Managing street vending: a Stackelberg-Nash game approach

Javiera Carrasco[a], Pablo Escalona[a,*], Sebastian Dávila-Gálvez[b,1]

[a] *Department of Industrial Engineering, Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso, Chile.*

[b] *Universidad de Santiago de Chile (USACH), Department of Industrial Engineering, Program for the Development of Sustainable Production Systems (LDSPS), Chile*

Abstract

Street vending represents one of the most persistent challenges to urban governance in cities of the Global South due to its resilience, spatial embeddedness, and adaptive capacity in response to law enforcement. The literature has exhaustively studied its causes and consequences in the urban environment. However, operational approaches based on public space intervention are limited. This paper addresses the operational implementation of an intervention strategy for public space captured by street vendors. The intervention strategy defines the frequency with which the local authority should intervene unauthorized sites to inhibit the public space capture and promote the use of authorized sites. The strategy implementation is done using an unpredictable intervention schedule, where the local authority chooses a schedule of n unauthorized sites to be intervened (one for each inspection team) each day with some probability avoiding any regularity that could be exploited by street vendors. The interaction between the local authority and the street vendors is modeled as a Stackelberg-Nash game, in which the street vendors location decision depends on the site attractiveness, the number of other vendors choosing the same site, and the seizure risk. The results using data from simulated environment highlight how the model can provide decision support in the street vending scope.

Keywords: Urban governance; Street vending; intervention strategy; unpredictable intervention schedule; Stackelberg-Nash game

1. Introduction

The proliferation of street vending in high urban density areas has been identified as a central challenge for urban governance due to its impact on the public space management, territorial order, and public safety [1]. In Cairo, Egypt, street vendors occupy between 60% and 90% of the pedestrian routes and sidewalks, as well as between 40% and 70% of urban infrastructure elements such as trees, lampposts, and building entrances [2]. This forces

*Corresponding author.
Email address: pablo.escalona@usm.cl (Pablo Escalona)

pedestrians to walk on the street [3], exposing them to the risk of being run over by public transportation buses and private cars. In Bafoussam, Cameroon, vendors have permanently occupied strategic locations such as roundabouts and major intersections, weakening the local authority's plans for land use [4]. Street vending not only affects Africa but is a phenomenon that primarily affects the Global South. For instance, in Cali, Colombia, the high concentration of vendors in areas of dense pedestrian traffic and weak institutional regulation has been associated with increased theft risk, extortion, and police harassment, affecting both the citizens integrity and the perception of safety in urban spaces [5]. In Latin America, it occurs in a regional context marked by high levels of labor informality. Approximately 45% of workers participate in the informal economy. This proportion in countries such as Colombia reaches nearly 50% [6].

The increasing presence of street vending in public spaces has motivated several control and regulation initiatives aimed at recovering urban functionality and ensuring pedestrian mobility. These initiatives are based on the *Crime Prevention Through Environmental Design* (CPTED), urban planning, and physical space interventions for street vending. The first one is based on architecture and urbanism, which focuses on redesigning the environment to prevent street vending [7]. Improving visibility, connectivity, and the orderly flow of people contribute to reducing the improper appropriation of public space [8]. The second initiative seeks to formalize and contain the activity in specific areas of the city [9, 10]. This initiative consists of spatially grouping vendors in authorized sites, facilitating enforcement and reducing their impact on urban infrastructure [9]. However, the results of this initiative have been heterogeneous, as its effectiveness depends both on the local authority's capacity to maintain consistent, and sustained law enforcement and on its social acceptance [11]. The third one is to intervene those sites with a high concentration of street vendors, where the term 'intervention" is defined as any action taken by the local authority that thwarts illegal street vending in public space. For instance, inspection, seizure, fines, and forced eviction from public space. The objective is to increase the perceived risk based on deterrence theory, which holds that incivilities can be prevented when potential offenders perceive that the costs of committing an infraction outweigh its expected benefits [12]. This approach has proven effective in reducing illicit behaviors in public spaces. According to Youth Endowment Fund [13], this initiative reduces violent crimes by 14% and general crime by 17%. Furthermore, it reduces violence in the intervened areas, with no evidence of crime displacement [14].

In this paper, we determine the risk level that space-based intervention initiatives should induce on street vendors to reduce the illegal occupation of public space and motivate as many street vendors as possible to move to authorized sites. The risk, measured as the intervention probability and interpreted as the frequency with which the authority must intervene the unauthorized sites occupied by street vendors, is determined using a Stackelberg-Nash game approach. In this game, the local authority (*the leader*) defines the spatial distribution of the intervention probabilities over unauthorized street vending sites, and street vendors (*the followers*) define their spatial distribution. We assume that followers' decisions are independent, simultaneous, and influenced by the seizure risk and the presence of other vendors. Thus, the spatial distribution of the street vendors is modeled using an *habitat selection game* that defines a Nash equilibrium [15], in which no vendor has an incentive to move to another site, as it does not offer a higher expected benefit. The problem of determining the spatial distribution of intervention probabilities, defined as the *intervention strategy* [16], is formulated as a single-level mixed-integer bilinear programming model.

A space-based intervention strategy is useful to the authority insofar as the intervention frequency defined by the strategy is achieved through the systematic use of an unpredictable schedule [16, 17]. An unpredictable intervention schedule consists of a set of intervention schedules, where each schedule defines the sites to be intervened (one for each intervention team) and an associated probability of being selected. We take advantage of the optimal intervention strategy to determine the unpredictable intervention schedule. More precisely, we use column generation to determine the unpredictable intervention schedule, where the pricing problem defines space-based intervention schedules, whereas the master problem determines the selection probabilities. Thus, the local authority chooses a different intervention schedule each day with some probability, avoiding any regularity that could be exploited by the street vendors.

The contributions of this paper are as follows. We present a framework for the management of street vending in urban areas based on spatial intervention. This framework allows us to determine the lowest number of intervention teams to achieve acceptable levels of public space informal occupation. Furthermore, we propose a mechanism to estimate the time horizon required to reach the intervention frequencies defined by the strategy that induces the largest number of street vendors to move to authorized sites defined by the local authority.

The remainder of this work is organized as follows. A related works review is discussed in Section 2. Section 3 formulates the space-based intervention strategy. Section 4 presents how to recover an unpredictable intervention schedule from the strategy. Section 5 describes the practical implementation of the strategy. Section 6 shows the numerical results associated with the performance of the models, managerial insights for the authority, and an illustrative example. Finally, Section 7 concludes the paper by highlighting the contributions and future research.

2. Literature review and problem relevance

Street vending has become a widespread phenomenon in urban areas of the Global South, driven by poverty, migration, and the systematic exclusion from the formal labor market [18]. Several studies conclude that impoverished immigrants lack the education and skills necessary to access formal employment. This naturally leads them to engage in informal economy activities, with street vending being one of its most visible expressions [18, 19]. More precisely, Sally Roever's [9] study collected data from 743 street vendors across five cities in Africa, Asia, and Latin America. According to this research, 68% of street vendors' households rely on this activity as their primary source of income. In contrast, less than 7% of these households report formal employment as their main income source, evidencing low integration into the regulated labor market. Furthermore, 62% of households have no source of income other than informal work.

Due to the high prevalence of informal activities and the difficulty in implementing intervention policies, this proliferation of street vending has created significant challenges for urban management led by the local authority. In fact, it has been empirically observed that street vendors respond intuitively to authority interventions through relocation, reorganization, and evasion, which has been described as a *cat-and-mouse game* between the authority and the street vendors [18]. Furthermore, the irregular arrangement of the vending stalls compromises the level of pedestrian service, forcing passersby to walk on the street, in certain high-density areas [3]. This situation has also been associated with perceptions of urban disorder and public discontent, particularly when vendors are perceived as part of an illegitimate occupation of public space that contributes to a sense of insecurity [11]. Furthermore, recurring conflicts between informal vendors and established merchants have been observed, arising from direct competition for urban space and the regulatory asymmetries faced by both groups [11]. These dynamics reinforce the need to design intervention strategies that consider both territorial congestion and adaptive street vendor responses to authority initiatives.

