Algorithm 1 Kernel Conditional Independence Test

Ensure the X, Y, Z are continuous random variables with domains x, y, z, respectively. Define a measurable, positive definite kernel Kx on x and denote the corresponding RKHS as Hx. Similarly define Ky, Hy, Kz, Hz.

$\langle g, \sum_{YX} f \rangle = E_{XY}(f(X)g(Y)) - E_X(f(X))E_Y(g(Y))$

$\sum_{XY|Z} = \sum_{YX} - \sum_{YZ} \sum_{ZZ}^{-1} \sum_{ZX}$ (1)

$\sum_{XY|Z} = 0 \Longleftrightarrow X \perp\!\!\!\perp Y|Z$ (2)

$\varepsilon_{YZ}^{'} = \{g^{'}|g^{'} = g^{'}(Y) - E(g^{'}|Z), g^{'} \in L_Y^2\}$ (3)

$\bar{f}(\ddot{X}) = f(\ddot{X}) - E(f|Z) = f(\ddot{X}) - h_f^*(Z)$ (4)

$T_{UI} = \frac{1}{n}Tr(K_x, K_y)$ - Test Statistic

It follows that

$T_{UI} = \frac{1}{n}Tr(K_{\ddot{X}|Z}, K_{\ddot{Y}|Z})$ for the conditional case

Compare conditional test statistic to bootstrapped null-distribution based on eigenvalues of Kernels of shuffled samples