COMPLETE LINEAR ALGEBRA

LINEAR ALGEBRA > **INTRODUCTION** > About This Course

- Python Setup, Basic Python, numpy, pandas and matplotlib
- Matrix Algebra
- Linear equations and transformations
- Vectors
- Vector Spaces
- Metric Spaces, Normed spaces, Inner Product Spaces
- Orthogonality
- Determinant and Trace Operator
- Matrix Decompositions (Eigen, SVD and Cholesky)
- Symmetric matrices and Quadratic Forms
- Left Inverse, Right Inverse, Pseudo Inverse

Connecting the dots 1
LINEAR REGRESSION

Connecting the dots 2
PRINCIPAL COMPONENT ANALYSIS

- $Tr(ABC) = Tr(BCA) = Tr(CAB)$
- $(ABC)^T = C^T B^T A^T$
- $P = P^T \quad \exists B / \quad B^{-1}PB = diag(\lambda_P) \quad B^T = B^{-1}$
- $A = U\Sigma V^T$
- $H \sqsubseteq V , B = Base(H) \qquad [Proj_H(y)]_E = Proj_H(y) = B\left(B^T B\right)^{-1} B^T y$

  $[Proj_H(y)]_B = \left(B^T B\right)^{-1} B^T y$

- Visualization
- Reduction of dimensions in order to increase computational efficiency without loosing much data.
- Noise removal

Determinant and Trace Operator | Metric Spaces, Normed spaces, Inner Product Spaces

Symmetric matrices and Quadratic Forms | Matrix Algebra | Orthogonality

Vectors | Vector Spaces | Matrix Decompositions (Eigen, SVD and Cholesky)

COMPLETE LINEAR ALGEBRA

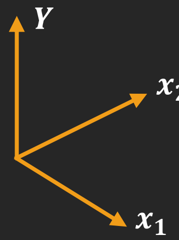
LINEAR ALGEBRA > Connecting the Dots > DIMENSIONALITY REDUCTION (PCA ALGORITHM)

$(x_1,\dots,x_m)\in R^n$ $\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \cdots & x_{m,n} \end{bmatrix}\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \longrightarrow \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,d} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,d} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \cdots & x_{m,d} \end{bmatrix}\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_m \end{array}$

$d<n$

$(x_1,\dots,x_m)\in R^d$

X_1	X_2
$x_{1,1}$	$x_{1,2}$
$x_{2,1}$	$x_{2,2}$
$\vdots$	$\vdots$
$x_{m,1}$	$x_{m,2}$

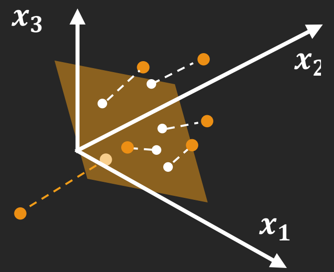


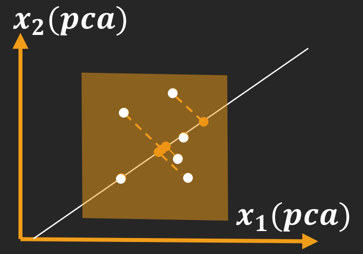
x_3 ?

$X_1(pca)$	$X_2(pca)$
$x(pca)_{1,1}$	$x(pca)_{1,2}$
$x(pca)_{2,1}$	$x(pca)_{2,2}$
$\vdots$	$\vdots$
$x(pca)_{m,1}$	$x(pca)_{m,2}$

$(X_1, X_2, X_3) \longrightarrow (X_1(pca), X_2(pca), X_3(pca))$ $\quad X_1(pca) > X_2(pca) > X_3(pca)$

$(X_1(pca))$ $\quad (X_1(pca), X_2(pca))$





$x_1(pca)$

$B=\{B_1,\dots,B_d\}$ $\quad H\sqsubseteq R^n$ $\quad B=base(H)$ $\quad x\in R^n$

$Proj_H(x) = BB^Tx = B\alpha = \alpha_1B_1+\cdots+\alpha_dB_d$ $\quad [Proj_H(x)]_B = \alpha$ $\quad \alpha = \begin{bmatrix} \propto_1 \\ \vdots \\ \propto_d \end{bmatrix}$

