r values of X.

$$\frac{\partial f_u}{\partial u} = \begin{pmatrix} \cdot & & \cdot \\ \frac{\partial f_{u_1}}{\partial u_p} & \cdots & \frac{\partial f_{u_p}}{\partial u_p} \end{pmatrix}$$

Let $g_u(x) = \| \text{proj}_u(x) \|_2^2$. $g_u : \mathbb{R}^p \to \mathbb{R}$

$$\frac{\partial g_u}{\partial u_j} = \frac{\partial}{\partial u_j} \| (x \cdot u) u \|_2^2$$

$$= \frac{\partial}{\partial u_j} \| \left(\sum_{i=1}^p x_i u_i \right) u \|_2^2$$

$$= \frac{\partial}{\partial u_j} \left(\sum_{i=1}^p x_i u_i \right)^2 \| u \|_2^2$$

$$= \frac{\partial}{\partial u_j} \left(\sum_{i=1}^p x_i u_i \right)^2$$

$$= 2 \left(\sum_{i=1}^p x_i u_i \right) \left(\sum_{i=1}^p \frac{\partial}{\partial u_j} x_i u_i \right)$$

$$= 2 (x \cdot u) x_j .$$

$$\Rightarrow \nabla_u g_u = 2 (x \cdot u) x = 2 x^T u x$$

$$\nabla_u g_u = \vec{0} \Rightarrow u = \vec{0} . \text{ xD}$$