 2 \\ 0 & 0 & -5 & 1 & -4 \\ 0 & 0 & 0 & -\frac{5}{3} & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} r_1 \\ r_2 \\ r_3 = r_3 - 2r_2 \\ r_4 = r_4 - r_2 \\ r_5 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 3 & \frac{1}{2} & 1 \\ 0 & 0 & 1 & -\frac{1}{5} & \frac{4}{5} \\ 0 & 0 & 0 & 1 & \frac{6}{5} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} r_1 = \frac{r_1}{3} \\ r_2 = \frac{r_2}{2} \\ r_3 = \frac{-r_3}{5} \\ r_4 = -\frac{3}{5} r_4 \\ r_5 \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{6}{15} \\ 0 & 1 & 0 & \frac{11}{10} & -\frac{7}{5} \\ 0 & 0 & 1 & 0 & \frac{26}{25} \\ 0 & 0 & 0 & 1 & \frac{6}{5} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} r_1 = r_1 - \frac{1}{3} r_4 \\ r_2 = r_2 - 3r_3 \\ r_3 = r_3 + \frac{1}{5} r_4 \\ r_4 \\ r_5 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{6}{15} \\ 0 & 1 & 0 & 0 & -\frac{136}{50} \\ 0 & 0 & 1 & 0 & \frac{26}{25} \\ 0 & 0 & 0 & 1 & \frac{6}{5} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} r_1 \\ r_2 = r_2 - \frac{11}{10} r_4 \\ r_3 \\ r_4 \\ r_5 \end{array}$$

Pivot Column(s) Free Column(s)

In Class Exercise 1.6

Find the RREF

$$\begin{bmatrix} 2 & 5 & 1 & 2 \\ 1 & 1 & 2 & 1 \\ 1 & 0 & 5 & 3 \end{bmatrix}$$

In Class Exercise 1.6
SOLUTION

$$\begin{array}{c}\begin{bmatrix}2&5&1&2\\1&1&2&1\\1&0&5&3\end{bmatrix}\begin{array}{l}r_1\\r_2\\r_3\end{array}\end{array} \sim \begin{bmatrix}1&0&5&3\\1&1&2&1\\2&5&1&2\end{bmatrix}\begin{array}{l}r_1=r_3\\r_2\\r_3=r_1\end{array}$$

$$\sim \begin{bmatrix}1&0&5&3\\0&1&-3&-2\\0&5&-9&-4\end{bmatrix}\begin{array}{l}r_1\\r_2=r_2-r_1\\r_3=r_3-2r_1\end{array}$$

$$\sim \begin{bmatrix}1&0&5&3\\0&1&-3&-2\\0&0&6&6\end{bmatrix}\begin{array}{l}r_1\\r_2\\r_3=r_3-5r_2\end{array}$$

$$\sim \begin{bmatrix}1&0&5&3\\0&1&-3&-2\\0&0&1&1\end{bmatrix}\begin{array}{l}r_1\\r_2\\r_3=\frac{r_3}{6}\end{array}$$

$$\sim \begin{bmatrix}1&0&5&3\\0&1&0&1\\0&0&1&1\end{bmatrix}\begin{array}{l}r_1\\r_2=r_2+3r_3\\r_3\end{array}$$

$$\sim \begin{bmatrix}1&0&0&-2\\0&1&0&1\\0&0&1&1\end{bmatrix}\begin{array}{l}r_1=r_1-5r_3\\r_2\\r_3\end{array}$$

$AA^{-1} = A^{-1}A = I_n$

$$A = \begin{array}{c}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 4\\0 & 6 & 2\end{bmatrix}\end{array}\begin{array}{l}r_1\\r_2\\r_3\end{array}$$

Operation 1(Op1)

$$\sim A_1 = \begin{bmatrix}1 & 2 & 3\\0 & 1 & 4\\0 & 0 & -22\end{bmatrix}\begin{array}{l}r_1\\r_2\\r_3 = r_3 - 6r_2\end{array}$$

Operation 2(Op2)

$$\sim A_2 = \begin{bmatrix}1 & 2 & 3\\0 & 1 & 4\\0 & 0 & 1\end{bmatrix}\begin{array}{l}r_1\\r_2\\r_3 = \frac{r_3}{-22}\end{array}$$

Operation 3(Op3)

$$\sim A_3 = \begin{bmatrix}1 & 0 & -5\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{array}{l}r_1 = r_1 - 2r_2\\r_2 = r_2 - 4r_3\\r_3\end{array}$$

Operation 4(Op4)

$$\sim A_4 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{array}{l}r_1 = r_1 + 5r_3\\r_2\\r_3\end{array}$$

$A \quad \sim A_1 \quad \sim A_2 \quad \sim A_3 \quad \sim A_4$

$\sim O_1(A) \quad \sim O_2(O_1(A)) \quad \sim O_3(O_2(O_1(A))) \quad \sim O_4\left(O_3\left(O_2(O_1(A))\right)\right) = I_n$

$$O_1 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & -6 & 1\end{bmatrix} \quad O_1A = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & -6 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 4\\0 & 6 & 2\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\0 & 1 & 4\\0 & 0 & -22\end{bmatrix}$$

$$O_4\left(O_3\left(O_2(O_1(A))\right)\right) = I_n \dashrightarrow O_4O_3O_2O_1A = I_n$$