Street vending has historically been addressed through direct intervention initiatives such as seizures, ground patrols, forced relocation, or mass evictions [19, 9]. In the case of Enugu, Nigeria, 80% of urban planners acknowledge that these measures fail to prevent vendors from returning to their original locations [19]. These actions, implemented in various cities in the Global South, follow a punitive logic aimed at imposing urban order through the expulsion of informal trade and are often enforced without offering effective alternatives to integrate vendors into regulated public spaces or to transition them out of the informal economy [20, 9]. For instance, in China, the authority has adopted restrictive policies toward street vendors to create an attractive investment climate and preserve an image of urban order [21]. Similarly, in cities such as Accra and Kumasi in Ghana, market modernization policies have led to the marginalization of informal vendors, without adequately considering their livelihoods [22]. It has also been documented that physical harassment, seizures, and chases of vendors are everyday practices in cities such as Lima, Durban, and Accra, even when these vendors pay usage fees or possess permits [9]. This situation reveals that control initiatives not only operate as a regulatory mechanism, but in practice restrict the informal use of public space as a means of livelihood and displace the discussion on the formalization of irregular trade. These approaches often overlook the daily dynamics of informal street vending, which prevents the construction of sustainable strategies in response to a territorially dispersed and socially resilient phenomenon [9].

Recent studies have incorporated mathematical models to represent the interaction between urban actors in street vending environments. Wang et al. [18] present a tripartite evolutionary model to explore the interactions between the local authority, informal vendors, and the citizens. They explicitly incorporate information diffusion through social networks and use official data and surveys to simulate an evolutionary game. Each agent updates its decisions through replication dynamics, while interacting in a two-dimensional spatial environment that allows for physical mobility, localized exchanges, and adaptation. The authors compare complete and partial information sharing scenarios, finding that the latter fosters more proactive behavior among agents and improves regulatory outcomes. However, the results arise from a scenario where behavioral patterns evolve endogenously through repeated interactions, without the presence of an actor making anticipatory decisions. Thus, the model does not capture governance contexts in which the local authority designs ex-ante intervention strategies as part of planning.

Radu et al. [23] propose a tripartite evolutionary model involving the local authority, vendors, and residents. Using a payoff matrix based on social and behavioral assumptions, they define a system of equations that allows equilibrium points to be determined and their stability to be analyzed using classical evolutionary dynamics methods. Although it does not incorporate mobility or social networks, the model explores the conditions for flexible law enforcement. To the best of our knowledge, only Radu et al. [23] and Wang et al. [18] have addressed the mathematical modeling of street vending as an urban phenomenon. Indeed, Wang et al. [18] acknowledge that no mathematical model has yet addressed street vending management from the authority's perspective, because representing the dynamics and adaptive decisions of street vendors is difficult.

3. The local authority´s intervention strategy formulation

Let us consider a commercial district in a city where street vending takes place at J sites ($j = 1, \ldots, |J|$). The local authority has defined $L \subsetneq J$ as authorized sites for street vending, where vendors are not subject to enforcement. Furthermore, the local authority knows the unauthorized sites $I \subsetneq J$ where informal street vending occurs, with $L \cup I = J$ and $L \cap I = \emptyset$. The economic attractiveness of the site $j \in J$ for street vendors is denoted by B_j, where the attractiveness of an unauthorized site is strictly greater than that of an authorized site, i.e., $B_j > B_r$ for any $j \in I$ and $r \in L$.

Let m be the number of street vendors. Each vendor chooses a single site $j \in J$, with capacity to accommodate up to k_j vendors. Thus, the spatial distribution of street vendors in the commercial district is represented by a vector of proportions $\mathbf{p} \in \mathcal{P}$, where $\mathcal{P} = \left\{(p_1, \ldots, p_{|J|}) : \sum_{j \in J} p_j = 1,\ p_j \geq 0\right\}$.

The local authority plans to implement a space-based intervention strategy at unauthorized street vending locations. The objective of the authority is to induce risk among street vendors located at unauthorized sites, encouraging them to move to authorized ones. The risk of site j is measured as the intervention probability $\mathbb{P}_j$.

As an illustrative example, Figure 1 shows the city of Valparaíso, Chile, where street vending has captured the commercial district. Furthermore, the local authority has defined authorized locations for street vending in the commercial district [24].

3.1. The Street Vendors Problem

We assume that street vendors act intuitively when choosing the site where they will carry out their activity. This intuition is based on the site's level of attractiveness, the number of vendors choosing the same site, and the seizure risk. Accordingly, we define the expected utility function $U_j(p_j | B_j, \mathbb{P}_j)$ for any $j \in J$. This function is strictly increasing in B_j, indicating that greater attractiveness implies higher benefits; non-increasing in p_j, since

Figure 1: Street vending in Valparaiso, Chile.

increased competition at a site reduces individual earnings; and non-increasing in risk, as higher risk implies lower expected utility.

A street vendor will locate at the site j that offers the highest expected utility. Consequently, under a competitive behavior, if vendors individually choose their locations, the site with the highest expected utility would become saturated, reducing utility as it must be shared among more vendors. Under this consideration, the distribution of street vendors constitutes a Nash equilibrium, as no vendor has an incentive to unilaterally change their location decision given the behavior of others, similar to a habitat selection game [15]. Thus, without loss of generality, the sites $j \in J$ are sorted according to their expected utility as follows,

$$U_1(p_1|B_1,\mathbb{P}_1) = \cdots = U_j(p_j|B_j,\mathbb{P}_j) > U_{j+1}(0|B_{j+1},\mathbb{P}_{j+1}) > \cdots > U_{|J|}(0|B_{|J|},\mathbb{P}_{|J|}). \tag{1}$$

The first vendor selects the site that they perceive as most attractive in terms of expected utility. As this site becomes congested, its utility decreases until it equals that of the next most attractive one, and both begin to be occupied. This process continues until all occupied sites offer the same expected utility. It should be noted that the intervention probability at an authorized site is zero, i.e., $\mathbb{P}_j = 0$ for any $j \in L$. Thus, the expected utility for each street vendor at equilibrium, γ, be expressed as a convex combination over J, i.e., $\sum_{j\in J} p_j U_j(p_j|B_j,\mathbb{P}_j) \leq \gamma$, with $\sum_{j\in J} p_j = 1$.

Sites $j \in J$ that are not occupied by street vendors offer a strictly lower utility than the equilibrium utility, i.e., if $p_j = 0$, then $U_j(0|B_j,\mathbb{P}_j) < \gamma$. In such cases, no vendor has an incentive to unilaterally change their location, since doing so would reduce their expected utility. The order expression (1) can be formalized as a Nash equilibrium over J, where no alternative distribution yields a higher average expected utility than the equilibrium one. Thus, a Nash equilibrium is defined as follows:

$$\mathbf{p}^T U(p^*|B_j, \mathbb{P}_j) \leq \mathbf{p}^{*T} U(p^*|B_j, \mathbb{P}_j) \qquad \forall \boldsymbol{p} \in \mathcal{P}, \tag{2}$$

where $\mathbf{p}^T U(p^*|B_j, \mathbb{P}_j) = \sum_{j\in J} p_j U_j(p_j^*|B_j, \mathbb{P}_j)$ represents the expected payoff of a street vendor using strategy $\mathbf{p}$, when vendors are distributed according to $\mathbf{p}^*$ under a Nash equilibrium. The expression $\mathbf{p}^{*T} U(p^*|B_j, \mathbb{P}_j)$ corresponds to the average individual payoff of vendors at the Nash equilibrium. Since (2) is linear with respect to $\mathbf{p}$, the following equilibrium conditions hold: $U_i(p_i^*|B_i, \mathbb{P}_i) = U_j(p_j^*|B_j, \mathbb{P}_j)$ for all $p_i^*, p_j^* > 0$, and $U_i(p_i^*|B_i, \mathbb{P}_i) \leq U_j(p_j^*|B_j, \mathbb{P}_j)$ if $p_i^* = 0$ and $p_j^* > 0$, with $i \neq j$ [25]. Thus, the Nash equilibrium (2) is equivalent to the following complementarity condition,

$$p_j \left(\gamma - U_j(p_j|B_j, \mathbb{P}_j)\right) = 0 \qquad \forall j \in J. \tag{3}$$

Under the assumption of rational behavior, street vendors choose their locations to maximize their expected utility. If a vendor chooses to locate at an unauthorized site $j \in I$, they face the seizure risk of their goods. Let $\bar{W}$ be the loss incurred by a street vendor when his products and resources are seized by the local authority. Thus, the expected utility function at site j is defined as:

$$U_j(p_j|B_j, \mathbb{P}_j) = B_j \left(1 - p_j \frac{m}{k_j}\right)(1 - \mathbb{P}_j) - \bar{W}\mathbb{P}_j \qquad \forall j \in J, \tag{4}$$

where the first term in (4) represents the expected profit that the street vendor obtains when choosing location j with no-seizure probability equal to $1 - \mathbb{P}_j$. The second term in (4) is the expected loss with seizure probability equal to $\mathbb{P}_j$. The expected utility function (4) is valid for any site $j \in J$ since $\mathbb{P}_j = 0$ for any $j \in L$.

