Spiking and Bursting in Josephson junction

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Abstract: We report neuron-like spiking and bursting in resistive-capacitive-inductive shunted Josephson junction (RCLSJ) model. Spiking oscillations namely, regular spiking (RS), intrinsically bursting (IB) and fast spiking (FS) as usually seen in rat's motor cortex are observed in a single junction under external DC bias. The spiking junction voltage is amplitude modulated as well as frequency modulated when forced by weak sinusoidal signal of frequency much lower than the junction resonant frequency. For stronger forcing, bursting oscillations are observed. The junction also shows autonomous bursting in the high inductive regime.

Index terms—Josephson junction, spiking, parabolic bursting, amplitude modulation, frequency modulation, sinusoidal orcing.

I. INTRODUCTION

The studies on the dynamics of Josephson junction under external weak magnetic field are important for its many applications. The Josephson junction is configured as a superconducting quantum interference device (SQUID), which is widely used to detect weak magnetism in geology and military. The SQUID is also used in biomedical tools like magnetoencephalograph (MEG) [1] for mapping changes in weak magnetic field in the human brain. In this respect, investigation on Josephson junction dynamics under external DC stimulus and the interaction of low frequency ac signals is always encouraging.

The resistive-capacitive-shunted junction (RCSJ) model of the superconducting device was mostly used in earlier studies [3-10]. A resistive-capacitive-inductive shunted (RCLSJ) model has been proposed [11-13] later to fit in some experimental inductive effect in shunt with the junction. The RCLSJ reveals hysteresis effect in its experimental current-voltage characteristics. The junction dynamics shows [11-14] limit cycle oscillations above a critical DC bias current and even chaotic behaviors in a selected parameter space. We report, in this paper, some interesting results on neuron-like spiking and bursting [2] behaviors found in a single Josephson junction.

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In biological neuron, the simplest ionic processes involved [15-16] in spiking are due to flow of Na^+ and K^+ ions across the cell membrane while bursting is observed when the fast spiking oscillations is controlled by a slow process of Ca^{++} ion induced by K^+ ion movement across the membrane. The spiking is defined as a repetitive firing state of oscillation while bursting is a recurrent switching between a firing state and a rest state [17]. Bursting is an important mechanism of information processing [2] in neurons where information is usually encoded in the interspike intervals. Various models [18-20] based on the ionic processes across a cell membrane are proposed after the Hodgkin-Huxley model [15] was first proposed in 1952. Since then many phenomenological models [21-24] of spiking and bursting are also derived, which are not strictly based on ionic processes yet they reproduce biologically plausible behaviors. Attempts are still being made to derive simpler dynamical models [2, 25] with slow-fast dynamics so as to reduce computational complexity in simulations of large assembly of neurons.

On the other hand, close similarities are noted in limit cycle, relaxation oscillations and even chaotic oscillation [14] of the superconducting junction in different parametric conditions with those of intracellular Ca^{++} oscillation [26] induced by neurotransmitters of different strength. Moreover, under the action of weak and low frequency sinusoidal signal, the junction chaos is found [14] amplitude modulated similar to what is seen in periodically forced squid giant axon [27]. The inherent slow-fast dynamics in RCLSJ has also been explored [28] recently under small parametric condition. It is known [17, 29-30] that spiking and bursting in neuron or any dynamical system are observed when a slow variable controls the fast dynamics. The obvious questions thus arise whether similar spiking and bursting behaviors could be found in Josephson junction.