X_1	X_2	X_3
1	0	1
1	2	2
2	1	4
1	1	0

$x_1 = \begin{bmatrix}1\\0\\1\end{bmatrix}$ $x_3 = \begin{bmatrix}2\\1\\4\end{bmatrix}$ $x_2 = \begin{bmatrix}1\\2\\2\end{bmatrix}$ $x_4 = \begin{bmatrix}1\\1\\0\end{bmatrix}$

$H\sqsubseteq R^3$

$B = base(H)$

$B=\{B_1,B_2\} = \left\{\begin{matrix} \frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \\ 0 \end{matrix}, \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}\right\}$

$x\in R^3$

$Proj_H(x) = BB^Tx$

$[Proj_H(x_1)]_B = B^Tx_1 = \begin{bmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}\frac{2}{\sqrt5}\\1\end{bmatrix}$ $\quad [Proj_H(x_1)]_E = B(B^Tx_1) = \begin{bmatrix} \frac{2}{\sqrt5} & 0 \\ \frac{1}{\sqrt5} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix}\frac{2}{\sqrt5}\\1\end{bmatrix} = \begin{bmatrix}\frac45\\\frac25\\1\end{bmatrix} = \begin{bmatrix}0.8\\0.4\\1\end{bmatrix}$

$[Proj_H(x_2)]_B = B^Tx_2 = \begin{bmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}\frac{4}{\sqrt5}\\2\end{bmatrix}$ $\quad [Proj_H(x_2)]_E = B(B^Tx_2) = \begin{bmatrix} \frac{2}{\sqrt5} & 0 \\ \frac{1}{\sqrt5} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix}\frac{4}{\sqrt5}\\2\end{bmatrix} = \begin{bmatrix}\frac85\\\frac45\\2\end{bmatrix} = \begin{bmatrix}1.6\\0.8\\1\end{bmatrix}$

$[Proj_H(x_3)]_B = B^Tx_3 = \begin{bmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}2\\1\\4\end{bmatrix} = \begin{bmatrix}\frac{5}{\sqrt5}\\4\end{bmatrix}$ $\quad [Proj_H(x_3)]_E = B(B^Tx_3) = \begin{bmatrix} \frac{2}{\sqrt5} & 0 \\ \frac{1}{\sqrt5} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix}\frac{5}{\sqrt5}\\4\end{bmatrix} = \begin{bmatrix}2\\1\\4\end{bmatrix}$

$[Proj_H(x_4)]_B = B^Tx_4 = \begin{bmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}\frac{3}{\sqrt5}\\0\end{bmatrix}$ $\quad [Proj_H(x_2)]_E = B(B^Tx_1) = \begin{bmatrix} \frac{2}{\sqrt5} & 0 \\ \frac{1}{\sqrt5} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix}\frac{3}{\sqrt5}\\0\end{bmatrix} = \begin{bmatrix}\frac65\\\frac35\\0\end{bmatrix} = \begin{bmatrix}1.2\\0.6\\0\end{bmatrix}$

X_1	X_2	X_3
1	0	1
1	2	2
2	1	4
1	1	0

$\longrightarrow$

$X_1(pca)$	$X_2(pca)$
$\frac{2}{\sqrt5}$	1
$\frac{4}{\sqrt5}$	2
$\frac{5}{\sqrt5}$	4
$\frac{3}{\sqrt5}$	0

$\boxed{B???}$

COMPLETE LINEAR ALGEBRA

LINEAR ALGEBRA > Connecting the Dots > DIMENSIONALITY REDUCTION (PCA ALGORITHM)

$$\sum_i \|x_i - [Proj_H(x_i)]_E\|_2^2 \longrightarrow 0 \qquad \boldsymbol{B = base(H)} \quad \boldsymbol{H \sqsubseteq R^n}$$

$$B_{optimal} = \underset{B}{argmin} \sum_i \|x_i - [Proj_H(x_i)]_E\|_2^2 \; \longrightarrow B_{optimal} = \underset{B}{argmin} \sum_i \|x_i - BB^T x_i\|_2^2$$

