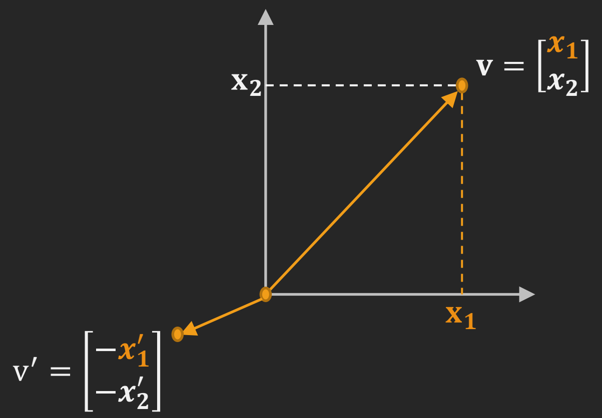
$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \epsilon \Re^n \quad A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,(n-1)} & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,(n-1)} & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{(m-1),1} & a_{(m-1),2} & \cdots & a_{(m-1),(n-1)} & a_{(m-1),n} \\ a_{m,1} & a_{m,2} & \cdots & a_{m,(n-1)} & a_{m,n} \end{bmatrix} \quad v_1 = \begin{bmatrix} a_{1,1} \\ a_{1,2} \\ \vdots \\ a_{1,n} \end{bmatrix} \quad v_2 = \begin{bmatrix} a_{1,1} \\ a_{2,1} \\ \vdots \\ a_{m,1} \end{bmatrix}$$

$$v = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad \text{a} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} \quad \text{b}= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{c} = \begin{bmatrix} 9 \\ 7 \\ 4 \\ 6 \end{bmatrix} \quad \text{d} = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix} \dashrightarrow v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 1 \\ 9 \\ 7 \\ 4 \\ 6 \\ 1 \\ 5 \\ 5 \end{bmatrix}$$

$$v = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix}$$



COMPLETE LINEAR ALGEBRA

LINEAR ALGEBRA > **VECTORS** > Vector Addition and Scalar Multiplication

$a = (a_i) \in \Re^n, b = (b_i) \in \Re^n, \propto \in \Re$

$$c = (a_i + b_i) \in \Re^n = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix} \in \Re^n$$

$$d = \propto a = \propto (a_i) \in \Re^n = \begin{bmatrix} \propto (a_1) \\ \propto (a_2) \\ \vdots \\ \propto (a_n) \end{bmatrix} \in \Re^n$$

$a + b = b + a$

$(a + b) + c = a + (b + c)$

$a + 0 = 0 + a = a$

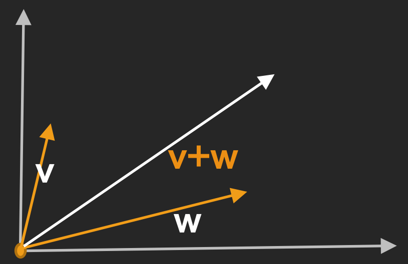
$a - a = 0$

$\propto (\beta a) = (\propto \beta) a$

$(\propto + \beta) a = (\propto) a + (\beta) a$

$\propto (a + b) = (\propto) a + (\propto) b$

$$a = \begin{bmatrix} 2 \\ 8 \\ 9 \\ 10 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 7 \\ 5 \\ 1 \end{bmatrix} \longrightarrow a + b = \begin{bmatrix} 2+2 \\ 8+7 \\ 9+5 \\ 10+1 \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 14 \\ 11 \end{bmatrix}$$



$$\longrightarrow a - b = a + (-1)b = \begin{bmatrix} 2 \\ 8 \\ 9 \\ 10 \end{bmatrix} + \begin{bmatrix} -2 \\ -7 \\ -5 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 4 \\ 9 \end{bmatrix}$$

$$a = \begin{bmatrix} 2 \\ 8 \\ 9 \\ 10 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} \longrightarrow a + b = Invalid$$

In Class Exercise 3.1

Find $a - 2b, 2b - a, 4(a - 3b)$

$$a = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 1 \\ -3 \\ 4 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 0 \\ 52 \\ 12 \\ -2 \\ -1 \end{bmatrix}$$

In Class Exercise 3.1 SOLUTION

$$\boldsymbol{a} - 2\boldsymbol{b} = \begin{bmatrix} -7 \\ 0 \\ -107 \\ -23 \\ 1 \\ 6 \end{bmatrix} \qquad \mathbf{2b - a = -(a - 2b)} = \begin{bmatrix} 7 \\ 0 \\ 107 \\ 23 \\ -1 \\ -6 \end{bmatrix} \qquad \mathbf{4(a - 3b)} = \begin{bmatrix} -44 \\ 0 \\ -636 \\ -140 \\ 12 \\ 28 \end{bmatrix}$$

