
Algorithm 1: Single Point Fast HHG Test

input : (1) sample point matrix $X \in \mathbb{R}^p$ of size N , $Y \in \mathbb{R}^q$ of size M
(2) single point $z_x \in \mathbb{R}^p$, $z_y \in \mathbb{R}^q$ (3) *test* = desired univariate independence test (4) *metric* = distance metric
output: test statistic T , dependent on chosen test, and p-value P

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1 function  $T, P = \text{SINGLEPOINTTEST}(X, Y, z_x, z_y)$ 
2   for  $i \in 1:N$  do /* compute distance-from-point matrix */
3      $D_x, D_y \leftarrow \text{POINTDISTANCE}(X, Y, z_x, z_y, \text{metric})$ 
4     /*  $D_x$  = 1D collection of distances of size N */
5     /*  $D_y$  = 1D collection of distances of size M */
6   end
7   if test = Kolmogorov-Smirnov then
8      $T, P = \text{KSTEST}(D_x, D_y)$ 
9   end
10  else if test = Cramer-Von Mises then
11     $T, P = \text{CMTEST}(D_x, D_y)$ 
12  end
13 end
```

Algorithm 2: Point Distance Function

input : (1) sample point matrix $X \in \mathbb{R}^p$ of size N , $Y \in \mathbb{R}^q$ of size M
(2) single point $z_x \in \mathbb{R}^p$, $z_y \in \mathbb{R}^q$ (3) Metric Type
output: 1D collection of distances D_x of size N and D_y of size M

```
1 function  $D_x, D_y = \text{POINTDISTANCE}(X, Y, z_x, z_y, \text{metric})$ 
2   for  $i \in 1:N$  do
3      $D_x[i] \leftarrow D(X_i, z_x)$  ; /* where D = valid distance metric */
4   end
5   for  $j \in 1:M$  do
6      $D_y[j] \leftarrow D(Y_j, z_y)$  ; /* where D = valid distance metric */
7   end
8 end
```

Algorithm 3: Kolmogorov-Smirnov Test

input : (1) One-dimensional collection of distances D_x of length N
and D_y of length M
output: test statistic T and p-value P

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1 function  $T, P = \text{KSTEST}(D_x, D_y)$ 
2    $T \leftarrow \sup_x (|EDF_{D_x}(x) - EDF_{D_y}(x)|)$ 
3    $P \leftarrow \Pr(K \leq \sqrt{\frac{N+M}{N \cdot M}} * T) ;$  /*  $K$  = Kolmogorov distribution */
4 end
```

Algorithm 4: Cramer-Von Mises Test

input : (1) One-dimensional collection of distances D_x of length N
and D_y of length M
output: test statistic T and p-value P

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1 function  $T, P = \text{CMTEST}(D_x, D_y)$ 
2    $RD_x \leftarrow D_x$  arranged in increasing order
3    $RD_y \leftarrow D_y$  arranged in increasing order
4    $RD_{xy} \leftarrow$  Combined sample of  $D_x$  and  $D_y$  in ranked order
5   Let  $r_1, r_2, \dots, r_N$  be ranks of  $D_x$  elements in  $RD_{xy}$ 
6   Let  $s_1, s_2, \dots, s_M$  be ranks of  $D_y$  elements in  $RD_{xy}$ 
7    $U \leftarrow N \sum_{i=1}^N (r_i - i)^2 + M \sum_{j=1}^M (s_j - j)^2$ 
8    $T \leftarrow \frac{U}{NM(N+M)} - \frac{4MN-1}{6(M+N)}$ 
9    $P \leftarrow$  approximated from limiting distribution of test statistic
10 end
```
