CSI 5137B Assignment 3

YANG XU, 300342009, yxu319@uottawa.ca

PEIZHOU ZHANG, 300400642, pzhan047@uottawa.ca

Q1

Q1.1

$$\bigwedge_{i=1}^{m}\left(\bigvee_{j=1}^{n} t_j\right)$$

Or expand to:

$$(c_1t_1 \vee c_1t_2 \vee \dots \vee c_1t_n) \wedge (c_2t_1 \vee c_2t_2 \vee \dots \vee c_2t_n) \wedge \dots \wedge (c_mt_1 \vee c_mt_2 \vee \dots \vee c_mt_n)$$

Q1.2

$$\bigwedge_{T' \subseteq T, |T'|=k+1}\left(\bigvee_{t_i \in T'} \neg t_i\right)$$

Or expand to:

$$(\neg t_1 \vee \neg t_2 \vee \neg t_3) \wedge (\neg t_1 \vee \neg t_2 \vee \neg t_4) \wedge (\neg t_1 \vee \neg t_3 \vee \neg t_4) \wedge (\neg t_2 \vee \neg t_3 \vee \neg t_4)$$

Q1.3

$$\bigwedge_{j=1}^{m}\left(\neg c_j \vee \bigvee_{t_i \in T(c_j)} t_i\right)$$

Or expand to:

$$(\neg c_1 \vee t_1 \vee t_2) \wedge (\neg c_2 \vee t_2 \vee t_3)$$

Q1.4

$$\left(\bigvee_{T' \subseteq T, |T'|=k} \bigwedge_{t_i \in T'} t_i\right) \wedge \left(\bigwedge_{T' \subseteq T, |T'|=k+1} \bigvee_{t_i \in T'} \neg t_i\right)$$

Or expand to:

$$\big((t_1 \wedge t_2) \vee (t_1 \wedge t_3) \vee (t_1 \wedge t_4) \vee (t_2 \wedge t_3) \vee (t_2 \wedge t_4) \vee (t_3 \wedge t_4)\big)$$
$$\wedge \big((\neg t_1 \vee \neg t_2 \vee \neg t_3) \wedge (\neg t_1 \vee \neg t_2 \vee \neg t_4) \wedge (\neg t_1 \vee \neg t_3 \vee \neg t_4) \wedge (\neg t_2 \vee \neg t_3 \vee \neg t_4)\big)$$

Q2.

We bring in new variables:

- Let $x_1 = q \wedge ((\neg p \wedge \neg r) \vee (p \wedge r)$
- Let $x_2 = (\neg p \wedge \neg r) \vee (p \wedge r)$
- Let $x_3 = \neg p \wedge \neg r$
- Let $x_4 = p \wedge r$

Now, we apply them:

- Apply $x_4 \Leftrightarrow p \wedge r$: $(x_4 \vee \neg p) \wedge (x_4 \vee \neg r) \wedge (\neg x_4 \vee p) \wedge (\neg x_4 \vee r)$
- Apply $x_3 \Leftrightarrow \neg p \wedge \neg r$: $(x_3 \vee p) \wedge (x_3 \vee r) \wedge (\neg x_3 \vee \neg p) \wedge (\neg x_3 \vee \neg r)$
- Apply $x_2 \Leftrightarrow x_3 \vee x_4$: $(x_2 \vee \neg x_3) \wedge (x_2 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4)$
- Apply $x_1 \Leftrightarrow q \wedge x_2$: $(x_1 \vee \neg q) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee q) \wedge (\neg x_1 \vee x_2)$

Thus, the CNF is:

$$
\begin{aligned}
&(p \vee x_1) \wedge (x_1 \vee \neg q) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee q) \wedge (\neg x_1 \vee x_2) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee x_2) \\
&\quad \wedge (\neg x_4 \vee x_2) \wedge (\neg x_3 \vee p) \wedge (\neg x_3 \vee r) \wedge (\neg x_4 \vee \neg p) \wedge (\neg x_4 \vee \neg r) \wedge (\neg x_4 \vee p) \\
&\quad \wedge (\neg x_4 \vee r)
\end{aligned}
$$

Q3.

$p \vee (q \wedge r)$

Transform:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

SAT Solver

- If p is True: then formula is always True
- If p is Not True: then formula is True when q is True and r is True

Thus, the formula is ***SATISFIABLE***

$\neg((p \wedge (p \to r)) \to r)$

Transform:

$$\neg((p \wedge (p \to r)) \to r) \equiv \neg((p \wedge (\neg p \vee r)) \to r) \equiv \neg((p \wedge r)) \to r) \equiv \neg(\neg p \vee r) \equiv p \wedge \neg r$$

SAT Solver

- If p is True and r is False, then formula is True

Thus, the formula is ***SATISFIABLE***

$\neg(p \wedge ((p \to q) \to q))$

Transform:

$$\neg(p \wedge ((p \to q) \to q)) \equiv \neg(p \wedge ((\neg p \vee q) \to q)) \equiv \neg(p \wedge (\neg(\neg p \vee q) \vee q))$$
$$\equiv \neg((p \wedge q) \vee (p \wedge \neg q)) \equiv \neg p$$