$A \epsilon \Re^{n}, B \epsilon \Re^{n},$

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad A \cdot B = (a_1 * b_1) + (a_2 * b_2) + \cdots + (a_n * b_n)$$

$A \epsilon \Re^{nx1}, B \epsilon \Re^{nx1}$

$$A^T B = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$= (a_1 * b_1) + (a_2 * b_2) + \cdots + (a_n * b_n)$$

$A \epsilon \Re^{n}, B \epsilon \Re^{n}, C \epsilon \Re^{n}\ \alpha \epsilon \Re,$

$$A \cdot B = B \cdot A$$

$$(A + B) \cdot C = A \cdot C + B \cdot C$$

$$(\alpha A) \cdot B = \alpha(A \cdot B) = A \cdot (\alpha B)$$

$$A \cdot A \geq 0$$

$$A \cdot A = 0 \leftrightarrow A = 0_{\Re^n}$$

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 8 \\ 5 \\ -2 \\ 0 \end{bmatrix} \quad v_1 \cdot v_2 = (1 * 8) + (2 * 5) + (2 * -2) + (1 * 0)$$

$$= 14$$

Magnitude, Length or Norm of Vectors

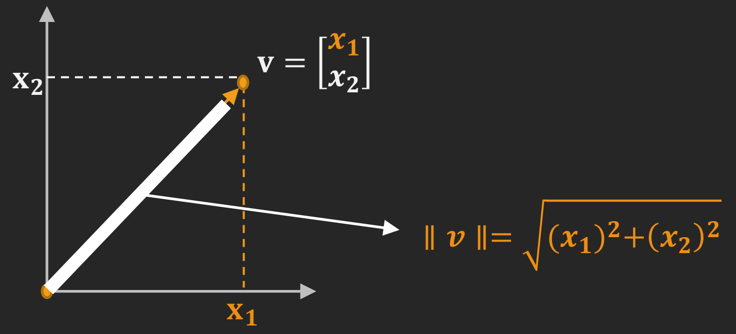
$$a = (a_i) \in \Re^n$$

$$\text{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in \Re^n$$

$$\| a \| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$$

$$a.a = a_1^2 + a_2^2 + \cdots + a_n^2$$

$$\longrightarrow \| a \|^2 = a \cdot a$$



$$\text{v}_1 = \begin{bmatrix} 3 \\ -8 \\ 0 \\ -3 \\ 5 \\ 4 \end{bmatrix} \quad \longrightarrow \| v_1 \| = \sqrt{3^2 + (-8)^2 + 0^2 + (-3)^2 + 5^2 + 4^2}$$

$$= 11.09$$

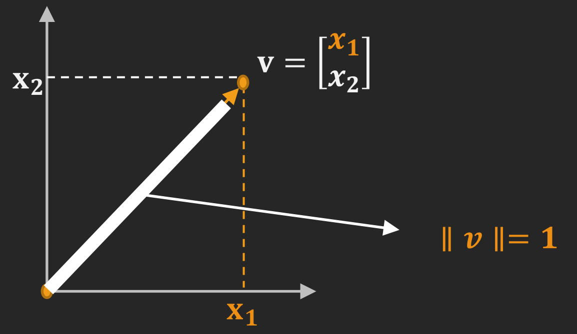
$$\text{v}_2 = \begin{bmatrix} 9 \\ 1 \\ 1 \end{bmatrix} \quad \longrightarrow \| v_2 \| = \sqrt{9^2 + 1^2 + 1^2}$$

$$= 9.11$$

$$a = (a_i) \in \Re^n$$

$$\| a \| = 1$$

$$\mathrm{u_a} = \frac{a}{\| a \|} \longrightarrow \mathrm{u_a} = \begin{bmatrix} \frac{a_1}{\| a \|} \\ \vdots \\ \frac{a_n}{\| a \|} \end{bmatrix}$$



$$a = \begin{bmatrix} 3 \\ -8 \\ 0 \\ -3 \\ 5 \\ 4 \end{bmatrix} \longrightarrow \| a \| = \sqrt{3^2 + (-8)^2 + 0^2 + (-3)^2 + 5^2 + 4^2}$$

$$= 11.09$$

$$\longrightarrow u_a = \begin{bmatrix} \frac{3}{11.09} \\ -\frac{8}{11.09} \\ \frac{0}{11.09} \\ -\frac{3}{11.09} \\ \frac{5}{11.09} \\ \frac{4}{11.09} \end{bmatrix} = \begin{bmatrix} 0.27 \\ -0.72 \\ 0 \\ -0.27 \\ 0.45 \\ 0.36 \end{bmatrix}$$

