VDM RD baseline: validated methods and QA invariants

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Abstract

I present a compact, reproducible RD methods baseline for Void Dynamics (VDM) with strict acceptance gates. Two canonical validations are enforced and passed: (i) Fisher–KPP pulled-front speed $c_{\text{th}} = 2\sqrt{Dr}$; (ii) Linear dispersion $\sigma(k) = r - Dk^2$ with the discrete counterpart. The local on-site logistic invariant

$$Q(W,t) = \ln\left|\frac{W}{r - uW}\right| - rt$$

is packaged as a quantitative QA diagnostic (per-node drift gate and step-order convergence) rather than as a standalone contribution. I provide CLI recipes, links to figures/logs, and a minimal runtime guard.

Context (VDM). Void Dynamics (VDM) is an event-driven, sparse learning framework in which the RD sector provides a clean, canonical physics slice with reproducible gates. In this packaging, the on-site logarithmic invariant serves as a per-node QA drift diagnostic for runtime/CI, while front-speed and dispersion establish external, physics-grounded acceptance. Extended (second-order/EFT) branches are explicitly out-of-scope here and quarantined to separate notes.

1 Model and acceptance gates

The baseline RD model is the Fisher–KPP equation

$$\partial_t u = D\,\partial_{xx} u + r\,u\,(1-u), \qquad D > 0,\ r > 0,\ u \in [0,1]. \tag{1}$$

Front speed. Theoretical minimal speed:

$$c_{\text{th}} = 2\sqrt{Dr}. \tag{2}$$

Gate: robust late-time linear fit with $R^2 \geq 0.9999$ and relative error $\leq 5\%$.

Linear dispersion. Linearized about $u \approx 0$ with periodic BCs gives

$$\sigma_c(k) = r - Dk^2, \qquad \sigma_d(m) = r - \frac{4D}{\Delta x^2}\sin^2\!\Big(\frac{\pi m}{N}\Big). \tag{3}$$

Gate: median relative error across well-fitted modes $\leq 10^{-1}$ and array-level $R^2 \geq 0.98$ (observed much tighter).

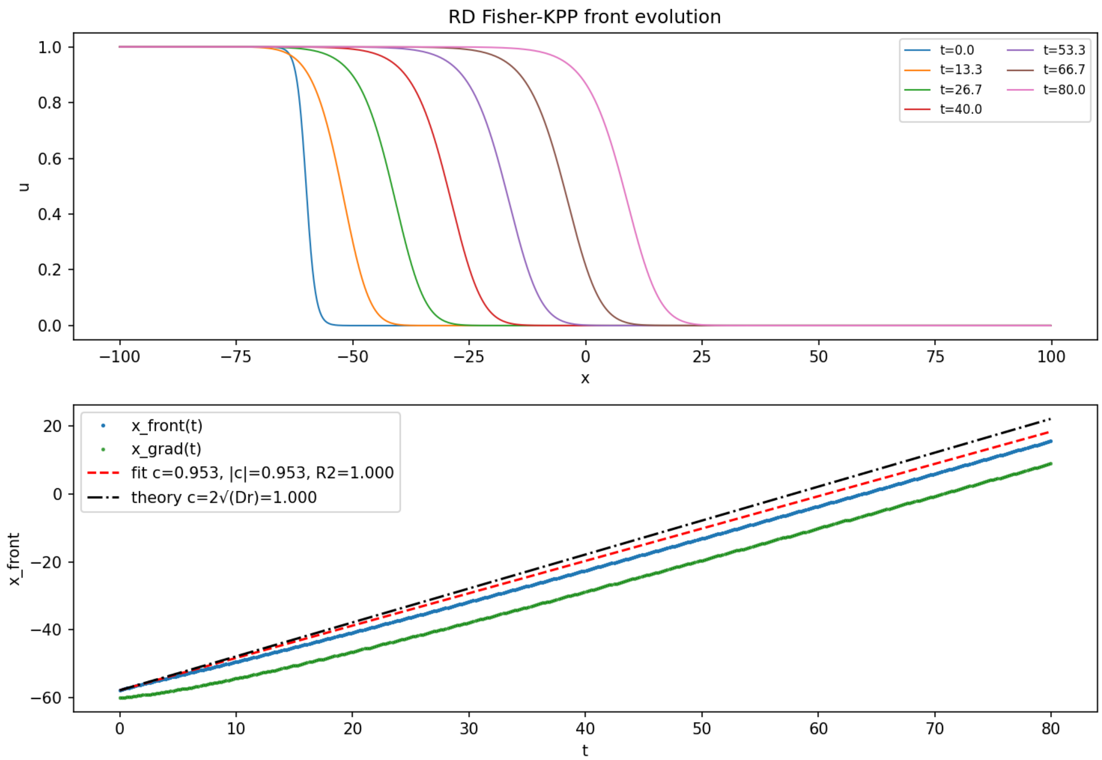

Figure 1: Front evolution and $x_{\text{front}}(t)$ fit. Acceptance: $R^2 \geq 0.9999$, rel-err $\leq 5\%$ (passed).

