Algorithm 1: Smooth Characteristic Function Test

Result: The test-statistic, $nW_n\Sigma_n^{-1}W_n$, and associated p-value

x $\leftarrow \mathbf{X} \in \mathbb{R}^{n\times m}$; y $\leftarrow \mathbf{Y} \in \mathbb{R}^{n\times m}$;

freq$\leftarrow r \in \mathbb{R}$; dim$\leftarrow m$;

wX $\leftarrow$ zeros $\in \mathbb{R}^n$, a placeholder array;

wY $\leftarrow$ zeros $\in \mathbb{R}^n$;

rand $\leftarrow \mathbf{Z} \in \mathbb{R}^{m\times r}$ where $\mathbf{Z}_{ij}$ are sampled from a normal distribution;

for $j = 0 : n$ **do**

  wX[j] $\leftarrow \|$x[j]$\|_2^2$, the norm of each row of X;

  wY[j] $\leftarrow \|$y[j]$\|_2^2$, the norm of each row of Y;

end

smoothX $\leftarrow \exp -\frac{\mathrm{wX}^2}{2}$;

smoothY $\leftarrow \exp -\frac{\mathrm{wY}^2}{2}$;

matX, matY $\leftarrow$ x $\cdot$ rand $\in \mathbb{R}^{n\times r}$, y $\cdot$ rand $\in \mathbb{R}^{n\times r}$, matrix multiplication;

smoothcfX $\leftarrow$ (sin(matX) $\cdot$ smoothX | cos(matX) $\cdot$ smoothX) $\in \mathbb{R}^{n\times 2\cdot r}$;

smoothcfY $\leftarrow$ (sin(matY) $\cdot$ smoothY | cos(matY) $\cdot$ smoothY) $\in \mathbb{R}^{n\times 2\cdot r}$;

diff $\leftarrow$ smoothcfX $-$ smoothcfY;

sigma $\leftarrow \frac{1}{n}\cdot$ diff $\cdot$ diff ^{T}, covariance matrix Σ_n;

mu $\leftarrow \frac{1}{n}\sum_i^n$diff[$i$], W_n;

stat $\leftarrow nW_n\Sigma_n^{-1}W_n$;