

Projective Indecomposable Modules and Quivers for a Class of Fountain Monoids with Applications to Descent Algebras

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A talk at the University of York is the most appropriate venue to use the term "Fountain Monoid" for the first time. These are the monoids that were formerly known as "semi-abundant semi-groups". There is nothing "semi" about John Fountain's contributions to the subject.

A major goal over the last ten years of studying algebras and representations of finite monoids is to widen the class of such for which we have nice descriptions of simple, projective and injective modules and other representation and homological parameters such as quivers and global dimension. Motivations come from monoids arising naturally in combinatorics.

We look at the class of finite right Fountain monoids, with the property that every regular D-class has a right invertible structure matrix. The most important class of such is the class of finite monoids with at most one idempotent in each R-class. This includes left regular bands and J-trivial monoids. In particular, the 0-Hecke monoid and hyperplane face monoids are important examples that have been intensively studied and have many applications.

We can completely describe the indecomposable projective and injective modules of the algebra of such a monoid as well as their quivers.

An important example is the cross product algebra of a Coxeter group and the left regular band algebra arising from its Coxeter hyperplane arrangement complex. Solomon's Descent Algebra is naturally embedded in such a cross product and we can describe its representation theory from our results.