

# Synopsis of the paper

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The manuscript studies probability calibration of predictive models and argues that widely used continuous-output recalibrators (e.g., Platt/temperature scaling) are less calibrated than reported because their true calibration error cannot be reliably measured with finite binning. It introduces the **scaling-binning calibrator**, which first fits a parametric scaling function and then bins its outputs to guarantee measurable calibration with improved sample complexity. The paper also analyzes calibration-error estimation and shows that a debiased estimator from meteorology yields tighter estimates than the common plugin estimator. Experiments on CIFAR-10 and ImageNet demonstrate lower (top-label and marginal) calibration error than histogram binning at comparable or fewer samples and more accurate estimation of ECE/CE. (Abstract; Sec. 3–5; Fig. 1–4, 7; Theorem 4.1; Theorems 5.3–5.4)

# Summary of Review

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Overall, the paper offers a clear problem formulation and practical algorithmic contribution with rigorous analysis and extensive empirical validation. The core insight—that binning underestimates calibration error for continuous methods and that discretized outputs enable verifiable calibration—is well supported (Sec. 3; Prop. 3.3; Fig. 2, 5–6). At the same time, certain assumptions and choices (e.g., well-balanced binning, injectivity/consistency of scaling family) could be stated more prominently with clearer implications for practitioners (Sec. 4; Lemma 4.3; Assumptions 1–4). The experimental section is strong but could benefit from additional baselines (e.g., Dirichlet/beta calibration, vector scaling) and ablations on bin count selection and data splits (Sec. 4.3; App. E–G). The mathematical presentation is mostly correct, though a few places would gain from tightened notation (e.g., use of CE vs.  $\ell_p$ -CE across statements, constants in bounds). (Sec. 3; App. B, D–F).

# Strengths

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- **Evidence-driven critique of continuous recalibration**
  - Demonstrates that binned estimates of calibration error monotonically increase with more bins, indicating underestimation for continuous methods (Sec. 3; Fig. 2a–b, 5–6). This is impactful because it challenges prevailing evaluation practice and motivates verifiable alternatives.
  - Provides constructive theory: Example 3.2 and Proposition 3.3 formally show how binning can hide true error and why coarse binning yields optimistic assessments (Sec. 3; App. B). This enhances technical soundness.
  - Clear separation of method vs. evaluation binning clarifies prior conflation (Sec. 3, 4.1). This improves clarity and reproducibility.
- **Novel scaling-binning calibrator with provable sample complexity**
  - Algorithm combining parametric scaling (Step 1) with uniform-mass binning and discretization (Steps 2–3) is well specified (Sec. 4.1; Fig. 1c; Steps 1–3). It offers a practical path to measurable calibration.

- Theorem 4.1 shows  $\text{CE}(\hat{g}_B)^2 \leq 2 \cdot \min_{\{g \in G\}} \text{CE}(g)^2 + \epsilon^2$  with  $n \gtrsim B \log B + \log(B)/\epsilon^2$  samples, establishing  $B+1/\epsilon^2$  scaling vs. histogram binning's  $B/\epsilon^2$  (Sec. 4.2; Thm. 4.1). This is a strong theoretical contribution.
- Lemma 4.3 (well-balanced binning) and Lemma D.2 (empirical binning convergence) add rigor and explain why discretization does not overly harm sharpness (Prop. D.4) (Sec. 4.2; App. D). This addresses technical soundness and practical impact.
- **Improved estimation of calibration error**
  - Defines plugin vs. debiased estimators and proves sample complexities: plugin  $O(B/\epsilon^2)$  vs. debiased  $O(\sqrt{B}/\epsilon^2)$  for estimating squared CE within a constant factor (Sec. 5; Thm. 5.3–5.4). This is novel and important for evaluation practice.
  - Empirical verification on CIFAR-10/ImageNet shows lower mean-squared deviation from ground-truth estimates for the debiased estimator, especially when  $B$  is large or  $n$  is small (Sec. 5.1; Fig. 4, 12–16). This demonstrates experimental rigor.
  - Extension to debiased ECE via Gaussian approximation provides a pragmatic improvement for  $\ell_1$  calibration metrics (App. G.1; Fig. 12–16). This broadens applicability.
- **Comprehensive experiments and reproducibility**
  - Multiclass marginal and top-label calibration across CIFAR-10 and ImageNet, with bootstrap CIs, ablations on bins/samples, and synthetic validations (Sec. 4.3; Fig. 3, 7–11; App. E). This indicates solid empirical coverage.
  - Open-source library and CodaLab/GitHub links for code/data (Page 10; “Reproducibility”). This supports replication.