Using equations (3) and (4), and considering that the expected utility is bounded by γ, the Nash equilibrium for the street vendors is defined by the following set of equality and inequality constraints:

$$\mathrm{NE}(\mathbb{P}): \quad p_j \left(\gamma - B_j \left(1 - p_j \frac{m}{k_j}\right)(1 - \mathbb{P}_j) - \bar{W}\mathbb{P}_j\right) = 0 \qquad \forall j \in J \tag{6}$$

$$B_j \left(1 - p_j \frac{m}{k_j}\right)(1 - \mathbb{P}_j) - \bar{W}\mathbb{P}_j \leq \gamma \qquad \forall j \in J \tag{7}$$

$$\sum_{j\in J} p_j = 1 \tag{8}$$

$$p_j \geq 0 \qquad \forall j \in J. \tag{9}$$

Since the function of the equality constraint (6) is nonlinear in the street vendors' decisions, $\mathbf{p}$ and γ, we introduce an auxiliary variable y_j equals to one if the site j is occupied by street vendors and zero otherwise, and a sufficiently large constant $M \gg 0$ (e.g.,

$M = \sum_{j \in J} B_j$). Thus, the street vendor problem is defined as the following mixed integer linear problem (MILP),

$$\text{SVP}(\mathbb{P}): \quad \min_{\boldsymbol{p},\boldsymbol{y},\gamma} \quad 0 \tag{10}$$

$$\text{s.t.} \quad M(1-y_j) + B_j\left(1 - p_j\frac{m}{k_j}\right)(1-\mathbb{P}_j) - \bar{W}\mathbb{P}_j \geq \gamma \quad \forall j \in J \tag{11}$$

$$p_j \leq y_j \quad \forall j \in J \tag{12}$$

$$y_j \in \{0,1\} \quad \forall j \in J \tag{13}$$

(7), (8), (9),

where (11), (12), and (13) are the linearization of (6).

The SVP($\mathbb{P}$) model is a feasibility problem because any feasible solution defines a Nash equilibrium for the street vendors. Furthermore, the Nash equilibrium is unique since $U_j(p_j|B_j,\mathbb{P}_j)$ is a non-increasing function in p_j [26].

3.2. The local authority Problem

The local authority intervention strategy consists of defining the spatial distribution of the intervention probabilities. Let n be the number of intervention teams, and let $\mathbb{1}_j$ be an indicator function equal to one if site $j \in I$ is disrupted, and zero otherwise.

We assume that the local authority deploys all intervention resources simultaneously over I. Thus, $\sum_{j \in J} \mathbb{1}_j \leq n$, since the local authority cannot intervene more than n unauthorized sites simultaneously. Taking the expected value, we obtain the following constraint,

$$\sum_{j \in J} \mathbb{P}_j \leq n, \tag{14}$$

where $\mathbb{P}_j = \mathbb{E}(\mathbb{1}_j)$ is the probability that the local authority will intervene on site j

The interaction between the police and the street vendors is modeled as a complete information game, where the local authority anticipates the reactions of vendors to any intervention pattern. The objective is to minimize the expected utility of street vendors, thus defining a zero-sum game. Since any feasible solution of SVP($\mathbb{P}$) is a Nash equilibrium, the local authority problem can be formulated as a single-level, as follows:

$$
\begin{aligned}
\text{SVIS:} \quad & \min_{\mathbf{p},\mathbf{y},\mathbb{P},\gamma} \quad \gamma && && (15)\\
\text{s.t.} \quad & \gamma \le M(1-y_j) + B_j\left(1 - p_j\frac{m}{k_j}\right)(1-\mathbb{P}_j) - \bar{W}\mathbb{P}_j && \forall j \in J && (11)\\
& \gamma \ge B_j\left(1 - p_j\frac{m}{k_j}\right)(1-\mathbb{P}_j) - \bar{W}\mathbb{P}_j && \forall j \in J && (7)\\
& y_j \ge p_j && \forall j \in J && (12)\\
& \sum_{j\in J} p_j = 1 && && (8)\\
& \sum_{j\in J} \mathbb{P}_j \le n && && (14)\\
& \mathbb{P}_j = 0 && \forall j \in L && (15)\\
& p_j \ge 0 && \forall j \in J && (9)\\
& \mathbb{P}_j \in [0,1] && \forall j \in J && (16)\\
& y_j \in \{0,1\} && \forall j \in J. && (13)
\end{aligned}
$$

The objective function (15) minimizes the expected utility of the street vendors who choose to locate in unauthorized sites. Constraints (7), (8), (9), (11), (12), and (13) represent the optimal reaction of the street vendor to the local authority's decision. Constraints (14) represent the local authority's budget constraint. Constraint (15) ensures that the authorized sites are not subject to intervention. The SVIS formulation is a mixed-integer bilinear programming (MIBLP) model that can be solved directly using commercial solvers.

Let $\mathbb{P}^*$, γ^*, and $\mathbf{p}^*$ be the optimal variables resulting from the SVIS model, where γ^* is the optimal expected utility of street vendors, $\mathbb{P}^*$ is the optimal intervention probability determined by the local authority, and $\mathbf{p}^*$ is the optimal spatial distribution of street vendors in the territory.

Using the optimal intervention strategy resulting from SVIS, the illegal occupation rate, defined as the proportion of street vendors who choose to locate in unauthorized sites, is determined as follows:

$$I_{OR} = \sum_{j\in I} p_j, \tag{17}$$

where $\{p_j\}_{j\in I}$ is the optimal variable of SVIS.

4. Recovering an unpredictable intervention schedule

The intervention strategy resulting from SVIS defines the frequency with which the authority should intervene unauthorized sites. However, the intervention strategy has limited practical value for the local authority unless it can be operationalized through an unpredictable intervention schedule. This section presents an approach to recover an unpredictable intervention schedule using the intervention probabilities resulting from SVIS.

Let $\mathcal{S}$ be the set of all intervention schedules. Each intervention schedule (or pure strategy) defines the n sites to be intervened (one for each intervention team). An intervention schedule $s \in \mathcal{S}$ is defined as the set $\{X_{j|s} : \sum_{j \in J} X_{j|s} \leq n\}$, where $X_{j|s}$ is equal to 1 if the local authority intervenes site $j \in I$ in schedule $s \in \mathcal{S}$, and 0 otherwise. Under a Stackelberg game approach, an unpredictable schedule also requires defining the probabilities associated with its selection (i.e., the mixed strategies). Let $\pi_s \in [0, 1]$ be the probability of selecting an intervention schedule $s \in \mathcal{S}$. Thus, an unpredictable schedule is defined as $\{(s, \pi_s) : \pi_s > 0\}_{s \in \mathcal{S}}$.

Assuming that the local authority knows all intervention schedules $s \in \mathcal{S}$, the probability that the authority intervenes site $j \in I$ can be expressed as a convex combination of the intervention team allocations, i.e., $\sum_{s \in \mathcal{S}} \pi_s X_{j|s}$ with $\sum_{s \in \mathcal{S}} \pi_s = 1$. Using the optimal strategy resulting from SVIS, we determine the selection frequency of each intervention schedule by solving the following linear problem (LP):

$$\text{UIS}(\mathbb{P}) : \quad \min_{\boldsymbol{\pi}, \boldsymbol{e}} \quad \sum_{j \in J} e_j \tag{18}$$

$$\text{s.t.} \quad e_j + \sum_{s \in S} \pi_s X_{j|s} = \mathbb{P}_j \qquad \forall j \in J \tag{19}$$

$$\sum_{s \in S} \pi_s = 1 \tag{20}$$

$$\pi_s \geq 0 \qquad \forall s \in S \tag{21}$$

$$e_j \geq 0 \qquad \forall j \in J, \tag{22}$$

where $\mathbb{P}_j$ is the optimal variable of SVIS, and each intervention schedule satisfies $\sum_{j \in J} X_{j|s} \leq n$, i.e., the number of unauthorized sites in which the local authority can simultaneously intervene is limited by the number of intervention teams.

It is clear that for a realistic urban territory, it is impossible to enumerate all intervention schedules, since $\mathcal{S}$ grows exponentially with the number of street vending sites. Thus, a column generation (CG) approach must be applied to solve UIS($\mathbb{P}$).

Column generation is defined such that a subset of the intervention schedule $\mathcal{S}_r \subset \mathcal{S}$ is first considered, and additional schedules are added as needed to reach optimality. The column generation procedure begins by defining a reduced master problem, denoted as RMP, by replacing $\mathcal{S}$ in UIS($\mathbb{P}$) with $\mathcal{S}_r$.