We report, in this paper, numerical evidences of spiking oscillations, namely, regular spiking (RS), intrinsic bursting (IB) and fast spiking (FS) in RCLSJ model under external DC bias. Such spiking is typically [25] seen in rat's motor cortex. The junction parameters, namely, the damping factor and the shunt inductor play important roles in such spiking behaviors. We also observed bursting oscillations, in both *nonautonomous* and *autonomous* cases. In *nonautonomous* case, a low frequency external periodic signal induces recurrent transition from the fast firing of the junction to a rest state near quiescence. The low frequency meant that it is much lower than the resonant frequency of the junction. In fact, the junction voltage is amplitude modulated [14] for weak and low frequency periodic forcing. For weaker forcing, the amplitude envelope of the junction voltage follows the forcing amplitude. Such amplitude modulation of high frequency electronic oscillator under the interaction of low frequency signal is well known in communications. However, we find, in forced Josephson junction, amplitude modulation (AM) as well as frequency modulation (FM) for comparatively stronger low frequency periodic forcing. In addition to AM, the interspike intervals of the spiking junction voltage inversely follow the instantaneous voltage level of sinusoidal forcing. The interspike intervals thus show periodic crowding or spreading with the voltage level of sinusoidal forcing. For further increase in forcing amplitude of low frequency periodic forcing we find bursting in junction dynamics. In *autonomous* case, we observed bursting for high junction inductance which act as a slow variable to induce a transition from firing to quiescence. It may be noted that spiking and bursting are described and classified here by comparison with geometrical representations in ref.17. No analytical approach is attempted at present.

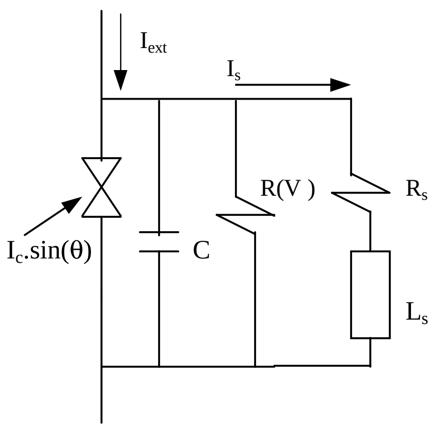

Fig.1(a). RCLSJ model of Josephson junction

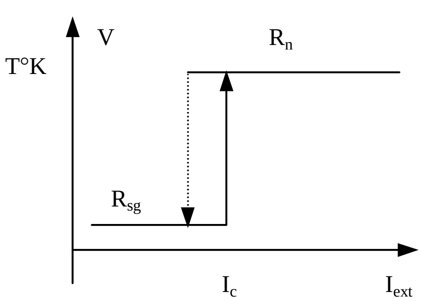

Fig.1(b). Current voltage characteristics of Josephson junction

This paper is structured as follows. In the next section, the RCL-shunted junction model is described. In section III, different autonomous spiking like rat's motor cortex are discussed. The AM/FM interactions and bursting oscillations are elaborated in section IV. Finally, the results are summarized in the last section with a conclusion.

II. RCLSJ MODEL

The circuit of the RCLSJ model is shown in Fig.1(a). The current I_s flows through the shunt inductance L and its leakage resistance R_s. The typical current-voltage (*I-V*) characteristic of the junction in Fig.1(b) at a particular temperature T°K shows a critical current I_C above which the nonlinear resistance R(V) switches from R_n to R_{sg}. R_n is the junction normal state resistance while R_{sg} is the subgap resistance. It shows a hysteresis as indicated by the dotted line in reverse direction. The Josephson phase or the phase difference of the superconductor pair is denoted by θ and C is the junction capacitance. In dimensionless form, the governing equation of the RCLSJ under DC bias and external periodic forcing is given by,

$$\beta_C \frac{d^2\theta}{d\tau^2} + g(\mathrm{v})\frac{d\theta}{d\tau} + \sin(\theta) + i_s = i_0 + i_1.\sin(\frac{\omega}{\omega_0}\tau) \quad (1a)$$

$$\beta_L \frac{di_s}{d\tau} = \mathrm{v} - i_s \quad (1b)$$

$$\frac{d\theta}{d\tau} = \mathrm{v} \quad (1c)$$

where

$$g(\mathrm{v}) = \left| \begin{array}{ll} \frac{R_s}{R_n} & \text{if } |\mathrm{v}| > \frac{V_g}{I_C.R_s} \\ \frac{R_s}{R_{sg}} & \text{if } |\mathrm{v}| \le \frac{V_g}{I_C.R_s} \end{array} \right.$$