In Class Exercise 3.2

Say whether the following vectors are unit vectors or not. If not find its(their) unit vector(s)

$$a = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, b = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

In Class Exercise 3.2
SOLUTION

– Yes

– No

$$u_b = \begin{bmatrix} \frac{3}{\sqrt{14}} \\ -\frac{2}{\sqrt{14}} \\ \frac{1}{\sqrt{14}} \end{bmatrix}$$

$$a = (a_i) \in \Re^n, b = (b_i) \in \Re^n$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in \Re^n, \; b = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in \Re^n$$

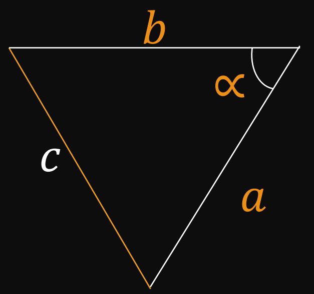
$$\| a - b \| = \left\| \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \right\| = \left\| \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \\ \vdots \\ a_n - b_n \end{bmatrix} \right\| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \cdots + (a_n - b_n)^2}$$

$$\| a \|^2 = a \cdot a \quad \longrightarrow \| a - b \|^2 = (a - b) \cdot (a - b) = (a - b)^T (a - b)$$

$$= a \cdot a - a \cdot b - b \cdot a + b \cdot b$$

$$= \boxed{\| a \|^2 + \| b \|^2 - 2 \boxed{a \cdot b}}$$

$\propto$



$$\| c \|^2 = \| a \|^2 + \| b \|^2 - 2 \| a \| \| b \| \cos(\propto)$$

$$\longrightarrow \| a - b \|^2 = \boxed{\| a \|^2 + \| b \|^2 - 2 \boxed{\| a \| \| b \| \cos(\propto)}}$$

$$\longrightarrow a \cdot b = \| a \| \| b \| \cos(\propto)$$

In Class Exercise 3.3

Deduce the following assertions

The angle between two vectors is 90 degrees, then $a.b = 0$

Two vectors are moving in same direction, then $a \cdot b = \| a \| \| b \|$

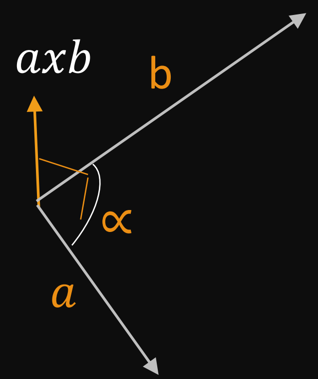
In Class Exercise 3.3
SOLUTION

$- \cos(90) = 0$

$- \cos(0) = 1$

$$axb = \| a \| \| b \| \sin(\propto)\, n$$

$$\| n \| = 1$$



In Class Exercise 3.4

Find aXb, hence deduce a general formula for finding the cross product of two vectors in R^3

$$a = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$a = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$$

$$axb = \left| \begin{array}{ccc} x_1 & x_2 & x_3 \\ \hline 2 & 3 & 5 \\ 4 & 0 & 1 \end{array} \right|$$

$$\begin{bmatrix} a_{11} & \cdots & \dots & a_{1n} \\ \vdots & a_{22} & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & \dots & a_{nn} \end{bmatrix}$$

$$= (+[(3*1) - (0*5)] , - [(1*2) - (4*5)] , + [(0*2) - (3*4)])$$

$$= \begin{bmatrix} 3 \\ 18 \\ -12 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 18 \\ -12 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = 3*2 + 18*3 - 12*5 = 0 \quad \begin{bmatrix} 3 \\ 18 \\ -12 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} = 3*4 + 18*0 - 12*1 = 0$$

In Class Exercise 3.4 SOLUTION

$$\text{axb} = \left| \begin{bmatrix} C_1 & C_2 & C_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \right|$$

$$\text{axb} = \begin{bmatrix} x_2y_3 - x_3y_2 \\ -(x_1y_3 - x_3y_1) \\ x_1y_2 - x_2y_1 \end{bmatrix}$$