Let Z^*_{RMP} denote the optimal objective value of RMP, and let β^*_j and θ^* be the values of the dual variables associated with constraints (19) and (20), respectively. Then, Z^*_{RMP} is an optimal solution to UIS($\mathbb{P}$) if all reduced costs are non-negative, i.e., $c_s = \sum_{j \in J} \beta^*_j Y_{j|s} + \theta^* \geq 0$. To generate additional intervention team allocations or to verify the optimality of the solution Z^*_{RMP}, the following pricing problem is solved,

$$\text{SP:} \quad \max_{\mathbf{X}} \quad c_s = \sum_{j \in J} \beta^*_j X_{j|s} + \theta^* \tag{23}$$

$$\text{s.t.} \quad \sum_{j \in J} X_{j|s} \leq n \tag{24}$$

$$X_{j|s} \in \{0, 1\} \qquad \forall j \in J. \tag{25}$$

If the optimal objective value of SP is positive, then the intervention schedule with the maximum reduced cost $\bar{c}_s$ is added to RMP as a new intervention schedule, and the updated RMP is solved again to optimality. Otherwise, Z^*_{RMP} is an optimal solution for UIS($\mathbb{P}$), i.e., $Z^*_{UIS} = Z^*_{RMP}$, where Z^*_{UIS} is the optimal objective function of UIS($\mathbb{P}$) model.

To improve the CG algorithm, we take advantage of the optimal variables resulting from SVIS. More precisely, let $\mathcal{J}_0 = \{j \in J : \mathbb{P}_j = 0\}$ be the set of no-intervention sites where $\mathbb{P}_j$ is the optimal variable of the SVIS model. Consequently, for any $j \in \mathcal{J}_0$ we have $\pi_s = 0$ or $X_{j|s} = 0$ for every $s \in \mathcal{S}$ because $\mathbb{P}_j = \sum_{s \in \mathcal{S}} \pi_s X_{j|s}$. Thus, an equivalent pricing problem is defined by including $j \in \mathcal{J}_0$ in the SP model, denoted RSP.

Solving UIS($\mathbb{P}$) through the CG procedure using RMP-RSP results in an unpredictable intervention schedule. Let $\mathcal{D} = \{(s, \pi_s) : \pi_s > 0\}_{s \in \mathcal{S}_r}$ be the unpredictable intervention schedule resulting from solving UIS($\mathbb{P}$), and $\hat{\mathcal{S}}_r = \{s \in \mathcal{S}_r : \pi_s > 0\}$ be the set of *useful intervention schedules*. The procedure for obtaining the unpredictable intervention schedule is summarized in Algorithm 1.

Algorithm 1 Unpredictable intervention schedule

```
1: Intervention strategy: {P_j}_{j∈J}  ⟵  solve SVLP model
2: Set J_0 = {j ∈ J : P_j = 0}
3: s = 0
4: S_r = {s}
5: Set c̄_s^(0) = ∞, X^(0) = 0
6: β^(0), θ^(0)  ⟵  solve RMP
7: while c̄_s^(s) > 0 do
8:     s = s + 1
9:     S_r = S_r ∪ {s}
10:    c̄_s^(s), X^(s)  ⟵  solve RSP
11:    β^(s), θ^(s)  ⟵  solve RMP
12: end while
13: return D = {(s, π_s) : π_s > 0}_{s∈S_r}, {X_{j|s} : Σ_{j∈J} X_{j|s} ≤ n}_{s∈S_r}
```

Given the unpredictable intervention schedule $\mathcal{D}$, it is reasonable to assume that the local authority is interested in knowing after how many periods the systematic implementation of $\mathcal{D}$ will achieve the illegal occupation rate defined in (17). We propose using Monte Carlo simulation to reproduce the local authority's daily choice of an intervention schedule $s \in \mathcal{D}$ with probability π_s, and compute the illegal occupation rate as a function of time. The illegal occupation rate after τ periods is:

$$I_{OR}(\tau) = \sum_{j \in I} p_{j,\tau} \tag{26}$$

where $p_{j,\tau}$ is the proportion of street vendors who locate at site $j \in I$ after τ periods. We compute $p_{j,\tau}$ solving the following street vendor Nash equilibrium problem.

$$\text{SVNE}(\tau): \quad \min_{\mathbf{p},\mathbf{y},\gamma} \quad 0 \tag{27}$$

$$\text{s.t.} \quad \gamma \leq M\left(1 - y_{j,\tau}\right) + B_j \left(1 - p_{j,\tau}\frac{m}{k_j}\right)\left(1 - \sum_{s \in \mathcal{S}_r} g_{s,\tau} X_{j|s}\right) - \bar{W} \sum_{s \in \mathcal{S}_r} g_{s,\tau} X_{j|s} \qquad \forall j \in J \tag{28}$$

$$\gamma \geq B_j \left(1 - p_{j,\tau}\frac{m}{k_j}\right)\left(1 - \sum_{s \in \mathcal{S}_r} g_{s,\tau} X_{j|s}\right) - \bar{W} \sum_{s \in \mathcal{S}_r} g_{s,\tau} X_{j|s} \qquad \forall j \in J \tag{29}$$

$$y_{j,\tau} \geq p_{j,\tau} \qquad \forall j \in J \tag{30}$$

$$\sum_{j \in J} p_{j,\tau} = 1, \tag{31}$$

where $g_{s,\tau}$ is the frequency with which the intervention schedule $s \in \mathcal{D}$ is selected after τ periods, i.e., $g_{s,\tau} = \frac{\sum_{t=1}^{\tau} w_s^t}{\tau}$ with w_j^t equal to one if s is selected in t, and zero otherwise.

5. The practical implementation of an intervention strategy

This paper has a practical orientation. The objective is to provide the local authority with an unpredictable intervention schedule that displaces the greatest number of street vendors towards authorized sites, considering the number of intervention teams. The practical implementation of the intervention strategy, summarized in Figure 2, is presented as follows:

- The first step consists of constructing a graph G representing a commercial district and defining the authorized and unauthorized sites where street vending may take place.
- The second step involves determining the intervention strategy that results from solving the SVIS model, i.e., the frequency with which the local authority intervenes at each unauthorized site in G. Using the intervention strategy, the illegal occupation rate is computed according to (17).

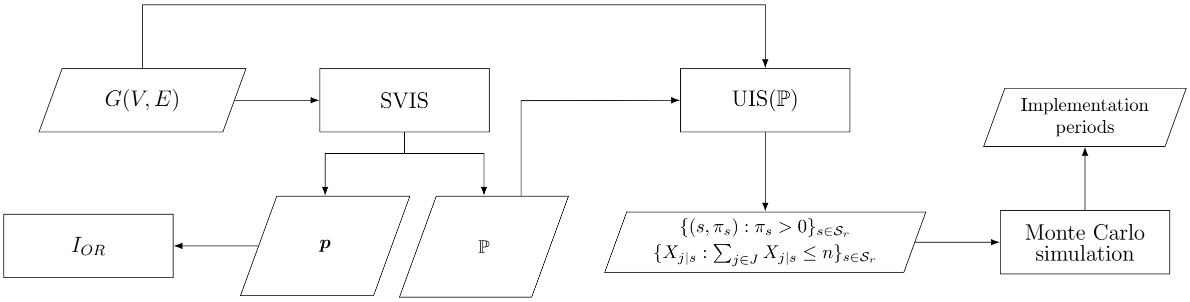


Figure 2: Practical implementation of the unpredictable intervention strategy

- The third step consists of solving UIS($\mathbb{P}$), from which the unpredictable intervention schedule $\mathcal{D}$ is determined. This corresponds to a set of intervention schedules, each associated with a probability of being selected. In each schedule, the unauthorized sites to be intervened are defined (one for each intervention team).
- The fourth step corresponds to the Monte Carlo simulation, which reproduces the local authority daily choice of an intervention schedule over time. The objective is to determine the number of periods required to reach the illegal occupation rate according to (17).

