MATHEMATICS - II
(MATH 1201)

Time Allotted : 3 hrs. **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If X be a random variable such that $E(X) = 1$ and $E(X^2) = 1$, then $E(X^{100})$ is equal to
(a) 0 (b) 1 (c) 2^{100} (d) $2^{100} + 1$.

(ii) If a connected simple graph has 8 vertices and e edges then,
(a) $7 \le e \le 26$ (b) $6 \le e \le 28$
(c) $7 \le e \le \infty$ (d) $7 \le e \le 28$

(iii) A matrix A can be factorized into lower and upper triangular matrix if all the principal minors of A are
(a) singular (b) non-singular
(c) zero (d) ± 1.

(iv) A fair coin is tossed 10 times. The probability that **ONLY** the first two tosses will yield heads is
(a) $\left(\frac{1}{2}\right)^2$ (b) $\left(\frac{1}{2}\right)^{10}$ (c) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\frac{1}{2}\right)^2$

(v) The Laplace transform of $f(t) = t^3 e^{-3t}$ is:
(a) $\frac{s}{(s-3)^4}$ (b) $\frac{6}{(s+3)^4}$ (c) $\frac{6}{(s-3)^4}$ (d) $\frac{s}{(s+3)^4}$

(vi) If G is a graph with parallel edges, which of the following representation is valid?
(a) Adjacency matrix with only Boolean entries
(b) Adjacency matrix where entries are integer
(c) Adjacency matrix cannot be formed
(d) Incidence matrix cannot be formed.

(vii) The Cauchy-Principal value of the integral $\int_{-1}^{1} \frac{1}{x^2} dx$ is:

(a) 2 (b) -2 (c) 3 (d) 0

(viii) The value of the integral $\int_2^{\infty} \frac{1}{x^2-1} dx$ is

(a) 1 (b) $\frac{1}{2}\log 3$ (c) $-\frac{1}{2}$ (d) $\sqrt{\pi}$.

(ix) Which of the following methods is an iterative method?

(a) Gauss Elimination method (b) Matrix inversion method
(c) Gauss-Seidel method (d) LU factorization method.

(x) $B\left(\frac{1}{2}, \frac{1}{2}\right) = ?$ (Here $B(m, n)$ represents beta function)

(a) π (b) $\sqrt{\pi}$ (c) π^2 (d) 0.

Group – B

2. (a) A random variable X has the following probability mass function :

x	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

(i) Find the value of k.
(ii) Obtain the distribution function $F(x)$.
(iii) Find $P(3 < x \leq 5)$.

[(MATH1201.1, MATH1201.2)(Understand/LOCQ)]

(b) Suppose that while answering a question in a multiple choice test, the probability that an examinee knows the answer is p and the probability that he guesses the answer is $1-p$. Assuming that the probability of answering a question correctly is unity for an examinee who knows the answer and $\frac{1}{m}$ for the examinee who guesses, where m is the number of multiple choice alternatives. Compute the probability that an examinee knows the answer to a question, given that he has correctly answered it.

[(MATH1201.1, MATH1201.2)(Apply/IOCQ)]

6 + 6 = 12

3. (a) Suppose the temperature during May is normally distributed with mean temperature 68° and standard deviation 6°. Find the probability that the temperature during May is

(i) between 70° and 80°
(ii) less than 60°.

[(MATH1201.1, MATH1201.2)(Evaluate/HOCQ)]

(b) A community is concerned about its power supply for the coming winter. There are three major sources of power supply, namely electricity, gas, and oil. The probabilities of shortage of electricity, gas, and oil are estimated to be 0.15, 0.1, and 0.2, respectively. Furthermore, assume that if there is a shortage in the oil supply, the probability of an electrical power shortage will be doubled. The shortage of gas may be assumed to be independent of shortages of oil and electricity.

(i) What is the probability that there will be a shortage of all three major sources of power supply?

(ii) What is the probability that a shortage will occur in at least one of the following sources: gas, electricity?

(iii) If there is a shortage of electricity, what is the probability that gas and oil also will be scarce?

[(MATH1201.1, MATH1201.2) (Understand/LOCQ)]

6 + 6 = 12

Group – C

4. (a) Find a positive real root of the equation $x + \ln x - 2 = 0$, using bisection method correct to two significant figures.