$$\|x_1 - [Proj_H(x_1)]_E\|_2^2 = \left\| \begin{bmatrix}1\\0\\1\end{bmatrix} - \boldsymbol{BB^T}\begin{bmatrix}1\\0\\1\end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix}1\\0\\1\end{bmatrix} - \begin{bmatrix}0.8\\0.4\\1\end{bmatrix} \right\|_2^2 = [(1-0.8)^2 + (0-0.4)^2 + (1-1)^2]$$

$$\|x_2 - [Proj_H(x_2)]_E\|_2^2 = \left\| \begin{bmatrix}1\\2\\2\end{bmatrix} - \boldsymbol{BB^T}\begin{bmatrix}1\\2\\2\end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix}1\\2\\2\end{bmatrix} - \begin{bmatrix}1.6\\0.8\\1\end{bmatrix} \right\|_2^2 = [(1-1.6)^2 + (2-0.8)^2 + (2-1)^2]$$

$$\|x_3 - [Proj_H(x_3)]_E\|_2^2 = \left\| \begin{bmatrix}2\\1\\4\end{bmatrix} - \boldsymbol{BB^T}\begin{bmatrix}2\\1\\4\end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix}2\\1\\4\end{bmatrix} - \begin{bmatrix}2\\1\\4\end{bmatrix} \right\|_2^2 = [(2-2)^2 + (1-1)^2 + (4-4)^2]$$

$$\|x_4 - [Proj_H(x_4)]_E\|_2^2 = \left\| \begin{bmatrix}1\\1\\0\end{bmatrix} - \boldsymbol{BB^T}\begin{bmatrix}1\\1\\0\end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix}1\\1\\0\end{bmatrix} - \begin{bmatrix}1.2\\0.6\\0\end{bmatrix} \right\|_2^2 = [(1-1.2)^2 + (1-0.6)^2 + (0-0)^2]$$

$$\|A - B\|_F^2 = \sum_{i,j} (a_{i,j} - b_{i,j})^2 \qquad = 0.2 + 2.8 + 0 + 0.8 = 3.8$$

$$\left\| \begin{bmatrix}1&1&2&1\\0&2&1&1\\1&2&4&0\end{bmatrix} - BB^T \begin{bmatrix}1&1&2&1\\0&2&1&1\\1&2&4&0\end{bmatrix} \right\|_F^2 = \left\| \begin{bmatrix}1&1&2&1\\0&2&1&1\\1&2&4&0\end{bmatrix} - \begin{bmatrix}0.8&1.6&2&1.2\\0.4&0.8&1&0.6\\1&1&4&0\end{bmatrix} \right\|_F^2$$

$$\longrightarrow B_{optimal} = \underset{B}{argmin} \|A^T - BB^TA^T\|_F^2 \qquad B^TB = I_n$$

$$= \underset{B}{argmin}\, Tr[(A^T - BB^TA^T)^T(A^T - BB^TA^T)] \;= \underset{B}{argmin}\, Tr[(A - ABB^T)(A^T - BB^TA^T)]$$

$$= \underset{B}{argmin}\, Tr[(AA^T - ABB^TA^T - ABB^TA^T + ABB^TBB^TA^T)]$$

$$= \underset{B}{argmin}\, Tr[(AA^T - ABB^TA^T - ABB^TA^T + ABI_nB^TA^T)] = \underset{B}{argmin}\, Tr[(AA^T - ABB^TA^T)]$$

$$= \underset{B}{argmin}\, Tr[(-ABB^TA^T)] \;= \underset{B}{argmax}\, Tr[(ABB^TA^T)] \qquad Tr(ABC) = Tr(BCA) = Tr(CAB)$$

$$= \underset{B}{argmax}\, Tr[(B^TA^TAB)] \qquad (A^TA)^T = (A^TA) \qquad \longrightarrow \exists B \;/\; B^TA^TAB = diag(\lambda_{A^TA})$$

$$\longrightarrow Tr(\lambda_{A^TA}) = Tr\left(\begin{bmatrix}\lambda_1&0&..&0\\0&\lambda_2&...&0\\\vdots&\vdots&\ddots&\vdots\\0&...&...&\lambda_n\end{bmatrix}\right) = (\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n)$$