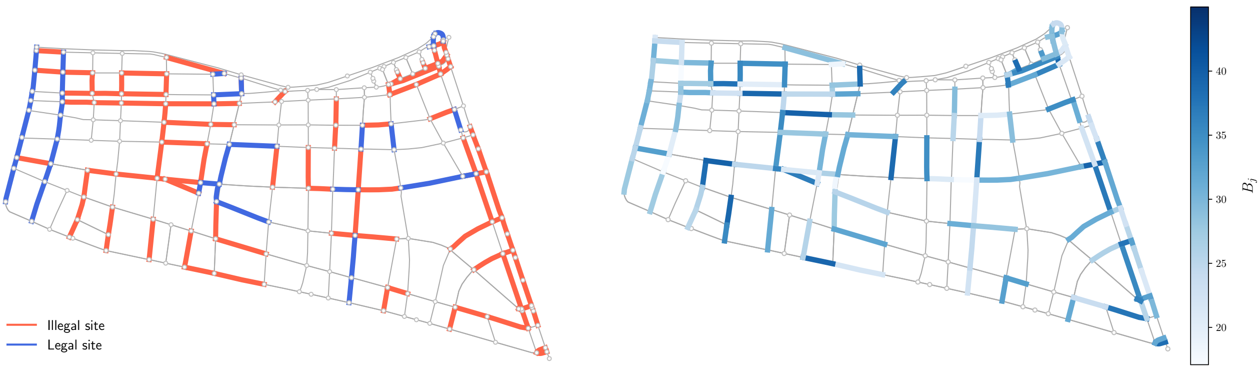
6. Numerical results

In this section, we present the numerical results. These include the performance of the models associated with the intervention strategy and the unpredictable intervention schedule. Then, we analyze managerial insights for the local authority and provide an illustrative example.

Computational experiments are performed on the Valparaiso commercial district, Chile. This area is heavily affected by illegal street vending. To determine the authorized and unauthorized sites in the Valparaiso commercial district, we relied on Ojeda and Pino [27], who identified street vending zones in Valparaiso through territorial observation, surveys, and municipal records. The Valparaiso commercial district constitutes a representative case for evaluating the performance of the proposed approach because there are designated sites for legal street vending and a high concentration of informal commerce.

We represent the Valparaiso commercial district as a graph $G = (V, E)$, where V is the set of nodes representing street intersections, and E is the set of edges representing the streets in the commercial district. It should be noted that $J \subsetneq E$. Based on the Valparaíso geographical reference system, we generate G with $|E| = 422$, where $|I| = 102$ are unauthorized edges and $|L| = 40$ are authorized edges. Figure 3a shows G, which represents the Valparaiso commercial district.

Twenty random test problems (test set) were generated, each considering $n \in \{0, \ldots, 13\}$ intervention teams, resulting in a total of 260 instances. The test problems have the following common parameters. The number of street vendors is uniformly distributed between 900



(a) Valparaíso commercial district

(b) Attractiveness distribution B_j

Figure 3: Valparaíso commercial district and Attractiveness distribution

and 1500, i.e., $m \sim U[900, 1500]$. The attractiveness B_j is uniformly distributed between 20 and 40 USD [28], i.e., $B_j \sim U[20, 40]$ with $B_j > B_r$ for any $j \in I, r \in L$. The loss incurred by a street vendor when his products are seized is estimated at 50 USD, i.e., $\bar{W} = 50$ for any $j \in I$ [29]. We compute the capacity of site j, k_j, considering its surface area and that a street vendor occupies between 4 and 9 square meters according to Ojeda and Pino [27]. The group of resilient street vendors, defined as those who continue to locate in unauthorized sites regardless of the effort made by the local authority to displace them to legal ones, represents 52% of the total street vendor population [28]. Using the first instance of the test set, Figure 3b shows the distribution of attractiveness over the Valparaiso commercial district.

The graph G was constructed in Python 3.11.5. The SVIS, RMP, RSP, and SVNE(τ) were solved to optimality using Gurobi 12.0.1. All tests are performed on a PC with an Intel Core i9 2.3 GHz processor and 16 GB RAM.

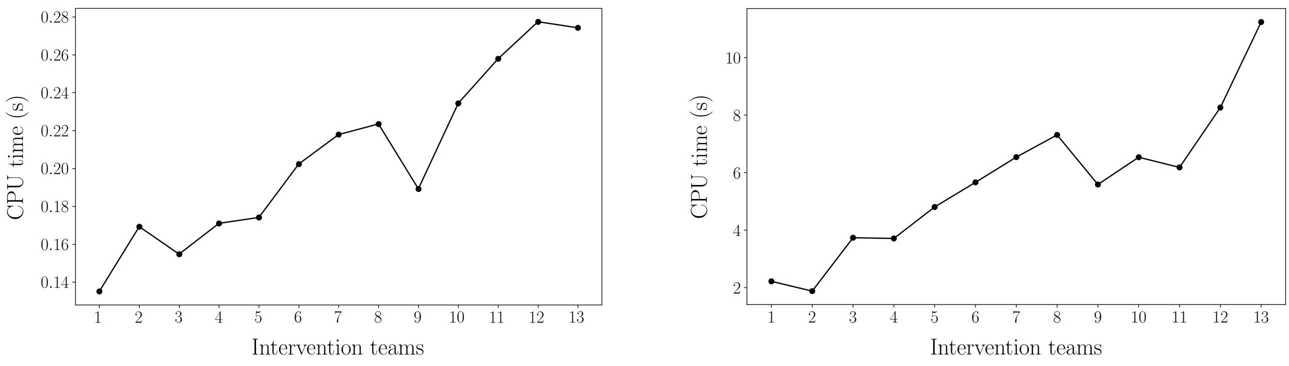
6.1. Performance of the spot strategy and the unpredictable intervention schedule

We compute the CPU time to solve the SVIS model and UIS($\mathbb{P}$) using Algorithm 1. Figures 4a and 4b show the average CPU time associated with solving SVIS and UIS($\mathbb{P}$), respectively, for all instances and intervention teams.

The CPU times tend to increase with the number of intervention teams. On the one hand, the CPU times of the SVIS model increase because it is an MIBLP, i.e., a higher number of intervention teams n results in more bilinear variables. On the other hand, the CPU time for the unpredictable intervention schedule increases because a greater number of intervention teams implies a greater combinatorics of allocations.

For each instance, the efficiency of Algorithm 1 is defined as the ratio between the number of useful intervention schedules and the total number of schedules generated using the CG procedure, i.e.,

$$\eta(\%) = 100 \times \frac{|\hat{\mathcal{S}}_r|}{|\mathcal{S}_r|}, \tag{32}$$



(a) Intervention strategy

(b) Unpredictable intervention schedule

Figure 4: Average CPU times

where $|\mathcal{S}_r|$ is the number of intervention schedules generated by Algorithm 1, and $|\hat{\mathcal{S}}_r|$ is the number of useful intervention schedules.

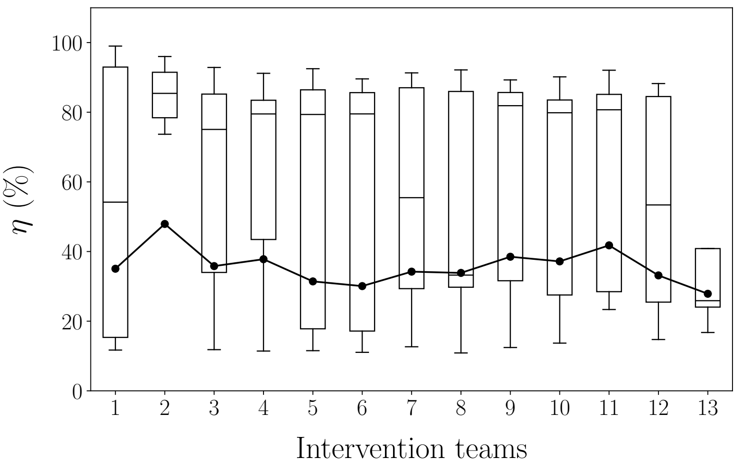


Figure 5: Efficiency of Algorithm 1

Figure 5 shows a boxplot to represent the efficiency of Algorithm 1 according to (32) for different numbers of intervention teams. The figure also reports the average efficiency for all instances and intervention teams. This efficiency reaches its highest average value at $n = 2$ and then tends to decrease and remain stable as the number of intervention teams increases. With a single intervention team, $n = 1$, an average of 64 intervention schedules are generated ($|\mathcal{S}_r|$), of which 22 are useful ($|\hat{\mathcal{S}}_r|$), corresponding to an efficiency of 34%. In the case of $n = 2$, efficiency rises to 47%, but from $n = 3$ onward, it stabilizes within 35% to 37% on average. Table 1 shows the average, minimum, and maximum number of generated and useful schedules, respectively.

Table 1: Generated and useful intervention schedules

n	Avg $\|\mathcal{S}_r\|$	Max $\|\mathcal{S}_r\|$	Min $\|\mathcal{S}_r\|$	Avg $\|\hat{\mathcal{S}}_r\|$	Max $\|\hat{\mathcal{S}}_r\|$	Min $\|\hat{\mathcal{S}}_r\|$
1	64	100	10	22	99	9
2	53	156	21	25	98	19
3	106	237	27	38	100	23
4	103	289	34	39	97	29
5	139	312	40	44	99	34
6	160	371	45	48	101	39
7	186	349	46	64	100	42
8	209	423	51	71	100	46
9	160	418	56	62	100	50
10	183	426	61	68	100	53
11	173	424	63	72	101	56
12	229	440	68	76	101	59
13	317	435	72	88	101	62

6.2. Managerial insights for the local authority

In this section, we present managerial insights for the local authority related to territorial intervention efficiency, the illegal occupation rate, and the systematic application of the unpredictable intervention schedule.