and $\tau=\omega_0 t$, $\omega_0=2\pi eI_C R_s/h$, $\beta_C=2\pi eI_C R_s^2 C/h$, $\beta_L=2\pi eI_C L/h$, $g(v)=R_s/R(v)$, $v=V/I_C R_s$, $i_s=I_s/I_C$. The external current $i_0=I_0/I_C$ is the DC bias current and $i_1=I_1/I_C$ is the amplitude of the sinusoidal forcing of angular frequency ω. As mentioned above, the step function g(v) approximates the I-V characteristic of the junction, which switches from $R_s/R_{sg}=0.061$ to $R_s/R_n=0.366$ when $|v|\geq 2.9$ at temperature T=4.22°K. This approximation does not compromise much with experimental junction behaviors. The typical junction resistance values are chosen from the experimental results in refs.11-13, which show close matching with numerical results in ref.14. The autonomous RCLSJ model under external DC bias showed chaotic behavior [14] in selected parameter space with moderate inductance and relaxation oscillation for large inductive effect.

In the next section we present our numerical results of regular spiking, fast spiking and intrinsic bursting as usually observed in rat's visual cortex under different stimuli. Finally, we will describe mainly two types of bursting in RCLSJ model under both *autonomous* and *nonautonomous* condition.

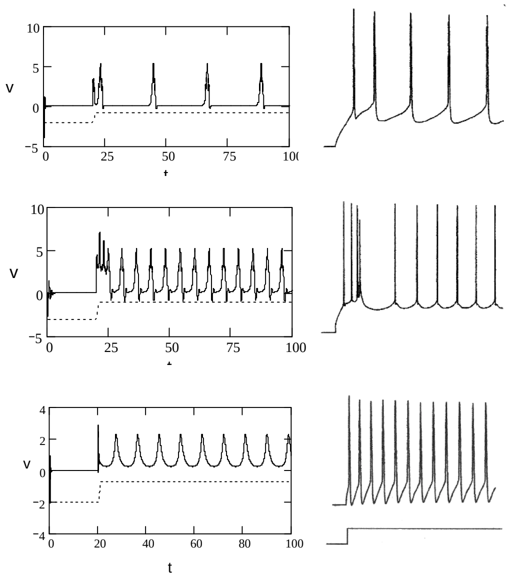

Fig.2. Time series of spiking oscillations. (a) regular spiking(RS) for moderate inductance and high damping β_L=5, β_C =0.007, i_0=1.25, (b) intrinsic bursting (IB) for large inductance and high damping, β_L=15, β_C=0.02, and i_0=2, (c) fast spiking (FS) for low inductance and high damping, β_L=0.25, β_C=0.007, i_0=1.25. Right column shows similar spiking sequence in rat's motor cortex (courtesy right column pictures reprinted from ref.25). R_s=0.01. Black solid trace for junction voltage and black dotted trace for DC bias.

III. AUTONOMOUS SPIKING AND BURSTING

The RCLSJ model shows spiking oscillation in response to external DC bias above a critical value as usually seen in rat's motor cortex [25]. The frequency of the spiking oscillation in RCLSJ increases (not shown here) with DC bias, similar to Morris-Lecar type *Class 1 Neural Excitability* neurons. On the contrary, the action potentials are generated in a frequency band and relatively insensitive to changes in the bias current in *Class II Neural Excitability* [17] neurons as represented by the Hodgkin-Huxley or FitzHugh-Nagumo models. The junction responds to both positive and negative bias current defined as excitatory and inhibitory stimulus respectively. For negative bias, the junction develops similar spiking behaviors but with a negative swing in contrast to positive bias.