$$\longrightarrow B_{optimal} = Eigen\ vectors\ of\ (A^TA), corresponding\ to\ biggest\ eigen\ values \qquad AA^T \quad A \in R^{nxm}$$

$$\boldsymbol{A} = \begin{bmatrix}1&0&1\\1&2&2\\2&1&4\\1&1&0\end{bmatrix} \longrightarrow \boldsymbol{A^TA} = \begin{bmatrix}1&1&2&1\\0&2&1&1\\1&2&4&0\end{bmatrix}\begin{bmatrix}1&0&1\\1&2&2\\2&1&4\\1&1&0\end{bmatrix} = \begin{bmatrix}7&5&11\\5&6&8\\11&8&21\end{bmatrix} \qquad \boldsymbol{P = A^TA}$$

$$\boldsymbol{det(P - \lambda I_3) = 0 \to det}\left(\begin{bmatrix}7&5&11\\5&6&8\\11&8&21\end{bmatrix} - \lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right) = 0 \to \boldsymbol{det}\left(\begin{bmatrix}7-\lambda&5&11\\5&6-\lambda&8\\11&8&21-\lambda\end{bmatrix}\right) = 0$$

$$\longrightarrow (7-\lambda)[(21-\lambda)(6-\lambda) - 64] - 5\,[(21-\lambda)(5) - 88] + 11[40 - (11)(6-\lambda)] = 0$$

$$\longrightarrow (7-\lambda)[(21-\lambda)(6-\lambda) - 64] - 5\,[(21-\lambda)(5) - 88] + 11[40 - (11)(6-\lambda)] = 0$$

$$\longrightarrow (\lambda - 30.64)(\lambda - 0.80)(\lambda - 2.55) = 0$$

$$\lambda_1 = 30.64, \qquad \lambda_2 = 2.55, \qquad \lambda_3 = 0.80$$

$$V_1 = \begin{bmatrix}0.45508391\\0.35717276\\0.81567573\end{bmatrix}, V_2 = \begin{bmatrix}-0.09647561\\-0.89085772\\0.44392002\end{bmatrix}, V_3 = \begin{bmatrix}0.88520681\\-0.28071363\\-0.37095656\end{bmatrix}$$

$$\longrightarrow B = \left\{\begin{bmatrix}0.45508391\\0.35717276\\0.81567573\end{bmatrix}, \qquad \right\}$$

$$\longrightarrow B = \left\{\begin{bmatrix}0.45508391\\0.35717276\\0.81567573\end{bmatrix}, \begin{bmatrix}-0.09647561\\-0.89085772\\0.44392002\end{bmatrix}\right\}$$

$$\longrightarrow B = \left\{\begin{bmatrix}0.45508391\\0.35717276\\0.81567573\end{bmatrix}\right\}$$

$$[Proj_H(x_1)]_B = B^T x_1 = \begin{bmatrix} 0.45508391 & 0.35717276 & 0.81567573 \\ -0.09647561 & -0.89085772 & 0.44392002 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.89900393 \\ 0.34744441 \end{bmatrix} \quad [Proj_H(x_1)]_E = B(B^T x_1) = \begin{bmatrix} 0.45508391 & -0.09647561 \\ 0.35717276 & -0.89085772 \\ 0.81567573 & 0.44392002 \end{bmatrix} \begin{bmatrix} 0.89900393 \\ 0.34744441 \end{bmatrix} = \begin{bmatrix} 0.375602312 \\ 0.011576180 \\ 0.887533216 \end{bmatrix}$$

$$[Proj_H(x_2)]_B = B^T x_2 = \begin{bmatrix} 0.45508391 & 0.35717276 & 0.81567573 \\ -0.09647561 & -0.89085772 & 0.44392002 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2.80078089 \\ -0.99035101 \end{bmatrix} \quad [Proj_H(x_2)]_E = B(B^T x_2) = \begin{bmatrix} 0.45508391 & -0.09647561 \\ 0.35717276 & -0.89085772 \\ 0.81567573 & 0.44392002 \end{bmatrix} \begin{bmatrix} 2.80078089 \\ -0.99035101 \end{bmatrix} = \begin{bmatrix} 1.370135036 \\ 1.882624483 \\ 1.844892356 \end{bmatrix}$$