Let $\mathcal{I}_0$ be the set of unauthorized sites without intervention, i.e., $\mathcal{I}_0 = \{j \in I : \mathbb{P}_j = 0\}$, where $\mathbb{P}_j$ is the optimal variable from SVIS. We compute for each instance the set $\mathcal{I}_0$ to determine the proportion of unauthorized sites that the intervention strategy defines not to be disrupted. Figure 6 shows the ratio between the number of unauthorized sites with zero probability of intervention and the total number of unauthorized sites.

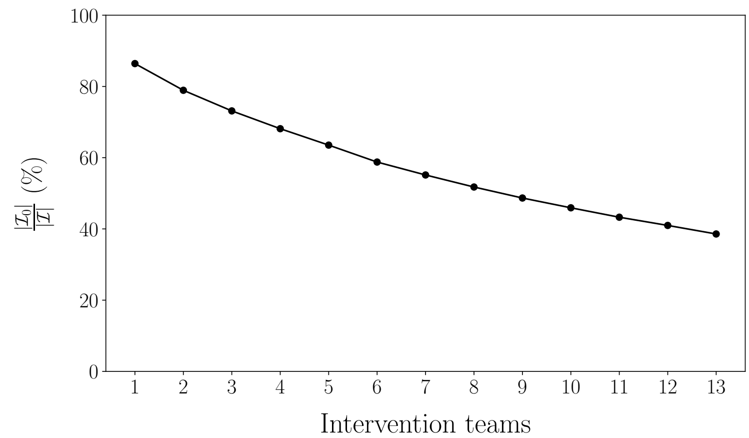


Figure 6: Percentage of nodes with zero probability of intervention.

The percentage of unauthorized sites with zero intervention probability decreases as the number of intervention teams increases. With a single intervention team, $n = 1$, the 86% of the unauthorized sites are not intervened, whereas with thirteen intervention teams, $n = 13$, this proportion drops below 40%. Thus, we observed that it is not necessary to intervene at all unauthorized sites in order to achieve an efficient intervention strategy.

For each instance, the illegal occupation rate is computed according to (17). Figure 7 shows how the decisions of street vendors change as the number of intervention teams increases.

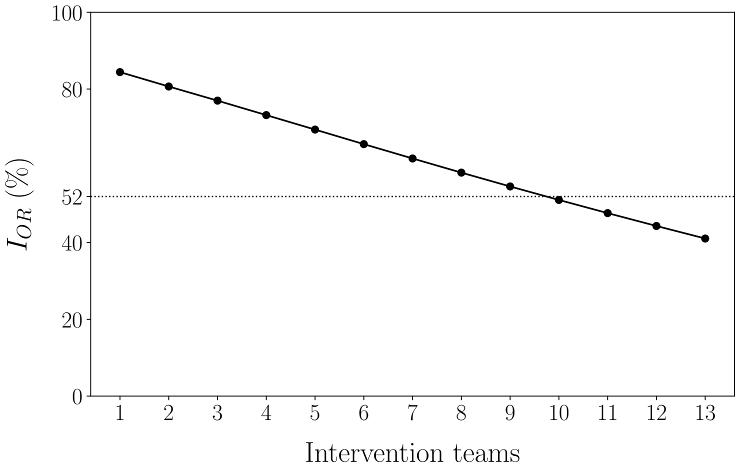


Figure 7: Illegal occupation rate

The illegal occupation rate is monotonically non-increasing, declining from 84% to 41% as intervention teams increases from $n = 1$ to $n = 13$, respectively. This shows the deterrent effect of intervention on the occupation of unauthorized sites. However, we assume that the proportion of resilient street vendors is 52%. As a practical result for the local authority, we observe that the smallest number of intervention teams inducing an illegal occupation rate less than or equal to 52% is achieved by deploying ten intervention teams.

It is also reasonable to assume that the local authority wants to know how many periods are needed to reach an illegal occupation rate equal to 52%. Using Monte Carlo simulation to reproduce the local authority's daily choice of an intervention schedule $s \in \mathcal{D}$ with probability π_s, we compute the illegal occupation rate after τ periods according to (26). Figure 8 shows the systematic implementation of the unpredictable intervention schedule over a 150-period horizon when ten intervention teams are deployed.

The illegal occupation rate after τ periods decreases as the unpredictable intervention schedule is systematically applied period after period. From Figure 8, we observe that after 58 periods, an illegal occupancy rate of 52% is reached. Thus, we conclude that the unpredictable intervention schedule can be operationally implemented in the medium term.

The Monte Carlo simulation selects an intervention schedule each period based on the selection probability π_s with $s \in \mathcal{D}$. We compute the empirical probability distribution function of the intervention schedule selection, i.e., $\sum_{s \in \mathcal{D}} \pi_s$, resulting from Algorithm 1 when ten inspection teams are deployed.

From Figure 9, we observe that 51% of the useful intervention schedules accumulate 80% of the selection probability. That is, half of the intervention schedules accumulate a high probability of being selected. This implies that the probability distribution function is flatter, indicating a high dispersion in schedule selection. This dispersion is due to the fact

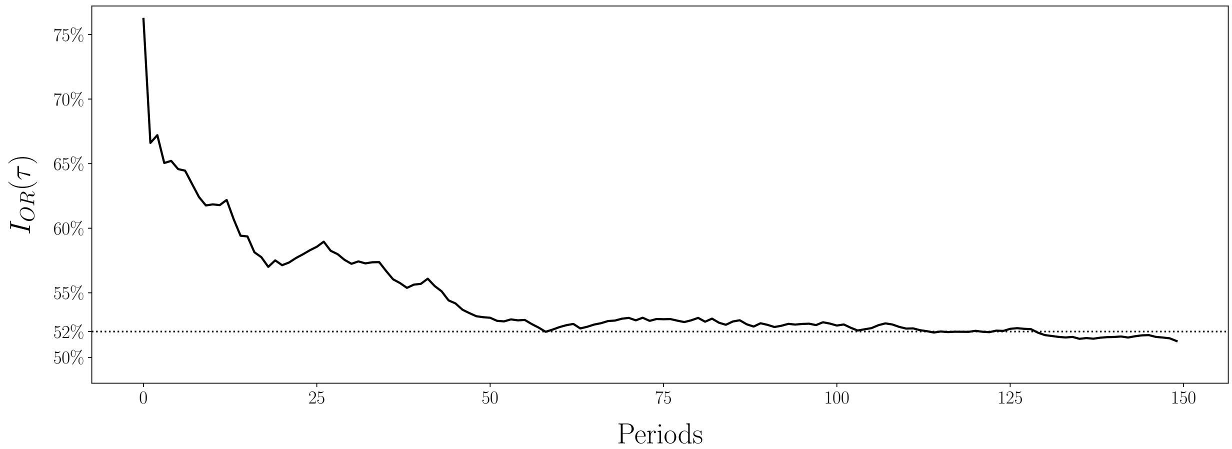


Figure 8: Unpredictable intervention schedule simulation with $n = 10$

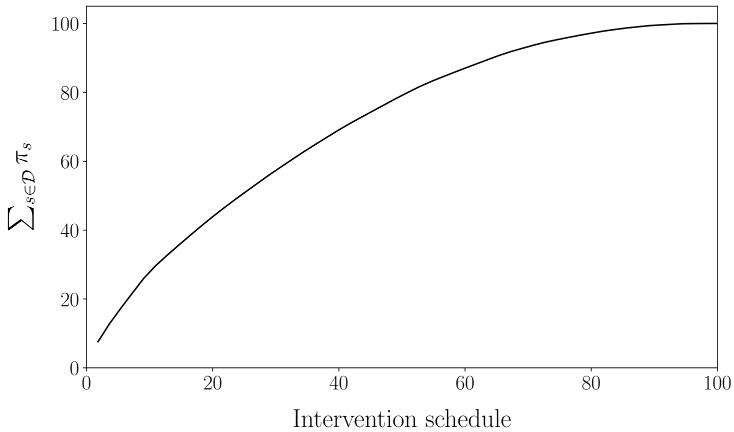


Figure 9: Probability distribution function of the intervention schedule selection with $n = 10$

that vendors have access to a wide variety of unauthorized sites where they carry out their activity. In summary, the city is still large enough to offer multiple spatial alternatives.