The RCLSJ model has three important parameters β_C, β_L and R_s external to an ideal junction. The R_s is not treated as a separate parameter but included in the β_C. the damping factor β_C and the inductive parameter β_L are changed to find various spiking oscillations namely regular spiking (RS), fast spiking (FS) and intrinsic bursting (IB) in the junction dynamics. The regular spiking (RS), shown in the upper row of Fig.2, usually starts when the DC bias is switched on for its typical values above a threshold. In FS, the system quickly adapts to high frequency. The interspike interval is initially low for RS (lower row) but quickly adapts to larger spiking period. While, in IB (middle row), a burst of spike precedes periodic spiking. Such spiking oscillations are typical of rat's motor cortex as shown in the right column of Fig.2. For negative bias as inhibitory stimulus, the spiking maintains (Fig.3) the basic spiking features (Fig.2) while the junction voltage shows a negative swing. However, the amplitude of RS and IB spikes are four times larger compared to those for positive bias.

Autonomous bursting is observed in the high shunt inductance regime of the junction. It may be observed in Fig.4(a) that the inductive current as well as the junction voltage as labeled by state variables i_s and v respectively are fast switching when inductance is moderate. However, for large inductance in Fig.4(b), the inductive current (i_s) is now slow enough in comparison to the fast junction voltage v so as to induce autonomous bursting of Hopf/homoclinic type for larger inductive effect in the junction. The junction voltage (black solid line) starts spiking via super-Hopf and stops firing via homolcinic bifurcation to saddle focus. This classification is made by comparison with similar geometrical representation of Hopf/Homoclinic bursting in Fig.71 of ref.17 for other models. The slow inductive current (black dashed line) controls the fast spiking junction voltage and thereby induces the related bursting. The junction phase is shown in black dotted line.

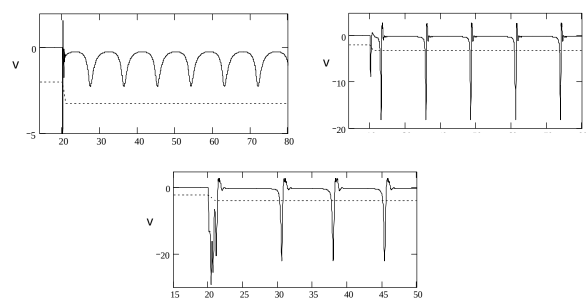

Fig.3. Time series of Junction voltage for inhibitory stimulus: upper left curve is FS for β_L=0.25, β_C=0.007, i_0=-1.25, upper right curve is RS for β_L=5, β_C=0.007, i_0=-1.25 and lower curve is IB for β_L=15, β_C=0.007, i_0=-1.75, black solid trace for junction voltage and black dotted trace for negative DC bias. R_s=0.01.

IV. BURSTING OSCILLATIONS

It appears from the neuron-like fast spiking activity in a single autonomous Josephson junction that bursting oscillations is a strong possibility if a slow motion interaction is induced to the RCLSJ. For low to moderate junction inductance, the inductive leakage current i_s is equally fast as the junction voltage and hence bursting is absent in such parametric space of the RCLSJ model. However, when a low frequency AC signal is forced into the junction as a slow motion, we observed bursting in the junction dynamics. The forcing frequency is again chosen much lower than the junction resonance frequency to induce a slow-fast interaction as a prerequisite to bursting. Similar to squid axon behavior

[27], the junction voltage is found to be amplitude modulated as shown in Fig.5 (upper plot, black solid line) for weak and low frequency forcing signal. The inductive current in black dotted line is also fast switching. The phase difference θ shows monotonic increase in time as shown in lower plot (black solid line). But when we increase the forcing amplitude keeping other parameter fixed including the forcing frequency we find amplitude modulation (AM) as well as frequency modulation (FM) as shown in Fig.6 (upper plot, black solid line). The junction oscillation amplitude still follows the forcing amplitude, over and above, the interspike period inversely follows the instantaneous voltage level of the sinusoidal signal. Thus a spreading and crowding of the spiking oscillations are observed as modulated by the instantaneous sinusoid voltage. It is interesting to note that such amplitude as well as frequency modulation is uncommon in other physical devices. For even larger amplitude, we find parabolic bursting as shown in Fig.7. The interspike period within a burst decreases with sinusoid voltage level. In other words, the interspike frequency decreases at both ends of a burst, which is defined as parabolic bursting in literature [17]. It is to be noted that the evolution of bursting oscillation in Figs.5-7 is shown in the parameter regime corresponding to RS mode. However, such AM, AM/FM and parabolic bursting may be observed for FS and IB modes also.