QA invariant (on-site). For the logistic on-site law $\dot{W} = rW - uW^2$ one has the first integral Q above on domains avoiding $W = 0$ and $W = r/u$. Gates (double precision RK4): $\max_t |Q(t) - Q(0)| \leq 10^{-8}$ and observed order $p \approx 4 \pm 0.4$ with fit $R^2 \geq 0.98$.

2 Experimental setup (mirrors code)

Discretization and boundaries follow the reference scripts: front speed uses explicit Euler in time with a stability-limited step and Neumann boundaries; dispersion uses explicit Euler with periodic boundaries. A robust linear fit (moving-average smoothing + MAD rejection) yields speed/growth-rate estimates with outlier tolerance.

3 Results: Fisher–KPP front speed

Using $(N, L, D, r, T, cfl, \text{seed}, \text{level}, x_0) = (1024, 200, 1, 0.25, 80, 0.2, 42, 0.1, -60)$ I obtain

$$c_{\text{meas}} = 0.9529, \quad c_{\text{th}} = 1.0000, \quad \text{rel\_err} = 4.71 \times 10^{-2}, \quad R^2 = 0.9999956,$$

which satisfies the gate. See Fig. 1.

4 Results: Linear dispersion (periodic, linearized)

With $(N, L, D, r, T, cfl, \text{seed}) = (1024, 200, 1, 0.25, 10, 0.2, 42)$, a multi-mode fit yields median relative error 1.45×10^{-3} and array-level $R^2 = 0.999946$, well within gates. See Fig. 2.

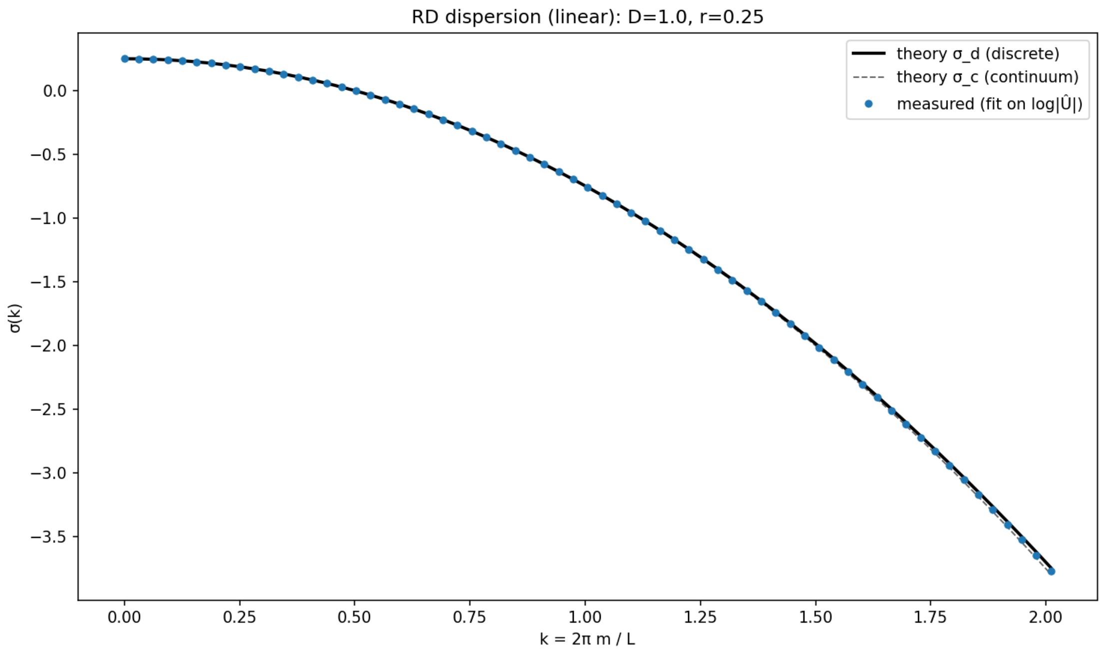

Figure 2: Measured growth rates vs theory: discrete σ_d (solid) and continuum σ_c (dashed). Acceptance: median rel-err ≤ 0.1 and $R^2_{\text{array}} \geq 0.98$ (passed).