6.3. Strategy and unpredictable schedule visualization.

In this section, we show the visualization of a strategy and its unpredictable intervention schedule that is easily assimilated by the local authority. We use the first instance of the test set as an illustrative example to show, on G, the intervention strategy and its unpredictable intervention schedule resulting from SVIS and UIS($\mathbb{P}$), respectively. Ten intervention teams are considered to hold the illegal occupation rate less than or equal to 52%.

Using a heat map on G, we show the spatial distribution of the intervention probabilities. The chromatic intensity in Figure 10 indicates the intervention probability in each unauthorized site of the Valparaiso commercial district.

Figure 10 shows that 54% of the unauthorized sites should be intervened by the local authority. Sites displayed in darker red on graph G should be intervened more frequently,

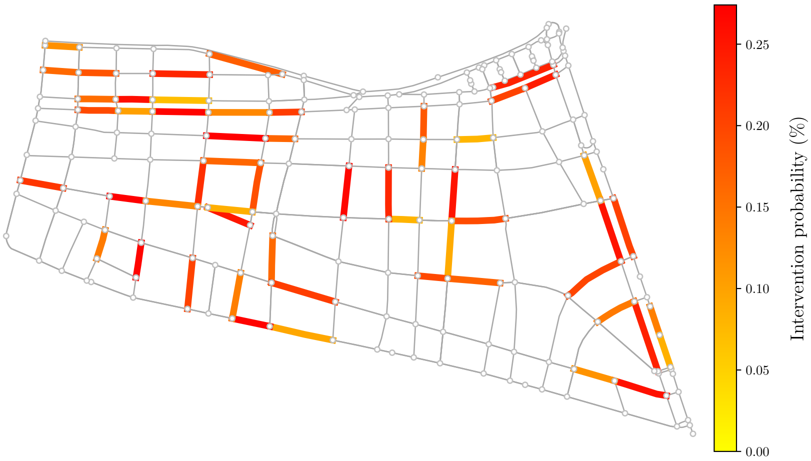


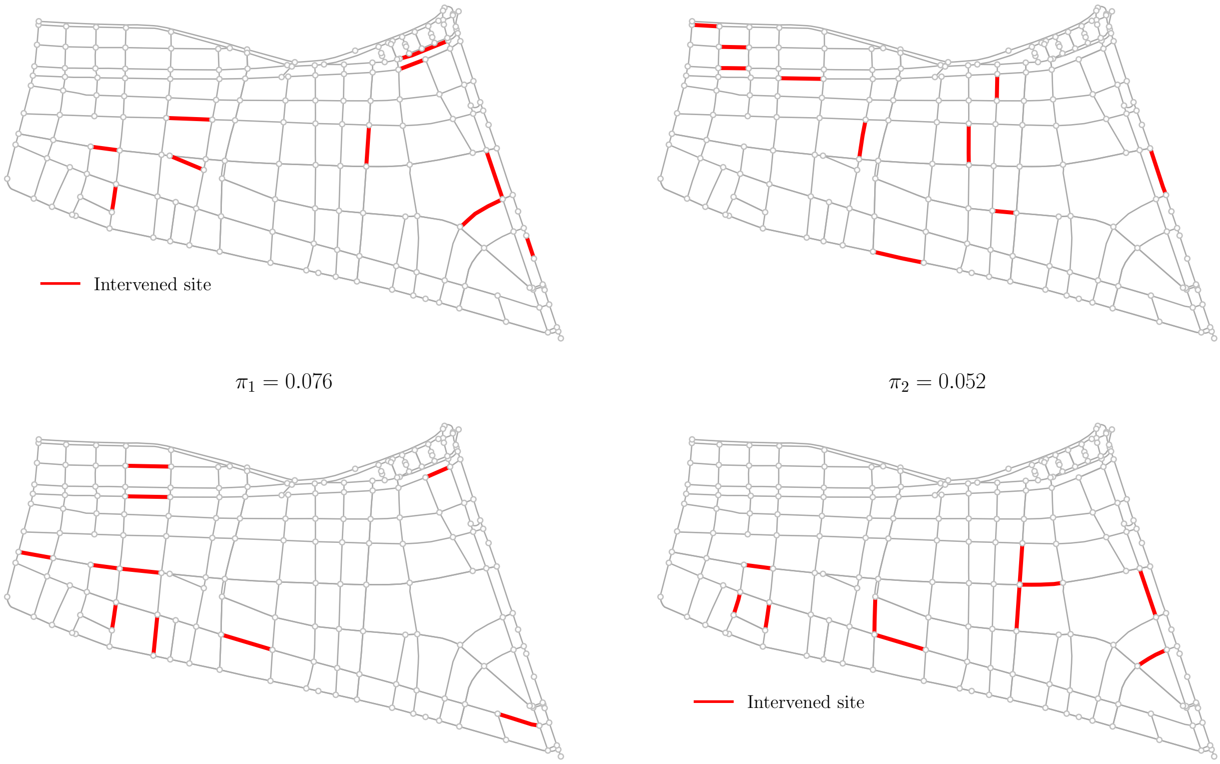
Figure 10: Intervention strategy in Valparaíso commercial district with $n = 10$.

and the lighter intensity sites should be intervened less frequently. The heat map effectively visualizes the optimal intervention strategy derived from the SVIS model, facilitating an intuitive interpretation by decision-makers. However, the practical implementation of this intervention strategy is done using the unpredictable intervention schedule resulting from solving UIS($\mathbb{P}$) using Algorithm 1. The algorithm generates 67 intervention schedules ($|\mathcal{S}_r| = 67$) of which 55 are useful ($|\hat{\mathcal{S}}_r| = 55$). Figure 11 shows the four intervention schedules most likely to be selected when deploying ten intervention teams.

Figures 11a, 11b, 11c, and 11d are the four intervention schedules most likely to be selected in the period-by-period implementation of the unpredictable intervention schedule. The selection probabilities associated with these schedules are $\pi_1 = 7.6\%$, $\pi_2 = 5.2\%$, $\pi_3 = 4.6\%$, and $\pi_4 = 4.4\%$, respectively. Over a 100-period horizon, these probabilities suggest that the schedules would be implemented approximately 8, 5, 5, and 4 times, respectively. These intervention schedules are consistent with the intervention strategy resulting from SVIS, as the selected sites in each case correspond to those with a high probability of intervention, as shown in Figure 10. Furthermore, the variation in the combination and spatial distribution of the selected sites across different schedules contributes to maintaining unpredictability from the perspective of street vendors.

7. Conclusion

This study proposes a framework for managing illegal street vending in densely populated urban contexts. The hierarchical interaction and physical distribution of street vendors are modeled as a Stackelberg-Nash game, in which the local authority determines the intervention probabilities, and the street vendors decide their spatial distribution in a commercial district. The resulting leader strategy is then used to generate an unpredictable intervention schedule, which is determined using column generation. The objective is to define intervention strategies that maximize deterrence effectiveness without requiring complete territorial coverage. Rather than relying on static or deterministic actions, this framework introduces



$\pi_1 = 0.076$ $\pi_2 = 0.052$

$\pi_3 = 0.046$ $\pi_4 = 0.044$

Figure 11: Intervention schedules in Valparaíso commercial district with $n = 10$.

a probabilistic and unpredictable deployment strategy whose systematic implementation promotes the relocation of street vendors from unauthorized to authorized sites.

From a practical point of view, the model allows the local authority to determine the spatial distribution of its resources and the level of illegal occupation that can be controlled with its available resources. For instance, numerical results indicate that deploying ten intervention teams reduces the illegal occupation rate to 52% within a time horizon of approximately 58 periods. It should be noted that the numerical experiments are based on the context of illegal street vending at Valparaíso, Chile.

The framework reveals that it is not necessary to intervene at all illegal vending sites to induce the relocation of street vendors occupying unauthorized sites. However, the high proportion of resilient street vendors suggests that this intervention mechanism must be complemented with additional measures if the local authority intends to comprehensively address informal street vending. Furthermore, the numerical results show that half of the intervention schedules accumulate 80% of the selection probability, meaning that half of the possible schedules are selected more frequently. This result is explained by the fact that street vendors have many alternatives for operating. Thus, the city remains large enough to sustain their activity, perpetuating a persistent "cat-and-mouse game" between local authorities and street vendors, where vendors continuously adapt to avoid intervention, taking advantage of the spatial flexibility of the urban environment.