As we tried to induce bursting in different spiking regimes, we find that the external forcing amplitude and frequency are the major player in such complex dynamics. However, irrespective of the spiking regime, the onset of bursting is either enhanced or delayed as decided by the autonomous spiking frequency. The autonomous spiking frequency is again decided by the ac bias. The spiking frequency is larger for larger bias current and vice versa. The larger the spiking frequency the larger is the forcing amplitude required for the onset of bursting.

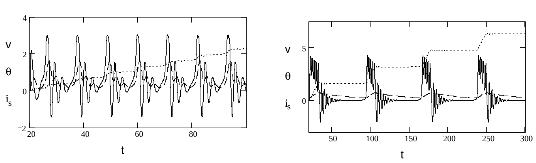

Fig.4. Autonomous bursting: junction voltage (v) in black solid line, and inductive current (i_s) in black dashed line, Josephson phase (θ) in black dotted lines, (a) left:periodic oscillation for moderate inductance and moderate damping, β_L=2.0, β_C=0.2, i_0=1.25 and (b) right: bursting for high inductance and moderate damping, β_L=50, β_C=0.2, i_0=1.25.

V.CONCLUSIONS

The inductively shunted Josephson junction model is found to show several features of spiking oscillations as typical of rat's motor cortex. The key point to such junction dynamics is that the switching mechanism of the superconducting device has similarities with the excitability of *Class I type neurons*. Interestingly, the junction shows amplitude modulation as well as frequency modulation under weak and low frequency sinusoidal forcing, which are usually seen in *Class I excitable* neurons only. On the other hand, when the low frequency AC signal forcing has comparatively larger amplitude, parabolic bursting are observed. However, detail bifurcation involved in such bursting are yet to be analyzed in future.

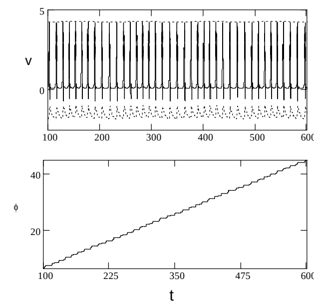

Fig.5. Amplitude Modulation: moderate inductance, high damping: weak and low frequency forcing, β_L=5, β_c=0.01, i_0=1.25, i_1=0.05 ω=0.05, no prominent FM. Time series of periodic signal in dotted line placed over the time series of junction voltage in solid line in upper plot. Inductive current is also shown in dotted line below the junction voltage. Lower plot is the time series of junction phase.

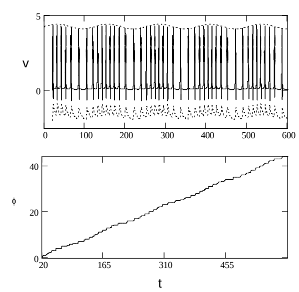

Fig.6. AM/FM interaction for weak low fequency forcing, β_L=5, β_c=0.01, i_0=1.25, ι_1=0.1, ω=0.05. Time series of periodic signal in black dotted line over the time series of junction voltage in balck solid line in upper plot. Inductive current is shown in dotted line below the junction voltage. Lower plot is the time series of junction phase.

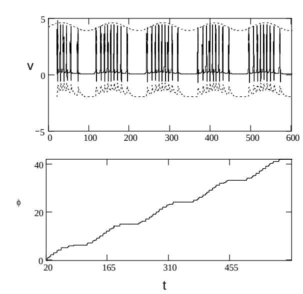

Fig.7. Parabolic Bursting: moderate inductance, high damping and weak low frequency forcing. AM is also prominent. β_L=5, β_c=0.01, ι_0=1.25, ι_1=0.35, ω=0.05. Time series of periodic signal in black dotted line over the time series of junction voltage in black solid line in upper plot. Inductive current is seen below the junction voltage. Lower plot in black solid line is the time series of junction phase.

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