[(MATH1201.3) (Remember/LOCQ)]

(b) Solve the following system of linear equations by matrix inversion method

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

[(MATH1201.3)(Apply/IOCQ)]

6 + 6 = 12

5. (a) Find a root of the equation $x\sin x + \cos x = 0$ using Regula-Falsi correct upto 4 significant figures. [(MATH1201.3) (Understand/LOCQ)]

(b) Find the value of $y(0.2)$ using Runge-Kutta method of fourth order with $h = 0.1$ given that $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$ [(MATH1201.3) (Apply/IOCQ)]

6 + 6 = 12

Group – D

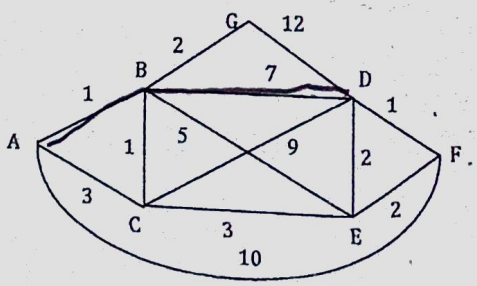
6. (a) Show that there is no simple graph with 12 vertices and 28 edges in which

(i) the degree of each vertex is either 3 or 4.

(ii) the degree of each vertex is either 3 or 6.

[(MATH1201.4) (Understand/LOCQ)]

(b) Using Dijkstra's algorithm find the shortest path and the length of the shortest path between vertices A and D in the following graph.



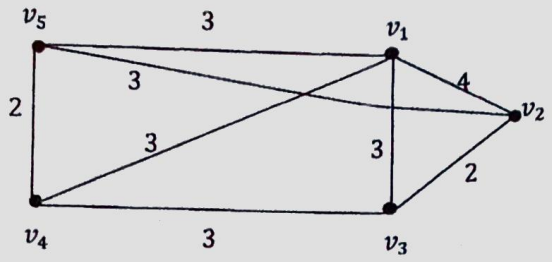
[(MATH1201.4) (Apply/IOCQ)]

(3 + 2) + 7 = 12

7. (a) Using graph theory, prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

[(MATH1201.4) (Analyze/IOCQ)]

(b) Find by Prim's algorithm a minimal spanning tree for the following graph:



[(MATH1201.4) (Apply/IOCQ)]

(c) Does there exist a simple graph with the given adjacency matrix? Justify your answer.

$$\begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

[(MATH1201.4) (Analyze/IOCQ)]

4 + 6 + 2 = 12

Group – E

8. (a) Using Laplace transform solve the following initial value problem:

$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = t^2e^{3t}, y(0) = 2, y'(0) = 6.$

[(MATH1201.5, MATH1201.6) (Evaluate/HOCQ)]

(b) Find the Laplace transform of $\frac{Sinat}{t}$.

Hence show that $\int_0^\infty \frac{Sint}{t}dt = \frac{\pi}{2}$

[(MATH1201.5, MATH1201.6) (Understand/LOCQ)]

7 + (3 + 2) = 12

9. (a) Express the function $F(t) = e^{-t}, 0 < t < 2$
$= 0, t \geq 2$

in terms of unit step function. Hence find $L\{F(t)\}$.

[(MATH1201.5, MATH1201.6) (Evaluate/HOCQ)]

(b) Evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$ [(MATH1201.5, MATH1201.6)(Evaluate/HOCQ)]

(2 + 4) + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	35.42%	38.54%	26.04%

Course Outcome (CO):

After the completion of the course students will be able to

MATH1201.1. Demonstrate the knowledge of probabilistic approaches to solve wide range of engineering problem.

MATH1201. 2. Recognize probability distribution for discrete and continuous variables to quantify physical and engineering phenomenon.

MATH1201. 3. Develop numerical techniques to obtain approximate solutions to mathematical problems where analytical solutions are not possible to evaluate.

MATH1201. 4. Analyze certain physical problems that can be transformed in terms of graphs and trees and solving problems involving searching, sorting and such other algorithms.

MATH1201. 5. Apply techniques of Laplace Transform and its inverse in various advanced engineering problems.

MATH1201. 6. Interpret differential equations and reduce them to mere algebraic equations using Laplace Transform to solve easily.

LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question