SAT Solver

- If p is False, then formula is True

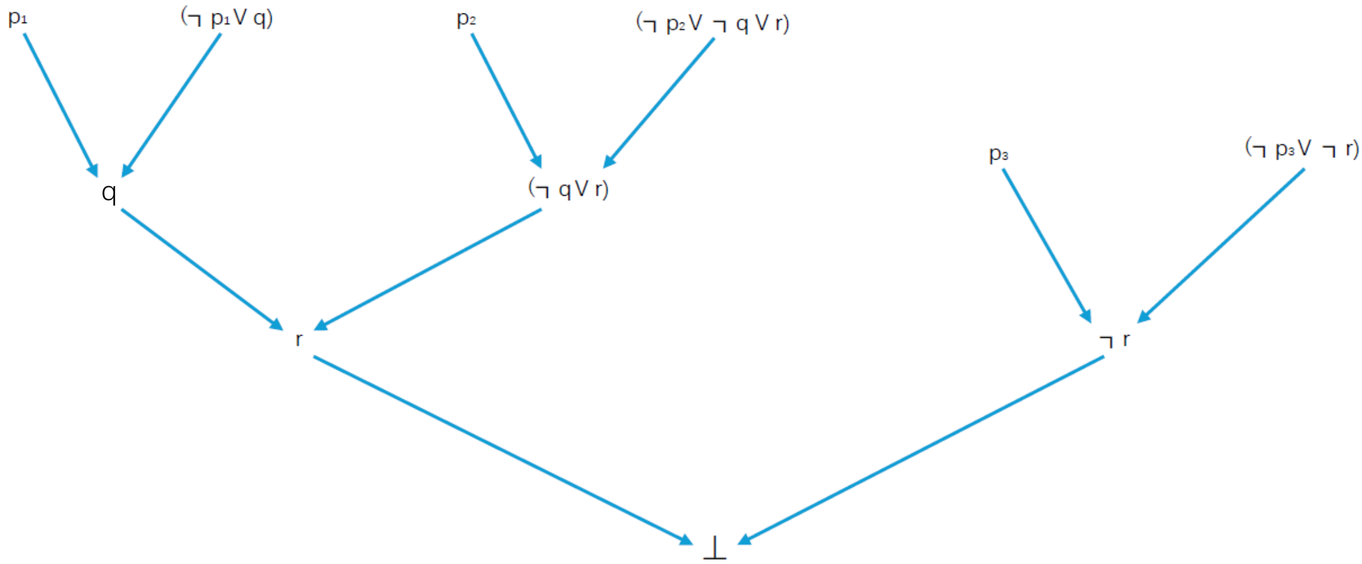
Thus, the formula is ***SATISFIABLE***

Q4.

The formula can be broken down into these parts:

$$p1, p2, p3, (\neg p1 \ \lor \ q), (\neg p2 \ \lor \ \neg q \ \lor \ r), (\neg p3 \ \lor \neg r)$$

Then the graph is:



Q5.

- Step 1: Initialization

Initially, each term is in its own equivalence class:

$$\{f(g(x))\}, \{g(f(x))\}, \{f(g(f(g(y))))\}, \{x\}, \{f(y)\}, \{g(f(x))\}, \{y\}$$

- Step 2: Apply $f(g(x)) = g(f(x))$

By the equality $f(g(x)) = g(f(x))$, merge $f(g(x))\ with\ g(f(x))$.

We get the following equivalence classes:

$$\{f(g(x)), g(f(x))\}, \{f(g(f(g(y))))\}, \{x\}, \{f(y)\}, \{y\}$$

- Step 3: Apply $f(g(f(g(y)))) = x$

By the equality $f(g(f(g(y)))) = x$, merge $f(g(f(g(y))))\ with\ x$.

We get the following equivalence classes:

$$\{f(g(x)), g(f(x))\}, \{f(g(f(g(y)))), x\}, \{f(y)\}, \{y\}$$

- Step 4: Apply $f(y) = x$

By the equality $f(y) = x$, merge $f(y)$ with x.

We get the following equivalence classes:

$$\{f(g(x)), g(f(x))\}, \{f(g(f(g(y)))), x, f(y)\}, \{y\}$$

- Step 5: Analyze $g(f(x)) != x$

At this stage:

$g(f(x))$ is in $\{f(g(x)), g(f(x))\}$ (from Step 2).

x is in $\{f(g(f(g(y)))), x, f(y)\}$ (from Step 4).

By propagating equivalences:

$f(g(x)) = g(f(x))$, so $g(f(x))$ connects to $f(g(f(g(y))))$.

$f(g(f(g(y)))) = x$, so equivalence forces $g(f(x))$ into the same class as x.

This violates the inequality $g(f(x)) \mathrel{!}= x$.

- Thus, this formula is ***UNSATISFIABLE***.