$$[Proj_H(x_3)]_B = B^T x_3 = \begin{bmatrix} 0.45508391 & 0.35717276 & 0.81567573 \\ -0.09647561 & -0.89085772 & 0.44392002 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 4.5300435 \\ 0.69187114 \end{bmatrix} \quad [Proj_H(x_3)]_E = B(B^T x_3) = \begin{bmatrix} 0.45508391 & -0.09647561 \\ 0.35717276 & -0.89085772 \\ 0.81567573 & 0.44392002 \end{bmatrix} \begin{bmatrix} 4.5300435 \\ 0.69187114 \end{bmatrix} = \begin{bmatrix} 1.994801218 \\ 1.001649393 \\ 4.002181989 \end{bmatrix}$$

$$[Proj_H(x_4)]_B = B^T x_4 = \begin{bmatrix} 0.45508391 & 0.35717276 & 0.81567573 \\ -0.09647561 & -0.89085772 & 0.44392002 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.81225667 \\ -0.98733333 \end{bmatrix} \quad [Proj_H(x_2)]_E = B(B^T x_1) = \begin{bmatrix} 0.45508391 & -0.09647561 \\ 0.35717276 & -0.89085772 \\ 0.81567573 & 0.44392002 \end{bmatrix} \begin{bmatrix} 0.81225667 \\ -0.98733333 \end{bmatrix} = \begin{bmatrix} 0.464898526 \\ 1.169689475 \\ 0.22421020 \end{bmatrix}$$

X_1	X_2	X_3
1	0	1
1	2	2
2	1	4
1	1	0

→

$X_1(pca)$	$X_2(pca)$
0.89900393	0.34744441
2.80078089	−0.99035101
4.5300435	0.69187114
0.81225667	−0.98733333

$$\|x_1 - [Proj_H(x_1)]_E\|_2^2 = \left\| \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.375602312 \\ 0.011576180 \\ 0.887533216 \end{bmatrix} \right\|_2^2 = [(1-0.38)^2 + (0-0.01)^2 + (1-0.88)^2] = 0.3989$$

$$\|x_2 - [Proj_H(x_2)]_E\|_2^2 = \left\| \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 1.370135036 \\ 1.882624483 \\ 1.844892356 \end{bmatrix} \right\|_2^2 = [(1-1.37)^2 + (2-1.88)^2 + (2-1.84)^2] = 0.1769$$

$$\|x_3 - [Proj_H(x_3)]_E\|_2^2 = \left\| \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 1.994801218 \\ 1.001649393 \\ 4.002181989 \end{bmatrix} \right\|_2^2 = [(2-2)^2 + (1-1)^2 + (4-4)^2] = 0$$

$$\|x_4 - [Proj_H(x_4)]_E\|_2^2 = \left\| \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.464898526 \\ 1.169689475 \\ 0.22421020 \end{bmatrix} \right\|_2^2 = [(1-0.46)^2 + (1-1.17)^2 + (0-0.22)^2] = 0.3689$$