## Weaknesses

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- **Assumption visibility and practitioner guidance**
  - Key regularity assumptions (finite parameters, injectivity, consistency, Lipschitz, twice differentiability) are relegated to App. D; clearer upfront guidance on when common calibrators (sigmoid, vector/Dirichlet/beta) satisfy these would aid users (App. D; Assumptions 1–4). This affects clarity and external validity.
  - The “2–well-balanced” binning property is crucial for both Theorem 4.1 and estimator guarantees, yet practical procedures for ensuring/diagnosing it are brief (Sec. 4.2; Lemma 4.3). More diagnostics would improve robustness.
  - Sensitivity of performance to bin count  $B$  and to merging of  $T1/T2/T3$  in practice is noted but not fully quantified in the main paper (App. E). This limits prescriptive guidance.
- **Scope of baselines and comparative positioning**
  - Baselines center on histogram binning vs. the proposed method; direct comparisons to **beta calibration** and **Dirichlet calibration** (NeurIPS’19) are only mentioned peripherally (App. E; Ref. [50], [25]) without empirical head-to-heads. This impacts claims of superiority across methods.
  - Vector scaling / class-wise Platt variants and post-hoc multiclass calibrators beyond per-class binning are not systematically evaluated (Sec. 2.2; App. E). This affects completeness.

- Evaluation primarily uses VGG16; diversity across architectures (e.g., ResNet, DenseNet) could reinforce generality (Sec. 4.3; App. E). No direct evidence found in the manuscript.
- **Mathematical presentation and notation consistency**
  - The switch between CE and  $\ell_p$ -CE (Sec. 3 vs. App. B) can be confusing; explicit notation conventions early on would help (Sec. 3; App. B). This affects clarity.
  - Some bounds hide constants/log factors ("e" notation) without concrete ranges (Prop. D.4; App. D). Explicit constants would aid reproducibility.
  - Minor ambiguities in algorithmic description (e.g., separate datasets T1/T2/T3 vs. merged in practice—App. E) could be clarified within the main text.
- **Generalization beyond image classifiers**
  - Claims emphasize calibration in classification; while CE/ECE definitions extend to regression (e.g., Brier score discussion in Sec. 2.2), experiments focus on image multiclass tasks. Broader domains (NLP, tabular risk models) are noted in related work but not evaluated (Sec. 6). This limits demonstrated external validity.

## Suggestions for Improvement

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- **Elevate assumptions & provide practitioner diagnostics**
  - Move Assumptions 1–4 (App. D) into the main text with concrete examples showing that common scaling families satisfy injectivity/regularity; add a brief checklist for users (Sec. 4). Mirror with a small table mapping families to assumptions. (Match the three sub-points above.)
  - Provide a practical test/heuristic for 2–well-balanced binning (e.g., empirical bin occupancy thresholds; QQ plots of bin mass) and report sensitivity when it is violated (Sec. 4.2). (Match the three sub-points above.)
  - Quantify sensitivity to B and to merging T1/T2/T3 via ablations in main text (not only App. E); include guidance for choosing B under sample constraints (Sec. 4.3). (Match the three sub-points above.)
- **Expand baselines & positioning**
  - Add empirical comparisons with **beta calibration** (App. E; Ref. [50]) and **Dirichlet calibration** (Ref. [25]) on CIFAR-10/ImageNet with identical splits; include vector scaling and multiclass post-hoc methods. (Match the three sub-points above.)
  - Evaluate on alternative architectures (e.g., ResNet-50, DenseNet-121) to demonstrate method robustness beyond VGG. (Match the three sub-points above.) No direct evidence found in the manuscript.
  - Summarize comparative takeaways in a positioning table (method class, measurability, sample complexity, CE/ECE results). (Match the three sub-points above.)
- **Tighten math and notation**
  - Unify CE vs.  $\ell_p$ -CE notation and state global conventions at the start of Sec. 2–3; when switching metrics, restate definitions inline (Sec. 3; App. B). (Match the three sub-points above.)

- Provide explicit constants (or ranges) hidden by  $\tilde{O}$ /e-notation in bounds (e.g., Prop. D.4) and annotate dependence on  $B$ ,  $n$ , and bin-balance parameters; add a short “constants and logs” appendix table (App. D). (Match the three sub-points above.)
- Clarify the algorithmic data split ( $T_1/T_2/T_3$ ) versus practical merging (App. E) within Sec. 4.1 and specify recommended splits for different  $n$ . (Match the three sub-points above.)
- **Broaden empirical scope**
  - Include at least one non-image task (e.g., tabular risk prediction or NLP classification) to demonstrate generality of scaling–binning and debiased estimation; report CE/ECE and MSE trade-offs (Sec. 6). (Match the three sub-points above.)

## References

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- Guo et al., “On calibration of modern neural networks” (ICML 2017) — cited in Sec. 2.3, 3 (for temperature scaling baseline).
- Kull et al., “Beyond sigmoids: ... beta calibration” (EJS 2017) — referenced in App. E for synthetic experiments’ scaling family.
- Kull et al., “Dirichlet calibration” (NeurIPS 2019) — referenced in References list and multiclass calibration context (Page 11; Ref. [25]).