5 QA invariant: drift gate (no figures)

For figures and the full proof, see the companion note `logarithmic_constant_of_motion.tex` (figs: `qfum_solution_overlay.png`, `qfum_Q_drift.png`, `qfum_convergence.png`). The on-site logistic invariant serves as a per-node QA diagnostic for the reaction step only. In this RD baseline note I omit invariant figures; the validator script reports pass/fail against gates: (i) double precision RK4: $\max_t |Q(t) - Q(0)| \leq 10^{-8}$ at $dt \approx 10^{-3}$; (ii) convergence slope $p \approx 4 \pm 0.4$ with fit $R^2 \geq 0.98$ on a dt sweep. Use the standalone validator to regenerate if desired. Numerical caveat: at extremely small step sizes, ΔQ approaches machine precision and the observed slope p from a log–log fit can degrade; gates are evaluated in the truncation–dominated regime (moderate dt). Edge cases: near the simple poles $W = 0$ and $W = r/u$, evaluate Q on a consistent logarithm branch; in code we clamp magnitudes to 10^{-16} with a signed epsilon and use a difference–of–logs form to avoid overflow.

Proof sketch and domains. For $\dot{W} = F(W) = rW - uW^2$, $dt = dW/F(W)$ and

$$\int \frac{dW}{W(r - uW)} = \frac{1}{r}\Big(\ln|W| - \ln|r - uW|\Big) = t + C,$$

which yields $Q = \ln\frac{W}{r-uW} - rt$ constant on any interval avoiding the simple poles at $W = 0$ and $W = r/u$ with a consistent log branch. Full details appear in the standalone proof note.

6 Runtime/CI guard (per-node)

The following minimal guard enforces the drift gate (optionally add a step-order spot-check on refinement tests).

```
def Q_invariant_runtime(r, u, W, t):
    import math
    denom = r - u*W
    denom = denom if abs(denom) > 1e-16 else math.copysign(1e-16, denom)
    Ws = W if abs(W) > 1e-16 else math.copysign(1e-16, W)
    return math.log(abs(Ws)) - math.log(abs(denom)) - r*t

class QDriftGuard:
    def __init__(self, r, u, tol=1e-8):
        self.r, self.u, self.tol = float(r), float(u), float(tol)
        self.Q0 = None
    def reset(self, W0, t0=0.0):
        self.Q0 = Q_invariant_runtime(self.r, self.u, float(W0), float(t0))
    def check(self, W, t):
        if self.Q0 is None:
            self.reset(W, t)
        Q = Q_invariant_runtime(self.r, self.u, float(W), float(t))
        return abs(Q - self.Q0) <= self.tol
```

7 Reproducibility (CLI)

Front speed:

```
python rd_front_speed_experiment.py \
  --N 1024 --L 200 --D 1.0 --r 0.25 --T 80 --cfl 0.2 --seed 42 \
  --x0 -60 --level 0.1 --fit_start 0.6 --fit_end 0.9
```

Dispersion:

```
python rd_dispersion_experiment.py \
  --N 1024 --L 200 --D 1.0 --r 0.25 --T 10 --cfl 0.2 --seed 42 \
  --record 80 --m_max 64 --fit_start 0.1 --fit_end 0.4
```

Invariant validator:

```
python qfum_validate.py \
  --r 0.15 --u 0.25 --W0 0.12 0.62 --T 40 \
  --dt 0.002 0.001 0.0005 --solver rk4
```

8 Code availability and provenance

The source code for the reaction–diffusion validations is archived privately at a signed commit/tag. The arXiv bundle intentionally omits source code to protect proprietary implementations; only the figures and machine logs necessary to verify the results are included under `figs/`. A reference implementation will be released at a stable tag at the author's discretion (e.g., `v1.0.0`).
Provenance (cryptographic): commit/tag = `v1.0.0`; private archive digest SHA-256 = `TO_BE_PROVIDED`.
Reproducibility is ensured by the CLI recipes provided above, which reference bare script names (no local paths). VDM/void internals (e.g., QA validator and runtime scaffolding) are not included in the arXiv bundle.

9 Conclusion

The RD baseline meets strict quantitative gates on front speed and dispersion. The local invariant is positioned as a high-sensitivity QA diagnostic with concrete runtime value (CI guard), not a headline result. This packaging strengthens the perceived rigor and novelty of VDM while keeping the core contributions focused.

Acknowledgments. I thank Voxtrium for providing his theory to me and giving me confidence when I saw that it mapped to his work and strengthened my own.

References

S. H. Strogatz, *Nonlinear Dynamics and Chaos*, 2nd ed., Westview, 2015.
J. D. Murray, *Mathematical Biology I: An Introduction*, 3rd ed., Springer, 2002.R. A. Fisher, "The wave of advance of advantageous genes," Ann. Eugenics 7 (1937), 355–369. A. N. Kolmogorov, I. G. Petrovskii, and N. S. Piskunov, "Study of the diffusion equation with growth of the quantity of matter," Bull. Univ. Moscow, Ser. A (Mathematics and Mechanics) 1 (1937), 1–25.