$$= 0.9447 < 3.8$$

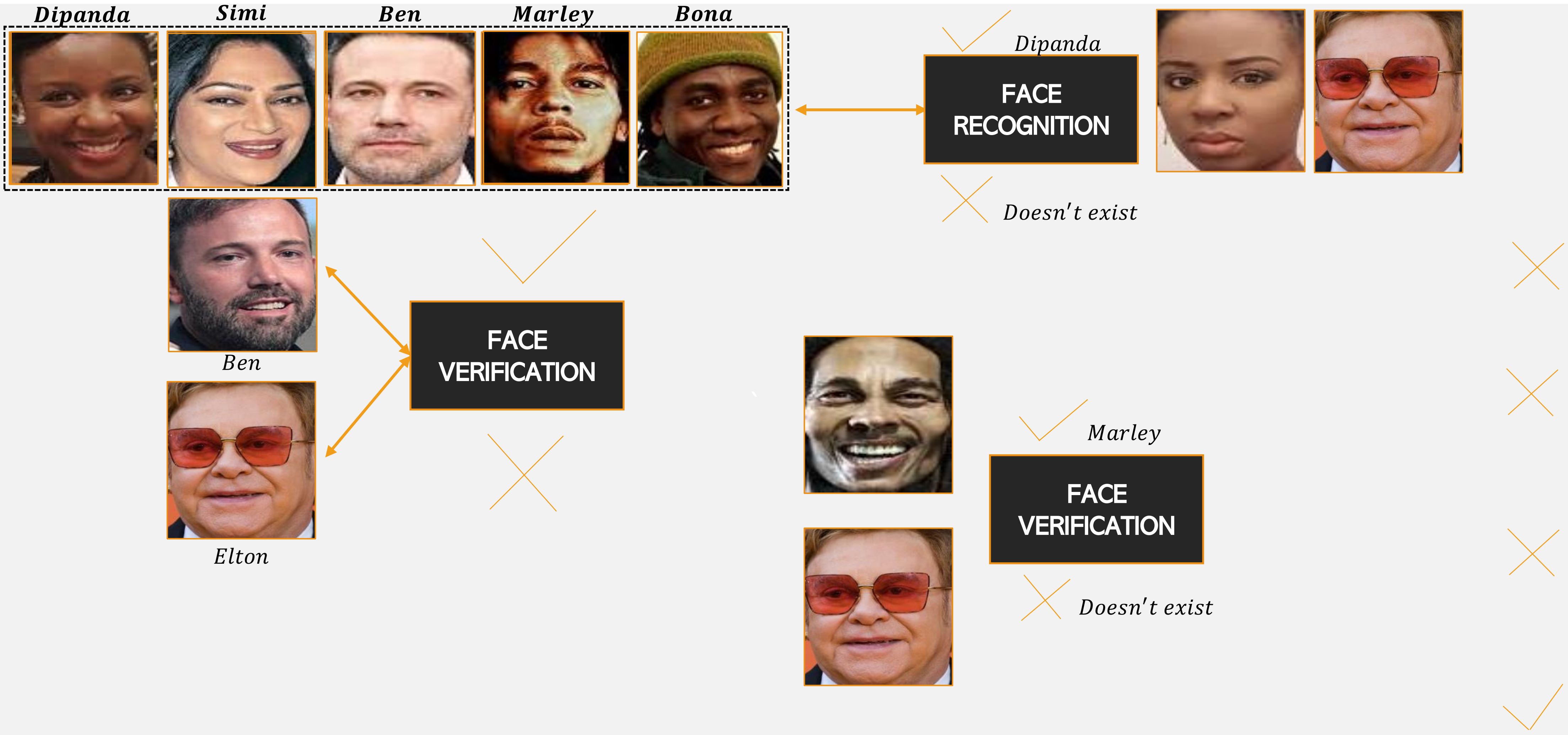
X_1	X_2	X_3
1	0	1
1	2	2
2	1	4
1	1	0

→

$X_1(pca)$
0.89900393
2.80078089
4.5300435
0.81225667

COMPLETE LINEAR ALGEBRA

LINEAR ALGEBRA > Connecting the Dots > DIMENSIONALITY REDUCTION (PCA ALGORITHM)



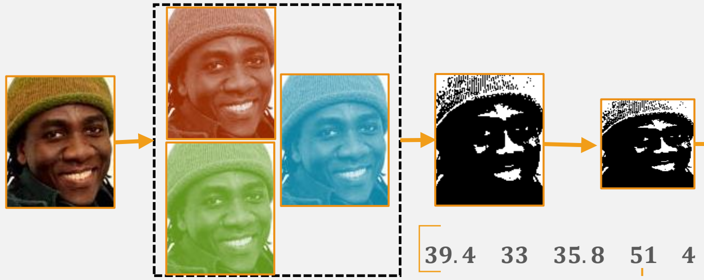
COMPLETE LINEAR ALGEBRA

LINEAR ALGEBRA > Connecting the Dots > DIMENSIONALITY REDUCTION (PCA ALGORITHM)

Get Pixels and Resize → Prepare data (Pixels) for classification → Flatten data → Standardize data → Project to a new space (PCA) → Train → Weights | PCA parametres

Get Pixels and Resize → Flatten data → Standardize data → Project to a new space (PCA) → Predict

