Independent Component Analysis for EEG Processing

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1 Introduction

Independent component analysis allows us to isolate distinct signal sources that cumulatively produce in the recorded EEG signal. This allows us to selectively remove EEG artifacts resulting from blinks, eye movement, and muscle movement. After an independent component analysis is completed, the resulting components can be manually analyzed, and subtracted out of the original signal, to ensure that the components being removed from the original signal do not contain relevant signal information.

2 Independent Component Analysis Fundamentals

2.1 Basics

The recorded EEG signal from one electrode can be mathematically represented by:

$$x = a_1 s_1 + a_2 s_2 + ... + a_n s_n$$

Where x is the recorded signal of the given electrode, s_n is the signal of the n^{th} independent component contributing to the signal, and a_n is the scaling value used to adjust the signal of the n^{th} component to accordingly contribute to the observed EEG signal. For vectors, this notation takes the form

$$x = As$$

Where x is a column vector of observed EEG signal values, s is a column vector of independent component signal values, and A is the mixing matrix. Since we know x, we must estimate A and s. After estimating A, we compute its inverse matrix (W), and then calculate the s components using:

$$s = Wx$$

1. ICA cannot determine true variances of independent sources (s), since they are unknown. Therefore, each independent source must have a variance value relative to the other independent sources. To do this, each source is assumed to have a unit variance of $\mathrm{E}\{s_i^2\}$=1
2. ICA cannot determine the order of independent components (the order of the independent components does not matter in equation)

$$x = a_1 s_1 + a_2 s_2 + ... + a_n s_n$$

2.2 NonGaussianity

Principle restriction of ICA: independent components cannot be Gaussian, because mixing matrix A then cannot be identified. If, for example, the mixing matrix is orthogonal and the independent components are Gaussian, then the observed signals are gaussian, uncorrelated, and of unit variance. Their joint density is:

$$p(x_1, x_2) = \frac{1}{2\pi} exp(-\frac{x_i^2 + x_2^2}{2})$$

and upon plotting this distribution, we see a symmetric shape (which does not contain information on directions of the columns of the mixing matrix A).

2.3 Centering

Center x by subtracting its mean vector

$$m = E\{x\}$$

from x. This makes the original signal (x) and the independent components (s) zero-mean variables. After the mixing matrix A is estimated (with the centered data), we can then add the mean vector of the independent components vector

$$= A^{-1}m$$

back to the centered estimates of s. This gives us the estimated mean.

2.4 Whitening

Whitening is useful because it creates a new, orthogonal mixing matrix $\tilde{A}$. As a result, instead of estimating n^2 parameters of the original mixing matrix (A), we only estimate the orthogonal mixing matrix ($\tilde{A}$). After centering, but before running ICA, we transform the vector of recorded EEG data (x) linearly so we can obtain a new vector of components ($\tilde{x}$) that are uncorrelated, and their variances equal unity. This means the covariance matrix of the new vector is equal to its identity matrix.

$$E\{\tilde{x}\tilde{x}^T\} = I$$

2.4.1 Eigenvalue Decomposition of Covariance Matrix

$$E\{xx^T\} = EDE^T$$

- E is the orthogonal matrix of eigenvectors of $E\{xx^T\}$
- D is the diagonal matrix of eigenvalues

We can now whiten the data using the formula:

$$\tilde{x} = ED^{-1/2}E^T x$$

And since we already discussed earlier that x = As, we can produce the equation for whitened EEG data.

$$\tilde{x} = ED^{-1/2}E^T As = \tilde{A}s$$

The importance of an orthogonal mixing matrix was discussed earlier, and it can be confirmed by the equation:

$$E\{\tilde{x}\tilde{x}^T\} = \tilde{A}E\{ss^T\}\tilde{A}^T = \tilde{A}\tilde{A}^T = I$$

2.4.2 Reducing Dimension of the Data

Remove all eigenvalues of $E\{xx^T\}$ that are too small - in an effort to remove noise.

3 FastICA

3.1 For One Unit

This method estimates only one projection pursuit direction (independent component). This is the direction (unit vector w) that produces the maximum nongaussianity of the projection $w^t x$. This nongaussianity is measured by the approximation of negentropy, given by the equation:

$$J(w^t x) \propto [E\{G(w^t x)\} - E\{G(v)\}]^2$$

- G is an arbitrary non-quadratic function
- v is a Gaussian variable of 0 mean and unit variance
- $w^T x$ is constrained to unity

This FastICA method can be generalized to the form:

1. Choose a random weight vector w
2. Let
$$\mathbf{w}^+ = E\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\} - E\{g'(\mathbf{w}^T\mathbf{x})\}\mathbf{w}$$
3. Let $\mathbf{w} = \frac{\mathbf{w}^+}{||\mathbf{w}^+||}$
4. If not converged, go back to step 2

3.2 For Multiple Units

We must run the one-unit FastICA algorithm using several units with weight vectors (w). We also want to stop different vectors from converging to the same maximum, therefore, we must de-correlate the outputs of w the weight vectors after each iteration. This can be achieved through one of three methods.

3.2.1 Deflation based on Gram-Schmidt-like decorrelation

With this method, we estimate independent components one by one. After estimating p independent components, we fun the one-unit fixed-point algorithm for w_{p+1}. After every iteration step, we must then subtract from w_{p+1} the projections of the previously estimated p vectors. Text, we re-normalize w_{p+1}:

1. $$\mathbf{w}_{p+1} = \mathbf{w}_{p+1} - \sum_{j=1}^{p} \mathbf{w}_{p+1}^T \mathbf{w}_j \mathbf{w}_j$$
2. $$\mathbf{w}_{p+1} = \frac{\mathbf{w}_{p+1}}{\sqrt{\mathbf{w}_{p+1}^T \mathbf{w}_{p+1}}}$$

3.2.2 Symmetric Decorrelation

In this method, no vectors are preferred over others

$$\mathbf{W} = (\mathbf{W}\mathbf{W}^T)^{-1/2}\mathbf{W}$$

- W is a matrix of vectors
- $(\mathbf{W}\mathbf{W}^T)^{-1/2}$ is found by eigenvalue decomposition where: $(\mathbf{W}\mathbf{W}^T)^{-1/2} = \mathbf{F}\lambda^{-1/2}\mathbf{F}^T$

3.2.3 Simplified Symmetric Decorrelation

1. $$\mathbf{w} = \frac{\mathbf{W}}{\sqrt{||\mathbf{W}\mathbf{W}^T||}}$$
2. Repeat this step until convergence
$$\mathbf{W} = \frac{3}{2}\mathbf{W} - \frac{1}{2}\mathbf{W}\mathbf{W}^T\mathbf{W}$$

4 Separation of Artifacts for EEG

1. Import a signal vector (x), consisting of the amplitudes of n signals (n=number of electrodes) at a given time point.
2. Whiten the data and decrease the dimensionality
3. Use FastICA to compute a subset of rows of the matrix W from the equation: $s = Wx$
4. After vector $\mathbf{w}_i$ has been generated, the ICA signal $s_i(t)$ is computed by: $s_i(t) = \mathbf{w}_i^T\mathbf{x}'(t)$ where $\mathbf{x}'(t)$ is the whitened and lower dimensional EEG signal vector

5. Plot the independent components separately, and attempt to identify the possible sources of each component (ie. eye movement, muscle movement, etc)
6. Run a high-pass filter to identify any remaining artifacts
7. Remove the components deemed noise by subtracting the component vector out from the original EEG signal vector