Summary of *Symmetric diffeomorphic image registration with cross-correlation: Evaluating automated labeling of elderly and neurodegenerative brain*

Author: B.B. Avants a,*, C.L. Epstein b, M. Grossman c, J.C. Gee a

1. Abstract:

This article developed a novel symmetric image normalization method (SyN) for maximizing the cross-correlation within the space of diffeomorphic maps and provide the Euler–Lagrange equations necessary for this optimization.

2. Some methods of registration

(1) Demons:

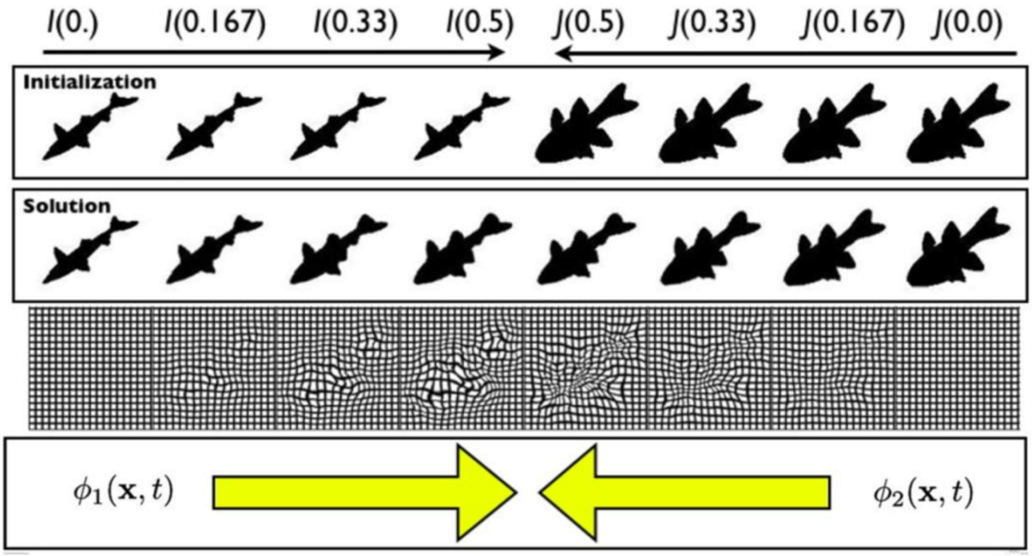
- Thirion,1996
- Uses an approximate elastic regularizer to solve an optical flow problem, where the moving image's level sets are brought into correspondence with those of a reference of fixed template image.
- Use Dice statistic to compare:

$$S(R1, R2) = \frac{2\sharp(R1 \cap R2)}{\sharp(R1) + \sharp(R2)},$$

Which measures both difference in size and location between two segmentations R1 and R2. The # counts the number of pixels in the region.

(2) Symmetric diffeomorphisms

- Define a diffeomorphism ϕ of domain Ω, for transforming image I into a new coordinate system by $\phi I = I \circ \phi(x, t = 1) = I\big(\phi(x, t = 1)\big)$, which indicates that I is warped forward by the map defined by $\phi(x, 1)$.
- This is a visualization of the components of ϕ.



- The top row are the original images, I and J, the second row is the SyN solution converging outputs. The deforming grids associated with these diffeomorphisms are shown in the bottom row.
- Set time t=0.5, then the forward and backward optimization problem is:

$$E_{\text{sym}}(I,J) = \inf_{\phi_1} \inf_{\phi_2} \int_{t=0}^{0.5} \left\{ \|\boldsymbol{v}_1(\mathbf{x},t)\|_L^2 + \|\boldsymbol{v}_2(\mathbf{x},t)\|_L^2 \right\} \mathrm{d}t + \int_\Omega |I(\phi_1(0.5)) - J(\phi_2(0.5))|^2 \mathrm{d}\Omega.$$

Subject to each $\phi_{\mathrm{i}} \in \boldsymbol{Diff}_0$ the solution of:

$$\mathrm{d}\phi_i(\mathbf{x},t)/\mathrm{d}t = \boldsymbol{v}_i(\phi_i(\mathbf{x},t),t) \text{ with } \phi_i(\mathbf{x},0) = \mathbf{Id} \text{ and } \phi_i^{-1}(\phi_i) = \mathbf{Id}, \phi_i(\phi_i^{-1}) = \mathbf{Id}.$$

Forward warp I:

$$\phi I = I \circ \phi(x, t=1) = I(\phi(x, t=1))$$

Backward warp $\phi^{-1}(x,1)$

L: the linear differential operator
t: time
x: a spatial coordinate
$v(x,t)$: a velocity field
ϕ: diffeomorphism
Ω: domain

$$\phi(x,1) = \phi(x,0) + \int_0^1 v(\underbrace{\phi(x,t)}_{y}, t)\,dt$$

$$D(\phi(x,0), \phi(x,1)) = \int_0^1 \|v(x,t)\|_L \, dt$$

$$L = a\nabla^2 + b\,Id$$

Facts: $\phi \to \phi_1, \phi_2$
$x : I$
$z : J$

$$D(Id, \phi_1(x, 0.5)) = D(Id, \phi_2(z, 0.5))$$

Costraint: $D(\phi_1(x, 0.5)) = D(\phi_2(x, 0.5))$

$\phi_1^{-1}(\phi_1) = Id$, $\phi_2^{-1}(\phi_2) = Id$

$\rightarrow \phi_1(x, 1) = \phi_2^{-1}(\phi_1(x, 0.5), 0.5)$