We next suggest future research prospects. The first one is to address the street vendors' heterogeneity. Heterogeneity can be exploited in two ways: (i) not all street vendors react in the same way to the seizure risk, and (ii) the street vendors can be grouped in several respects. The second one is to address the implementation periods' management of the unpredictable intervention schedule. Management can be done using several approaches, e.g., (i) determining the schedules that best contribute to reducing the illegal occupation rate in the fewest number of periods, or (ii) determining the sites to be authorized so that the greatest number of street vendors decide to relocate in the fewest number of periods. Finally, one is to validate the models in the Valparaiso commercial district.

Acknowledgments

Pablo Escalona and Sebastian Davila are grateful for the support of the National Agency for Research and Development (ANID) Chile through grant FONDECYT 1250126 and FONDECYT 11251766, respectively.

References

[1] S. Roever, Street Trade in Latin America: Demographic Trends, Legal Issues and Vending Organizations in Six Cities, Routledge, 2010, pp. 208–240. doi:10.4324/9780203150542-9.

[2] M. A. Elsayed, A. Ashour, A. Elshater, M. A. Elfayoumi, S. Mahmoud, Mapping street vendors and informal economy effects in public spaces, in: IOP Conference Series: Earth and Environmental Science, Vol. 1056, IOP Publishing, 2022, p. 012043. doi:10.1088/1755-1315/1056/1/012043.
URL https://doi.org/10.1088/1755-1315/1056/1/012043

[3] K. G. Hagos, M. Adnan, A. ul Haque Yasar, Effect of sidewalk vendors on pedestrian movement characteristics: A microscopic simulation study of addis ababa, ethiopia, Cities 103 (2020) 102769. doi:https://doi.org/10.1016/j.cities.2020.102769.
URL https://www.sciencedirect.com/science/article/pii/S0264275119323959

[4] A. Yemmafouo, et al., Street vending power relationships and governance of public spaces in bafoussam, west cameroon, Current Urban Studies 6 (04) (2018) 611. doi:10.4236/cus.2018.64032.

[5] L. Martínez, J. R. Short, The informal city: Exploring the variety of the street vending economy, Sustainability 14 (12) (2022). doi:10.3390/su14127213.
URL https://www.mdpi.com/2071-1050/14/12/7213

[6] L. Martínez, J. R. Short, D. Estrada, The diversity of the street vending: A case study of street vending in cali, Cities 79 (2018) 18–25. doi:10.1016/j.cities.2018.02.018.

[7] J. S. Lee, S. Park, S. Jung, Effect of crime prevention through environmental design (cpted) measures on active living and fear of crime, Sustainability 8 (9) (2016) 872. doi:10.3390/su8090872.
URL https://www.mdpi.com/2071-1050/8/9/872

[8] P. Navarrete-Hernandez, A. Luneke, R. Truffello, L. Fuentes, Planning for fear of crime reduction: Assessing the impact of public space regeneration on safety perceptions in deprived neighborhoods, Landscape and Urban Planning 237 (2023) 104809. doi:https://doi.org/10.1016/j.landurbplan.2023.104809.
URL https://www.sciencedirect.com/science/article/pii/S0169204623001287

[9] S. Roever, Informal economy monitoring study: Street vendors, Tech. rep., WIEGO (2014).
URL https://www.wiego.org/wp-content/uploads/2019/09/IEMS-Sector-Full-Report-Street-Vendors.pdf

[10] M. A. Amalia Fikriya Putri, Ahmad Mujani, Legal challenges in regulating and supervising street vendors in banten province, West Science Law and Human Rights 1 (04) (2023) 157–165. doi:10.

58812/wslhr.v1i04.322.
URL https://wsj.westsciences.com/index.php/wslhr/article/view/322

[11] R. Bromley, Street vending and public policy: A global review, International Journal of Sociology and Social Policy 20 (1/2) (2000) 1–28. doi:10.1108/01443330010789052.
URL https://doi.org/10.1108/01443330010789052

[12] B. Turchan, A. A. Braga, The effects of hot spots policing on violence: A systematic review and meta-analysis, Aggression and Violent Behavior 79 (2024) 102011. doi:https://doi.org/10.1016/j.avb.2024.102011.
URL https://www.sciencedirect.com/science/article/pii/S1359178924001010

[13] Youth Endowment Fund, Hot spots policing (2023).
URL https://youthendowmentfund.org.uk/toolkit/hot-spots-policing/

[14] A. Braga, A. Papachristos, D. Hureau, The effects of hot spots policing on crime: An updated systematic review and meta-analysis, Justice Quarterly 2012 (05 2012). doi:10.1080/07418825.2012.673632.

[15] V. Krivan, E. Sirot, Habitat selection by two competing species in a two-habitat environment, The American naturalist 160 (2002) 214–34. doi:10.1086/341016.

[16] P. Escalona, L. Brotcorne, B. Fortz, M. Ramirez, Fare inspection patrolling under in-station selective inspection policy, Annals of Operations Research 332 (2023) 1–22. doi:10.1007/s10479-023-05670-2.

[17] L. Brotcorne, P. Escalona, B. Fortz, M. Labbé, Fare inspection patrols scheduling in transit systems using a stackelberg game approach, Transportation Research Part B: Methodological 154 (2021) 1–20. doi:https://doi.org/10.1016/j.trb.2021.10.001.
URL https://www.sciencedirect.com/science/article/pii/S0191261521001855

[18] P. Wang, D. Fang, G. Cao, Q. Luo, Does information sharing affect the regulation of street vending: An analysis based on the dynamic tripartite evolutionary game in social networks, Cities 141 (2023) 104494. doi:https://doi.org/10.1016/j.cities.2023.104494.
URL https://www.sciencedirect.com/science/article/pii/S0264275123003062

[19] V. A. Onodugo, N. H. Ezeadichie, C. A. Onwuneme, A. E. Anosike, The dilemma of managing the challenges of street vending in public spaces: The case of enugu city, nigeria, Cities 59 (2016) 95–101. doi:https://doi.org/10.1016/j.cities.2016.06.001.
URL https://www.sciencedirect.com/science/article/pii/S0264275116302244

[20] N. Peimani, H. Kamalipour, Informal street vending: A systematic review, Land 11 (6) (2022). doi:10.3390/land11060829.
URL https://www.mdpi.com/2073-445X/11/6/829

[21] A. Hanser, Street politics: Street vendors and urban governance in china, The China Quarterly 226 (2016) 363–382. doi:10.1017/S0305741016000278.

[22] V. Okoye, Street vendor exclusion in "modern" market planning: A case study from kumasi, ghana, WIEGO Resource Document 15, WIEGO (Women in Informal Employment: Globalizing and Organizing) (2020).
URL https://www.wiego.org/research-library-publications/street-vendor-exclusion-modern-market-pla

[23] V. Radu, C. Dumitrescu, E. Vasile, A. I. Tăbîrcă, M. C. Stefan, L. Manea, F. Radu, Modeling and prediction of sustainable urban mobility using game theory multiagent and the golden template algorithm, Electronics 12 (6) (2023). doi:10.3390/electronics12061288.
URL https://www.mdpi.com/2079-9292/12/6/1288

[24] L. Ojeda, A. Pino, Spatiality of street vendors and sociospatial disputes over public space: The case of valparaíso, chile, Cities 95 (2019) 102275. doi:10.1016/j.cities.2019.02.005.
URL https://doi.org/10.1016/j.cities.2019.02.005

[25] V. Krivan, R. Cressman, C. Schneider, The ideal free distribution: A review and synthesis of the game-theoretic perspective, Theoretical population biology 73 (2008) 403–25. doi:10.1016/j.tpb.2007.12.009.

[26] R. Cressman, V. Krivan, Migration dynamics for the ideal free distribution, The American naturalist 168 (2006) 384–97. doi:10.1086/506970.

[27] L. Ojeda, A. Pino, Valparaíso y su comercio callejero: ¿espacialidad esporádica y/o espacialidad satu-

rada?, AUS - Arquitectura / Urbanismo / Sustentabilidad (25) (2019) 11–19. doi:10.4206/aus.2019.n25-03.
URL http://revistas.uach.cl/index.php/aus/article/view/4209

[28] Centro de Políticas Públicas UC, Estudio de caracterización del comercio ambulante en la región metropolitana, https://politicaspublicas.uc.cl/publicacion/estudio-de-caracterizacion-del-comercio-ambulante-en-la-region-metropolitana/ (2023).

[29] R. A. A. Saavedra, Los trabajadores informales urbanos en chile neoliberal: El caso de los trabajadores ambulantes en la comuna de santiago, 1978–2004, tesina de Licenciatura en Historia, Universidad de Chile (2005).
URL https://repositorio.uchile.cl/handle/2250/110209