Classifier



$$x = \frac{X - mean}{std}$$

39.4	33	35.8	51	4	101	110	108	65.8

$$A = \begin{bmatrix} -0.1 & -0.09 & 0.45 & -0.18 & -0.02 & -0.40 & -0.34 & 0.56 & -0.06 \\ 0.32 & 0.44 & -0.1 & -0.1 & -0.01 & -0.2 & -0.41 & -0.36 & -0.25 \\ -0.13 & -0.1 & -0.13 & -0.18 & 0.05 & 0.58 & 0.56 & 0.06 & 0.20 \\ -0.14 & -0.12 & 0.09 & 0.59 & -0.01 & -0.19 & -0.30 & 0.06 & -0.13 \\ 0.04 & -0.12 & -0.13 & -0.11 & -0.02 & -0.2 & 0.5 & -0.33 & 0.65 \end{bmatrix} \in R^{5x9}$$

$AA^T \in R^{5x5}$ $\quad U \in R^{5x5}$ $\quad V \in R^{9x9}$

$A^TA \in R^{9x9}$ $\quad A^TA \in R^{10000x10000}$ $\quad A = U\Sigma V^T$

$$\begin{bmatrix} -0.1 & -0.09 & 0.45 & -0.18 & -0.02 & -0.40 & -0.34 & 0.56 & -0.06 \\ 0.32 & 0.44 & -0.1 & -0.1 & -0.01 & -0.2 & -0.41 & -0.36 & -0.25 \\ -0.13 & -0.1 & -0.13 & -0.18 & 0.05 & 0.58 & 0.56 & 0.06 & 0.20 \\ -0.14 & -0.12 & 0.09 & 0.59 & -0.01 & -0.19 & -0.30 & 0.06 & -0.13 \\ 0.04 & -0.12 & -0.13 & -0.11 & -0.02 & -0.2 & 0.5 & -0.33 & 0.65 \end{bmatrix} \in R^{5x9}$$

$B \epsilon R^{9x4}$

$$V = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & & & & & & & & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

$$AB = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \epsilon R^{5x4}$$

$R^{5x10000} \longrightarrow R^{5x4}$

$U_r \in R^{5x4}$ $\quad \Sigma_r \in R^{4x4}$ $\quad V_r \in R^{9x4}$

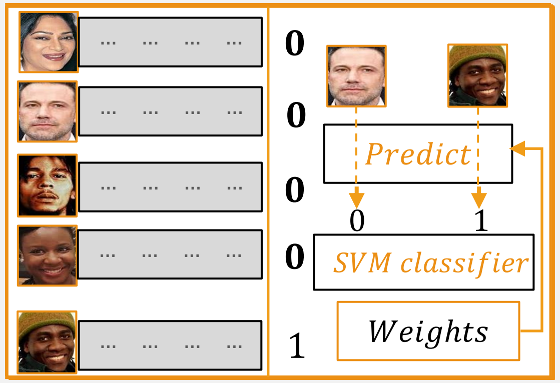
$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} 1.88 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.863 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.533 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.512 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cdots & \cdots & \cdots \\ \vdots & & \vdots \\ \cdots & \cdots & \cdots \end{bmatrix}^T$$

$A = U_r\Sigma_r V_r^T$

$$A = U_r\Sigma_r V_r^T \longrightarrow U_r^T A = U_r^T U_r \Sigma_r V_r^T = \Sigma_r V_r^T$$

$$\longrightarrow \Sigma_r^{-1} U_r^T A = \Sigma_r^{-1}\Sigma_r V_r^T \longrightarrow V_r^T = \Sigma_r^{-1} U_r^T A$$

$$\longrightarrow V_r = (V_r^T)^T = (\Sigma_r^{-1} U_r^T A)^T = A^T U_r (\Sigma_r^{-1})^T = A^T U_r \Sigma_r^{-1}$$



LINEAR ALGEBRA > DIMENSIONAL REDUCTION WITH PCA

Then complete practicals for all the topics
Also complete exercises for all the topics

LINEAR ALGEBRA: MATRIX ALGEBRA

RATIONALE

SDFDF
ASDFDFSDF
ASDDFDFSAFDSF