$$\dashrightarrow OA = I_n$$

$$\dashrightarrow O^{-1}OA = O^{-1}I_n$$

$$\dashrightarrow A = O^{-1}I_n = O^{-1}$$

$$\dashrightarrow A^{-1} = (O^{-1})^{-1}$$

$$\dashrightarrow A^{-1} = O$$

$$\dashrightarrow A^{-1} = OI_n$$

$$A=\begin{bmatrix}1&0&2&0\\1&1&0&0\\1&2&0&1\\1&1&1&1\end{bmatrix}$$

$$[A|I_n]\sim\left[\begin{array}{cccc|cccc}1&0&2&0&1&0&0&0\\1&1&0&0&0&1&0&0\\1&2&0&1&0&0&1&0\\1&1&1&1&0&0&0&1\end{array}\right]\begin{array}{l}r\\r_2\\r_3\\r_4\end{array}$$

$$\sim\left[\begin{array}{cccc|cccc}1&0&2&0&1&0&0&0\\0&1&-2&0&-1&1&0&0\\0&2&-2&1&-1&0&1&0\\0&0&1&1&0&-1&0&1\end{array}\right]\begin{array}{l}r_1\\r_2=r_2-r_1\\r_3=r_3-r_1\\r_4=r_4-r_2\end{array}$$

$$\sim\left[\begin{array}{cccc|cccc}1&0&2&0&1&0&0&0\\0&1&-2&0&-1&1&0&0\\0&0&1&1&0&-1&0&1\\0&2&-2&1&-1&0&1&0\end{array}\right]\begin{array}{l}r_1\\r_2\\r_3=r_4\\r_4=r_3\end{array}$$

$$\sim\left[\begin{array}{cccc|cccc}1&0&2&0&1&0&0&0\\0&1&0&2&-1&-1&0&2\\0&0&1&1&0&-1&0&1\\0&0&2&1&1&-2&1&0\end{array}\right]\begin{array}{l}r_1\\r_2=r_2+2r_3\\r_3\\r_4=r_4-2r_2\end{array}$$

$$\sim\left[\begin{array}{cccc|cccc}1&0&2&0&1&0&0&0\\0&1&0&2&-1&-1&0&2\\0&0&1&1&0&-1&0&1\\0&0&0&1&-1&0&-1&2\end{array}\right]\begin{array}{l}r_1\\r_2\\r_3\\r_4=-r_4+2r_3\end{array}$$

$$\sim\left[\begin{array}{cccc|cccc}1&0&2&0&1&0&0&0\\0&1&0&0&1&-1&2&-2\\0&0&1&0&1&-1&1&-1\\0&0&0&1&-1&0&-1&2\end{array}\right]\begin{array}{l}r_1\\r_2=r_2-2r_4\\r_3=r_3-r_4\\r_4\end{array}$$

$$\sim\left[\begin{array}{cccc|cccc}1&0&0&0&-1&2&-2&2\\0&1&0&0&1&-1&2&-2\\0&0&1&0&1&-1&1&-1\\0&0&0&1&-1&0&-1&2\end{array}\right]\begin{array}{l}r_1=r_1-2r_3\\r_2\\r_3\\r_4\end{array}$$

$$\longrightarrow A^{-1}=\begin{bmatrix}-1&2&-2&2\\1&-1&2&-2\\1&-1&1&-1\\-1&0&-1&2\end{bmatrix}$$

In Class Exercise 1.7

$A_1=\begin{bmatrix}3&2&1\\1&7&11\\5&13&3\end{bmatrix}, Find\ A_1^{-1}$

$A_2=\begin{bmatrix}a&b\\c&d\end{bmatrix}, Find\ A_2^{-1}, hence\ deduce\ the\ inverse\ of\ a\ 2\ by\ 2\ matrix$

In Class Exercise 1.7
SOLUTION

- $A_1^{-1} = \begin{bmatrix} 0.429 & -0.024 & -0.052 \\ -0.183 & -0.014 & 0.112 \\ 0.077 & 0.102 & -0.066 \end{bmatrix},$

- $[A_2|I_2] \sim \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \begin{array}{c} r_1 \\ r_2 \end{array}$

$$\sim \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & \dfrac{ad-bc}{a} & -\dfrac{c}{a} & 1 \end{array}\right] \begin{array}{l} r_1 \\ r_2 = r_2 - \left(\dfrac{c}{a}\right) r_1 \end{array}$$

$$\sim \left[\begin{array}{cc|cc} a & 0 & \dfrac{ad}{ad-bc} & \dfrac{-ab}{ad-bc} \\ 0 & \dfrac{ad-bc}{a} & -\dfrac{c}{a} & 1 \end{array}\right] \begin{array}{l} r_1 = r_1 - \left(\dfrac{ab}{ad-bc}\right) r_2 \\ r_2 \end{array}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \dfrac{d}{ad-bc} & \dfrac{-b}{ad-bc} \\ 0 & 1 & -\dfrac{c}{ad-bc} & \dfrac{a}{ad-bc} \end{array}\right] \begin{array}{l} r_1 = (\dfrac{1}{a}) r_1 \\ r_2 = \left(\dfrac{a}{ad-bc}\right) r_2 \end{array}$$

$$\boxed{\longrightarrow A_2^{-1} = (\frac{1}{ad-bc}) \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

$$A = \begin{bmatrix} 1 & 4 \\ 8 & 7 \end{bmatrix} \longrightarrow A^{-1} = (\frac{1}{(1*7)-(4*8)}) \begin{bmatrix} 7 & -4 \\ -8 & 1 \end{bmatrix}$$