3^{rd} level = MA-level => $k = 1, \dots, L$ studies
↳ M

2^{nd} lvl = group => $s = 1, \dots, N$ subjects
↳ G

1^{st} lvl = subject => $j = 1, \dots, T$ time points
↳ W

↳ 3^{rd} lvl: true parameter values

$$Y_M = X_m \beta_m + \varepsilon_M$$

=> $\beta = \{0, 3\}$ ↝ i.e. effect at population level

X = vector of length L with 1's

$$\varepsilon_k \sim N(0, \underbrace{\sigma_m^2})$$

↳ i.e. between-study variability

Y_M = vector of length L with study-level values

↳ 2

↳ 2^e lvl: $Y_M = \{\hat{\beta}_1, \dots, \hat{\beta}_L\}$

$$Y_G = X_G \beta_G + \varepsilon_G$$ for each study k

⇒ X_G = vector of length N with 1's

$\beta_G = \beta_k$ with $k = 1, \dots, L$

$\varepsilon_G \sim N(0, \sigma^2_G)$

↳ i.e. between-subject variability

Y_G = vector of length N with subject-level values for each study k

↳ 1st level: $Y_G = \{\hat{\beta}_1, \dots \hat{\beta}_N\}$ for each study k

$$Y_s = X\beta + \varepsilon_s$$

$\Rightarrow$ X = vector of length T time points

β = vector: $[\beta_0, \beta_s]$ for each $s = 1, \dots, N$

$$\varepsilon_s \sim N(0, \sigma_w^2)$$

↳ within subject-variance

Y_s = vector of length T with BOLD response

True parameter values

$\beta_M = \{0, 3\}$

Cohen's $d = \{0.14 ; 0.55 ; 1.02\}$

↳ percentile 10, 50 & 90 over voxels in ROI

from $N = +/- 180$ subjects group analysis

$$\hookrightarrow d = \frac{\beta}{\sqrt{VAR(Y_G)}}$$

$$= \frac{\beta}{\underbrace{\sqrt{c'(X'X)^{-1}c\,\sigma_W^2 + \sigma_G^2}}_{\sigma_G^{2*}}}$$

$$\boxed{d} = \frac{\boxed{\beta}}{\sigma_G^*} \Rightarrow \text{fixed}$$

$d = [0.14, .55, 1.02]$

$\beta = [0, 3]$

$$\hookrightarrow \sigma_G^* = \frac{\beta}{d} \Rightarrow \text{assume:}$$

$$\frac{\sigma_G^2}{c'(X'X)^{-1}\sigma_w^2} = 0.5$$

$$\sigma_G^2 = \frac{\sigma_G^{2*}}{3}$$

$$\sigma_w^2 = \left[\frac{2 * \sigma_G^{2*}}{3}\right] / c'(X'X)^{-1}$$

$I^2 = \{0, 0.6261, 0.8728\}$

↳ 0^{th}, 50^{th} & 100^{th} percentile in data base

⇒ assume:

$$I^2 = \frac{\sigma_M^2}{\sigma_W^2 + \sigma_G^2 + \sigma_M^2}$$

$$\Rightarrow \sigma_m^2 = \frac{I^2 \sigma_W^2 + I^2 \sigma_G^2}{1 - I^2}$$

↳ combine: B, d, $\mathbf{L}$, σ_m^2

↳ $L = 5, \ldots, 50$